A Calculation of Baryon Diffusion Constant in Hot and Dense Hadronic Matter Based on an Event Generator URASiMA

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February 4, 2008

Abstract

We evaluate thermodynamical quantities and transport coefficients of a dense and hot hadronic matter based on an event generator URASiMA (Ultra-Relativistic AA collision Simulator based on Multiple Scattering Algorithm). The statistical ensembles in equilibrium with fixed temperature and chemical potential are generated by imposing periodic boundary condition to the simulation of URASiMA, where energy density and baryon number density is conserved. Achievement of the thermal equilibrium and the chemical equilibrium are confirmed by the common value of slope parameter in the energy distributions and the saturation of the numbers of contained particles, respectively. By using the generated ensembles, we investigate the temperature dependence and the chemical potential dependence of the baryon diffusion constant of a dense and hot hadronic matter.

PACS : 13.75.-n, 25.75, 33.15.V, 28.20.G
Keyword:Dense Hadronic Matter, Event Generator, Molecular Dynamics, Diffusion, Transport Phenomena.

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1 Introduction

Physics of a high density and high temperature hadronic matter has been highly attracting in the context of both high energy nuclear collisions and cosmology as well as theoretical interest[1]. In the recent ultra-relativistic nuclear collisions, though the main purpose should be confirmation of Quark-Gluon Plasma(QGP) state, physics of hot and/or dense hadronic state dominates the system. Hence, thermodynamical properties and transport coefficients of a hadronic matter are essentially important for the phenomenological description of the space-time evolution of the produced exited region. In the cosmology, in addition to the global evolution of the early universe, baryon diffusion would play an important roll in the nucleosynthesis problem.

Because of the highly non-perturbative property of a hot and dense hadronic state, investigation on the thermodynamical properties and transport coefficients has been hardly investigated. Numerical simulation based on Lattice gauge theory is a very powerful tool for the analysis of finite temperature QCD. Recently, transport coefficient of hot gluonic matter has been investigated [2]. But even for the modern high-performance supercomputer, lattice QCD evaluation of the transport coefficients of hadronic matter is very difficult, especially below T_c. Furthermore, at finite density, present numerical scheme of lattice QCD is almost useless since inclusion of chemical potential makes lattice action complex although there are several new approaches have been proposed[3][4]. In this paper, we evaluate the transport coefficients by using statistical ensembles generated by Ultra-Relativistic A-A collision simulator based on Multiple Scattering Algorithm

(URASiMA). Originally, URASiMA is an event generator for the nuclear collision experiments based on the Multi-Chain Model(MCM) of the hadrons[5]. Some of us(N. S and O. M) has already discussed thermodynamical properties of a hot-dense hadronic state based on a molecular dynamical simulations of URASiMA with periodic condition[6]. Recently, some groups have been performed similar calculation with use of the different type of event-generator UrQMD[7], where Hagedorn-type temperature saturation is reported. We improve URASiMA to recover detailed balance at temperature below two hundred MeV. As a result, Hagedorn-type behavior in the temperature disappears[8]. This is the first calculation of the transport coefficient of a hot and dense hadronic matter based on an event generator.

In section 2, we review URASiMA and explain how to make ensembles with finite density and finite temperature. Section 3 is devoted to the calculation for nucleon diffusion constant through the first-kind fluctuation dissipation theorem. Section 4 is concluding remarks.

2 URASiMA for Statistical Ensembles

URASiMA is a relativistic event generator based on hadronic multi-chain model, which aims at describing nuclear-nuclear collision by the superposition of hadronic collisions. Hadronic 2-body interactions are fundamental building blocks of interactions in the model, and all parameters are so designed to reproduce experimental data of hadron-hadron collisions. Originally, URASiMA contains 2-body process (2 incident particle and 2 out-going particles), decay process (1 incident particle and 2 out-going particles), resonance (2 incident particles and 1 out-going particle) and production process

(2 incident particles and n (≥ 3) out going particles). The production process is very important for the description of the multiple production at high energies. On the other hand re-absorption processes (n (≥ 3) incident particles and 2 out-going particles) thought to be unimportant in the collisions since system quickly expands and they have not been included in the simulation. On the other hand, in the generation of statistical ensembles in equilibrium, detailed balance between processes is essentially important. Lack of re-absorption process leads one-way conversion of energy into particle production rather than heat-up. As a result, artificial temperature saturation occurs.

Therefore, role of re-absorption processes is very important and we should take into account it. However exact inclusion of multi-particle re-absorption processes is very difficult. In order to treat them effectively, multi-particle productions and absorptions are treated as 2-body processes including resonances with succeeding decays and/or preceding formations of the resonances. Here two body decay and formation of resonances are assumed. For example, $NN \rightarrow NN\pi$ is described as $NN \rightarrow NR$ followed by decay of $R \rightarrow N\pi$, where R denotes resonance. The reverse process of it is easily taken into account. In this approach, all the known inelastic cross-sections for baryon-baryon interactions up to $\sqrt{s} < 3$GeV, are reproduced.

Table 1: Baryons, mesons and their resonances included in the URASiMA.

nucleon	N_{938}	N_{1440}	N_{1520}	N_{1535}	N_{1650}	N_{1675}	N_{1680}	N_{1720}
Δ	Δ_{1232}	Δ_{1600}	Δ_{1620}	Δ_{1700}	Δ_{1905}	Δ_{1910}	Δ_{1950}	
meson	π	η	σ_{800}	ρ_{770}				

For the higher energy, $\sqrt{s} > 3$GeV, in order to give appropriate total cross section, we need to take direct production process into account. Only this point, detailed balance is broken in our simulation, nevertheless, if temperature is much smaller than 3 GeV, the influence is negligibly small. For example , if the temperature of the system is 100 MeV , occurrence of such process is suppressed by factor of $\exp(-30)$ and thus time scale to detect violation of detailed balance is very much longer than hadronic scale.

In order to obtain equilibrium state, we put the system in a box and impose periodic condition to URASiMA as the space-like boundary condition. Initial distributions of particles are given by uniform random distribution of baryons in a phase space. Total energy and baryon number in the box are fixed at initial time and conserved through-out simulation. Though initial particles are only baryons, many masons are produced through interactions. After thermalization time-period about 100 fm/c, system seems to be stationary. In order to confirm the achievement of equilibrium, we calculate energy distributions and particle numbers. Slope parameters of energy distribution of all particles become the same value in the accuracy of statistics(Fig. 1). Thus, we may call this value as the temperature of the system. The fact that numbers of species saturate indicates the achievement of chemical equilibrium(Fig. 2). Running URASiMA many times with the same total energy and total baryons in the box and taking the stationary configuration later than $t = 150$ fm/c, we obtain statistical ensemble with fixed temperature and fixed baryon number(chemical potential).

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fig.1

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fig.2

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By using the ensembles obtained through above mentioned manner, we can evaluate thermodynamical quantities and equation of states[8].

3 Diffusion Constant

According to the Kubo's Linear Response Theory, the correlation of the currents stands for admittance of the system(first fluctuation dissipation theorem) and equivalently, random-force correlation gives impedance(Second fluctuation dissipation theorem) [9]. As the simplest example, we here focus our discussion to the diffusion constant. First fluctuation dissipation theorem tells us that diffusion constant D is given by current(velocity) correlation,

$$D = \frac{1}{3}\int_0^\infty < \boldsymbol{v}(t) \cdot \boldsymbol{v}(t+t') > dt'. \tag{1}$$

Average $< \cdots >$ is given by,

$$< \cdots >= \frac{1}{\text{number of ensembles}} \sum_{\text{ensemble}} \frac{1}{\text{number of particle}} \sum_{\text{particle}} \cdots. \tag{2}$$

If the correlation decrease exponentially, i.e.,

$$< \boldsymbol{v}(t) \cdot \boldsymbol{v}(t+t') > \propto \exp{(-\frac{t'}{\tau})}, \tag{3}$$

with τ being relaxation time, diffusion constant can be rewritten in the simple form,

$$D = \frac{1}{3} < \boldsymbol{v}(t) \cdot \boldsymbol{v}(t) > \tau. \tag{4}$$

Usually, diffusion equation is given as,

$$\frac{\partial}{\partial t} f(t, \boldsymbol{x}) = D \nabla^2 f(t, \boldsymbol{x}), \tag{5}$$

and diffusion constant D has dimension of $[L^2/T]$. Because of relativistic nature of our system, we should use $\boldsymbol{\beta} = \frac{\boldsymbol{v}}{c} = \frac{\boldsymbol{p}}{E}$ instead of $\boldsymbol{v}$ in eq.(1) and D is obtained by,

$$D = \frac{1}{3} \int_0^{\infty} < \boldsymbol{\beta}(t) \cdot \boldsymbol{\beta}(t + t') > dt' c^2. \tag{6}$$

$$= \frac{1}{3} < \boldsymbol{\beta}(t) \cdot \boldsymbol{\beta}(t) > c^2 \tau. \tag{7}$$

$$= \frac{1}{3} < \left(\frac{\boldsymbol{p}(t)}{E(t)} \right) \cdot \left(\frac{\boldsymbol{p}(t)}{E(t)} \right) > c^2 \tau \tag{8}$$

with c being the velocity of light. Figure 3 shows correlation function of the velocity of baryons. The figure indicates that exponential damping is very good approximation. Figure 4 displays the our results of baryon diffusion constant in a hot and dense hadronic matter.

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fig.3

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fig.4

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Our results show clearer dependence on the baryon number density while dependence on energy density is mild. This result means importance of baryon-baryon collision process for the random walk of the baryons and thus non-linear diffusion process of baryons occurs. In this sense, we can state that baryon number density in our system is still high. In the inhomogeneous big-bang nucleosynthesis scenario, baryon-diffusion plays an important roll. The leading part of the scenario is played by the difference between proton diffusion and neutron diffusion[10]. In our simulation, strong interaction dominates the system and we assume charge independence in the strong interaction, hence, we can not discuss difference between proton and neutron. However obtained diffusion constant of baryon in our simulation can give some kind of restriction to the diffusion constants of both proton and neutron.

From diffusion constant, we can calculate charge conductivity[11]. Figure 5 shows baryon number conductivity σ_{B},

$$\sigma_{\mathrm{B}} = \frac{n_{\mathrm{B}}}{k_{\mathrm{B}}T} D, \tag{9}$$

where n_{B} is baryon number density, T is temperature and k_{B} is Boltzmann constant(put as unity through out this paper), respectively.

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fig.5

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Therefore, if we want, we can discuss Joule heat and entropy production in the *Baryonic circuit* based on the above baryon number conductivity.

Because fundamental system in URASiMA is high energy hadronic collisions, we use relativistic notations usually. However, diffusion equation (5) is not Lorentz covariant and is available only on the special system i.e. local rest frame of the thermal medium. For the full-relativistic description of the space-time evolution of a hot and dense matter, we need to establish relativistic Navier-Stokes equation[12]. Taking correlation of appropriate currents, we can easily evaluate viscosities and heat conductivity in the same manner [13] .

4 Concluding Remarks

Making use of statistical ensembles obtained by an event generator URASiMA, we evaluate diffusion constants of baryons in the hot and dense hadronic matter. Our results show strong dependence on baryon number density and weak dependence on temperature. The temperature in our simulation is limited only small range, i.e., from 100 MeV to 200 MeV, and this fact can be one of the reasons why the change of diffusion constant of temperature is not clear. Strong baryon number density dependence indicates that, for the baryon diffusion process, baryon plays more important roll than light mesons. In this sense our simulation corresponds to high density region and non-linear diffusion process occurs. Calculation of the diffusion constants is the simplest examples of first fluctuation dissipation theorem. In principle, taking correlation of appropriate currents, i.e. energy flow, baryon number current, stress-tensor, etc., we can evaluate any kinds of transport coefficients. However, in relativistic transport theory, there exist several delicate points, e.g., relativistic property makes difference of mass and energy meaningless and, as a result, meaning of the "flow" of the fluid and "heat flow" become ambiguous[12][14]. The choice of the current depends on the macroscopic phenomenological equations which contain transport coefficients. Once we establish phenomenological equations for the high temperature and high density hadronic matter, we can evaluate the appropriate transport coefficients in the same manner. Detailed discussion will be reported in our forthcoming paper[13].

Acknowledgment

The authors would like to thank prof. T. Kunihiro for the discussion. This work is supported by Grant-in-Aid for scientific research number 11440080 by Monbusho. Calculation has been done at Institute for Nonlinear Sciences and Applied Mathematics, Hiroshima University.

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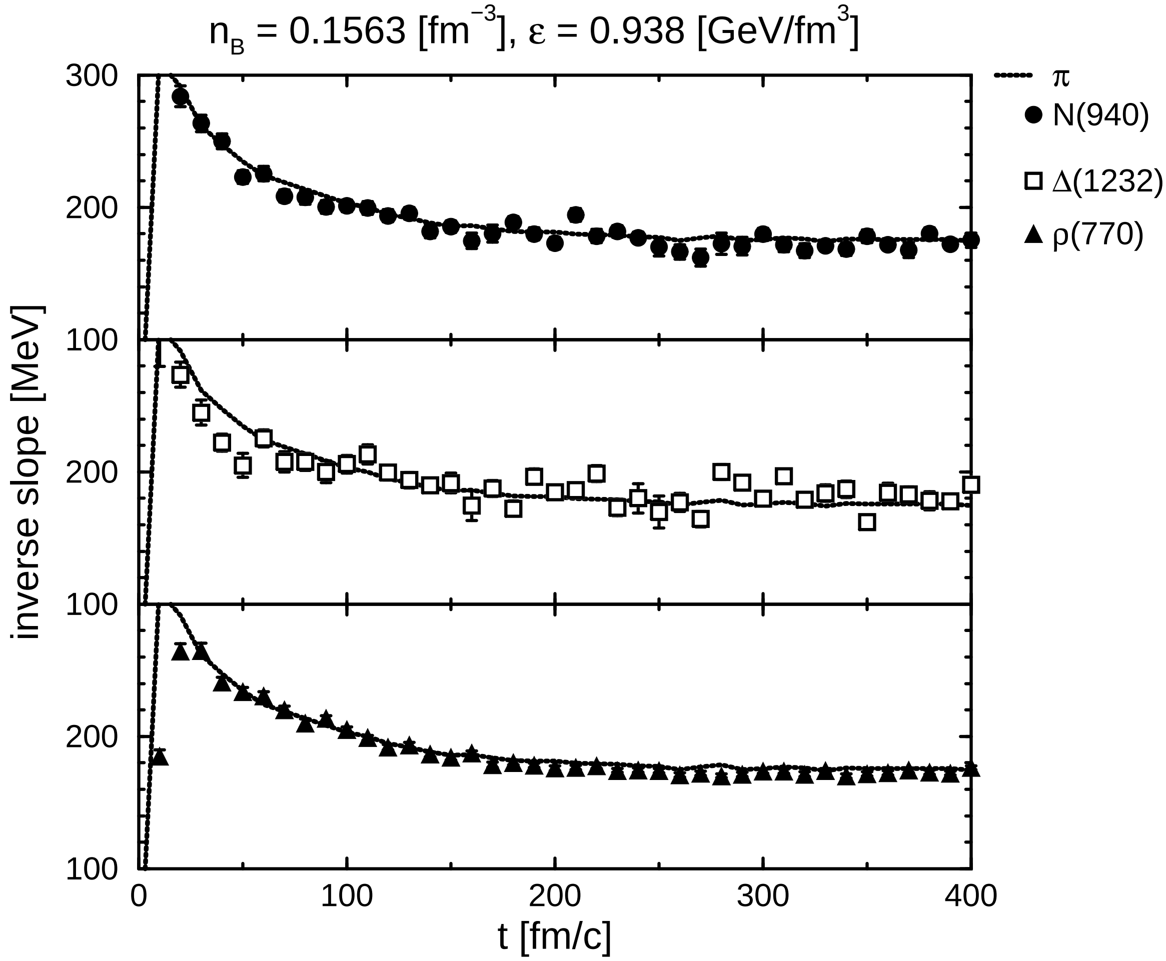

Figure 1: The time evolution of the inverse slopes β^{-1} of N_{938}, Δ_{1232}, ρ_{770} and π at $n_B = 0.1563$ fm^{-3} and $\varepsilon_{tot} = 0.938$ GeV/fm^3. β^{-1} is obtained by the energy distributions, $\frac{dN}{d^3\vec{p}} = \frac{dN}{4\pi EpdE} = C\exp(-\beta E)$. The dotted line stands for the β^{-1} of π.

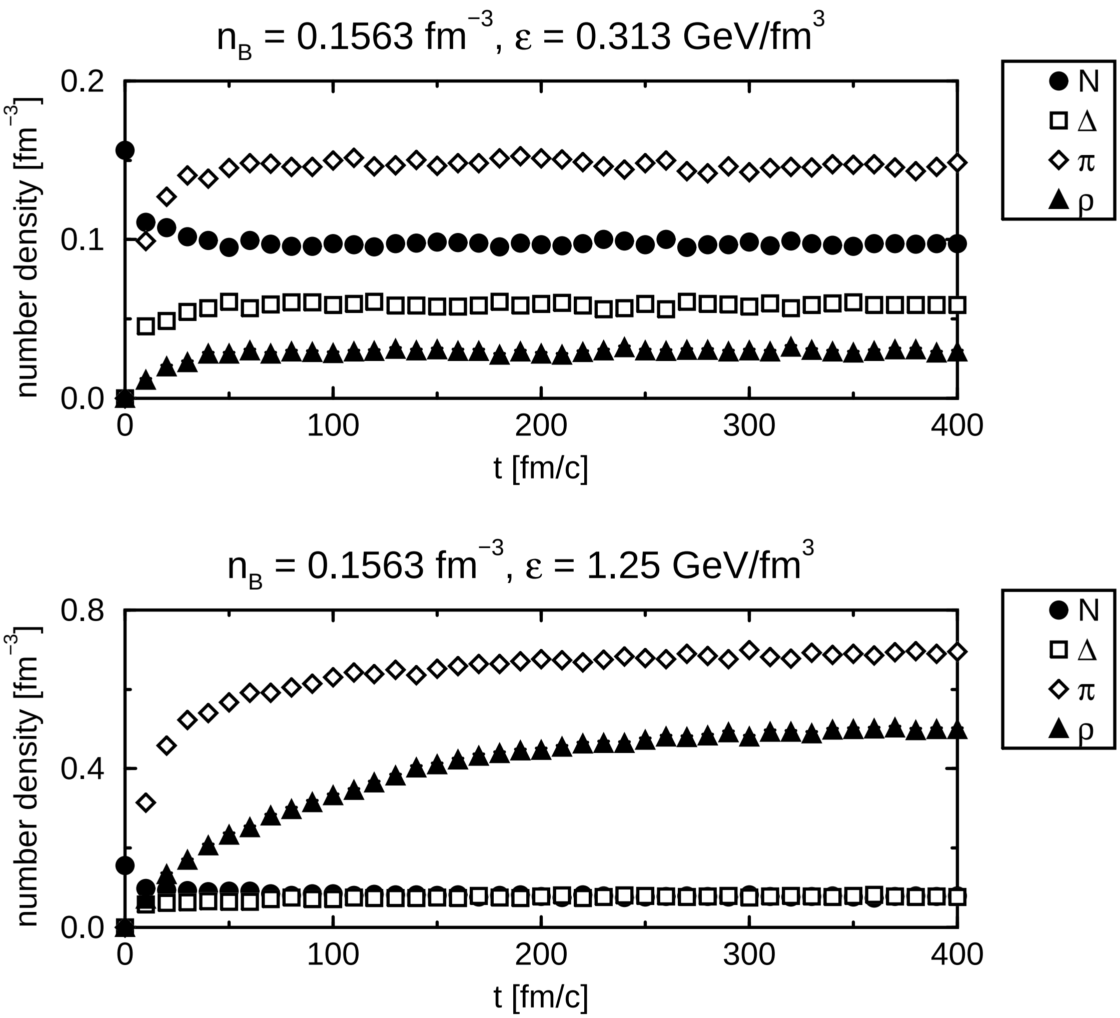

Figure 2: The time evolution of number densities: (a) $\varepsilon_{tot} = 0.313$ [GeV/fm^3] and (b) $\varepsilon_{tot} = 1.25$ [GeV/fm^3].

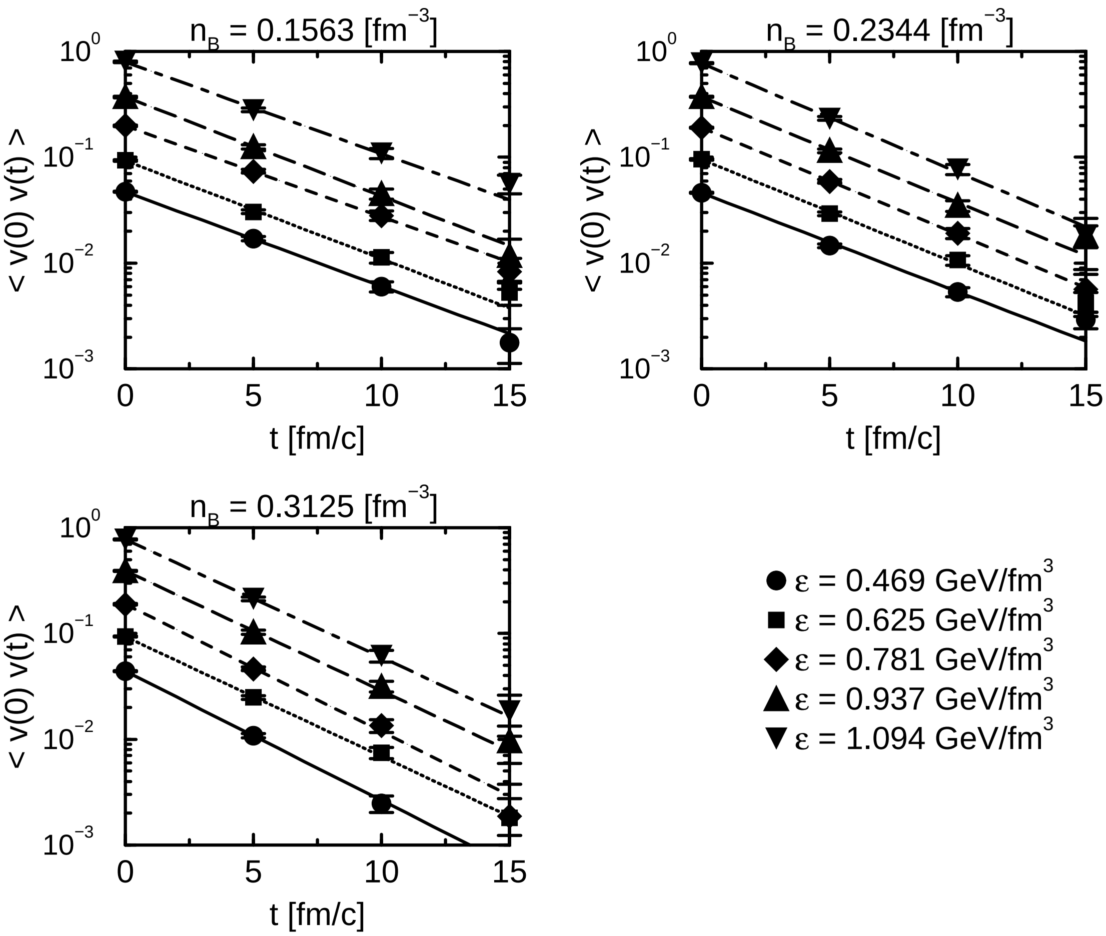

Figure 3: Velocity correlation of the baryons as a function of time. Lines correspond to the fitted results by exponential function. Normalizations of the data are arbitrary.

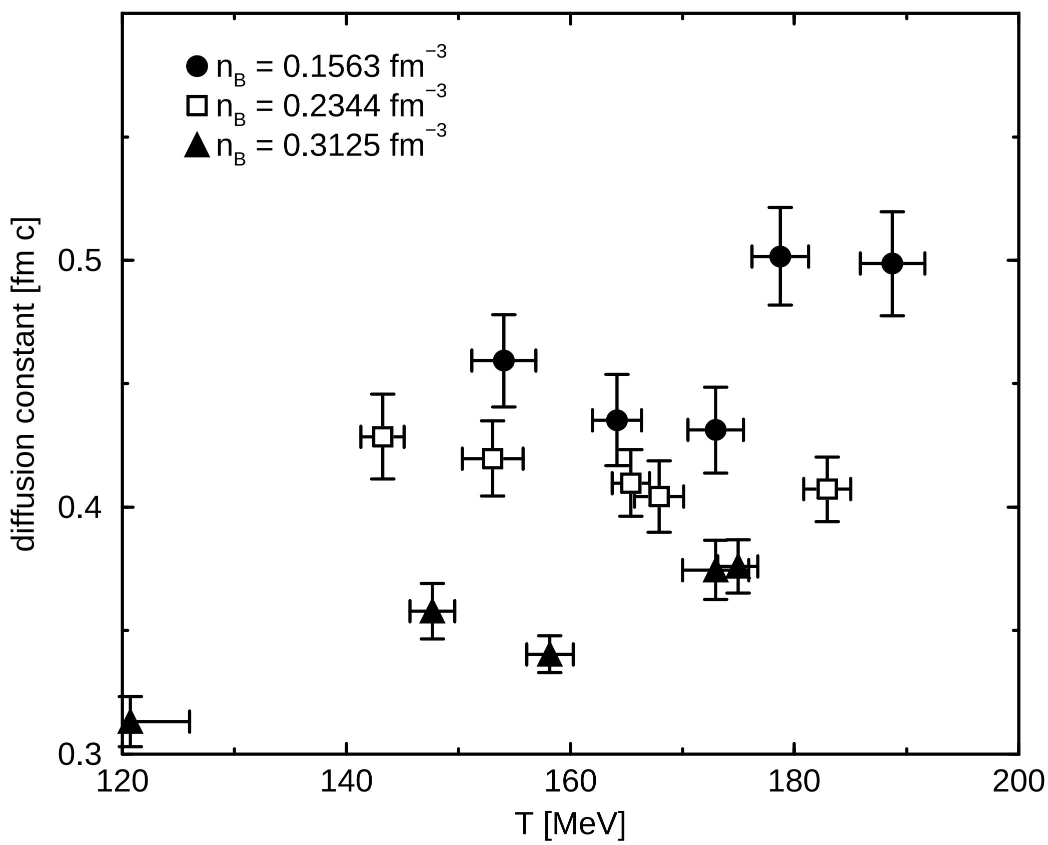

Figure 4: Diffusion constant of baryons.

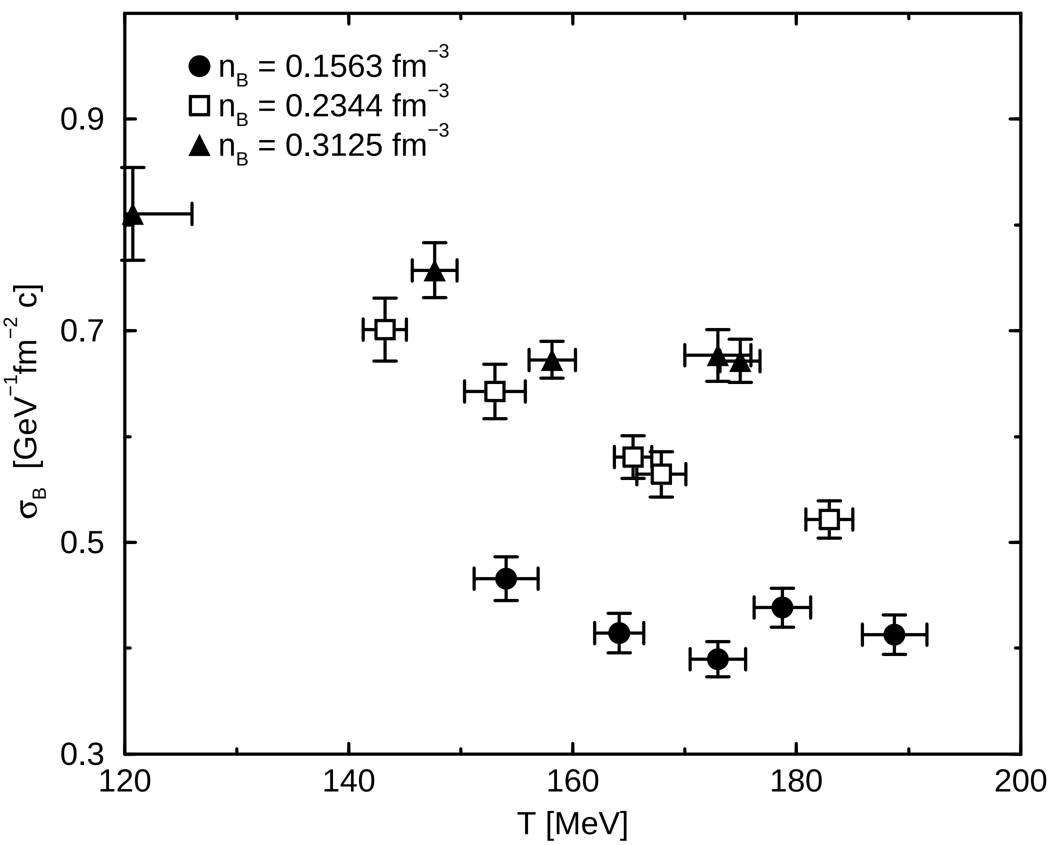

Figure 5: Baryon charge conductivity.