F-spin as a Partial Symmetry

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Abstract

We use the empirical evidence that F-spin multiplets exist in nuclei for only selected states as an indication that F-spin can be regarded as a partial symmetry. We show that there is a class of non-F-scalar IBM-2 Hamiltonians with partial F-spin symmetry, which reproduce the known systematics of collective bands in nuclei. These Hamiltonians predict that the scissors states have good F-spin and form F-spin multiplets, which is supported by the existing data.

21.60Fw, 21.10.Re, 21.60.Ev, 27.70.+q

The interacting boson model (IBM-2) [1–3] describes collective low-lying states in even-even nuclei in terms of monople (s_ρ) and quadrupole (d_ρ) proton ($\rho = \pi$) and neutron ($\rho = \nu$) bosons. Microscopic, shell-model-based interpretation of the model [2,3] suggests that the number of bosons of each type (N_ρ) is fixed and is taken as the sum of valence proton and neutron particle and hole pairs counted from the nearest closed shell. The proton-neutron degrees of freedom are naturally reflected in the IBM-2 via an SU(2) F-spin algebra [2] with generators $\hat{F}_+ = s^\dagger_\pi s_\nu + d^\dagger_\pi \cdot \tilde{d}_\nu$, $\hat{F}_- = (\hat{F}_+)^\dagger$, $\hat{F}_0 = (\hat{N}_\pi - \hat{N}_\nu)/2$. The basic F-spin doublets are $(s^\dagger_\pi, s^\dagger_\nu)$, and $(d^\dagger_{\pi\mu}, d^\dagger_{\nu\mu})$, with F-spin projection $+1/2$ $(-1/2)$ for proton (neutron) bosons. In a given nucleus, with fixed N_π, N_ν, all states have the same value of $F_0 = (N_\pi - N_\nu)/2$, while the allowed values of the F-spin quantum number F range from $|F_0|$ to $F_{max} \equiv (N_\pi + N_\nu)/2 \equiv N/2$ in unit steps. F-spin characterizes the π-ν symmetry properties of IBM-2 states. States with maximal F-spin, $F \equiv F_{max}$, are fully symmetric and correspond to the IBM-1 states with only one type of bosons [1]. There are several arguments, *e.g.*, the empirical success of IBM-1, the identification of F-spin multiplets [4-7] (series of nuclei with constant F and varying F_0 with nearly constant excitation energies), and weakness of M1 transitions, which lead to the belief that low lying collective states have predominantly $F = F_{max}$ [8]. States with $F < F_{max}$, correspond to 'mixed-symmetry' states, most notably, the orbital magnetic dipole scissors mode [9] has by now been established experimentally as a general phenomena in deformed even-even nuclei [10].

Various procedures have been proposed to estimate the F-spin purity of low lying states [8]. These involve exploiting the data on M1 transitions (which should vanish between pure $F = F_{max}$ states), extracting the difference in proton-neutron deformations from pion charge exchange [11], using ratios of γ and ground band magnetic moments [12] and the experimental g-factors of 2^+_1 states [13], and considering the excitation energy of mixed symmetry states. In the majority of analyses the F-spin admixtures in low lying states are found to be of a few percents ($< 10\%$), typically $2\% - 4\%$ [8]. In spite of its appeal, however, F-spin cannot be an exact symmetry of the Hamiltonian. The assumption of F-spin scalar Hamiltonians is at variance with the microscopic interpretation of the IBM-2, which necessitates different effective interactions between like and unlike nucleons. Furthermore, if F-spin was a symmetry of the Hamiltonian, then *all* states would have good F-spin and would be arranged in F-spin multiplets. Experimentally this is not case. As noted in an analysis [5,6] of rare earth nuclei, the ground bands are in F-spin multiplets, whereas the vibrational β bands and some γ bands do not form good F-spin multiplets. The empirical situation in the deformed Dy-Os region is portrayed in Table I and Fig. 1. From Table I it is seen that, for $F > 13/2$, the energies of the $L = 2^+$ members of the γ bands vary fast in the multiplet and not always monotonically. The variation in the energies of the β bands is large and irregular. Thus both microscopic and empirical arguments rule out F-spin invariance of the Hamiltonian. F-spin can at best be an approximate quantum number which is good only for a selected set of states while other states are mixed. We are thus confronted with a situation of having 'special states' endowed with a good symmetry which does not arise from invariance of the Hamiltonian. These are precisely the characteristics of a "partial symmetry" for which a non-scalar Hamiltonian produces a subset of special (at times solvable) states with good symmetry. Such a symmetry notion [14] was recently applied to nuclei [15], to molecules [16] and to the study of mixed systems with coexisting regularity and chaos [17]. Previously determined [11] non-F-scalar Hamiltonians were shown to have solvable ground bands with good F-spin. It is the purpose of this Letter to analyze in detail these Hamiltonians and to show that their partial F-spin symmetry reproduces the known systematics of ground and excited bands. In particular, we find F-spin multiplets in only selected bands, and observe common collective signatures for the ground and scissors bands in deformed nuclei, *e.g.* the same F-spin purity and equal moments of inertia. We further test a prediction for the existence of F-spin multiplets of scissors states.

The ground band in the IBM-2 is represented by an intrinsic state which is a product of a proton condensate and a rotated neutron condensate with N_π and N_ν bosons respectively [18]. It depends on the quadrupole deformations, β_ρ, γ_ρ, $(\rho = \pi, \nu)$ of the proton-neutron equilibrium shapes and on the relative orientation angles Ω between them. For $\beta_\rho > 0$, the intrinsic state is deformed and members of the rotational ground-state band are obtained from it by projection. It has been shown in [11] that the intrinsic state will have a well defined F-spin, $F = F_{max}$, when the proton-neutron shapes are aligned and with equal deformations. The conditions ($\beta_\pi = \beta_\nu$,$\gamma_\pi = \gamma_\nu$, $\Omega = 0$) are weaker than the conditions for F-spin invariance, which makes it possible for a non-F-scalar IBM-2 Hamiltonian to have an equilibrium intrinsic state with pure F-spin. Since the angular momentum projection operator is an F-spin scalar, the projected states of good L will also have good $F = F_{max}$. A non-F-spin scalar Hamiltonian which has the above equilibrium condensate as an eigenstate is therefore guaranteed to have a ground band with good F-spin symmetry. Such explicit construction of an IBM-2 Hamiltonian with partial F-spin symmetry was presented in [11] for the most likely situation, namely, aligned axially symmetric (prolate) deformed shapes ($\beta_\rho = \beta$, $\gamma_\rho = \Omega = 0$). In this case, the equilibrium deformed intrinsic state for the ground band with $F = F_{max}$ has the form

$$\begin{aligned} |c; K=0\rangle \equiv |N_\pi, N_\nu\rangle &= (N_\pi! N_\nu!)^{-1/2} (b^\dagger_{c,\pi})^{N_\pi} \, (b^\dagger_{c,\nu})^{N_\nu} |0\rangle \ , \\ b^\dagger_{c,\rho} &= (1+\beta^2\,)^{-1/2} (\, s^\dagger_\rho + \beta\, d^\dagger_{\rho,0}\,) \ , \end{aligned} \tag{1}$$

where K denotes the angular momentum projection on the symmetry axis. The relevant IBM-2 Hamiltonian with partial F-spin symmetry can be transcribed in the form

$$H = \sum_i \sum_{L=0,2} A^{(L)}_i R^\dagger_{i,L} \cdot \tilde{R}_{i,L} + \sum_{L=1,2,3} B^{(L)} W^\dagger_L \cdot \tilde{W}_L + C^{(2)} \Big[\, R^\dagger_{(\pi\nu),2} \cdot \tilde{W}_2 + H.c. \Big] \ , \tag{2}$$

where $H.c.$ means Hermitian conjugate and the dot implies a scalar product. The $R^{\dagger}_{i,L}$ ($L = 0, 2$) are boson pairs with $F = 1$ and $(F_0 = 1, 0, -1) \leftrightarrow (i = \pi, (\pi\nu), \nu)$, and $W^{\dagger}_{L}$ ($L = 1, 2, 3$) are F-spin scalar ($F = 0$) boson pairs defined as

$$
\begin{aligned}
R^{\dagger}_{\rho,0} &= d^{\dagger}_{\rho} \cdot d^{\dagger}_{\rho} - \beta^2 (s^{\dagger}_{\rho})^2, & R^{\dagger}_{(\pi\nu),0} &= \sqrt{2}\,(\, d^{\dagger}_{\pi} \cdot d^{\dagger}_{\nu} - \beta^2 s^{\dagger}_{\pi} s^{\dagger}_{\nu} \,) \\
R^{\dagger}_{\rho,2} &= \sqrt{2}\,\beta\, s^{\dagger}_{\rho} d^{\dagger}_{\rho} + \sqrt{7} (d^{\dagger}_{\rho} d^{\dagger}_{\rho})^{(2)}, & R^{\dagger}_{(\pi\nu),2} &= \beta(\, s^{\dagger}_{\pi} d^{\dagger}_{\nu} + s^{\dagger}_{\nu} d^{\dagger}_{\pi} \,) + \sqrt{14} (d^{\dagger}_{\pi} d^{\dagger}_{\nu})^{(2)} \\
W^{\dagger}_{L} &= (d^{\dagger}_{\pi} d^{\dagger}_{\nu})^{(L)} \;\; (L = 1, 3), & W^{\dagger}_{2} &= s^{\dagger}_{\pi} d^{\dagger}_{\nu} - s^{\dagger}_{\nu} d^{\dagger}_{\pi}
\end{aligned}
\tag{3}
$$

with $\rho = \pi, \nu$ and $\tilde{R}_{i,L,\mu} = (-1)^{\mu} R_{i,L,-\mu}$, $\tilde{W}_{L,\mu} = (-1)^{\mu} W_{L,-\mu}$. The pair operators satisfy $R_{i,L,\mu}|c\rangle = W_{L,\mu}|c\rangle = 0$ and consequently, the condensate is a zero energy eigenstate of H for *any* choice of parameters $A^{(L)}_{i}$, $B^{(L)}$, $C^{(2)}$ and any *any* N_{π}, N_{ν}. When $A^{(L)}_{i}$, $B^{(L)}$, $A^{(2)}_{\pi\nu} B^{(2)} - (C^{(2)})^2 \geq 0$, the above Hamiltonian is positive-definite and hence $|c\rangle$ is its exact ground state with $F = F_{max}$. H, however, is an F-spin scalar only when $A^{(L)}_{\pi} = A^{(L)}_{\nu} = A^{(L)}_{\pi\nu}$, $(L = 0, 2)$ and $C^{(2)} = 0$. We thus have a non-F-spin scalar Hamiltonian with a solvable (degenerate) ground band with $F = F_{max}$. The degeneracy can be lifted by adding to the Hamiltonian (F-spin scalar) SO(3) rotation terms which produce $L(L+1)$ type splitting but do not affect the wave functions. States in other bands can be mixed with respect to F-spin, hence the F-spin symmetry of H is partial. H trivially commutes with $\hat{F}_0$ but not with $\hat{F}_{\pm}$. However, $[\, H, \hat{F}_{\pm} \,]|c\rangle = 0$ does hold and therefore H will yield F-spin multiplets for members of ground bands. On the other hand, states in other bands can have F-spin admixtures and are not compelled to form F-spin multiplets. These features which arise from the partial F-spin symmetry of the Hamiltonian are in line with the empirical situation as discussed above and as depicted in Table I and Fig. 1. It should be noted that the partial F-spin symmetry of H holds for any choice of parameters in Eq. (2). In particular, one can incorporate realistic shell-model based constraints, by choosing the $A^{(2)}_{\rho}$ ($\rho = \pi, \nu$) terms (representing seniority-changing interactions between like nucleons), to be small. For the special choice $A^{(2)}_{i} = C^{(2)} = 0$ and $B^{(1)} = B^{(3)}$, H of Eq. (2) becomes SO(5) scalar which commutes, therefore, with the SO(5) projection operator and hence produces F-spin multiplets with good SO(5) symmetry. Such multiplets were reported in the Yb-Os region of γ-soft nuclei [7].

The same conditions ($\beta_\rho = \beta$, $\gamma_\rho = \Omega = 0$) which resulted in $F = F_{max}$ for the condensate of Eq. (1), ensure also $F = F_{max} - 1$ for the intrinsic state representing the scissors band

$$\begin{aligned} |sc; K = 1\rangle &= \Gamma^\dagger_{sc} |N_\pi - 1, N_\nu - 1\rangle \ , \\ \Gamma^\dagger_{sc} &= b^\dagger_{c,\pi} d^\dagger_{\nu,1} - d^\dagger_{\pi,1} b^\dagger_{c,\nu} \ . \end{aligned} \tag{4}$$

Here $\Gamma^\dagger_{sc}$ is a $F = 0$ deformed boson pair whose action on the condensate with $(N - 2)$ bosons produces the scissors mode excitation. Furthermore, the scissors intrinsic state (4) is an exact eigenstate of the following Hamiltonian, obtained from Eq. (2) for the special choice $C^{(2)} = 0$ and $B^{(1)} = B^{(3)} = 2B^{(2)} \equiv 2B$

$$H' = \sum_i \sum_{L=0,2} A_i^{(L)} R^\dagger_{i,L} \cdot \tilde{R}_{i,L} + B \hat{\mathcal{M}}_{\pi\nu} \tag{5}$$

The last term in Eq. (5) is the Majorana operator [1], related to the total F-spin operator by $\hat{\mathcal{M}}_{\pi\nu} = [\, \hat{N}(\hat{N} + 2)/4 - \hat{F}^2 \,]$, with eigenvalues $k(N - k + 1)$ for states with $F = F_{max} - k$. The Hamiltonian H' is non-F-scalar but is rotational invariant. If we add to it an SO(3) rotation term, $H' + \lambda \hat{L}^2$, ($\hat{L} = \hat{L}_\pi + \hat{L}_\nu$), the resulting Hamiltonian will have a subset of *solvable* states which form the $K = 0$ ground band ($L = 0, 2, 4, \ldots$) with $F = F_{max}$, and the $K = 1$ scissors band ($L = 1, 2, 3 \ldots$) with $F = F_{max} - 1$. The resulting spectrum is

$$\begin{aligned} E_g(L) &= \lambda\, L(L+1) & \qquad (F = F_{max}) \\ E_{sc}(L) &= B\, N + \lambda\, L(L+1) & \qquad (F = F_{max} - 1) \end{aligned} \tag{6}$$

where the Majorana coefficient B may depend on the boson numbers and deformation [8,19,20]. It follows that for such Hamiltonians, with partial F-spin symmetry, both the ground and scissors band have good F-spin and have the same moment of inertia. The latter derived property is in agreement with the conclusions of a recent comprehensive analysis of the scissors mode in heavy even-even nuclei [19], which concluded that, within the experimental precisions ($\sim 10\%$), the moment of inertia of the scissors mode are the same as that of the ground band. It is the partial F-symmetry of the Hamiltonian (5) which is responsible for the common signatures of collectivity in these two bands.

The Hamiltonian H' of Eq. (5) is not F-spin invariant, however, $[\,H', \vec{F}\,]\,|c; K=0\rangle = [\,H', \vec{F}\,]\,|sc; K=1\rangle = 0$. This implies that members of both the ground and scissors bands are expected to form F-spin multiplets. For ground bands such structures have been empirically established [4-7]. The prediction for F-spin multiplets of scissors states requires further elaboration. Although the mean energy of the scissors mode is at about 3 MeV [20], the observed fragmentation of the M1 strength among several 1^+ states prohibits, unlike ground bands, the use of nearly constant excitation energies as a criteria to identify F-spin multiplets of scissors states. Instead, a more sensitive test of this suggestion comes from the summed ground to scissors B(M1) strength. The IBM-2 M1 operator $(\hat{L}_\pi - \hat{L}_\nu)$ is an F-spin vector $(F=1, F_0=0)$. Its matrix element between the ground state $[L=0^+_g, (F=F_{max}, F_0)]$ and scissors state $[L=1^+_{sc}, (F'=F-1, F_0)]$ is proportional to an F-spin Clebsch Gordan coefficient $C_{F,F_0} = (F, F_0; 1, 0|F-1, F_0)$ times a reduced matrix element. It follows that the ratio $B(M1; 0^+_g \to 1^+_{sc})/(C_{F,F_0})^2$ does not depend on F_0 and should be a constant in a given F-spin multiplet. In Table II we list *all* F-spin partners for which the summed B(M1) strength to the scissors mode has been measured todate [21,22]. It is seen that within the experimental errors, the above ratio is fairly constant. The most noticeable discrepancy for ^{172}Yb (F=8), arises from its measured low value of summed B(M1) strength. The latter should be regarded as a lower limit due to experimental deficiencies (large background and strong fragmentation [10]). These observations strengthen the contention of high F-spin purity and formation of F-spin multiplets of scissors states.

As noted in [5,6] and shown in Table I and Fig. 1, for nuclei with $F = 6, 6.5,$ also members of the γ bands display constant excitation energies and seem to form good F-spin multiplets. This empirical observation has a natural explanation within the family of Hamiltonians with partial F-spin symmetry. For the choice $\beta = \sqrt{2}$ and $A^{(2)}_{\pi} = A^{(2)}_{\nu} = A^{(2)}_{\pi\nu}$ in Eq. (5), H' will have both F-spin and SU(3) partial symmetries. In such circumstances, the ground ($K = 0$), scissors ($K = 1$), symmetric-γ ($K = 2$), and antisymmetric-γ ($K = 2$) bands are solvable and have good SU(3) and F-spin symmetries: $[(\lambda,\mu),F] = [(2N,0),F_{max}]$, $[(2N-2,1),F = F_{max}-1]$, $[(2N-4,2),F = F_{max}]$ and $[(2N-4,2),F = F_{max}-1]$ respectively. The intrinsic states for the symmetric-γ or antisymmetric-γ bands are obtained by F-spin coupling the $F = 1$ pair $R^{\dagger}_{i,2,\mu=2}$ to the $(F = F_{max} - 1)$ condensate $|N_{\pi} - 1, N_{\nu} - 1\rangle$ with $(N - 2)$ bosons to form a N-boson intrinsic state with $F = F_{max}$ or $F = F_{max} - 1$. Since, in this case, the commutator $[\,H', \vec{F}\,]$ vanishes when it acts on the solvable intrinsic states, the projected states are ensured to have good F-spin and form F-spin multiplets. At the same time, since the Hamiltonian is not F-spin scalar, the β bands can have F-spin admixtures and need not form F-spin multiplets.

In summary, we have examined in detail IBM-2 Hamiltonians with partial F-spin symmetry. The latter are not F-spin scalars, yet have a subset of solvable eigenstates with good F-spin symmetry. In particular, the corresponding ground bands form F-spin multiplets with $F = F_{max}$, but excited bands can be mixed, which is in line with the empirically observed F-spin multiplets [4-7]. A class of IBM-2 Hamiltonians with partial F-spin symmetry predict the occurrence of F-spin multiplets of scissors states, with a moment of inertia equal to that of the ground band. This prediction is in agreement with recent analyses of the empirical systematics of excitation energy and M1 strength of the scissors mode in even-even nuclei [19,20]. All the above findings illuminate the potential useful role of F-spin (and other) partial symmetries in nuclear spectroscopy and motivate their further study.

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TABLES

TABLE I. Energies (in MeV) of 2^+ levels of the ground (g), γ and β bands in F-spin multiplets. The mass numbers are $A = 132 + 4F$.

F	Energy	^{A}Dy	$^{A+4}$Er	$^{A+8}$Yb	$^{A+12}$Hf	$^{A+16}$W	$^{A+20}$Os
6	$E(2_g^+)$	0.14	0.13	0.12	0.12	0.12	0.14
	$E(2_\gamma^+)$	0.89	0.85	0.86	0.88		0.86
	$E(2_\beta^+)$	0.83	1.01	1.07	1.06		0.74
13/2	$E(2_g^+)$	0.10	0.10	0.10	0.10	0.11	0.13
	$E(2_\gamma^+)$	0.95	0.90	0.93	0.96		
	$E(2_\beta^+)$	1.09	1.17	1.14	0.99		
7	$E(2_g^+)$	0.09	0.09	0.09	0.10	0.11	0.13
	$E(2_\gamma^+)$	0.97	0.86	0.98	1.08		0.87
	$E(2_\beta^+)$	1.35	1.31	1.23	0.95		0.83
15/2	$E(2_g^+)$	0.08	0.08	0.08	0.09	0.11	
	$E(2_\gamma^+)$	0.89	0.79	1.15	1.23	1.11	
	$E(2_\beta^+)$	1.45	1.53	1.14	0.90	1.08	
8	$E(2_g^+)$	0.07	0.08	0.08	0.09		
	$E(2_\gamma^+)$	0.76	0.82	1.47	1.34		
	$E(2_\beta^+)$		1.28	1.12	1.23		
17/2	$E(2_g^+)$	0.08	0.08	0.08			
	$E(2_\gamma^+)$	0.86	0.93	1.63			
	$E(2_\beta^+)$	1.21	0.96	1.56			

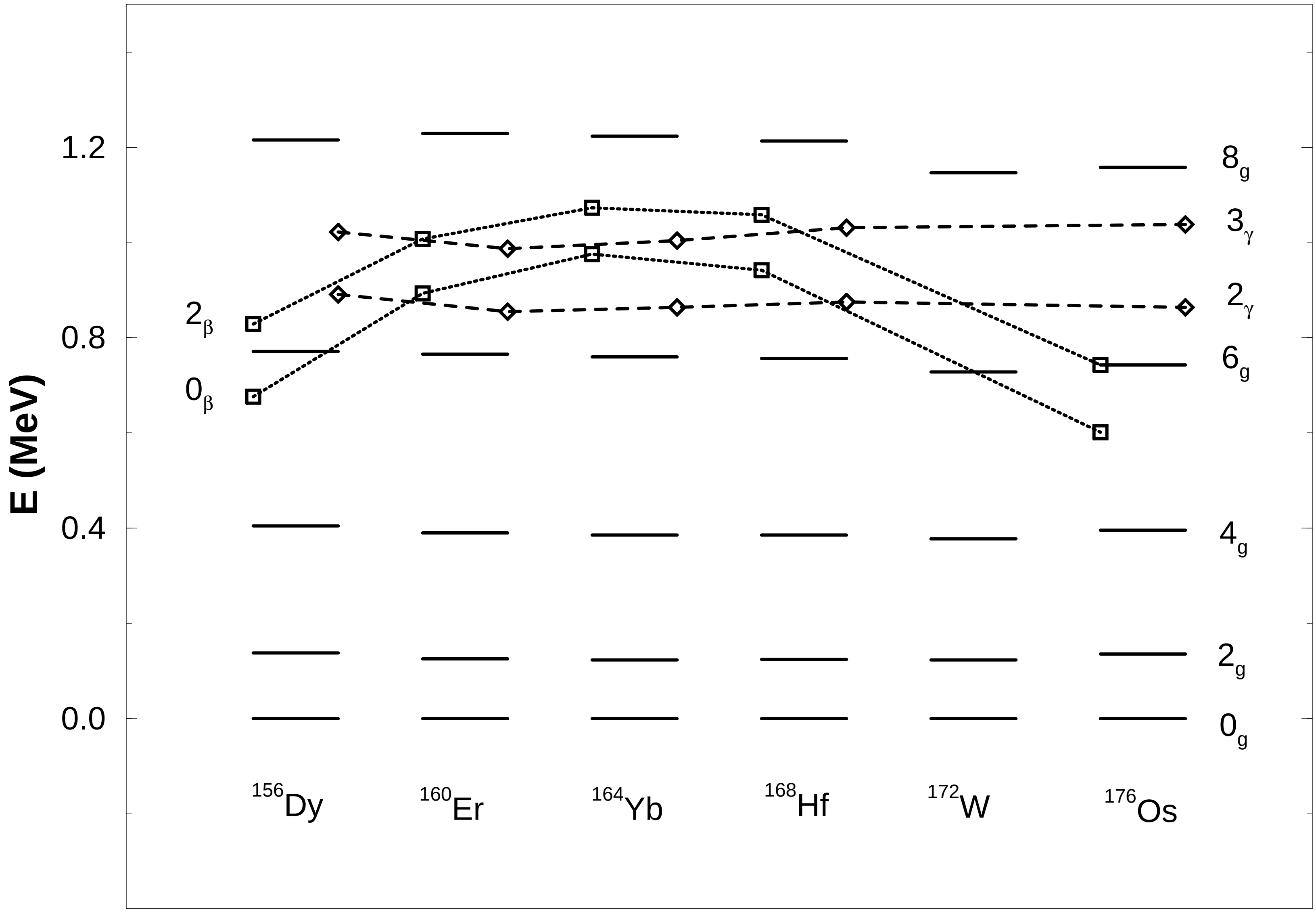
TABLE II. The ratio $R = \sum B(M1)\uparrow /(C_{F,F_0})^2$ for members of F-spin multiplets. Here $\sum B(M1)\uparrow$ denotes summed M1 strength to the scissors mode and $C_{F,F_0} = (F, F_0; 1, 0|F-1, F_0)$. Data taken from [21,22].

Nucleus	F	F_0	$\sum B(M1)\uparrow$ $[\mu_N^2]$	$(C_{F,F_0})^2$	R
^{148}Nd	4	1	0.78 (0.07)	5/12	1.87 (0.17)
^{148}Sm		2	0.43 (0.12)	1/3	1.29 (0.36)
^{150}Nd	9/2	1/2	1.61 (0.09)	4/9	3.62 (0.20)
^{150}Sm		3/2	0.92 (0.06)	2/5	2.30 (0.15)
^{154}Sm	11/2	1/2	2.18 (0.12)	5/11	4.80 (0.26)
^{154}Gd		3/2	2.60 (0.50)	14/33	6.13 (1.18)
^{160}Gd	7	0	2.97 (0.12)	7/15	6.36 (0.26)
^{160}Dy		1	2.42 (0.18)	16/35	5.29 (0.39)
^{162}Dy	15/2	1/2	2.49 (0.13)	7/15	5.34 (0.28)
^{166}Er		$-1/2$	2.67 (0.19)	7/15	5.72 (0.41)
^{164}Dy	8	0	3.18 (0.15)	8/17	6.76 (0.32)
^{168}Er		-1	3.30 (0.12)	63/136	7.12 (0.26)
^{172}Yb		-2	1.94 (0.22)[a)]	15/34	4.40 (0.50)
^{170}Er	17/2	$-3/2$	2.63 (0.16)	70/153	5.75 (0.35)
^{174}Yb		$-5/2$	2.70 (0.31)	66/153	6.26 (0.72)

[a)] The low value of $\sum B(M1)\uparrow$ for ^{172}Yb has been attributed to experimental deficiencies [10].

FIGURES

FIG. 1. Experimental levels of the ground γ and β bands in an F-spin multiplet $F = 6$ of rare earth nuclei. Levels shown are up to $L = 8_g^+$ for the ground band, $L = 2_\gamma^+, 3_\gamma^+$ for the γ band (diamonds connected by dashed lines) and $L = 0_\beta^+, 2_\beta^+$ for the β band (squares connected by dotted lines).


E (MeV)
1.2
0.8
0.4
0.0
2β
0β
8g
3γ
2γ
6g
4g
2g
0g
156Dy
160Er
164Yb
168Hf
172W
176Os