RELATIVITY

Q1. What do you mean by an inertial and non inertial frame of reference.

A Reference frame:-

Three space coordinates x, y, z and time t define a reference frame. All physical phenomenons need a reference frame with respect to which they can be studied.

Inertial frame:-

If a reference frame is either at rest or moving with a uniform velocity then it is called an inertial frame of reference. Example: A person standing on railway station and aperson standing in a train that is moving withconstant velocity. Both persons are in inertialframes.

Non inertial frame:-

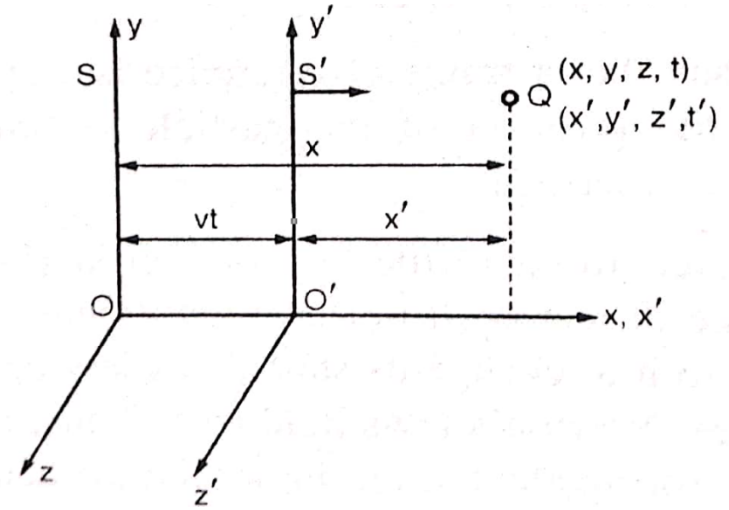
An accelerating frame of reference is called non inertial frame of reference. Example: If the train in previous example does not move with constant velocity then trainwould have been example of non inertialframe of reference.

Q2. Discuss the Galilean transformations for space and time.

Different inertial frames may have different values of physical quantities, but laws of physics do not depend on the choice of the frame. Galilean transformations are used to relate the physical quantities in one inertial frame to those in another inertial frame.

Let S and S' be Representations of two inertial frames with co-ordinate system (x, y, z) and (x' y' z') respectively. Let S' be moving with uniform velocity v with respect to S reference frame. Let an event take place at point P. The coordinates of point P with respect to individual frame is S (x, y, z, t) and S'(x', y', z', t') for simplicity let us assume that the x, y and z axes of coordinate systems are parallel to each other. We start counting from the time when origin of S i.e. O and origin of S'i.e. O'coincide.

After lapse of time 't', S' moving with velocity 'v' would have covered a distance 'd = v t'. Let co-ordinates system move along x axis with respect to each other. Hence, the relation between co-ordinates of P in one frame is related to those in other frame by the equations called the Galilean transformation equations given as:-



$x' = x - vt$

$y' = y$

$z' = z$

$t' = t$

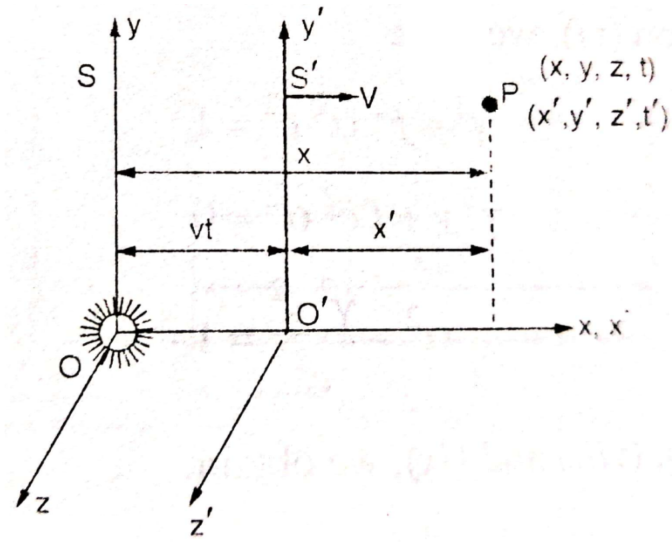
Q3. State the fundamental postulates of Special theory of relativity

1. The Principle of Relativity
   All physical laws are the same in all inertial frames that are moving relative to each other with constant velocity.
2. Principal of independence of velocity of light
   The speed of light in free space has the same value 'c' in all inertial reference frames.

Q4. Derive the Lorentz transformation equations for space and time

The Galilean transformations relate space time co-ordinates in one inertial frame to those in the other frame. But these transformations are not valid for the case where velocity of motion of one frame with respect to other frame approaches 'c' i.e. velocity of light.

The transformations that apply to all speeds and are valid up to 'c' i.e. velocity of light are known as Lorentz transformations.



Consider two inertial frames S and S' where standards of measuring distance and time are same. Let S be stationery and S' be moving with a constant velocity 'v' with respect to frame S along x' and x axes of frames S' and S respectively. Let us assume x and x' are the same line while y and y' also z and z' are parallel. Let at initial time t=t'=0 and this is when Origin O and O' of frames S and S' coincide.

As space and time are regarded homogeneous. The relation between their coordinate and time in different frames is linear

$x'=ax+bt$..(1)

$t'=fx+gt$..(2)

Equation (1) can be written as :

$x'=a(x+\frac{b}{a}t)$....................................(3)

After time t, origin O' of frame S' is at position x=vt with respect to frame S

And x' for origin O' corresponds to x'=0, putting these values in Equation (3)

$0=a(vt+\frac{b}{a}t)$ this gives ,

$v=-\frac{b}{a}$ ....(4)

Substituting (4) in Equation (3) we get,

$x'=a(x-vt)$............................(5)

Similarly we can write, $x=a(x'+vt)$ ....(6)

Since S' moves along x axis only, the y and z coordinates do not change , hence:

$y=y'$ and $z'=z$ ....(7)

We know that for light velocity ‘c’,

x’ =ct’ , whereas x = ct putting these two in Equation (5) we have:

c t’=a (c t - v t)

:. c t’ = a c t (1-$\frac{v}{c}$)

t’ = a t (1- $\frac{v}{c}$) ....(8)

Hence for t we can write, t = a t’ (1+ $\frac{v}{c}$) ....(9)

Using Equation (9) in Equation (8);

t’ = a a t’ (1+ $\frac{v}{c}$) (1- $\frac{v}{c}$) this becomes

$$a^2=\frac{1}{1-\frac{v^2}{c^2}}$$

More often the constant ‘a’ is denoted by $'\gamma'$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \text{ ....(10)}$$

Using Equation (10) in Equation (8) and Equation (5),

$$x' = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}} \text{ ....(11)}$$

$$t' = \frac{t\left(1-\frac{v}{c}\right)}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{t-\frac{vtc}{c\,c}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{t-\frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{(using ct=x) ....(12)}$$

Thus, the Lorentz transformation of coordinate from system S’ to frame S are

$$x' = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

y’= y; z’= z

$$t' = \frac{t-\frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}\text{. . ...(13)}$$

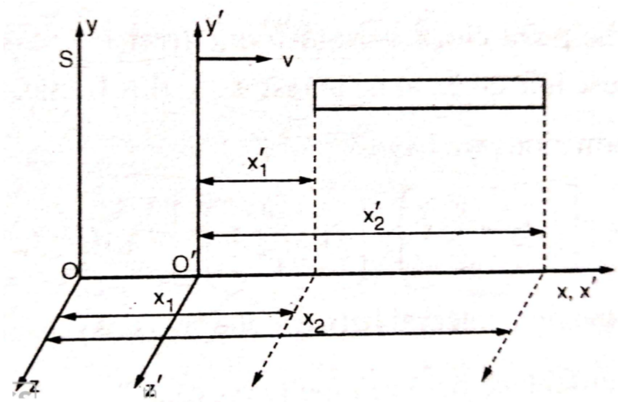
Similarly, the Inverse Lorentz transformation equations can be written as:

$$x = \frac{x' + vt}{\sqrt{1-\frac{v^2}{c^2}}}; \quad y = y'; \quad z = z'; \quad t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \quad \ldots\ldots\ldots\ldots(14)$$

Q5. Explain length contraction and time dilation

Relativistic effects appear to conflict commonsense. This is due to the fact that the velocities that we have observed in our everyday experience are very small as compared to velocity of light 'c'. Special theory of relativity predicts observers will measure different time and length in different inertial frames only when the frames are moving with a velocity comparable to 'c' with respect to each other. Length contraction and time dilation are two relativistic effects explained below:

1. Length Contraction:
   Consider two inertial frames S and S', where S' is moving with a constant velocity 'v' with respect to S. Let x_1' and x_2' be two ends of a rod at rest as seen by an observer in S' frame. Let L be the length of the rod measured by observer in S frame.



$$L = x_2 - x_1$$

$$L' = x_2' - x_1'$$

Both measurements are done simultaneously $\therefore t_2 = t_1 = t$

Using Lorentz transformation equations we get:

$$L' = x_2' - x_1' = \gamma\, [\,(x_2 - vt) - (x_1 - vt)]$$

$$L' = \gamma\,(x_2 - x_1)$$

$$L' = \gamma\, L$$

The Rod is at rest in S' $\therefore$ L' =Lo true length of the rod and the rod appears to be in motion to an observer in S frame so L= apparent length

$$L = \frac{L_o}{\gamma} = L_o\sqrt{1 - \frac{v^2}{c^2}}$$

So length contraction is explained by the equation $L=L_o\sqrt{1-\frac{v^2}{c^2}}$

2. Time dilation:
   Let a clock be at rest in S frame at point x. Suppose it produces two ticks at times t_1' and t_2' in frame S'. The time interval between these two ticks may be given by,

   $$\Delta t' = t_2' - t_1'$$

   Using Lorentz transformation equations,

   $$\Delta t' = t_2' - t_1' = \gamma\left[\left(t_2 - \frac{v}{c^2}x\right) - \left(t_1 - \frac{v}{c^2}x\right)\right\}$$

   $$\Delta t' = \gamma\,(t_2 - t_1) = \gamma\,\Delta t$$

   Let $\Delta t = t_o$ as the clock was at rest in the S frame
   $\Delta t' = t$ as the apparent time interval measured in the S' frame

   $$\therefore t = \gamma\, t_o$$

   So the time dilation is explained by equation $t = \frac{t_o}{\sqrt{1-\frac{v^2}{c^2}}}$

Q6. Derive Einstein's Mass energy relation.

Relativistic mass:

Mass of a body is supposed to be independent of its velocity. Due to momentum conservation we require that momentum of isolated system be conserved.

Relativistically for an isolated system to conserve momentum it is observed that mass must depend on velocity and the relation that govern this dependence is,

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} \text{..........................(1)}$$

Where,

m= moving mass; m_o= rest mass; v= velocity of motion; c = velocity of light

Relativistic momentum:

$$P = mv = \frac{m_o \times v}{\sqrt{1-\frac{v^2}{c^2}}} \text{.......................}(2)$$

This is the resultant relativistic momentum of the particle after substituting the relativistic mass that is dependent on velocity of the particle 'v'.

Kinetic energy:

Newton second law states that force is equal to rate of change of momentum.

$$F = \frac{d\,(mv)}{dt} = m\frac{d\,(v)}{dt} + v\frac{d\,(m)}{dt} \text{ ....}(3)$$

Kinetic energy of a moving body is force into displacement,

$$dE_k = F.dx = \left(m\frac{d\,(v)}{dt} + v\frac{d\,(m)}{dt} \right).dx$$

$$dE_k = mv\,dv + v^2\,dm \text{ .. ..}(4).\text{using } \frac{dx}{dt} = v$$

We know that, $m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$ and hence

$$dm = m_o \left(\frac{-1}{2}\right)\left(1-\frac{v^2}{c^2}\right)^{\frac{-3}{2}} \left(\frac{-2v}{c^2}\,dv\right)$$

$$dm = \frac{m_0\, v dv}{c^2\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}}$$

m_o can be replaced by $m\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}$.....using (1)

$$\therefore dm = \frac{mvdv}{c^2\left(1-\frac{v^2}{c^2}\right)} = \frac{mvdv}{(c^2 - v^2)} \text{ ………..............................}(5)$$

Rearranging (5) we can write,

$$c^2\,dm - v^2 dm = m\,vdv \text{ ....}(6)$$

Comparing Equations (6) and (4) we get

$$dE_k = c^2 dm$$

$E_k = \int_{o}^{E_k} dE_k = c^2 \int_{m_0}^{m} dm$

$E_k = c^2 (m - m_o)$

Thus, relativistic kinetic energy of a body is equal to the gain in mass multiplied by square of speed of light.

:.$m_o c^2$ is energy of body at rest.

Total energy of a body : $E = E_k +$ rest energy

$E = c^2 (m - m_o) + m_o c^2$

This is the Energy mass relation of Einstien i.e. **$E = m c^2$**