DIFFRACTION

Q1a. What do you mean by diffraction? State its types and differentiate between them. **(M.U. May 09, 11; Dec. 2009, 11, 15) (3 m)**

When waves encounter obstacles (or opening), they bend round the edges of the obstacles if the dimensions of the obstacles are comparable to the wavelength of the waves. The bending of waves around the edges of an obstacle (or opening) is called Diffraction.

Basically, the diffraction phenomenon has two main types:

(A) Fresnel Diffraction
(B) Fraunhoffer Diffraction

Difference between Fresnel diffraction and Fraunhoffer diffraction

Fresnel Diffraction	Fraunhoffer Diffraction
Both the source and slits are at finite distance from the slit.	Both the source and slits are at infinite distance from the slit.
The wavefront incident on the aperture is either spherical or cylindrical.	The wavefront incident on the aperture is plane.
The incident and diffracted rays are divergent.	The incident and diffracted rays are parallel.
Lenses are not required in actual experiment.	Lenses are used in experiment to achieve parallel wavefront.
Path difference between the rays forming the diffraction pattern depends on distance of slit from source as well as the screen and the angle of diffraction. Hence mathematical treatment is complicated.	Path difference between the rays forming the diffraction pattern depends only on the angle of diffraction. Hence mathematical treatment is comparatively easier.

Q1b. Why is diffraction not evident in daily life?

(M.U. May 2008) (3 m)

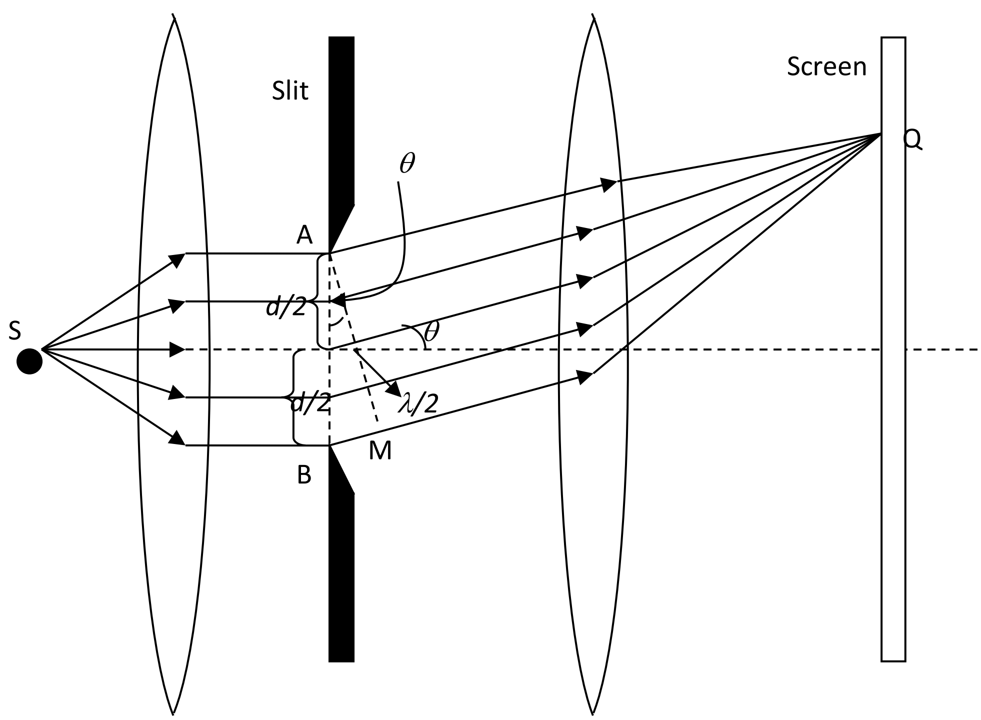
In daily life the objects that we come across are very small. Objects of size of the order of wavelength of light i.e. around 0.1 micro meter are needed for diffraction to be observed, hence it is not evident in daily life easily.

Q2. Explain Fraunhoffer Diffraction at a single slit, obtain expression for the resultant intensity and derive expressions for maxima and minima for a single slit.

(M.U. May 2007) (5 m)

Let *AB* be a single narrow slit of width ***d*** on which a parallel beam of monochromatic light is incident as a plane wave front.

The incident wave front is diffracted by the slit and is then focused on the screen by the lens *L*. According to Huygens principle, each point on *AB* acts as a source of secondary waves.



As all the points on *AB* are in phase, the point sources will be coherent. Hence the light from one portion of the slit can interfere with light from another portion and the resultant intensity on the screen will depend on the direction θ of the diffracted waves.

When the waves traveling in straight direction without diffraction, they are in the same phase and after covering equal optical path lengths their superposition produces zero order central maxima where the dotted line meets the screen. Lets refer this point as P.

Let us consider another point *Q* on the screen. The waves that leave the slit at an angle θ reach the point Q. The point *Q* will be dark or bright depending upon the path difference between the waves arriving at Q from different points on the wave front. The total path difference between the waves that are travelling from point A and point B on the slit is :-

path difference = $d \sin\theta$

This path difference corresponds to a total phase difference of :-

$$\phi = \frac{2\pi}{\lambda} d \sin\theta \qquad \text{..............(1)}$$

Derivation for Intensity variation :-

In order to determine the intensity distribution for the single slit diffraction pattern, we follow the graphical approach.

Consider the slit is divided into a large number (N) of narrow strips of equal width Δy .

Each strip acts as a source of coherent radiation and the light emanating from it can be represented by a short phasor.

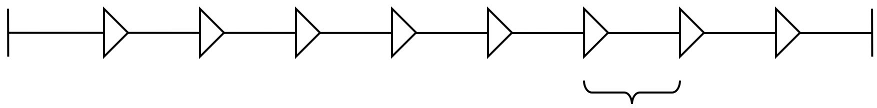
The path difference between the rays diffracted from its upper and lower edges :-

path difference for a strip= $dy.\sin\theta$

hence the corresponding phase difference between them is,

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta y \sin\theta \qquad \text{..............(2) such that } \phi = N\Delta\phi$$

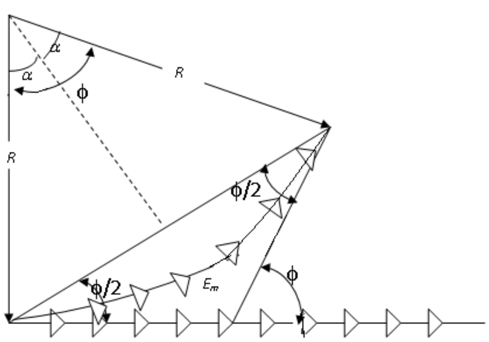
(i) At the centre of diffraction pattern θ=0 hence net phase difference is zero, and the phasors in this case are laid end to end



The amplitude of the resultant has its maximum value E_m given as :-

$$E_m = N\Delta E \quad \text{(slit is divided into N no. of parts)}$$

(ii) At a value of θ other than zero, $\Delta\phi$ has a finite value. The amplitude E_θ of the resultant is the vector sum of phasors, and hence given by the length of chord. It can be seen that E_θ is less than E_m.



The resultant amplitude E_θ equals the length of chord and E_m is the length of an arc as shown in the figure and ϕ is the measure of total phase difference between initial and final phasors and as opposite angles of quadrilateral are supplementary ϕ is also the angle of the sector formed between the two radii R.

Inside this sector the perpendicular drawn from the centre of the circle to the chord bisects the chord, hence

$$\sin\phi/2 = \frac{E_\theta/2}{R}$$

$$\therefore E_\theta = 2R\sin\phi/2$$

$$\text{But } \phi = \frac{E_m}{R} = \frac{arc}{radius}$$

$$E_\theta = 2\left(\frac{E_m}{\phi}\right)\sin\frac{\phi}{2}$$

Using all the above we can write expression for resultant amplitude at pt Q

$$\boxed{E_\theta = E_m\left[\frac{\sin\phi/2}{\phi/2}\right]} \quad .... \text{ (3)}$$

The intensity I_θ is proportional to amplitude square E_θ^2,

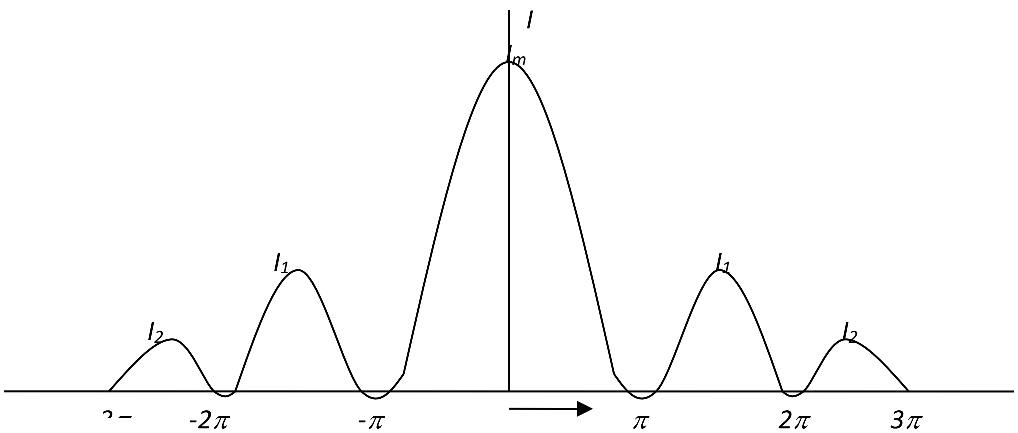
$$I_\theta = I_m\left[\frac{\sin\phi/2}{\phi/2}\right]^2 \quad ....\text{(4)}$$

Let $\frac{\phi}{2} = \alpha = \frac{\pi d\sin\phi}{\lambda}$ ....(5)

Hence,

$$\boxed{I_\theta = I_m\left[\frac{\sin^2\alpha}{\alpha}\right]} \quad ....\text{(6)}$$

The following graph shows the intensity variation with respect to angle ϕ in a single slit diffraction.



Conditions for maxima and minima :-

(a) Principle Maximum:

The resultant amplitude in diffraction pattern is given by

$$E_\theta = E_m\left[\frac{\sin\alpha}{\alpha}\right]$$

For E_θ to be maximum, we need $\alpha = 0$ i.e $\alpha = \frac{\pi}{\lambda} d \sin\theta = 0$

$\therefore \sin\theta = 0$ i.e. $\theta = 0$.

Thus principal maxima is obtained at $\theta = 0$.

(b) Minimum intensity positions (minima):

The intensity $I_\theta = I_m\left[\frac{\sin^2\alpha}{\alpha^2}\right]$ will be zero where $\sin\alpha = 0$ but $\alpha \neq 0$

The values of α which satisfy that equation are :-

$\alpha = n\pi$ where $n = \pm1, \pm2, \pm3, \ldots\ldots.$

$$\alpha = \frac{\pi}{\lambda} d \sin\theta = n\pi$$

Hence, the condition for minima is

$d \sin\theta = n\lambda$, where $n = \pm1, \pm2, \pm3, \ldots\ldots.$

(Since θ becomes zero which corresponds to the principal maximum. The positions of minima are on either side of principal maximum.)

(c)Secondary maxima

Analysis shows that the secondary maxima lie approximately half way between the minima. i.e.

$$\alpha = \pm\left(n + \frac{1}{2}\right)\pi \qquad n = 1, 2, 3, \ldots\ldots.$$

$$d \sin\theta = (2n+1)\frac{\lambda}{2}$$

Substituting this value of α in I_θ.

$I_\theta = I_m\left[\frac{\sin^2\alpha}{\alpha^2}\right]$ we get,

$$\frac{I_\theta}{I_m} = \left[\frac{\sin\left(n+\frac{1}{2}\right)\pi}{\left(n+\frac{1}{2}\right)\pi}\right]^2$$

$$= \frac{1}{\left(n+\frac{1}{2}\right)^2 \pi^2} \qquad m = 1, 2, 3, \ldots\ldots$$

$$\frac{I_\theta}{I_m} = 0.045, 0.016, 0.0083, \ldots\ldots$$

Thus the successive maxima decrease in intensity rapidly.

Q3. What is Diffraction Grating ? Explain the construction of diffraction grating. Determination of Wavelength of Light using Grating.

(M.U. May 2008, 11, 13, 16, 17; Dec. 2009, 12, 17, Nov. 2018) (5 m)

An arrangement consisting of large number of parallel slits of equal width and separated from one another by equal opaque spaces is called the diffraction grating.

A diffraction grating is formed by ruling a plane glass plate with fine lines using a diamond point. Ideally a diffraction grating will have 5000 to 15000 lines per inch.

The diffraction formula for a principal maxima in a grating's diffraction is given by

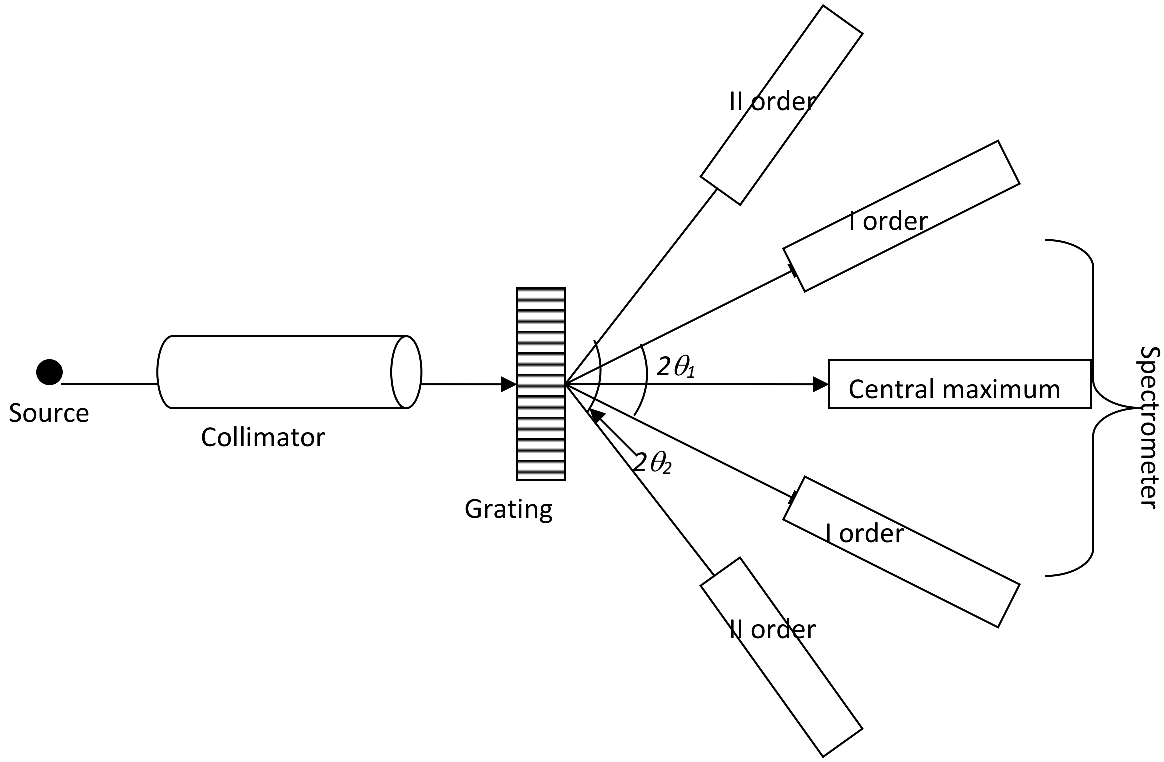
$$(a+b)\sin\theta = n\lambda$$

Where; (a+b) = grating element

n = order of spectrum

λ= wavelength of incident light.

A diffraction grating is often used in laboratories to determine the unknown wavelength of light. The grating spectrum of the given source of monochromatic light is obtained by using a spectrometer. The arrangement is as shown in Figure

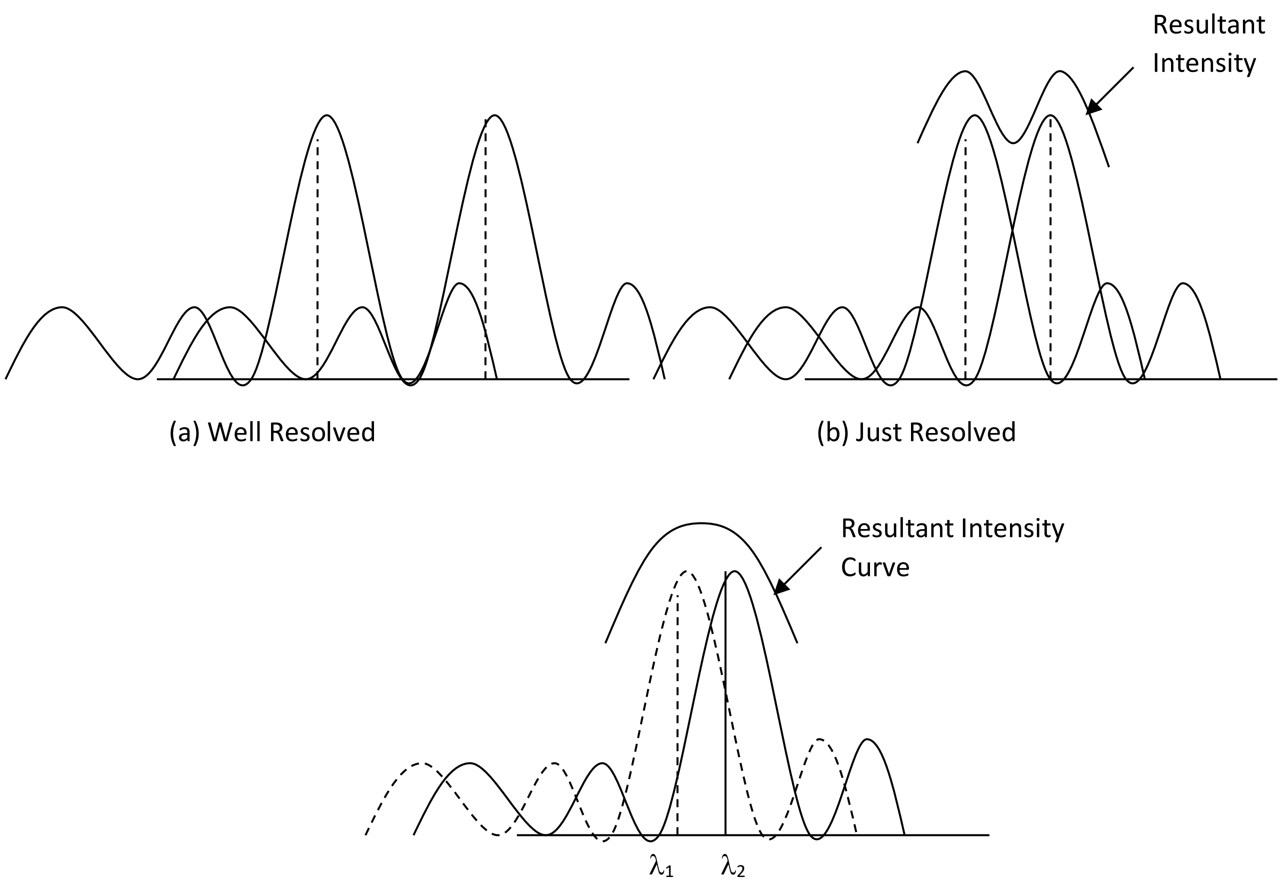


1. The spectrometer is first adjusted for parallel rays the grating is then placed on the prism table and adjusted for normal incidence.
2. In the same direction as that of the incident light, the direct image of the slit or the zero-order spectrum can be seen in the telescope.
3. The angle of diffraction θ for a particular order *n* of the spectrum is measured.
4. The number of lines per inch of grating are written over it by the manufacturers this is noted as 'N'
5. Hence grating element is calculated using following formula
   a. *(a+b) in cm* = 1/(Number of lines/cm)
6. The unknown wavelength is calculated using the grating formula:

$$(a+b)\sin\theta = n\lambda$$

Q4. Explain Rayleigh's Criterion of Resolution .

(M.U. May 2010, 11, 13, 14, 15; Dec. 2016, 17) (3 m)



(a) Well Resolved

(b) Just Resolved

(c) Objects are not resolved

Rayleigh's criterion of resolution states that :

Two closely spaced point sources of light are said to be **just resolved** by an optical instrument only if the central maximum of diffraction pattern one falls over the 1^{st} minimum in the diffraction pattern of the other and vice versa.

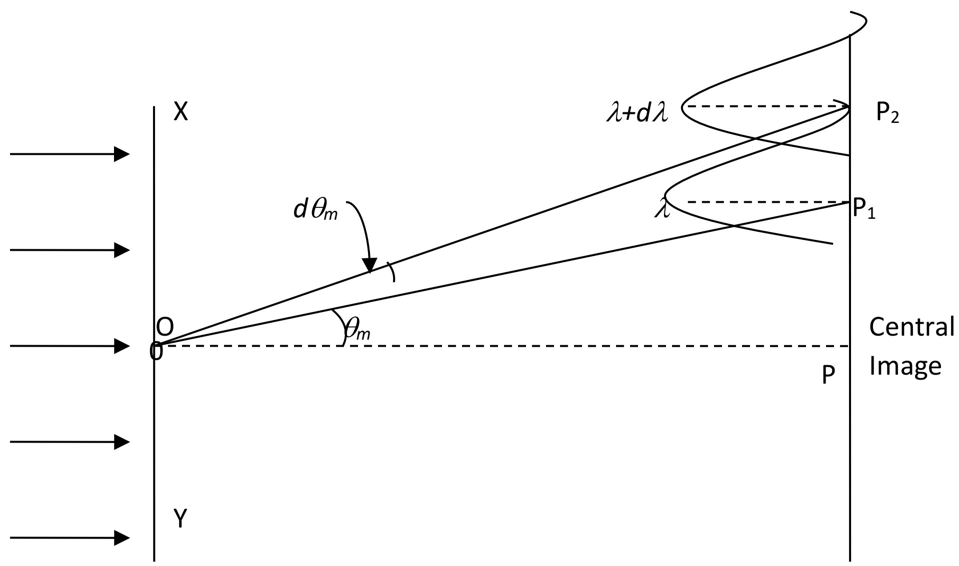
Q5. What is Resolving Power of an optical Instrument ? Obtain an expression for resolving power of a diffraction grating .

(M.U. May 2010, 11, 13, 14, 15; Dec. 2016, 17) (3 m)

An optical instrument is said to be able to resolve two point objects if the corresponding diffraction patterns are distinguishable from each other. The ability of the instrument to produce just separate diffraction pattern of two close objects is known as its **Resolving power**.

Resolving Power of a grating :

A grating is capable of resolving the image of slit formed by the two spectral lines of wavelength λ and $(\lambda + d\lambda)$.



The resolving power of a grating is defined as the smallest wavelength $d\lambda$ for which the spectral lines can be first resolved at the wavelength λ, is mathematically given as $R.P = \dfrac{\lambda}{d\lambda}$ ---------(1a)

Let XY be the grating surface and MN is the field of view of the telescope.

P_1 is the mth primary maximum of spectral line wavelength λ at an angle of diffraction θ_m.

P_2 is the m^{th} primary maximum of a second spectral line of wavelength $\lambda+d\lambda$ at an angle of diffraction $\theta_m+d\theta_m$.

Conditions for maxima P_1 and P_2 are:

$$(a+b)\sin(\theta_m+d\theta_m) = m(\lambda+d\lambda) \text{--------(1)}$$

$$(a+b)\sin(\theta_m) = m\lambda \text{------(2)}$$

Distance between two maxima can be calculated by substracting (1)-(2) we get:

$$P1\ \mathrm{P2} = (a+b)\sin(\theta_m+d\theta_m) - (a+b)\sin(\theta_m) = m(\lambda+d\lambda) - m\lambda$$

Which results into

$$P1\ \mathrm{P2} = m(d\lambda) \text{-------(3)}$$

According to Rayleigh's criterion $P1\ \mathrm{P2}$ will also be equal to the distance between the principal maxima and first minima of source λ

Principal Maxima condition : $(a+b)\sin(\theta_m) = 0$---------(4)

First Minima condition: $(a+b)\sin(\theta_m) = \frac{\lambda}{N}$----------(5)

Thus we get $P1\ \mathrm{P2}$ by subtracting (5)-(4):

$$P1\ \mathrm{P2} = \frac{\lambda}{N} \text{-------(6)}$$

Comparing equation (6) and (3):

$$md\lambda = \frac{\lambda}{N}$$

Then by the definition of resolving power of grating in equation (1a) we obtain the expression for resolving power of grating in terms of number of lines N for the grating as:

$$\therefore R.P = \frac{\lambda}{d\lambda} = mN$$