PHYSICAL MODEL OF SCHRODINGER ELECTRON. HEISENBERG CONVENIENT WAY FOR DESCRIPTION OF ITS QUANTUM BEHAVIOUR

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Abstract

The object of this paper is to discuss the physical interpretation of quantum behaviour of Schrodinger electron (SchEl) and bring to light on the cause for the Heisenberg convenient operator way of its describing, using the nonrelativistic quantum mechanics laws and its mathematical results. We describe the forced stochastically diverse circular harmonic oscillation motion, created by force of the electrical interaction of the SchEl's elementary electric charge (ElmElcChrg) with the electric intensity (ElcInt) of the resultant quantum electromagnetic field (QntElcMgnFld) of the existing StchVrtPhtns, as a solution of Abraham-Lorentz equation. By dint of this equation we obtain that the smooth thin line of a classical macro particle is rapidly broken of many short and disorderly orientated lines, owing the continuous dispersion of the quantum micro particle (QntMicrPrt) on the StchVrtPhtns. Between two successive scattering the centers of diverse circular oscillations with stochastically various radii are moving along this short disordered line. These circular harmonic oscillations lie within the flats, perpendicular to same disordered short line, along which are moving its centers. In a result of same forced circular harmonic oscillation motion the smooth thin line of the LrEl is roughly spread and turned out into some cylindrically wide path of the SchEl. Hence the dispersions of different dynamical parameters, determining the state of the SchEl, which are results of its continuously interaction with the resultant QntElcMgnFld of the StchVrtPhtns. The absence of the smooth thin line trajectory at the circular harmonic oscilation moving of the QntMicrPrt forces us to use the matrix elements (Fourier components) of its roughly spread wide cylindrical path for its description.

1 Introduction

We assume that the vacuum fluctuations (VcmFlcs) through zero-point quantum electromagnetic field (QntElcMgnFld) perform an important role in a behaviour of the micro particles (MicrPrts). As thing turned out that if the Brownian stochastic motion (BrnStchMtn) of some classical micro particle (ClsMicrPrt) is a result of fluctuating deviations of averaged values of all having an effect forces on a ClsMcrPrt, coming from many molecule blows from a surround environment, then the quantized stochastic dualistic wave-particle behaviour of every QntMicrPrt is a result of the continuous uncontrolled electromagnetic interaction (ElcMgnIntAct) between its well spread (WllSpr) elementary electric charge (ElmElcChrg) of the charged one (a Schrodinger's electron (SchEl)) or its magnetic dipole moment (MgnDplMmn) for uncharged one (as a neutron), and the averaged electric intensity for charged MicrPrts or the averaged magnetic intensity for uncharged one, of the resultant quantized electromagnetic field (QntElcMgnFld) of all stochastic virtual photons (StchVrtPhtns), excited within the FlcVcm and existing within its neighborhood, which exercises a very power influence on its state and behaviour. Consequently the continuously scattering of the well spread (WllSpr) elementary electric charge (ElmElcChrg) of the SchEl on the StchVrtPhtns at its creation powerfully broken the smooth thin line of the classical trajectory of many short and very disorderly orientated small lines and the powerfully its interaction (IntAct) with the electric intensity (ElcInt) or magnetic intensity (MgnInt) of the resultant QntElcMgnFld of the existing StchVrtPhtns forced it to make the circular harmonic oscillations with various radii and the centers, lying over the small disordered lines. In a result of this complicated motion the narrow smooth line of the classical trajectory is turned out into some wide rough cylindrically spread path of the QntMcrPrt. Although in further we will give the necessary calculations, we wish to repeated, that as a result of the continuously scattering of the QntMicrPrt on the StchVrtPhtns at their creations the smooth thin line of the classical trajectory is turned out into powerfully often broken of many small and very disorderly orientated short lines. The uninterrupted ElcMgnIntAct of the ElmElcChrg or of the MgnDplMmn of the QntMicrPrt with the ElcInt or the MgnInt of the resultant QntElcMgnFld of the StchVrtPhtns, existent within the fluctuating vacuum (FlcVcm) between two consecutive scatterings forced the QntMicrPrt to carry out the stochastic circular oscillation motion, which exercise an influence of its behavior within a neighborhood of the smooth classically line into the cylindrically spread by different radii wide path. It isn't allowed us to forget that the broken of the smooth thin line of very short and very disorderly orientated small line is a result of its continuously scattering on the StchVrtPhtns, over which are found the centers of the forced stochastic circular oscillation motion of the QntMicrPrt, owing a result of the ElcMgnIntAct of its WllSpr ElmElcChrg and MgnDplMmn with the intensities of the resultant electric field (RslElcFld) and resultant magnetic field (RslMgnFld) of the stochastic virtual photons (StchVrtPhtns). The WllSpd ElmElcChrg of the SchEl is moving at its circular oscillations of different radii within the flats, which are perpendicular to the very short and very disorderly orientated small lines, obtained in a result of its continuously scattering on the StchVrtPhtns, at its Furtian quantized stochastic circular harmonic oscillation motion through the fluctuating vacuum (FlcVcm). Therefore in our transparent survey about the physical model (PhsMdl) of the nonrelativistic quantized SchrEl one will be regarded as some WllSpr ElmElcChrg), participating simultaneously in two different motions: A) The classical motion of a classical Lorentz' electron (LrEl) along an well contoured smooth thin trajectory, realized in a consequence of some known interaction (IntAct) of its over spread (OvrSpr) ElmElcChrg, MgnDplMnt or bare mass with the intensity of some external classical fields (ClsFlds) as in the Newton nonrelativistic classical mechanics (NrlClsMch) and Maxwell nonrelativistic classical electrodynamics (ClsElcDnm). B) The isotropic three-dimensional nonrelativistic quantized (IstThrDmnNrlQnt) Furthian stochastic boson circular harmonic oscillations motion (FrthStchBsnCrcHrmOscMtn) of the SchEl as a natural result of the permanent ElcIntAct of its WllSpr ElmElcChrg with the ElcInt of the resultant QntElcMgnFld of a large number StchVrtPhtns. This ElcIntAct between the WllSpr ElmElcChrg and the FlcVcm (zero-point ElcMgnFld) is generated by dint of StchVrtPhtns exchanged between the fluctuating vacuum (FlcVcm) and the WllSpr ElmElcChrg during a time interval of their life. As soon as this Furthian quantized stochastic wave-particle behaviour of the SchEl is very similar to known Brownian classical stochastic behaviour of the ClsMacrPrt, therefore the QntMicrPrt cannot has the classical sharp contoured smooth and thin trajectory but has a cylindrical broad rough path, obtained as a sum of circular oscillations motions of different radii and centers, lying on accidental broken short lines, strongly disordered within a space. Hence the often broken trajectory of the moving QntMicrPrt present itself a sum of small parts from some circumferences with different radii and centers, lying within flats, which are perpendicular to accidental broken short lines, strongly disordered in space. Therefore in a principle the exact description of the resultant behaviour of the SchEl owing of its joint participation in both mentioned above motions could be done only by means of the NrlQntMch's and nonrelativistic ClsElcDnm's laws.

It is known of many scientists the existence of three different ways [5], [8] and [9], [7] and [18], [19] for the description of the quantum behaviour [4] of the nonrelativistic SchEl. It is turned out that there is some possibility enough to show by means of the existence intrinsic analogy between the quadratic differential wave equation in partial derives (QdrDfrWvEqtPrtDrv) of Schrodinger and the quadratic differential particle equation in partial derivative (QdrDfrPrtEqtPrtDrv) of Hamilton-Jacoby that the addition of the kinetic energy of the Furthian stochastic boson circular harmonic oscillation of some QntMicrPrt to the kinetic energy of such ClsMacrPrt determines their dualistic wave-particle quantized behaviour. It turns out the stochastic motion over the powerfully break up the sharp contoured smooth thin classical line of the in many shortly and very disorderly (stochastically orientated) small lines. As in such a natural way we have ability enough to obtain the minimal value of the dispersion product, determined with the Heisenberg uncertainty relation. Science there exists an essential analogy between the registration forms of the quadratic differential diffusive equation (QdrDfrDfsEqt) of Focker-Plank for the distribution function $P(r,t)$ of a probability density (DstFncPrbDns) of the free Brownian ClsMacrPrt (BrnClsMacrPrt) in a motionless coordinate system in a respect to it and quadratic differential wave equation in partial derivative (QdrDfrWvEqtPrtDrv) of Schrodinger for the orbital wave function (OrbWvFnc) of some free Furthian QntMicrPrt (FrthQntMicrPrt) in a motionless coordinate system in a respect to it we come to an essential conclusion that there are also some possibility enough to describe the quantized stochastic behaviour of the SchEl by means of the analogy between the classical Wiener continual integral and the quantized Feynman continual integral. Feynman has used for the description of transition between two OrbWvFncs of some free FrthQntMicrPrt with different coordinates and times some formula, analogous of such the formula, which early had been used by Einstein [13], [16], Smoluchowski [14] and Wiener [17] for the description of same transition between two DstFncsPrbDns of the free BrnClsMacrPrts. In this way we understand why the behaviour of the QntMicrPrt must be described by the OrbWvFnc Ψ , although the behaviour of the ClsMacrPrt may be described by a line.

2 Mathematical description of the physical cause ensuring the display of the QntMicrPrt behaviour.

The object of this paper is to discuss the fundamental problems of the physical interpretation of the nonrelativistic quantized behaviour of the SchEl and bring to light for understanding the cause,securing the existence of this uncommon state of each the QntMicrPrt. It is necessary to understand why the QntMicrPrt has no classical smooth thin trajectory and why its behaviour must be described by the Heisenberg matrix of the convenient operator way, using the laws of the NrlQntMch and its effective mathematical results. The PhsMdl of the SchEl is built by means of the equation of the forced motion of the dumping classical oscillator under the force action of electric interaction (ElcIntAct) between its WllSpr ElmElcChrg and the ElcInt of the

RslQntElcMgnFld of the StchVrtPhtns, created in the FlcVcm. The unusual behaviour of the SchEl may be described by the following motion equation in Maxwell nonrelativistic classical electrodynamics (ClsElcDnm):

$$\ddot{r_j} + \omega_o^2 r_j = -(\frac{e}{m})\{E_j + E_j^i\} = \frac{e}{mC}\frac{\partial A_j}{\partial t} + \frac{2e^2}{3mC^3}\dddot{r_j}, \tag{1}$$

where E_j^i and E_j denote the ElcInt of both the ElcFld E_j^i of radiative friction, that is to say of LwEng unemitted longitudinal (Lng) VrtPhtn (VrtLngPht), and ElcInt E_j of the LwEng-VrtPhtn in the FlcVcm. In accordance of the relation (1) the ElcInt E_j of an external QntElcMgnFld may be described by means of its A_j, having the following analytical presentation:

$$A_j = \frac{i}{L}\sum_q \sqrt{\frac{2\pi\hbar C}{L\,q}}\, I_{jq}\left[a_{jq}^+ e^{i(t\omega - qr)} - a_{jq}\, e^{-i(t\omega - qr)}\right], \tag{2}$$

Indeed the ElcInt E_j of StchVrtFtn could be obtained by taking of a particle derivative of the expression (2) relatively for the

$$E_j = \frac{1}{L}\sum_q \sqrt{\frac{2\pi\hbar\omega}{L}}\, I_{jq}\left[a_{jq}^+ e^{i(t\omega - qr)} + a_{jq}\, e^{-i(t\omega - qr)}\right], \tag{3}$$

There is a necessity to note here that we have exchanged the signs in eqs. (2) and (3). Indeed, in order to get the necessary correspondences between operator expressions of the $\hat{p}_j$ and $\hat{A}_j$, it is appropriate to use the sign (-) in the eq.(2) and the sign (+) in an eq.(3). The helpful of this exchange of signs will be letter seen in following expressions (5) of $\hat{r_j}$ and (6) of $\hat{p_j}$. Hence by substituting the eq.(2) in the eq.(1) and transposition of same term in its left-hand side one can obtain motion equation in Lorentz-Abrahams nonrelativistic presentation (LAP):

$$\ddot{r_j} - \tau\,\dddot{r_j} + \omega_o^2 r_j = -(\frac{e}{m})\,E_j, \tag{4}$$

The temporary dependence of r_j contains two frequencies ω_o and ω. In a spite of $\omega_o \geq \omega$, then the very greatest magnitude of the term $\tau\,\dddot{r_j}$ is -$\tau\,\omega_o^2\,\dot{r_j}$. Although of that the term $\tau\,\dddot{r_j}$ still presents itself by -$\tau\,\omega^2\,\dot{r_j}$. Indeed, the general solution of eq.(4) is given by sum of the general solution of the homogeneous equation and a particular solution to the inhomogeneous equation. At $\omega\,\tau = \frac{2e^2}{3mC^2}\frac{\omega}{C} = \frac{\pi}{3}(\frac{2e^2}{C\hbar})\frac{\hbar}{mC}\frac{2}{\lambda} \leq 1$, the general solution of the homogeneous equation has a form of a relaxing oscillation of a frequency ω. The particular solution has a form of a forced oscillation of a frequency ω. Therefore we may rewrite eq. (4) in the following form :

$$\ddot{r_j} + \tau\,\omega^2\,\dot{r_j} + \omega_o^2 r_j = -(\frac{e}{m})\,E_j(r,t), \tag{5}$$

From eq.(5) it is easily seen that the motion dumping of the SchEl is caused by well-known Lorentz' dumping force owing to radiation friction of its moving WllSpr ElmElcChrg. In a rough approximation of the Maxwell nonrelativistic ClsElcDnm the minimum time interval for an emission or absorption of a real photon (RlPhtn) by the WllSpr ElmElcChrg of the SchEl may be evidently determined by the parameter of Lorentz-Abrahams :

$$\tau = \frac{2e^2}{3mC^3}, \tag{6}$$

The particular solution of the motion eq.(5), describing the forced quantized stochastic circular harmonic motion of the QntMicrPrt, have been written by Welton [20], Kalitchin [21] and Sokolov and Tumanov [22], citeAAS by the way of the operator division in the following analytical form :

$$\hat{r_j} \,=\, \sum_q \frac{e\,q}{m\,L}\sqrt{\frac{2\pi\hbar\omega}{L\,q}}\,I_{jq}\left[\frac{a^+_{jq}\,\exp\{i\,t\omega\,-\,i\,qr\}}{\omega_o^2\,-\,\omega^2\,+\,i\tau\omega^3} + \frac{a_{jq}\,\exp\{-\,i\,t\omega\,+\,i\,qr\}}{\omega_o^2\,-\,\omega^2\,-\,i\tau\omega^3}\right], \tag{7}$$

$$\hat{P_j} \,=\, i\sum_q \frac{e\,\omega_o^2}{C\,L}\sqrt{\frac{2\pi\hbar\omega}{L\,q}}\,I_{jq}\left[\frac{a^+_{jq}\,\exp\{\,i\,t\omega\,-\,i\,qr\}}{\omega_o^2\,-\,\omega^2\,+\,i\tau\omega^3} - \frac{a_{jq}\,\exp\{-\,i\,t\omega\,+\,i\,qr\}}{\omega_o^2\,-\,\omega^2\,-\,i\tau\omega^3}\right], \tag{8}$$

The analytical presentation (8) of the SchEl's momentum components have been calculated through using the relation known from Maxwell ClsElcDnm :

$$\hat{P_j} \,=\, m\,\dot{r}_j \,-\, (\frac{e}{C})\left[A_j \,+\, A^i_j\right], \tag{9}$$

Further they have calculated the well-known Heisenberg's commutation relations (HsnCmtRlts) between the operators of the dynamic variables $\hat{r}_j$ (7) and $\hat{P}_j$ (8) by virtue of the following definition :

$$\hat{P}_j\,\hat{r}_k \,-\, \hat{r}_k\,\hat{P}_j \,\approx\, -i\,\hbar\,\delta_{jk} \tag{10}$$

Since then it is easily to understand by means of the upper account that if the ClsMacrPrt's motion is occurred along a clear definite smooth thin trajectory in the NrlClsMch, then the QntMicrPrt's motion is performed in a form of the RndTrmMtn along a pete very small line, stochastically orientated in the space near the clear-cut smooth thin trajectory in the NrlQntMch. As a result of that we can suppose that the QntStchBhv of the QntMicrPrt can be described by means of the following physical quantities in the NrlQntMch :

$$r_j \,=\, \bar{r}_j \,+\, \delta r_j \quad ; \quad p_j \,=\, \bar{p}_j \,+\, \delta p_j \quad ; \tag{11}$$

3 Mathematical description of the minimal dispersions of some dynamical parameters of a QntMicrPrt

Indeed,because of the eqs.(11) the values of the averaged physical parameters in the NrlQntMch $\langle p_j^2\rangle$ is different from the values of the same physical parameters in the NrlClsMch $\bar{p}_j^2$ as it is seen :

$$\langle r_j^2\rangle \,=\, \bar{r}_j^2 \,+\, \langle\delta r_j^2\rangle\,; \quad \langle p_j^2\rangle \,=\, \bar{p}_j^2 \,+\, \langle\delta p_j^2\rangle\,; \tag{12}$$

In spite of that the averaged value of the orbital (angular) mechanical momentum of the QntMicrPrt has the following value :

$$\langle L^2\rangle \,=\, \sum_j(\bar{L}_j)^2 \,+\, \sum_j\langle(\delta L_j)^2\rangle \,=\, (\bar{L}_x)^2 \,+\, \langle(\delta L_x)^2\rangle \,+\, (\bar{L}_y)^2 \,+\, \langle(\delta L_y)^2\rangle \,+\, (\bar{L}_z)^2 \,+\, \langle(\delta L_z)^2\rangle\,; \tag{13}$$

or at the $(\bar{L}_x)^2 \,=\, 0$ and $(\bar{L}_y)^2 \,=\, 0$ we must obtain :

$$\langle L^2\rangle \,=\, (\bar{L}_z)^2 \,+\, \langle(\delta L_z)^2\rangle \,+\, \langle(\delta L_y)^2\rangle \,+\, \langle(\delta L_x)^2\rangle \tag{14}$$

As both the value of the $\langle(\delta L_x)^2\rangle$ and $\langle(\delta L_y)^2\rangle$ are equal of the $\frac{\bar{L}_z \hbar^2}{2}$ and the value of the $\langle(\delta L_z)^2\rangle$ is equal of the $\frac{\hbar^2}{4}$. Therefore :

$$\langle L^2\rangle = l^2\hbar^2 + l\hbar^2 + \frac{\hbar^2}{4} = (l + \frac{1}{2})^2\hbar^2 ; \tag{15}$$

The realized above investigation assists us to come to the conclusion that the dispersions of the dynamical parameters of the QntMicrPrt are natural results of their forced stochastic oscillation motions along the very small line stochastically orientated in space near to the classical clear-cut smooth thin line of the corresponding dynamical parameters values of the ClsMacrPrt, owing to ElcMgnIntAct of its OvrSpr ElmElcChrg or MgnDplMm with the intensities of the RslElcFld or RslMgnFld of the QntElcMgnFlds of the StchVrtPhtns at its motion through the FlcVcm. It is turned out that the kinetic energy of the IstThrDmnNrlQnt FrthStchBsnCrcHrmOscs, which the QntMicrPrt takes from the FlcVcm, called as its localized energy, one ensures the stability of the SchEl in its ground state in the H-like atom. We have the ability to obtain the minimal value of the dispersion product, determined by the Heisenberg uncertainty relation.

In a consequence of what was asserted above in order to obtain the QntQdrDfr WvEqn of Sch we must add to the kinetic energy $\frac{(\nabla_l S_1)^2}{2m}$ of the NtnClsPrt in the following ClsQdrDifPrtEqt of Hml-Jcb :

$$- \frac{\partial S_1}{\partial t} = \frac{(\nabla_j S_1)^2}{2m} + U ; \tag{16}$$

the kinetic energy $\frac{(\nabla_l S_2)^2}{2m}$ of the BrnClsPrt. In such the natural way we obtain the following analytic presentation of the QntQdrDfrWvEqt of Sch :

$$- \frac{\partial S_1}{\partial t} = \frac{(\nabla_j S_1)^2}{2m} + \frac{(\nabla_j S_2)^2}{2m} + U ; \tag{17}$$

The purpose of our investigation in henceforth is to obtain the eq. (17) by means of physically obvious and mathematically correct proof. Therefore we could desire a voice of a supposition that all uncommon ways of the SchEl's behaviour in the NrlQntMch or of other QntMicrPrts in the micro world are natural consequences of unconstrained stochastic joggles on account of continuously accidently exchanges of LwEnr-StchVrtPhtn between its WllSpr ElmElcChrg and the VcmFlc. In consequence of the absence of SchEl's trajectory within the NrlQntMch as within the NrlClsMch and the stochastical character of its random trembling motion together with the probably interpretation of the SchEl's OrbWvFnc module square are naturally consequences of the continuous ElcMgnIntAct between the SchEl's WllSpr ElmElcChrg and EfcElcInt E_j of existent LwEnr-VrtPhtns, stochastically generated by fluctuating energy within FlcVcm through continuous incident exchange of LwEnr-StchVrtPhtns, which are either emitted or adsorbable by either the VcmFlcs or the Schel's Wllspr ElmElcChrg. Really, a deep understanding of the physics of the random trembling motion, in accordance with the description of the Brownian stochastic behaviour of BrnCslPrts we can determine both as the value V^- of the SchEl's velocity before the moment t of the scattering time of some LwEnr-StchVrtPhtns from its WllSpr ElmElcChrg, so the value V^+ after the same moment t of the scattering time by means of the following definitions :

$$V_j^- = lim_{\Delta t \to o} \left\{ \frac{r(t)_j - r(t - \Delta t)_j}{\Delta t} \right\} = (V_j - i\, U_j) ; \tag{18}$$

$$V_j^+ \,=\, lim_{\Delta t \to o} \left\{ \frac{r(t+\Delta t)_j \,-\, r(t)_j}{\Delta t} \right\} \,=\, (\, V_j \,+\, i\, U_j\,)\,; \tag{19}$$

In addition we may determine two new velocities V_j and U_j by dint of the following equations :

$$2\,V_j \,=\, V_j^+ \,+\, V_j^- \qquad \text{and} \qquad 2\,i\,U_j \,=\, V_j^+ \,-\, V_j^-\,, \tag{20}$$

In conformity with the eq.(20) it is obviously followed that the current velocity V describes the regular drift of the SchEl and the osmotic velocity U describes its nonrelativistic quantized stochastic bozon oscillations. Afterwards by virtue of the well-known definition equations :

$$2\,m\,V_j \,=\, m\,(\,V_j^+ \,+\, V_j^-\,) \,=\, 2\,\nabla_j\,S_1 \tag{21}$$

and

$$2\,i\,m\,U_j \,=\, m\,(\,V_j^+ \,-\, V_j^-\,) \,=\, 2\,i\,\nabla_j\,S_2 \tag{22}$$

one can obtain the following presentation of the SchEl's OrbWvFnc $\psi(r,t)$:

$$\psi(r,t) \,=\, \exp\{\,i\,\frac{S_1}{\hbar} \,-\, \frac{S_2}{\hbar}\,\} \,=\, B\,\exp\{\,i\,(\frac{S_1}{\hbar}\} \tag{23}$$

It is easily to verify the results (20), (21) (22). In an effect ones may be obtained by means of the following natural equations :

$$m\,V_j^+\,\psi(r,t) \,=\, -\,i\,\hbar\,\nabla_j\,\exp\{\frac{i\,S_1}{\hbar} \,-\, \frac{S_2}{\hbar}\} \,=\, (\,\nabla_j S_1 \,+\, i\,\nabla_j S_2\,)\,\psi(r,t) \tag{24}$$

and

$$m\,V_j^-\,\psi(r,t)^+ \,=\, +\,i\,\hbar\,\nabla_j\,\exp\{\frac{i\,S_1}{\hbar} \,-\, \frac{S_2}{\hbar}\} \,=\, (\,\nabla_j S_1 \,-\, i\,\nabla_j S_2\,)\,\psi(r,t)^+ \tag{25}$$

Indeed,

$$\begin{aligned} 2\,m\,V_j \,=\, m\,(\,V_j^+ \,+\, V_j^-\,) \,=\, \\ \{\,(\,\nabla_j S_1 \,+\, i\,\nabla_j S_2\,) \,+\, (\,\nabla_j S_1 \,-\, i\,\nabla_j S_2\,)\,\} \quad \text{or} \quad 2\,m\,V_j \,=\, 2\,\nabla_j S_1 \end{aligned} \tag{26}$$

and

$$\begin{aligned} 2\,i\,m\,U_j \,=\, m\,(\,V_j^+ \,-\, V_j^-\,) \,=\, \\ \{\,(\,\nabla_j S_1 \,+\, i\,\nabla_j S_2\,) \,-\, (\,\nabla_j S_1 \,-\, i\,\nabla_j S_2\,)\,\} \quad \text{or} \quad 2\,i\,m\,U_j \,=\, 2\,i\,\nabla_j S_2 \end{aligned} \tag{27}$$

In consequence we could assume that the module square of the SchEl's OrbWvFnc $\psi(r,t)$ describes the probability density of its location close by the space point r at the time moment t in the good light of our obvious interpretation. Further in order to obtain the partial differential equation of the continuity we are going to calculate one by virtue of its well-known definitions :

$$\begin{aligned} \frac{\partial|\,\psi\,|^2}{\partial t} \,+\, \nabla_j(V_j^+\,|\,\psi\,|^2\,) \,=\, \frac{\partial(\exp\{-\,2\,\frac{S_2}{\hbar}\})}{\partial t} \,+\, \nabla_j\left[(\nabla_j\frac{S_1}{m} \,+\, i\,\nabla_j\frac{S_2}{m})\,\exp\{-\,2\,\frac{S_2}{\hbar}\}\right] \,=\, \\ \left[-\frac{2}{\hbar}\,\frac{\partial S_2}{\partial t} \,+\, \frac{1}{m}\,(\nabla_j)^2 S_1 \,+\, \frac{i}{m}\,(\nabla_j)^2 S_2 \,-\, \frac{2}{m\hbar}\,\nabla_j S_1\,\nabla_j S_2 \,-\, \frac{2i}{\hbar}\,\nabla_j S_2\,\nabla_j S_2\right]\,\left[\exp\{\,-\,2\,\frac{S_2}{\hbar}\}\right] \end{aligned} \tag{28}$$

$$\frac{\partial |\,\psi\,|^2}{\partial t} + \nabla_j (V_j^- \mid \psi \mid^2) = \frac{\partial (\exp\{-2\,\frac{S_2}{\hbar}\})}{\partial t} + \nabla_j \left[(\nabla_j \frac{S_1}{m} - i\,\nabla_j \frac{S_2}{m}) \exp\{-2\,\frac{S_2}{\hbar}\} \right] =$$
$$\left[-\frac{2}{\hbar}\frac{\partial S_2}{\partial t} - \frac{1}{m}(\nabla_j)^2 S_1 - \frac{i}{m}(\nabla_j)^2 S_2 - \frac{2}{m\hbar}\nabla_j S_1 \nabla_j S_2 + \frac{2i}{\hbar}\nabla_j S_2 \nabla_j S_2 \right] \left[\exp\{-2\,\frac{S_2}{\hbar}\} \right] \tag{29}$$

With the purpose to calculate the last expressions of the continuity equations (28) and (29) we are going to turn the expression (23) of the SchEl's OrbWvFnc $\psi(r,t)$ in the quadratic differential wave equation in partial derivatives of Schrodinger :

$$i\,\hbar\,\frac{\partial \psi(r,t)}{\partial t} = -\frac{\hbar^2}{2}\,\frac{(\nabla_j)^2}{m}\,\psi(r,t) + U(r,t)\,\psi(r,t) \tag{30}$$

Further we are able to obtain the following result :

$$\left(-\frac{\partial S_1}{\partial t} - i\,\frac{\partial S_2}{\partial t} \right) \psi(r,t) =$$
$$\left\{ \frac{(\nabla_j S_1)^2}{2m} - \frac{(\nabla_j S_2)^2}{2m} + \frac{\hbar}{2m}(\nabla_j)^2 S_2 - i\,\frac{\hbar}{2m}(\nabla_j)^2 S_1 + \frac{i}{m}\nabla_j S_1\,\nabla_j S_2 + U(r,t) \right\} \psi(r,t) \tag{31}$$

As there exist both the real and imaginary parts in the complex valued eq. (31) , it is obviously that from this one follows two quadratic differential equations in partial derivatives :

$$\frac{\partial S_2}{\partial t} = \frac{\hbar}{2m}(\nabla_j)^2 S_1 - \frac{1}{m}(\nabla_j S_1)(\nabla_j S_2) \tag{32}$$

and

$$-\frac{\partial S_1}{\partial t} = \frac{1}{2\,m}(\nabla_j S_1)^2 - \frac{1}{2\,m}(\nabla_j S_2)^2 + \frac{\hbar}{2\,m}(\nabla_j)^2 S_2 + U(r,t) \tag{33}$$

Inasmuch as it is well-known from the NrlQntMch the continuity partial differential equation can be obtained by means of the eqs.(32), (21 and (23) in the following form :

$$\frac{\partial |\psi|^2}{\partial t} + \nabla_j \left(V_j\,|\psi|^2 \right) = \frac{\partial \exp\{-2\,\frac{S_2}{\hbar}\}}{\partial t} + \frac{1}{m}\nabla_j \left(\nabla_j S_1 \exp\{-2\,\frac{S_2}{\hbar}\} \right) = 0; \tag{34}$$

Thence the eq.(28) and eq.(29) can be simplified by means of the eqs.(34) and (32). In a result of such substitutions the following continuity partial differential equations could be obtained :

$$\frac{\partial |\psi|^2}{\partial t} + \nabla_j \left(V_j^+\,|\psi|^2 \right) = \frac{i}{m}\nabla_j \left(\nabla_j S_2 \exp\{-2\frac{S_2}{\hbar}\} \right) \tag{35}$$

$$\frac{\partial |\psi|^2}{\partial t} + \nabla_j \left(V_j^-\,|\psi|^2 \right) = -\frac{i}{m}\nabla_j \left(\nabla_j S_2 \exp\{-2\frac{S_2}{\hbar}\} \right) \tag{36}$$

In order to calculate the value of the expressions in the brackets in the right-hand side of the eqs.(35) and (36) we will determine the relation between the values of both integrals :

$$\int\int_{V_R}\int {\nabla_j}^2\, S_2 \exp\{-2\frac{S_2}{\hbar}\}\, dV \quad \text{and} \quad \int\int_{V_R}\int (\nabla_j S_2\,)^2 \exp\{-2\frac{S_2}{\hbar}\}\, dV \tag{37}$$

The first integral in (37) may be calculated through integration by parts. In this easily way we could obtain :

$$\int\int_{V_R}\int (\nabla_j)^2\, S_2\, \exp\{-2\frac{S_2}{\hbar}\}\, dV \;=\; \int\int_{S_R} \nabla_j S_2\, \exp\{-2\frac{S_2}{\hbar}\}\, dS_j \;- \\ \int\int_{S_o} \nabla_j S_2\, \exp\{-2\frac{S_2}{\hbar}\}\, dS_j \;+\; \frac{2}{\hbar}\int\int_{V_R}\int (\nabla_j S_2)^2\, \exp\{-2\frac{S_2}{\hbar}\}\, dV \qquad (38)$$

From above it is evidently that the second two-dimensional integral over the surface S_o cannot exist in the case when the integrational domain V_R of the three-dimensional integral has the form of one-piece-integrity domain. Indeed, the three-multiple integral in the left-hand side of eq. (38) has an integration domain of the volume V_R,then the both two-multiple integrals (the first and second ones on the right handside of the same equation) have a integration domain in form of surface of same volume (the outer skin S_R and the inter skin S_o of the volume V_R). Inasmuch as we don't take into account the creation and annihilation of the FrthQntMicrPrt in the NrlQntMch, than the SchEl's OrbWvFnc $\psi(r,t)$ may have no singularity within the volume V_R. Therefore the three-multiple integrals have the one-piece integrity domain of an integration without its inter skin surface S_o. Hence it is easily seen that both two-multiple integrals are canceled in the case when R go to ∞ and at the absence of any kind of singularity in the SchEl's OrbWvFnc. Consequently eq.(37) becomes the form :

$$\int\int_{V_\infty}\int (\nabla_j)^2\, S_2\, \exp\{-2\frac{S_2}{\hbar}\}\, dV \;=\; \frac{2}{\hbar}\int\int_{U_\infty}\int (\nabla_j S_2)^2\, \exp\{-2\frac{S_2}{\hbar}\}\, dV \qquad (39)$$

Then in a result of the existence of the eqt.(39) we may suppose the existence of the following equations between the values of both integrand functions :

$$\text{the first}\;:\quad (\nabla_j)^2\, S_2\, \exp\{-\,2\,\frac{S_2}{\hbar}\,\} \;=\; \frac{2}{\hbar}\,(\nabla_j S_2)^2\, \exp\{-\,2\,\frac{S_2}{\hbar}\,\} \qquad (40)$$

$$\text{and the second}\;:\quad (\nabla_j)^2 S_2 \;=\; \frac{2}{\hbar}\,(\nabla_j S_2)^2 \qquad (41)$$

Hence it is obviously seen that in a line with the existence of the eq. (41) the equation (33) could been rewritten in the following transparent form :

$$-\,\frac{\partial S_1}{\partial t} \;=\; \frac{1}{2\,m}(\nabla_j S_1)^2 \;+\; \frac{1}{2\,m}(\nabla_j S_2)^2 \;+\; U(r,t) \qquad (42)$$

In such a way it is evidently that the right-hand side expressions of the equations of the continuity (35) and (36) are canceled by the virtue of the eq:(41).Consequently we had an opportunity to shoe that the continuity partial differential equations are satisfied not only in the form (34), but they are satisfied also in the forms (35) and (36). Furthermore the expression of the eq.(42) might been interpreted from my new point of view, that the kinetic energy E_k of the SchrEl is formed by two differential parts. Really, if the first part $\frac{(\nabla_j S_1)^2}{2\,m}$ describes the kinetic energy of its regular translation motion along some clear-cut thin smooth classical trajectory in an accordance with the laws of the NrlClsMch and ClsElcDnm with its current velocity $V_j \;=\; \frac{1}{m}\,\nabla_j S_1$, then the second part $\frac{(\nabla_j S_2)^2}{2\,m}$ mouth describe the kinetic energy of its Furthian quantum stochastic motion of the FrthQntMicrPrt with its probable velocity $U_j \;=\; \frac{1}{m}\,\nabla_j S_1$ in a total analogy with the Brownian classical stochastic motion of the

BrnClsMicrPrt with its osmotic velocity.Therefore it is very helpfully to rewrite the expression (42) in the following well-known form :

$$E = \frac{m V^2}{2} + \frac{m U^2}{2} = \frac{(\langle \bar{P} \rangle)^2}{2m} + \frac{\langle (\Delta P)^2 \rangle)}{2m} \tag{43}$$

Indeed, some new facts have been brought to light. Therefore the upper investigation entitles us to make the explicit assertion that the most important difference between the quadratic differential wave equation in partial derivative of Schrodinger and the quadratic differential particle equation in partial derivative of Hamilton-Jacoby is exhibited by the existence of the kinetic energy of the QntMicrPrt's Furthian trembling circular oscillations harmonic motion in the first one.

$$- \frac{\partial S_1}{\partial t} = \frac{(\nabla_j S_1)^2}{2m} + \frac{(\nabla_j S_2)^2}{2m} + U\,; \tag{44}$$

As we can observe by cursory comparison there is a total coincidence of eq.(16) with eq.(44). Hence we are able to proof that the QdrDfrPrtEqt with PrtDrv of Schrodinger may be obtained from the QdrDfrPrtEqt with PrtDrv of Hamilton-Jacoby by addition the part of the kinetic energy of the Furthian stochastic circular harmonic oscillations motion. Indeed, it is obviously to understand that the first term $\frac{(\nabla_l S_1)^2}{2m}$ in the eq.(44) describes the kinetic energy of the regular translation motion of the NtnClsPrt with its current velocity $V_l = \frac{\nabla_l S_1}{m}$ and the second term $\frac{(\nabla_l S_2)^2}{2m}$ describes the kinetic energy of the random trembling circular harmonic oscillations motion (RndTrmMtn) of the FrthQntPrt in a total analogous with BrnClsPrt with its osmotic velocity $U_l = \frac{\nabla_l S_2}{m}$. Therefore we can rewrite the expression (44) in the following form :

$$E_t = \frac{m V^2}{2} + \frac{m U^2}{2} + U = \frac{\langle \bar{P} \rangle^2}{2\,m} + \frac{\langle (\Delta P)^2 \rangle}{2\,m} + U\,; \tag{45}$$

After elementary physical obviously suppositions some new facts have been brought to light. Therefore the upper investigation entitles us to make the explicit assertion that the most important difference between the QntQdrDfr WvEqt with PrtDrv of Schodinger and the ClsQdrDfrPrtEqt with PrtDrv of Hamilton-Jacoby is exhibited by the existence of the kinetic energy of the FrthRndTrmCrcHrmOscsMtn in the first one. Therefore when the SchEl is appointed in the Coulomb's potential of the atomic nucleus spotted like (SptLk) elementary electric charge (ElmElcChrg) Ze its total energy may be written in the following form :

$$\langle E_t \rangle = \frac{1}{2\,m} \left[(\langle P_r \rangle)^2 + \frac{(\langle L \rangle)^2}{(\langle r \rangle)^2} \right] + \frac{1}{2\,m} \left[\langle (\Delta P_r)^2 \rangle + \frac{\langle (\Delta L)^2 \rangle}{\langle r \rangle^2} \right] - \frac{Ze^2}{\langle r \rangle} \tag{46}$$

As any SchEl has eigenvalues $n_r = 0$ and $l = 0$ in a case of its ground state, so it follows that $\langle P_r \rangle = 0$ and $\langle L \rangle = 0$. As a consistency with the eq.(48) the eigenvalue of the SchEl's total energy E_t^o in its ground state in some H-like atom is contained only by two parts :

$$\langle E_t^o \rangle = \frac{1}{2\,m} \left[\langle (\Delta P_r)^2 \rangle + \frac{\langle (\Delta L)^2 \rangle}{\langle (r)^2 \rangle} \right] - \frac{Ze^2}{\langle r \rangle} \tag{47}$$

Further the values of the dispersions $\langle (\Delta P_r)^2 \rangle$ and $\langle (\Delta L)^2 \rangle$ can be determined by virtue of the Heisenberg Uncertainty Relations (HsnUncRlt) :

$$\langle (\Delta P_r)^2 \rangle \times \langle (\Delta r)^2 \rangle \geq \frac{\hbar^2}{4} \tag{48}$$

$$\langle(\Delta L_x)^2\rangle \times \langle(\Delta L_y)^2\rangle \geq \frac{\hbar^2}{4}\langle(\Delta L_z)^2\rangle \tag{49}$$

Thence the dispersion $\langle(\Delta P_r)^2\rangle$ will really have its minimal value at the maximal value of the $\langle(\Delta r)^2\rangle == \langle r\rangle^2$.In this way the minimal dispersion value of the $\langle(\Delta P_r)^2\rangle$ can be determined by the following equation :

$$\langle(\Delta P_r)^2\rangle = \frac{\hbar^2}{4\langle r^2\rangle} \tag{50}$$

As the SchEl's ground state has a spherical symmetry at $l = 0$,then the following equalities take place :

$$\langle(\Delta L_x)^2\rangle = \langle(\Delta L_y)^2\rangle = \langle(\Delta L_z)^2\rangle ; \tag{51}$$

Hence we can obtain minimal values of the dispersions (51) through division of the eq.(48) with the corresponding equation from the eq. (51).In that a way we obtain the following result :

$$\langle(\Delta L_x)^2\rangle + \langle(\Delta L_y)^2\rangle + \langle(\Delta L_z)^2\rangle = \frac{3\hbar^2}{4} \tag{52}$$

Just now we are in a position to rewrite the expression (48) in the handy form as it is well-known :

$$E_t^o = \frac{1}{2\,m}\left[\frac{\hbar^2}{4r^2} + \frac{3\hbar^2}{4r^2}\right] - \frac{Z\,e^2}{r} = \frac{1}{2}\frac{\hbar^2}{m\,r^2} - \frac{Z\,e^2}{r} ; \tag{53}$$

It is extremely important to note here that we have used undisturbed ElcInt E_j (3) of the QntElcMgnFld of StchVrtPhtns from the FlcVcm by dint of the equations (2) and (3) in order to obtain constrain of dynamical mutual conjugated quantities r_j (7) and P_x (8) from the NrlClsMch in their operator forms $\hat{r}_j$ and $\hat{P}_j$ within NrlQntMch. The quantum behaviour of the SchEl within NrlQntMch is caused by the ElcIntAct between its WllSpr ElmElcChrg and the ElcInt E_j of the undisturbed QntElcMgnFld of StchVrtPhtns from the FlcVcm. So in consequence of the continuous ElcIntAct of the SchEl's WllSpr ElmElcChrg with the ElcInt of the QntElcMgnFld of StchVrtPhtns one participates in the Furthian quantized stochastic motion (FrthQntStchMtn), which is quite obviously analogous of the Brownian classical stochastic motion (BrnClsStchMtn). As it is well-known the BrnClsPrts have no classical wave properties (ClsWvPrp), but the FrthQntPrts have QntWvPrps and display them every where. The cause of this distinction consists of indifference between the liquid and FlcVcm. Indeed, if atoms and molecules within liquid have no ClsWvPrps,all excitations of the FlcVcm and one itself have QntWvPrp. Therefore the FlcVcm transfers its QntWvPrp over the SchEl at ones ElcIntAct with its WllSpr ElmElcChrg.

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