# TOPIC: ELECTRODYNAMICS

**Co-ordinate Systems**

In electromagnetics, most of the quantities are functions of space and time. In order to describe the spatial variations of these quantities, all the points in space must be defined uniquely using an appropriate coordinate system.

We will discuss the most useful three coordinate systems, namely,

1. Cartesian, or rectangular, coordinates

2. Cylindrical, or circular, coordinates

3. Spherical, or polar, coordinates.

**1. Cartesian or Rectangular Coordinates (x, y, z)**

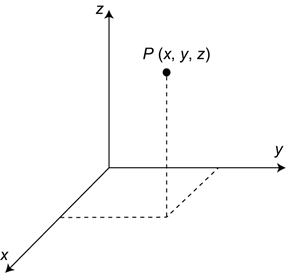
A point P in Cartesian coordinates is represented as P(x, y, z).

The ranges of coordinate variables are



From Figure b, it is understood that any point in a rectangular coordinate system is the intersection of three planes:

1. Constant X-Plane.
2. Constant Y-Plane. And
3. Constant Z-Plane, Which are mutually perpendicular



A vector in the Cartesian coordinate system is written as,

Where,

Are the unit vectors along the x,y and z directions respectively

**2. Cylindrical or Circular Coordinates(r, ϕ, z)**

A point P in cylindrical coordinates is represented as P(r, ϕ, z),

Here,

r=Radius of the cylinder passing through P=Radial Distance from the Z-axis

ϕ=Angle measured from the X-axis in the xy-plane, known as azimuthal angle

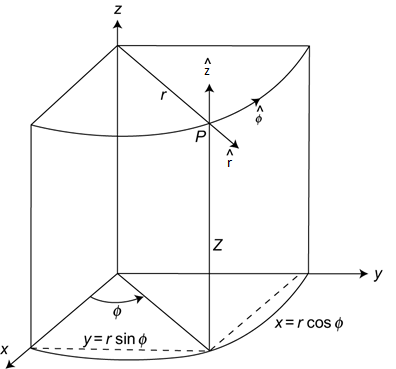
z= same as in Cartesian coordinates

The Ranges of Coordinate variables are,



From the figure (b), it is understood that any point in the cylindrical coordinates in an intersection of three planes viz.

1. Constant ‘r’ plane (a circular cylinder)
2. Constant ϕ plane (semi-infinite plane with its edges along the z-axis ) and
3. Constant Z-plane (Parallel to xy-plane)



A vector A in cylindrical the coordinate system is written as,

Where and

Are the unit vectors along the r, ø and z directions respectively.

Relations between Cartesian (x, y, z) and Cylindrical (r, ø, z) Coordinates

The relationships between Cartesian (x, y, z) and cylindrical (r, ø, z) coordinates are obtained from Fig. (a) And are written as,



And



**3.Spherical or Polar Coordinates (r,ϴ,ø)**

A point P in spherical coordinates is represented as P(ρ,ϴ,ø).

Here,

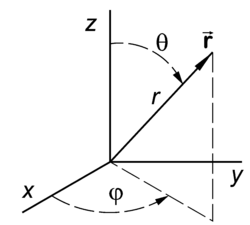
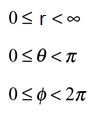
r= Distance of the point from the origin

ϴ = Angle between the z-axis and the position vector P, known as colatitudes, and

ø =Angle measured from the x-axis in the xy-plane, known as azimuthal angle (same as in cylindrical

coordinates)

The ranges of coordinate variables are,



From Fig. we can se that any point in spherical coordinates is an intersection of three planes, viz,

1. Constant''*r*" plane (a sphere with its centre at the origin),
2. constant ϴ -plane (circular cone with z-axis as its axis and the origin at its vertex),
3. constant ø -plane (semi-infinite plane as in cylindrical coordinates).

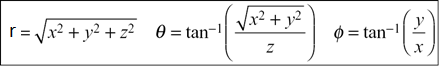
A vector in the spherical coordinate system is written as,

Where,

are the unit vectors along the r,ϴ,and ø directions respectively.

Relation between Cartesian(x,y,z) and Spherical (r,ϴ,ϕ)Coordinates

The relationships between Cartesian (x, y, z) and cylindrical (r,ϴ,ϕ) coordinates are written as



And



**Scalar and Vector fields:**

Fields: Behaviour of a physical quantity in a given region is described by its value at each point in the region.

**Del Operator**

1. The collection of partial derivative operators is called DEL operator. Hence DEL can be viewed as the derivative in multi-dimensional space.  
 2. DEL operator is defined as a vector differential operator.  
  
3. A DEL operator is not a [vector](http://www.emtmadeeasy.com/2009/12/vector-algebra-introduction.html) in itself, but when acts on a [scalar](http://www.emtmadeeasy.com/2009/12/vector-algebra-introduction.html) function, it becomes a vector.  
  
4. Del is not simply a [vector](http://www.emtmadeeasy.com/2009/12/vector-algebra-introduction.html); it is a vector operator. Whereas a vector is an quantity with both a magnitude and direction, DEL does not have a precise value for either until it is allowed to operate on something.  
  
5. In [Cartesian coordinate system](http://www.emtmadeeasy.com/2009/11/cartesian-co-ordinate-system.html) Del operator is given as:

This operator is useful or significant in defining

* Gradient of a scalar V: ()
* [Divergence of a vector A : (](http://www.emtmadeeasy.com/2009/12/divergence-of-vector-field-div.html.))
* [Curl of a vector A: (](http://www.emtmadeeasy.com/2009/12/curl-of-vector-field-curl-defination.html.))

**1.Gradient of a Scalar T (grad T)**

1. The gradient of a scalar field provides a vector field that states how the scalar value is changing throughout space – a change that has both a magnitude and direction.  
  
2.The physical meaning of the gradient of a scalar is that it represents the steepness of the slope or line. For example, height is a scalar quantity; gradient of the height would be a vector pointing upwards. The length of the [vector](http://www.emtmadeeasy.com/2009/12/vector-algebra-introduction.html) is proportional to the steepness of the slope.  
Gradient of a scalar T for [Cartesian coordinate system](http://www.emtmadeeasy.com/2009/11/cartesian-co-ordinate-system.html) is given as:

**2.Curl Of a Vector Field (Curl A)**

1.Circulation of vector field  around a closed path L is defined as:

2.The direction of the curl is the axis of rotation, as determined by the right hand thumb rule and the magnitude of the curl is the magnitude of rotation.  
  
3.If the curl of a vector field is zero, then the vector field is said to be irrotational or potential (if =0). In such cases, the circulation of  around a closed path is zero; it implies that the line integral of is independent of the chosen path. Hence an irrotational field is also known as a conservative field.   
  
4.Curl of a vector  in [Cartesian Coordinate system](http://emtmadeeasy.blogspot.com/2009/11/cartesian-co-ordinate-system.html) is given as:

### 3.Divergence Of a Vector Field ( div )

1. A divergence is applied to a vector as a function of position, yielding a [scalar](http://www.emtmadeeasy.com/2009/12/vector-algebra-introduction.html). The divergence actually measures how much the vector function is spreading out.

2. Divergence of a vector field is a measure of how much a vector field converges to or diverges from a given point. In simple terms it is a measure of the outgoingness of a vector field.  
  
 3. A vector field with constant zero divergence is called solenoid or divergence less or incompressible (= 0). In such cases no net flow can occur across any closed surface.   
  
4. Divergence of a vector field results in a scalar field that represents the sources of the vector field.  
  
5. Divergence of a vector field in [Cartesian coordinate system](http://www.emtmadeeasy.com/2009/11/cartesian-co-ordinate-system.html) is given as:

**Important Theorem for electrodynamics.**

**1.Divergence Theorem.**

This theorem states that volume integral of divergence of Vector taken over volume ‘V’ equals surface integral of taken over surface ‘S’ enclosing the volume.

**2. Stokes Theorem**

The Surface integral of curl of vector over an open surface ‘S’ equals line integral of vector over the enclosed curve ‘l’ bounding surface area ‘S’.

**Maxwell’s Equations**

**1.Maxwell’s First Equation (Gauss Law for Electric Field).**

Electric flux passing through any closed surface ‘S’ is equal to the total charge enclosed by the surface.

= ø = /

And we know that =

Where, =

By Divergence Theorem, =

Therefore, =

Hence***,***

**2.Maxwell’s Second Equation(Gauss Law for Magnetic Field).**

In a magnetic field, the magnetic lines are closed on themselves. Hence, total magnetic flux is zero.

= = 0

Where, B is magnetic flux density

By Divergence Theorem, =

Therefore = 0

Hence ***= 0***

**3. Faraday’s Law(Maxwells third equation for steady fields).**

In static electric field work done in moving a test charge around a closed path is equal to zero.

Conservative Field

= 0

By Stoke’s Law, =

Therefore,  ***= 0***

**Maxwell’s Equations:**

Four equations that are foundation of electromagnetic theory. There are extensions of work of Gauss,Faraday and Ampere.

**Two classifications of Maxwell’s equations:**

1. Static and time varying
2. Integral form and differential form

**Static electric and magnetic field:**

1. Gauss law for electric field

STATEMENT- The electric flux passing through any closed surface is equal to charge enclosed by it.

D is flux density

Q=

:.

Divergence theorem,

Therefore,

1. Gauss law for magnetic field

STATEMENT-In magnetic field the magnetic lines are closed on themselves. Hence the total outgoing magnetic flux is zero.

Divergence theorem,

Therefore,

1. Faradays law

STATEMENT: in static electric fileds work done in moving a test charge around a closed path is equal to zero. Such fields are called conservative fields.

Stroke’s theorem

Hence,

1. Ampere’s law for static Magnetic field:

STATEMENT: The line integral of magnetic field around a closed path is exactly equal to the direct current enclosed by that path.

But we know that,

-by strokes theorem

Therefore,

**Maxwell’s Equations for Static Physical Quantities**

|  |  |  |  |
| --- | --- | --- | --- |
| **S.NO.** | **DIFFERENTIAL FORM** | **INTEGRAL FORM** | **SIGNIFICANCE** |
| 1. |  | =Q/ | Gauss Law of Electrostatics |
| 2. | = 0 | = 0 | Gauss Law of Magneto statics |
| 3. | = 0 | = 0 | Faraday’s Law |
| 4. | = | = | Ampere’s Circuital Law |

**4.Ampere’s Circuital Law (Maxwells fourth equation for steady fields).**

The line integral of magnetic field ‘H’ around a closed path is exactly equal to direct current ‘I’ enclosed by that path.

= I

By Stoke’s Theorem, =

And I =

Therefore, =

Hence, ***=***

***Electrostatics and magnetostatics***

|  |  |  |  |
| --- | --- | --- | --- |
| Sr No. | Differential Form | Integral form | Significance |
| 1 |  |  | Gauss law of electrostatics |
| 2 |  |  | Magnetostatics |
| 3 |  |  | Faraday’s law |
| 4 |  |  | Ampere’s circuital law |

***Time Varying Electric and Magnetic Fieds:***

1. **Maxwell’s Third Equation with time varying field:**

STATEMENT: A time varying magnetic field produces an electromotive force which may establish a current in a suitable closed circuit.

Electromotive force induced in losed loop is negative rate of change of magnetic flux ø

e= -

ø =

Therefore, e = -

The electromotive force is the work done in carrying unit charge around a closed loop.

Therefore, e =

= -

By Stoke’s Theorem, =

Hence, = -

1. **Ampere’s law in time varying field.**

* Consider a small volume element located inside a conducting medium the current density has the direction of current flow.
* There is no source or sink of charge inside current in steady and continuous so,

--------- (1)

Divergence theorem,

* It is not steady. Difference between the current flowing into the volume and that flowing out of the volume must equal to rate of change of electric charge inside the volume.

Net flow of current out is positive

Rate of decrease of charge this is expressed by continuity equation

Ampere’s law,

Divergence with respect to is always zero.

Therefore,

This contradicts continuity equation

Therefore, suppose G is unknown

Therefore, continuity equation

Gauss law,

G=

Therefore,

**Maxwell’s Equations for Time-Varying Fields**

|  |  |
| --- | --- |
| **S.NO.** | **DIFFERENTIAL FORM** |
| 1 |  |
| 2 | = |

**NOTE:** We assumed that magnetisation and polarisation of the medium is zero.

If these are not zero for a particular medium,

**Significance of Maxwells Equations:**

1. Maxwells equations are set of 4 complicated equations that describe the world of electromagnetics in a concise way.
2. Maxwells equations describe how electric and magnetic field propogate, interact and how are they influenced by objects.
3. Maxwells equations units electromagnetism and optics. Since Maxwells equations electricity, magnetism and light are understood as aspect of single objective the electromagnetic field.
4. In short Maxwells equations for the first time summarized the fundamentals of electricity and magnetism in the most elegant way, forming a theory of electrodynamic.

* Beauty of these equations makes these one of the greatest intellectual achievements of mankind.

1. Maxwells equations are critical in understanding working of antennas, waveguide and satellite communication.

**APPLICATIONS OF ELECTRODYNAMICS**

**1.DESIGN OF ANTENNA**

Antenna is used for wireless communication to transmit or receive signals .During transmission, electrical power is converted into radio waves while during reception radio wave energy is converted into electrical energy.

The basic principle on which antenna works is that radiation is produced by accelerated charges or time varying current, which gives rise to oscillating magnetic & electric fields in the space surrounding the antenna.

**2.WAVEGUIDES**

These are used for propagating the electromagnetic waves. The intensity of the electromagnetic energy falls rapidly with the distance as per the inverse square law. A waveguide reduces the loss by guiding he waves along a certain path.

Optical fibres, coaxial cables & hollow metal pipes are used as waveguides. To analyse the mode propagation in the waveguide, Maxwell equations are used.

**3.SATELLITE COMMUNICATION**

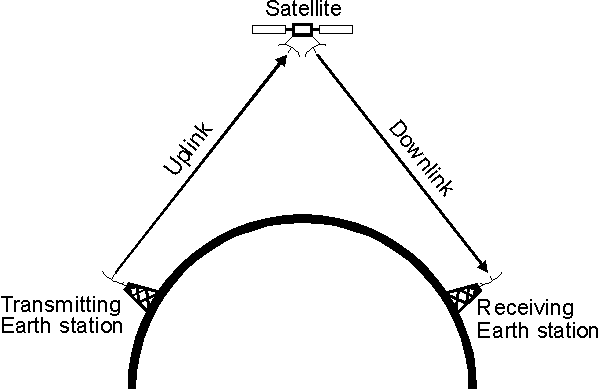


Figure above represents basic satellite communication system consisting of transmitting antenna, a satellite transponder and a receiving antenna.

For satellite communication, frequencies above 3 MHz are used which pass through the ionosphere .Therefore to utilize these frequencies forcommunication, geostationary satellites are used which are located at a height of 36000 Km that is outside the ionosphere. These satellites orbit around the earth with the same speed as that of earth’s rotational speed about its axis & in the same direction hence these satellites appear stationary with respect to earth that is why they are called ‘geostationary’.

The high frequency waves are from transmitting antenna known as uplink .They are received by the satellite transponder, translates them into lower frequency signal ,amplifies it & then transmits them towards the receiving antenna on the earth which is known as the downlink.

The uplink signal is required to have large power so that the satellite receives significant part of it.

To prevent swamping of the weaker downlink signal by the strong uplink signal, different frequencies are used for the uplink & downlink.

Initially communication satellites used uplink & downlink of 4GHz and 6GHz respectively but now higher frequencies are used.

|  |  |  |  |
| --- | --- | --- | --- |
| **CO-ORDINATE SYSTEMS →** | **Cartesian**  **Co-Ordinate System** | **Cylindrical**  **Co-Ordinate System** | **Spherical**  **Co-Ordinate System** |
| **PARAMETERES ↓** |
| **Co-Ordinates** | x,y,z | r,ø,z | r,ɵ,ø |
| **Position Vector** |  |  | = |
| **Length Element** |  |  |  |
| **Area Element** |  |  |  |
| **Volume Element** | dV=dxdydz | dV =(dr)(rdø)(dz) | dv=(dr)(rdɵ)(r sinɵdø) |
| **Gradient**  **()** |  | \_ | \_ |
| **Divergence**  **()** |  |  |  |
| **Curl**  **()** |  |  |  |

***FORMULAE:***

|  |  |
| --- | --- |
|  | Divergence |
| Cartesian |  |
| Cylindrical | ++ |
| Spherical |  |
|  | Vector representation |
| Cartesian |  |
| Cylindrical |  |
| Spherical |  |
|  | Differential line Element |
| Cartesian |  |
| Cylindrical |  |
| Spherical |  |
|  | Area Element |
| Cartesian |  |
| Cylindrical |  |
| Spherical |  |
|  | Volume Element |
| Cartesian |  |
| Cylindrical |  |
| Spherical |  |