Q3. Find the divergence and curl of the field $F = 30\hat{\imath} + 2xy\hat{\jmath} + 5xz^2\hat{k}$ in Cartesian coordinates.

Given:- $F = 30\hat{\imath} + 2xy\hat{\jmath} + 5xz^2\hat{k}$

Formula:-Divergence $(\vec{\nabla}.\vec{A}) = \hat{\imath}\frac{\partial}{\partial x}A_x + \hat{\jmath}\frac{\partial}{\partial y}A_y + \hat{k}\frac{\partial}{\partial z}A_z$

Curl: $(\vec{\nabla} \times \vec{A}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$

Solution:-

Divergence, $\vec{\nabla}.\vec{F} = \left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right).(30\hat{\imath} + 2xy\hat{\jmath} + 5xz^2\hat{k})$

$$= \frac{\partial}{\partial x}(30) + \frac{\partial}{\partial y}(2xy) + \frac{\partial}{\partial z}(2xz^2) = 2x + 10xz$$

$$\vec{\nabla}.\vec{F} = 2x(1 + 5z)$$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 30 & 2xy & 5xz^2 \end{vmatrix} = -5z^2\,\hat{\jmath} + 2y\,\hat{k}$$

Ans:- Divergence of field is $2x(1 + 5z)$ and its curl $= -5z^2\,\hat{\jmath} + 2y\,\hat{k}$