Relativity

Inertial reference frame:-

A Reference frame :- Three coordinates x,y z and time t define a reference frame All phenomenon need a reference frame with respectbto which they can be studied.

If a reference frame is either a rest of moving with a uniform velocity then it is called an inertial frame of reference

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Example:- A person standing on railway station and aperson standing in a train that is moving with constant velocity Both persons are in inertial frames.

Physical quantities measured wrt frame may vary but physical laws remain lame.

Ie. a person is running on the platform his velocity measured by first person stationary on the platform will be different from the velocity measured by passion inside the moving train but in both cases velocity = dislance /time the law remains unchanged.

Accelerated frame of reference:-

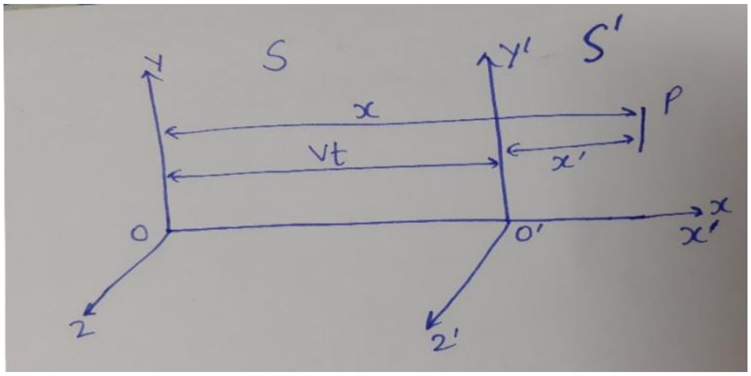
An accelerating frame of reference is called non inertial frame of reference.

Example;:- if the train in previous example does not Move with constant velocity then train would have been example of non inertial of reference

Galilean transformations :-

Different inertial frames may have different values of physical quantities, but laws of physics do not depend on the choice of the frame galilean transformation to relate the physical quantities in one inertial frame to those in another inertial frame.

Let S and S' be Representation of two inertial frames with co-ordinate system (x, y, z) and (x' y' z') respectively. Let S' be moving with uniform velocity v with respect to S reference frame.



let an event take place at point P. The coordinate of point P with wrt individual frame is S(x, y, z) and S'(x ,y, z,) for simplicity let us assume that the x y and z axis of coordinate systems are parallel to each other. We start counting from the time when origin of S ie. O and origin of S' ie. O' coincide.

After lapse of time 't' S' moving with velocity V would have covered a distance vt. Coordinates system move along x axis with respect to each other. Hence the relation between coordinate of P in one frame are related to those in other frame by these equation called Galilean transformation equation:-

$X' = x-vt$

$Y' = Y$

$Z' = Z$

$t' = t$

postulates of Special theory of relativity:-

The Principle of Relativity

1. All physical laws are the time in all inertial frames that are moving relative to each other with constant velocity.

2. principal of independence of velocity of light

The speed of light in free space has the same value c in all inertial reference frame.

Lorentz transformation:-

The Galilean transformations studied in section relate space time coordinatesbin one inertial frame to those in the other frame. But these transformations area not valid for the case where velocity of motion one frame with respect to other frame approaches C ie velocity of light.

The transformations that apply to all speeds and are valid upto C ie velocity of light are known as Lorentz frame transformations They were developed in 1890 by german physics H.A. Lorentz(1853-1928)

Consider two identical trains S and S’ where standards of measuring distance and time are same, let S be stationery and S’ with a constant velocity with respect to frame S along x’ and x axes of frames S’ and S   respectively . Let us assume x and x’ are the same time while Y and Y’ also Z and Z’ are parallel .Let at initial time t=t’=0 and this is when Origin O and O’ of frame S and S’ coincide.

As space and time are regarded homogeneous . The relation between their coordinate and time in different frames is linear

x^2 =a x +by...1

t^2 =f x +g t...2

Equation 1 can be written then as

x^2=a(x+ $\frac{b}{a}$ t’)....................................3

At time t, O’ of frame S’ is at position X=vt

x' for O’ is x'=0

Putting all this in equation 3

0=a (v t + $\frac{b}{a}$ t)

V= - $\frac{b}{a}$ 4

Putting 4 and 3 gives.,

X’'=a (x -v t) 5

Similarly, x= a (x’+vt)

Since S’ moves along x axis only y and z coordinates do not change.

Y = Y’

Z’= Z

We know that for light velocity C,

X’ =Ct’

For t’ we can write c t’=a (c t -v t)

:. X’ = c t’ = a c t (1- $\frac{b}{a}$)

X= c t=ac t’ (1+ $\frac{b}{a}$)

t’=at(1- $\frac{b}{a}$)

t_o=a^2t (1- $\frac{b}{a}$)

a^2=(1/(1-v^2/c^2))

More often a is denoted by r

r = $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ 10

Using 5 and 10,

X’ = $\frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}}$

$T^2 = \frac{x^2}{c} = \frac{\frac{x}{c}-\frac{vt}{c}}{\sqrt{1-\frac{v^2}{c^2}}}$

Thus, Lorentz transformation of coordinate from system S1 to frame S2 are

x’ = r(x -vt)

y’= y

z’= z

t’ = r (t - $\frac{vx}{c^2}$)............................13

Inverse transformation,

x=r(x’-vt’)

y = y’

z = z’

t = r (t’ - $\frac{vx'}{c^2}$)...............................14

Velocity transformation:-

Suppose that a body moving with a constant velocity u is observed in the references frame S at x_1' at time t_1' and x_2' at time t_2' . It’s speed measured in S' Ux' is given by

Ux'= $\frac{x'_2 - x'_1}{t'_2 - t'_1} = \frac{dx'}{dy'}$ 15

We know that x'=$\frac{x-v\,t}{\sqrt{1-\frac{v^2}{c^2}}}$

dx' =$\frac{dx- vdt}{\sqrt{1-\frac{v^2}{c^2}}}$

similarly, dt'= $\frac{dt-\frac{v}{c^2}dx}{\sqrt{1-\frac{v^2}{c^2}}}$

$\frac{dx'}{dt'} = \frac{\frac{dx}{dt} - v}{1-(\frac{v}{c^2})\frac{dx}{dt}}$

Ux'= $\frac{Ux- v}{1-U_x\,\frac{v}{c^2}}$

Similarly inverse transformation is,

Ux = $\frac{Ux'+ v}{1+ U_x\,\frac{v}{c^2}}$

The above is the case when S' moves along x axis . In case if the motion is along y or z direction we find,

Uy' = $\frac{U_y\sqrt{1-\frac{v^2}{c^2}}}{1- U_x\frac{v}{c^2}}$

We arrive at this equation as follows,

$y' = y$

$dy' = dy$

$$dt' = \frac{dt - \frac{v}{c^2}dx}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{dy\prime}{dt\prime} = \frac{\frac{dy}{dt} - v}{1-(\frac{v}{c^2})U_x}$$

$$Uy' = \frac{U_y\sqrt{1-\frac{v^2}{c^2}}}{1-U_x\frac{v}{c^2}} \quad \text{...18}$$

$$Uz' = \frac{U_z\sqrt{1-\frac{v^2}{c^2}}}{1-U_x\frac{v}{c^2}} \quad \text{...19}$$

Conclusion drawn from Lorentz transformation:-

Consequences of Special Relativity –

Relativistic effect appear to conflict commonsense . This is due to the fact that the velocities that we have observe in our everyday experience are very small as compared to velocity of light. Special theory of relativity predicts observes will measure different times and length in different inertial frames only when the frames are moving with velocity comparable to C with respect to each other.We will now study two effects of relativity in detail.

1} Length Contraction:-

Two frames S and S' where S' is moving with a constant velocity v with respect to S . Let x_1' and x_2' be two ends of a rod at rest as seen by an observer in S' frame. Let L be the length of the rod measured by observer in S frame.

L=$x_2 - x_1$

$L' = x_2' - x_1'$

Both measurements are done simultaneously :. $t_2 = t_1 = t$

$L' = x_2' - x_1' = r\,[\,(x_2 - v\,t) - (x_1 - v\,t)\,]$

$L' = r\,(x_2 - x_1)$

$L' = rL$

Rod is at rest in S' :. L' =Lo true lengthy of rod.

$L = \frac{L_o}{r} = L_o \sqrt{1 - \frac{v^2}{c^2}}$....................................20

2} Time dilation :-

Let a clock be at rest in S frame at point x. Suppose it produces two ticks at times t_1' and t_2' in frame S_2'.The time interval between these two ticks may be given by , $\Delta t' = t_2' + t_1'$

Using Lorentz transformation equation,

$\Delta t' = t2' - t1' = r\,[\,(t_2 - \frac{v}{c^2} x) - (t_1 - \frac{v}{c^2}\ x)\}$

$\Delta t' = r\,(t_2 - t_1)$

$= r\,\Delta t$

Δt =to as the clock was at rest in S frame

Δt' = t as the time interval is measured in S'

:. t = r to

$t = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$ 21

:. Time gets dialated.

Mass energy relation:-

<u>Relativistic mass:</u>- mass of a body is supposed to be independent velocity Due to momentum conservation we require that momentum of isolated system be conserved.

Relativistically for an isolated system to conserve momentum it is observed that mass must depend on velocity and the relation that govern this dependence is,

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} \text{ ....22}$$

Where m= moving mass

m_o= rest mass

V= velocity of motion

C = velocity of light

<u>Relativistic momentum:-</u>

$$P= mv = \frac{m_o \times v}{\sqrt{1-\frac{v^2}{c^2}}} \text{ ....23}$$

This is the relativistic momentum of the particle.

Kinetic energy:- Newton second law states that force is equal to rate of change of momentum. $F= \frac{d\,mv}{dt}$ 24

Kinetic energy of a moving body as force into displacement,

$$Ex=\int_0^s F ds = \int_0^s (mv) ds$$

$$dEx = F.dx = m\,(\frac{dv}{dt})dx + v\,(\frac{dm}{dt})dc$$

$$dEx= mv\,dv+ v^2\,dm$$

We know that, $m= \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$

$$dm = m_o\,(\frac{-1}{2})(1-\frac{v^2}{c^2})^{\frac{-3}{2}}\,(\frac{-2v}{c^2}\,dv)$$

$$dm= \frac{m_0\,vdv}{c^2\,(1-\frac{v^2}{c^2})^{\frac{3}{2}}}$$

m_o can be replaced by $m(1 - \frac{v^2}{c^2})^{\frac{3}{2}}$

:.$dm = \frac{m\, vdv}{c^2\ (1 - \frac{v^2}{c^2})} = \frac{m\, vdv}{(c^2 - v^2)}$ ……………………………3

3 can be rearranged as,

$c^2\ dm - v^2\ dm = m\ v\ dv$ ……………………………4

$dE_k = m\ v\ dv + v^2\ dm$

$dE_k = c^2\ dm$

$E_k = \int_o^{E_k} dE_k = c^2 \int_{m_0}^{m} dm$

$E_k = c^2\ (m - m_o)$

Thus, relativistic kinetic energy of a body is equal to the gain in mass multiplied by square of speed of light.

:. $m_o c^2$ is energy of body at rest.

<u>Total energy of </u>:-

$E = E_k$ + rest energy

$E = c^2\ (m - m_o) + m_o\ c^2$

Energy mass relation $\mathbf{E = m\ c^2}$