Pion abundance and entropy in the hydrodynamical description of relativistic nuclear collisions

Frédérique Grassi and Otavio Socolowski Jr.
Instituto de Física, Universidade de São Paulo, C. P. 66318, 05315-970 São Paulo-SP, Brazil

We show that a hydrodynamical model with continuous particle emission instead of sudden freeze out can explain both the strange particle abundances and pion abundance from NA35 without extra assumption (e.g., sequential freeze out, modified equation of state, sudden plasma hadronization,...). In this scenario, the observation of a larger pion abundance is natural and does not imply a higher initial entropy and early plasma phase.

The main purpose of the ongoing and future heavy ion programs at the high energy laboratories CERN (Switzerland) and Brookhaven National Laboratory (U.S.A.) is to investigate the formation of hot dense matter and the possible transition from hadronic matter to quark gluon plasma. Various possible signatures of the appearence of a quark gluon plasma (thereafter QGP) have been suggested: entropy increase (due to the release of new degrees of freedom, namely color), strangeness increase (due to enhanced strange quark production and faster equilibration), J/ψ supression (due to color screening), production of leptons and photons (emitted from a thermalized QGP and unaffected by strong interactions), etc. These signals have been studied extensively experimentally (see for exemple [1]). In this paper, we concentrate on two signatures: strangeness increase which has been observed between p-p and nucleus-nucleus at a fixed energy and entropy production which is studied via pion measurements.

A major problem to trace back any signature unambiguously to a quark gluon phase is that it is still unknown which theoretical description describes best high energy nuclear collisions. On one extreme, one might use a microscopic model. Hadronic microscopic models fail to reproduce simultaneously strange and non-strange particle data in nucleon-nucleon collisions and central nucleus-nucleus collisions at SPS energies (see [2,3]). Partonic microscopic models are expected to work at energies higher than SPS (however see [4] and references therein).

On the other extreme, one might use a thermal or hydrodynamical model. In such models, it is assumed that a fireball (region filled with dense hadronic matter or QGP in local thermal and chemical equilibrium) is formed in a high energy heavy ion collision and evolves. Hydrodynamical models have been used successfully to describe various kinds of data at AGS and SPS. In particular they are able to account for strangeness data but in their simplest version, fail to predict big enough pion abundances.

Since as already mentioned, both strangeness and pion or entropy productions are expected to be modified by the appearence of a quark gluon plasma, looking for a joint explaination (with or without plasma) of the relevant data is crucial. In the case of the thermal and hydrodynamical models mentioned above, various ways out of the pion problem have been proposed (see next section). In this paper, we study another possibility: use a more accurate emission mecanism, namely continuous emission, rather than standard freeze out, in a hydrodynamical description. This explaination has the advantage that no extra assumption is needed: once the initial conditions of the hydrodynamical expansion are fixed, both strangeness and pion yields come out with the right magnitude.

Hydrodynamical or thermal description with (standard) freeze out emission - First we remind what is the status of the standard hydrodynamical or thermal description of relativistic nuclear collisions. In this kind of description, hadrons are kept in chemical or thermal equilibrium until some decoupling criterion has become satisfied. An exemple of freeze out criterion often used is that a certain temperature and baryonic potential have been reached.

Since abundances are fixed by the chemical freeze out, the chemical freeze out parameters can be extracted by analyzing experimental particle abundances. This has been done by many groups [5]. The models have some variations between them, but as a general rule, while they can reproduce strange particle abundances, they underpredict the pion abundance. This was first noted by [6] in a study of NA35 data and emphasized by [7,8] in an analysis of the WA85 strange particle ratios and EMU05 specific net charge $D_q \equiv (N^+ - N^-)/(N^+ + N^-)$ (with N^+ and N^-, the positive and negative charge multiplicity respectively).

Various possible mecanisms have been suggested so that a hadronic gas could yield both the correct strange particle ratios and pion multiplicity: sequential freeze out [9] or separate chemical and thermal freeze outs, hadronic equation of state with excluded volume corrections [10–12], non-zero pion chemical potential [6,11], equilibrated plasma undergoing sudden hadronization and immediate decoupling [7,8,13].

All the physical points suggested to salvage the standard hadronic gas model might need to be contemplated in a precise hydrodynamical description of relativistic nuclear collisions, however it is somewhat suprising that one has to go to such kind of details to reconcile the strange particle and pion data. *Is it not possible to build a simple hydrodynamical model that yield the correct abundances without extra assumption?* We discuss this question in the next section [14].

Hydrodynamical description with continuous emission - In the standard hydrodynamical models, one assumes that the freeze out occurs on a sharp three-dimensional surface (defined for example by $T(x, y, z, t) = \text{const}$). Before crossing it, particles have a hydrodynamical behavior, and after, they free-stream toward the detectors, keeping memory of the conditions (flow, temperature) of where and when they crossed the three dimensional surface. The Cooper-Frye formula [15] gives the invariant momentum distribution in this case

$$Ed^3N/dp^3 = \int_\sigma d\sigma_\mu p^\mu f(x, p). \quad (1)$$

$d\sigma_\mu$ is a normal vector to the freeze out surface σ and f the distribution function of the type of particles considered. This is the formula implicitly used in all standard thermal and hydrodynamical model calculations of the previous section.

The notion that particle emission does not necessarily occur on a three dimensional surface but may be continuous was incorporated in a hydrodynamical description in [16]. In this model, the fluid is assumed to have two components, a free part plus an interacting part and its distribution function reads

$$f(x, p) = f_{\text{free}}(x, p) + f_{\text{int}}(x, p). \quad (2)$$

f_{free} counts all the particles that last scattered earlier at some point and are at time x^0 in $\vec{x}$. f_{int} describes all the particles that are still interacting (i.e., that will suffer collisions at time $> x^0$ and change momentum). The invariant momentum distribution is then

$$Ed^3N/dp^3 = \int d^4x\, D_\mu[p^\mu f_{\text{free}}(x, p)]. \quad (3)$$

$D_\mu[p^\mu f_{\text{free}}(x, p)]$ is a covariant divergence in general coordinates and d^4x is the invariant volume element. A priori formula (3) is sensitive to the whole fluid history and not just to freeze out conditions as in formula (1).

To compare particle abundances in the continuous emission and freeze out scenarios, we use a simplified framework to describe the fluid expansion, namely we suppose longitudinal expansion only and longitudinal boost invariance [17]. This approximation allows to carry out some calculations analytically and turns the physics involved more transparent. It is implicit however that this description applies at best to the midrapidity region. We will therefore concentrate on midrapidity abundances, precisely data from NA35.

In this simplified framework, in the case of a fluid with freeze out at a constant temperature and chemical potential, the Cooper-Frye formula (1) can be rewritten ignoring transverse expansion as [18]

$$\left.\frac{dN}{dyp_\perp dp_\perp}\right|_{y=0} = \frac{gR^2}{2\pi}\tau_{\text{fo}}\left(T_{\text{fo}}, T_0, \tau_0\right) m_\perp \times \sum_{n=1}^{\infty} (\mp)^{n+1} \exp\left(\frac{n\mu_{\text{fo}}}{T_{\text{fo}}}\right) K_1\left(\frac{nm_\perp}{T_{\text{fo}}}\right). \quad (4)$$

(The plus sign corresponds to bosons and minus, to fermions.) It depends on the conditions at freeze out: T_{fo} and $\mu_{\text{fo}} = \mu_{b\,\text{fo}}B + \mu_{S\,\text{fo}}S$, with B and S the baryon number and strangeness of the hadron species considered, and $\mu_{S\,\text{fo}}(\mu_{b\,\text{fo}}, T_{\text{fo}})$ obtained by imposing strangeness neutrality. So the experimental spectra of particles teach us in that case what the conditions were at freeze out.

For continuous emission, we can approximate equation (3) as [16]

$$\left.\frac{dN}{dyp_\perp dp_\perp}\right|_{y=0} \approx \frac{2g}{(2\pi)^2} \times \int_{\mathcal{P}=0.5} d\phi d\eta \frac{m_\perp \cosh\eta\tau_F\rho d\rho + p_\perp \cos\phi\rho_F\tau d\tau}{\exp\left[(m_\perp\cosh\eta - \mu)/T\right] \pm 1}, \quad (5)$$

where $\mathcal{P}$ is the probability to escape without collision calculated with a Glauber formula, τ_F (resp. ρ_F) is solution of $\mathcal{P}(\tau_F, \rho, \phi, \eta; v_\perp) = 0.5$ (resp. $\mathcal{P}(\tau, \rho_F, \phi, \eta; v_\perp) = 0.5$). In (5), various T and $\mu = \mu_b B + \mu_S S$ appear (again μ_S is obtained from strangeness neutrality), reflecting the whole fluid history, not just T_{fo} and $\mu_{b\,\text{fo}}$. This history is known by solving the hydrodynamical equations of a hadronic gas with continuous emisison. It depends only on the initial conditions T_0 and μ_{b0}. Therefore (5) only depends on the initial conditions.

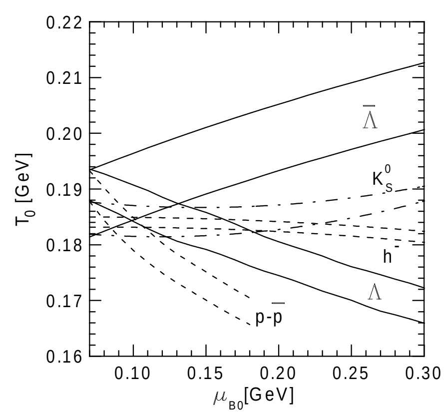

FIG. 1. Allowed region for the initial conditions determined from the NA35 data.

Once the spectra are known, they can be integrated to get abundances. Figure 1 shows the allowed region of initial conditions that lead to the experimental NA35 midrapidity values [19] $\Lambda = 1.26 \pm 0.22_{p_\perp > 0.5\text{GeV}}$, $\overline{\Lambda} = 0.44 \pm 0.16_{p_\perp > 0.5\text{GeV}}$, $K^0_S = 1.30 \pm 0.22_{p_\perp > 0.62\text{GeV}}$, $h^- = 27 \pm 1$ and $p - \overline{p} = 3.2 \pm 0.4$. We do not use the K^+ and K^- abundances because they were measured outside the mid-rapidity region. For heavy particles, initial conditions dominate [20] so when looking at their abundances in finite $p_\perp$ windows, transverse flow (assumed zero initially) can be neglected as we did. For negatives, the experimental abundance is for all $p_\perp$ so tranverse flow does not affect their abundance either. The allowed window corresponds to $T_0 \approx 185$ MeV, $\mu_{b0} \approx 100$ MeV , for an ideal gas equation of state and the strangeness saturation factor $\gamma_s = 1.3$

Using a more sofisticated equation of state, the value of T_0 might be decreased [20] by some 10-15%, i.e., to 155-165 MeV, compatible with (i.e., below) QCD lattice values for the phase transition temperature from QGP to hadronic matter. Our value of γ_S is above 1 and this might looked surprising. However, its value is decreased by some 15% when looking at a more realistic equation of state. In addition, we have imposed strangeness neutrality, it is possible that this is a too strong constraint when analyzing data taken in a very restricted rapidity region (see [21] where a similar problem was encountered).Using a larger value of γ_s, the size of the allowed window for initial conditions in figure 1 increases. There are other factors that influence the precise location and size of the window: values of the cross section needed to compute $\mathcal{P}$ (taken constant and equal for simplicity here), value of the cutoff in $\mathcal{P} = 0.5$, etc. However the important point is that it is possible to find initial conditions of the hydrodynamical expansion such that strange and non-strange particle abundances can be reproduced simultaneously without extra assumption.

To illustrate why the continuous emission is able to reproduce both strangeness and pion data, in table 1, we compare results from the continuous emission scenario and the freeze out model. We took $T_0 = 185$ MeV and $\mu_{b0} = 100$ MeV for the continuous emission case, $T_{\rm fo} = 185$ MeV and $\mu_{b\,\rm fo} = 100$ MeV for the freeze out case. We used $\gamma_s = 1.3$ for both [22]. We expect roughly similar results for both models for heavy particles: in the continuous emission case, due to thermal suppression, they are mostly emitted early [20], i.e., in similar conditions than in the freeze out model. Pions in the freeze out case are too few as discussed previously. Pions in the continuous emission case, on the other side, are emitted early and then on, so we expect to have more of them. This is precisely what we see in the table.

The initial conditions that we discussed so far correspond to a hadron gas, starting its hydrodynamical evolution. However the present continuous emission scenario is in fact probably compatible with the possibility that a QGP was created before for the following reason. Continuous emission is possible from a QGP but is inhibited by two factors: 1) only color singlet objects (not single quarks) can escape from it 2) the QGP core is surrounded by a dense hadronic region that the color singlets would have to cross *without* collisions to modify the previous results. In this case, particle emission would occur mostly in the hadronic phase. Numerical estimates are however necessary to back up these qualitative arguments.

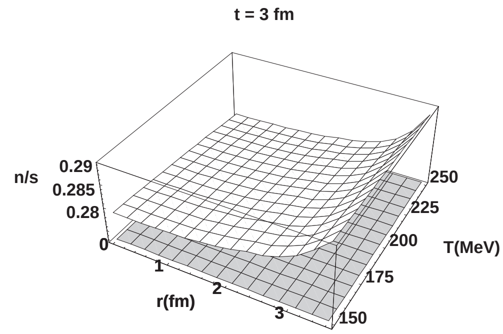

FIG. 2. Massless pion gas: value of the pion density over entropy density as function of the initial temperature and position for $t = 3$ fm in the continuous emission scenario (white surface). For comparison the value for the freeze out case is shown (gray surface).

Relation between pion number and entropy - In a hydrodynamical model without shocks and dissipation, entropy is conserved. In the usual freeze out scenario, this is an important point because the initial entropy can be determined from the final multiplicity. To illustrate this connection, let us consider a massless pion gas. Statistical mechanics yields the following relationship for the entropy density and the pion density $s = 3.6 n_\pi$. In the longitudinal boost invariant model, the entropy conservation equation gives: $s\tau = $ const. Using $dV = \tau dy \pi R^2$ at $y = 0$, this can be written: $dS(\tau)/(\pi R^2 dy) = $ const. So we can rewrite

$$\frac{dS}{dy}(\tau) = \frac{dS}{dy} = 3.6 \frac{dN_\pi}{dy}. \tag{6}$$

Therefore, experimental knowledge of dN_π/dy, permits to extract dS/dy, which are both independent of time.

In the continuous emission case, $s \neq 3.6 n_\pi$ because the distribution function needed to compute the pion density now depends on the escape probability $\mathcal{P}$, embodied in $f_{\rm free}$ (cf. eq. (2)). In fact, using similar methods than in [16], one can show that

$$s = \left[1 + \frac{3\beta - \alpha}{4(1+\alpha)}\right] 3.6 n_\pi, \tag{7}$$

where $\alpha(t,\rho) = (4\pi)^{-1} \int d\phi d\theta \sin\theta \left[\mathcal{P}/(1-\mathcal{P})\right]_{z=0}$ and $\beta(t,\rho) = (4\pi)^{-1} \int d\phi d\theta \sin\theta \cos^2\theta \left[\mathcal{P}/(1-\mathcal{P})\right]_{z=0}$. This relationship is plotted in figure 2. For all radii ρ, time τ and inicial condition T_0, $s(\tau,\rho) \leq 3.6 n_\pi(\tau,\rho)$. One then gets in the longitudinal boost invariant model:

$$\frac{dS}{dy} \leq 3.6 \frac{dN_\pi(\tau)}{dy}, \tag{8}$$

in other words, for a fixed entropy, there are more pions in the continuous emission case than in the freeze out case (for all times τ). This is in fact expected (even using a more realistic equation of state and including pions from decays): since pions are emitted continuously, there are more copious than in the the usual freeze out case for a given dS/dy (itself fixed by the initial conditions).

As a consequence, a large experimental value of dN_π/dy should not necessarily be associated to a large dS/dy and be considered a hint of QGP formation as usually done in freeze out models [7]. A larger dN_π/dy and not large dS/dy is a natural consequence of continuous emission, compared to freeze out. This result is in contrast for exemple with [7,8].

Conclusion - In this paper, we discussed data on strange and non-strange particles by NA35, from an hydrodynamical point of view. The standard model with sudden freeze out can reproduce the strange particle data but underpredicts the pion abundance, if no extra assumption is made. We showed that a hydrodynamical model with a more precise emission process, continuous emission, can reproduce both the strange and non-strange particle data without extra assumption.

This indicates the necessity of doing a more accurate description of particle emission in hydrodynamics, else some problems might artificially appear, such as a too low predicted pion abundance as discussed here or a too high freeze out density [20].

This point is reinforced by the fact that a large pion number is usually associated with a large entropy and QGP formation. Here we showed that a large pion number can be generated by continuous emission without modifying the entropy. (The larger pion emission will cause a faster cooling and shorter fluid lifetime.) In other words, a better understanding of particle emission in the hydrodynamical regime is also necessary to assess the possibility of QGP formation in relativistic heavy ion collisions.

There is a growing tendency to use hydrodynamical models to describe relativistic nuclear collisions, there is also a growing concern to modelize freeze out better [23,24]. However it seems that the very notion of particle emission during the hydrodynamical expansion needs to be put under more scrutiny.

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TABLE I. Comparison of experimental particle abundances with continuous emission and freeze out predictions.

	Experimental value	Continuous emission	Freeze out
Λ	1.26±0.22	1.05	0.98
$\bar{\Lambda}$	0.44±0.16	0.28	0.46
$p-\bar{p}$	3.2±0.4	3.6	1.52
h^-	27±1	27.6	16
K^0_S	1.30±0.22	1.27	1.08