Dressing the Nucleon in a Dispersion Approach

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Abstract

We present a model for dressing the nucleon propagator and vertices. In the model the use of a K-matrix approach (unitarity) and dispersion relations (analyticity) are combined. The principal application of the model lies in pion-nucleon scattering where we discuss effects of the dressing on the phase shifts.

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I. INTRODUCTION

The properties of unitarity and analyticity have been exploited in various theoretical approaches to pion-nucleon scattering. Unitarity is usually implemented either by solving a relativistic wave equation for the scattering amplitude [1–5] or using a K-matrix formalism [6–9]. Analytic properties, which are related to the condition of causality, have allowed one to derive useful dispersion relations for various amplitudes [10,11].

In the present work we have developed an approach in which constraints due to both unitarity and analyticity are incorporated by using both a K-matrix method [6,9] and dispersion relations [10,12]. The approch used consists of two separate stages. In the first effective 2- and 3-point Green's functions (i.e. propagators and vertices) are built which incorporate non-perturbative dressing due to regular (non-pole) parts of loop diagrams. In the second stage a K-matrix formalism is used to calculate the T-matrix, where the kernel, the K-matrix, is constructed from tree-level diagrams using the dressed vertices and propagators calculated in the first stage. Through the use of the K-matrix formalism the pole contributions are taken into account which were left out in the first stage. The T-matrix obtained from thus constructed K-matrix will contain the principal value parts of loop integrals, which is important for implementing analyticity in the K-matrix framework. The model as presented is geared to the calculation of pion-nucleon scattering and includes a consistent dressing of interacting nucleons, expanding the method of Ref. [13]. Since the dressing is formulated in terms of effective vertices and propagators through the use of form factors and self-energies, a broader application might be possible.

In the calculation of the propagators and vertices a technique based on the use of dispersion relations is used. This allows us to arrange the dressing of the nucleon such that the only πNN vertices needed throughout the procedure are those with one virtual external nucleon line (half-off-shell vertices). For the construction of the K-matrix we also need only these πNN vertices. Being interacting 3-point Green's functions, such vertices are not measurable quantities. In particular, they depend on the representation of interpolating fields in the Lagrangian. A wide class of field transformations leave the S-matrix (and therefore the observables) invariant while changing the interaction Lagrangian and hence the vertices [14–17]. In this work all calculations are performed in a particular representation in which self-energies and vertex modifications are calculated consistently with the K-matrix. In particular, 4- and higher-point Green's functions are absent in this representation. In Section V we show that the field ambiguity can be taken advantage of to change to a different representation in which the nucleon self-energy vanishes and no 4- (or higher-) point vertices are introduced. While leading to the same observables in virtue of the equivalence theorem [14,16], this representation is convenient for interpreting the effects of nucleon dressing in terms of effective πNN vertices. Rather soft form factors in 3-point vertices are dynamically generated through the dressing.

In the dressing it is essential to include the low-lying meson and baryon degrees of freedom, in particular, beside the pion, the ρ– and σ– mesons and the Δ–resonance. The associated coupling parameters were fixed by considering phase shifts for pion-nucleon scattering. A good agreement could be obtained for pion energies exceeding 400 MeV, adjusting only 5 parameters. The requirement that the dressing procedure converges for a given bare πNN form factor puts additional constraints on the allowed range of these parameters.

In discussing the calculated phase shifts, we focus primarily on effects of the dressing. These can be regarded as effects of the the explicit inclusion of the principal value parts of loop integrals (which are omitted in the usual approximation for the K-matrix [6,8,9]). Effects of principal value parts were also considered in Ref. [1] in a calculation based on three-dimentional reductions of the Bethe-Salpeter equation. In our approach we have kept the most general Lorentz structure for the πNN vertex. Another difference with [1] is that we obtain the principal value parts through the use of dispersion integrals.

A general description of the model is given in Section II. Details of the calculation of the main building blocks of the K-matrix – dressed vertices and propagators – are given in Sections III and IV, respectively. In Section V we construct a representation which is convenient for interpreting results of the dressing. Effects of the dressing on calculated phase shifts for pion-nucleon scattering are discussed in Section VI. Concluding remarks are made in Section VII.

II. OUTLINE OF THE MODEL

We start by giving the main formulae of the K-matrix approach in the context of pion-nucleon scattering. The S-matrix S is expressed in terms of the scattering amplitude $\mathcal{T}$ (the T-matrix) by

$$S = 1 + 2i\mathcal{T} \; . \tag{1}$$

In principle, the T-matrix can be obtained by solving the Bethe-Salpeter equation,

$$\mathcal{T} \;=\; V \;+\; V\,\mathcal{G}\,\mathcal{T}, \tag{2}$$

where the kernel (potential) V is the sum of irreducible diagrams describing the scattering, and $\mathcal{G}$ is a dressed πN propagator.

In general, any integral over the 4-momenta in $\mathcal{G}$ can be split into its pole and principal value (regular) parts,

$$\mathcal{G} \;=\; i\delta + \mathcal{G}^{R}, \tag{3}$$

where the pole part $i\delta$ is the contribution of the real (on-mass-shell) πN state, and $\mathcal{G}^{\mathcal{R}}$ corresponds to the propagation of virtual (off-shell) nucleon and pion. According to Cutkosky rules [19], $i\delta$ contains the imaginary parts of the invariant functions which parametrize $\mathcal{G}$, and $\mathcal{G}^{\mathcal{R}}$ contains the real parts.

It is this separation of the pole and principal value parts of propagators that is exploited in the K-matrix formalism. Namely, on defining the K-matrix by the equation

$$K \;=\; V \;+\; V\,\mathcal{G}^{R}\,K\,, \tag{4}$$

Eq. (2) can be written in the form

$$\mathcal{T} \;=\; K \;+\; K\,i\delta\,\mathcal{T}. \tag{5}$$

This can be formally solved, yielding the central equation of the K-matrix method [18]:

$$\mathcal{T} = \frac{1}{1 - Ki\delta} K. \tag{6}$$

The S-matrix can be obtained in a two step approach: 1) given a hermitian potential V, calculate K according to Eq. (4), and 2) solve Eq. (5) to calculate the amplitude $\mathcal{T}$ and use Eq. (1) to calculate the S-matrix. This scheme is equivalent to solving the Bethe-Salpeter equation Eq. (2) and as such should provide the full unitary and analytic S-matrix. Given a hermitian K-matrix, Eq. (5) can be solved relatively easily (for instance, by expanding K in partial waves and using Eq. (6)), and a unitary S-matrix is obtained. The simplicity of Eq. (5) is due to the fact that it involves integrals only over the mass-shells of internal particles. The problem of solving Eq. (4) is harder since one has to integrate over off-shell 4-momenta of $\mathcal{G}^R$. For this reason one usually avoids solving Eq. (4) in K-matrix models, setting $K = V$ [6–9]. The main drawback of this approximation is that the principal value parts of the integrals are completely ignored, i. e. $\mathcal{G}^R = 0$, and therefore analyticity of the amplitude cannot be fulfilled.

We take the potential V as a sum of tree diagrams. These include the s- and u-channel diagrams with an intermediate nucleon and Δ-resonance plus the t-channel diagrams with an intermediate ρ- and σ-meson. The tree diagrams are calculated using the free propagators and bare vertices. Note that we require that 4- and higher-point vertices do not participate in the construction of this potential. In other words, the bare interaction Lagrangian is assumed to contain 3-point vertices only. According to Eq. (4), the K-matrix should be obtained by dressing V with the principal value parts of loop integrals. We construct the K-matrix as the sum of skeleton diagrams shown in Fig. (1). It has the same form as V, except that it contains dressed πNN vertices and nucleon propagators and dressed propagators of the Δ, ρ and σ.

Once the K-matrix is constructed, the T-matrix is calculated from Eq. (6) using a partial wave decomposition as in Refs. [7,9]. The form of the K-matrix in Fig. (1) implies that physical one-nucleon – one-pion intermediate states are explicitly included in the unitarization procedure. Thus, the S-matrix obtained from Eq. (1) obeys $1N - 1\pi$ unitarity exactly.

A. Dressing procedure

The calculation of the dressed πNN vertex and the nucleon propagator is based on a system of integral equations, shown diagrammatically in Fig. (2). In the equation for the vertex, the external on-shell lines for the outgoing nucleon (on the right) and the pion, as well as the off-shell line for the incoming nucleon, are stripped away, as indicated by dashes on these lines. The solution is obtained in an iteration procedure and is described in detail in Ref. [13] (for the case including the nucleon and pion only). Here we shall repeat the main points.

Every iteration step (say, step number n) proceeds as follows. The imaginary or pole contributions of the loop integrals are obtained by applying cutting rules [19,20] to both the propagators and the vertices. Since the outgoing nucleon and the pion are on-shell, the only kinematically allowed cuts for the vertex loops are those shown by the curved lines in Fig. (2). In calculating these pole contributions, we retain only real parts of the loop integrals for vertices and self-energies from the previous step $n - 1$, as dictated by Eq. (4).

The real parts of the form factors and self-energy functions are calculated at every iteration step by applying dispersion relations [10,12] to the imaginary parts. For example, for the form factors at the iteration step n we have

$$Re\, G_i^n(p^2) = G_i^0(p^2) + \frac{\mathcal{P}}{\pi}\int_{(M+\mu)^2}^{\infty} dp'^2\, \frac{Im\, G_i^n(p'^2)}{p'^2 - p^2}\,, \tag{7}$$

where i labels the structure of the vertex, pseudoscalar (S) or pseudovector (V), see Eq. (8). M and μ are the nucleon and pion masses. $G_i^0(p^2)$ are the form factors in a bare πNN vertex, the first term on the right-hand side of Fig. (2).

This procedure is repeated until a converged solution is reached. We use a normalized root-mean-square difference d_n between two subsequent iteration steps n and $n+1$ for the form factors and self-energy functions. The convergence criterion is that $d_n < 10^{-4}$ for at least a hundred iteration steps. As zeroth iteration step the bare πNN vertex and the free nucleon propagator are taken. The solution of the equations in Fig. (2) is equivalent to a dressing of the potential V with the principal value parts of loop integrals.

Despite the explicit use of dispersion integrals, analyticity in the model is obeyed only "approximately". The violation of analyticity comes in at the level of bare form factors needed as part of the regularization of dispersion integrals. Strictly speaking, singularities of a bare form factor should give rise to additional residue contributions to the dispersion relation. To evaluate such a residue, one would have to know the behaviour of the function to which the dispersion relation is applied at the singularity of the bare form factor. In general however, this behaviour is not known. One way to circumvent this difficulty is to choose the bare form factor with singularities that are as remote from the region of physical interest as possible, implying a large width in general. This is supported by the fact that the width of the form factor should be larger than the masses of mesons included explicitly.

We remark that – as a consequence of Liouville's theorem – the difficulty with additional singularities in the complex plane is inherent in any approach where phenomenological form factors are used.

III. VERTICES

A. The πNN vertex

The general πNN vertex can be parametrized in terms of four Lorentz-invariant functions (form factors) [21]. The form factors may depend on the 4-momenta squared of each of the three external lines. For the K-matrix in this model we need πNN vertices with one virtual nucleon (4-momentum p), while the other nucleon (p') and the pion are on the mass-shell. Such vertices are conventionally called "half-off-shell" vertices. The general Lorentz and isospin covariant form of this vertex can be written [21] [1]

$$\tau_\alpha\, \Gamma(p) = \tau_\alpha\, P_+(p')\gamma^5 \left[G_S(p^2) + P_+(p) G_V(p^2)\right], \tag{8}$$

[1]Here and throughout the paper, we use the notation of Ref. [22].

for an incoming virtual nucleon. Here $G_S(p^2)$ and $G_V(p^2)$ are pseudoscalar and pseudovector form factors, $\tau_\alpha, \alpha = 1, 2, 3$, are Pauli isospin matrices, and $P_+(p) \equiv (\not{p} + M)/(2M)$. In the course of the dressing procedure the most general structure Eq. (8) of the πNN vertex is maintained.

The bare vertex in the dressing procedure is chosen as

$$G_V(p^2) = f_N\,(1-\chi)\,G^0(p^2) \quad , \; G_S(p^2) = f_N\,\chi\,G^0(p^2) \tag{9}$$

with

$$G^0(p^2) = \exp\left[-\ln 2 \frac{(p^2 - M^2)^2}{\Lambda_N^4}\right], \tag{10}$$

Here Λ_N^2 is the half-width of the bare form factor, the parameter χ is the amount of pseudoscalar admixture in the bare vertex, and f_N is a bare coupling constant. The latter is fixed from the renormalization condition imposed on the dressed vertex at the on-shell point,

$$\overline{u}(p')\,\Gamma(M)\,u(p) = \overline{u}(p')\,\gamma^5\,g_{\pi NN}\,u(p)\;, \tag{11}$$

where $g_{\pi NN}$ is the physical pion-nucleon coupling constant (for which we take the value 13.02 [9]) and $u(p)$ is the positive-energy nucleon spinor. The renormalization conditions for the nucleon propagator are described in Section IV. Because of the coupled structure of the equations in Fig. (2), the renormalization of the vertex and that of the propagator are not independent of each other.

The role of the bare πNN vertex is two-fold. On the one hand, it serves as the driving vertex at the zeroth iteration step. On the other hand, it is used for regularization of the dispersion integrals. The bare vertex is supposed to encapsulate the physics due to degrees of freedom not included explicitly in the dressing.

B. Vertices with Δ, ρ and σ

In principle, the dressing procedure should also apply to vertices describing the coupling to the Δ-resonance and to the ρ- and σ-mesons. In such an approach one would have to solve a system of 10 coupled equations, instead of the system of two equations in Fig. (2). Clearly, pursuing this is hardly feasible.

For all other vertices except πNN we ignore the dressing and restrict ourselves to one particular Lorentz covariant form. With each vertex a form factor is associaled which is required for regularization of the loop integrals. The propagators of the Δ-resonance and the ρ- and σ-mesons are dressed by the standard summation of loop insertions as discussed in Section IV.

The $\sigma\pi\pi$ and $\rho\pi\pi$ vertices for a σ or ρ meson with 4-momentum $p = q + q'$ are taken as

$$(\Gamma_{\rho\pi\pi})^{\;\nu}_{\alpha\beta\gamma} = (\hat{e}_{\alpha\beta\gamma})\,ig_{\rho\pi\pi}F_\rho(p^2)\left[k^\nu - \frac{(p\cdot k)}{p^2}p^\nu\right]\;, \tag{12}$$

$$(\Gamma_{\sigma\pi\pi})_{\,\alpha\beta} = -i\,\frac{g_{\sigma\pi\pi}}{\mu}F_\sigma(p^2)\delta_{\alpha\beta}\,(q\cdot q')\;, \tag{13}$$

where q and q' are the 4-momenta of the pions with isospin indices α and β respectively, $k = q - q'$ and $(\hat{e}_{\alpha\beta\gamma}) = -i\epsilon_{\alpha\beta\gamma}$. The ρ-meson carries the isospin index γ and the Lorentz vector index ν. $g_{\rho\pi\pi}$ and $g_{\sigma\pi\pi}$ are coupling constants (the values of all coupling constants will be given later).

For the vertices discussed in this section a generic form factor F_r is introduced whose functional form is similar to that of the bare πNN form factor given in Eq. (10):

$$F_r(p_r^2) = \exp\left[-\ln 2\frac{(p_r^2 - \widetilde{m}_r^2)^2 - (m_r^2 - \widetilde{m}_r^2)^2}{\Lambda^4}\right] . \tag{14}$$

normalized to unity at the on-shell point $p_r^2 = m_r^2$ with a half-width of Λ^2, the latter taken the same for all vertices considered in this subsection. For the $\rho\pi\pi$ and the $\sigma\pi\pi$ vertices, $\widetilde{m}_r$, the position of the maximum of the form factor, is set equal to the mass of the meson, $\widetilde{m}_r = m_r$.

The $\rho\pi\pi$ vertex, Eq. (12) chosen such that it vanishes when contracted with the 4-momentum p of the ρ-meson. As a consequence, the spin-0 part of the ρ propagator does not contribute to any matrix element (because the projection operator on the spin-0 component is $\mathcal{P}^0_{\mu\nu}(p) = p_\mu p_\nu / p^2$). The apparent singularity at $p^2 = 0$ of the vertex Eq. (12) lies outside the kinematical range covered in the calculations. In any case, the $1/p^2$-pole behaviour could be compensated by choosing in Eq. (12) a form factor with a zero at $p^2 = 0$.

The ρNN and σNN vertices are taken as

$$(\Gamma_{\rho NN})^{\nu}_{\gamma} = -i\, g_{\rho NN} F_N(p_N^2) \frac{\tau_\gamma}{2}\left[\gamma^\nu + i\,\kappa_\rho \frac{\sigma^{\nu\lambda} q_\lambda}{2M}\right] , \tag{15}$$

$$\Gamma_{\sigma NN} = -i\, g_{\sigma NN} F_N(p_N^2) , \tag{16}$$

where q is the (incoming) momentum of the ρ-meson, and $g_{\rho NN}$, κ_ρ and $g_{\sigma NN}$ are coupling constants. The form factor $F_N(p_N^2)$, where p_N is the 4-momentum of the off-shell nuceon, is given in Eq. (14) with $\widetilde{m}_N = M$, the nucleon mass.

The $\pi N\Delta$ vertex used in this calculation can be written as

$$(\Gamma_{\pi N\Delta})^{\nu}_{\alpha} = i\frac{g_{\pi N\Delta}}{\mu^2}\, T_\alpha\, F_\Delta(p^2)\, F_N(p_N^2)\, [\not{p} q^\nu - (p\cdot q)\gamma^\nu] , \tag{17}$$

where p is the (incoming) 4-momentum of the Δ-resonance and $p_N = p - q$ is the (outgoing) nucleon 4-momentum, $g_{\pi N\Delta}$ is a coupling constant and T_α, $\alpha = 1, 2, 3$, are isospin 3/2 to 1/2 transition operators. The form factors F_Δ and F_N are taken as in Eq. (14) with $\widetilde{m}_N = M$ and $\widetilde{m}_\Delta^2 < M_\Delta^2$, the mass squared of the Δ, to obtain a reasonable description of the $P33$-phase shift in pion-nucleon scattering (see the discussion of results below). Indications in favour of a $\pi N\Delta$ form factor slightly assymmetric with respect to the Δ mass have been also found in other works [2,8], even though $\pi N\Delta$ vertices different to Eq. (17) have been used there. The dependence of the form factor in Eq. (17) on p_N^2 turns out to be neccesary to regularize the contribution of the third loop diagram on the righ-hand side of the equation in Fig. (2).

The reason for the particular structure Eq. (17) of the $\pi N\Delta$ vertex is that it possesses the property $p \cdot (\Gamma_{\pi N\Delta})_\alpha = 0$. As a consequence, the "sandwich" of the spin-1/2 part of the Rarita-Schwinger Δ propagator between two $\pi N\Delta$ vertices vanishes since every term in the spin-1/2 part of the Δ propagator is proportional to either p_μ or p_ν. Thus only the spin-3/2 part of the Δ propagator gives rise to non-vanishing matrix elements [23], and it suffices to calculate only the spin-3/2 part of the Δ self-energy.

IV. DRESSED PROPAGATORS

The inverse of the dressed nucleon propagator can be written as

$$S^{-1}(p) = \not{p} - M - \Sigma(p), \tag{18}$$

with the self-energy given by

$$\Sigma(p) = \Sigma_L(p) - (Z_2 - 1)(\not{p} - M) - Z_2\,\delta M. \tag{19}$$

Here $\Sigma_L(p)$ is the contribution of pion loops,

$$\Sigma_L(p) = A(p^2)\not{p} + B(p^2)M, \tag{20}$$

parametrized by the "self-energy functions" $A(p^2)$ and $B(p^2)$. The field and mass renormalization constants Z_2 and δM are fixed by requiring that the propagator have a simple pole with a unit residue at $\not{p} = M$. This yields

$$Z_2 = 1 + Re\,A(M^2) + 2M^2 \frac{d}{d(p^2)}\left[Re\,A(p^2) + Re\,B(p^2)\right]\Big|_{p^2=M^2}, \tag{21}$$

$$\delta M = \frac{M}{Z_2}\left[Re\,A(M^2) + Re\,B(M^2)\right] . \tag{22}$$

For dressing the Δ-propagator only a one πN loop approximation is used. Similar to the nucleon self-energy, the imaginary parts of the resonance self-energy are calculated using cutting rules and the real parts are obtained from dispersion integrals.

As explained in the previous Section, the choice of the $\pi N\Delta$ vertex Eq. (17) allows us to retain the spin-3/2 part of the Δ propagator only,

$$P_{\mu\nu}(p) = \frac{1}{\not{p} - M_\Delta - \Sigma_\Delta(p)}\,\mathcal{P}^{3/2}_{\mu\nu}(p), \tag{23}$$

with the spin-3/2 projection operator [23]

$$\mathcal{P}^{3/2}_{\mu\nu}(p) = g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3p^2}(\not{p}\gamma_\mu p_\nu + p_\mu\gamma_\nu\not{p}) . \tag{24}$$

Due to the elimination of the spin-1/2 components the Δ self-energy can be parametrized by only two Lorentz invariant functions ($A_\Delta(p^2)$ and $B_\Delta(p^2)$) instead of 10 functions which would be needed in the general case with the spin-1/2 part present. The structure of the self-energy is thus the same as for the nucleon, Eq. (20). The counter-term contribution to the self-energy [20] contains the real constants Z_2^Δ and δM_Δ fixed by the renormalization as described for the nucleon. The term $\sim 1/p^2$ in Eq. (24) does not lead to a singularity if the $\pi N\Delta$ vertices from Eq. (17) are used.

The meson propagators are dressed through the insertion of a $\pi\pi$ loop as described in Section II A. The pion propagator thus remains undressed.

The dressed propagator of the σ-meson has the form

$$D(p^2) = \frac{1}{p^2 - m_\sigma^2 - \Pi_\sigma(p^2)}, \tag{25}$$

where m_σ is the physical mass of the meson and $\Pi_\sigma(p^2)$ is its self-energy. The latter can be written as a sum of the loop and counter-term contributions,

$$\Pi_\sigma(p^2) = \Pi_{\sigma,L}(p^2) - (Z_\sigma - 1)(p^2 - m_r^2) - Z_\sigma \delta m_\sigma^2, \tag{26}$$

where Z_σ and δm_σ^2 play the role of the field and mass renormalization constants. These constants are fixed by requiring that the expansion of $Re\, \Pi_\sigma(p^2)$ contain only second and higher powers of $(p^2 - m_\sigma^2)$ [20]. In other words,

$$\frac{1}{p^2 - m_\sigma^2 - Re\, \Pi_\sigma(p^2)} \tag{27}$$

is required to have a simple pole with a unit residue at $p^2 = m_\sigma^2$. This yields Z_σ and δm_σ^2 in terms of $Re\, \Pi_{\sigma,L}(p^2)$:

$$Z_\sigma = 1 + \frac{d}{d(p^2)} Re\, \Pi_{\sigma,L}(p^2)\Big|_{p^2 = m_\sigma^2}, \tag{28}$$

$$\delta m_\sigma^2 = \frac{Re\, \Pi_{\sigma,L}(m_\sigma^2)}{Z_\sigma}. \tag{29}$$

Following similar arguments as for the Δ, the structure of the vertex Eq. (12) has been chosen such that only the spin-1 part of the dressed ρ propagator can be retained,

$$D_{\mu\nu}(p) = \frac{1}{p^2 - m_\rho^2 - \Pi_\rho(p^2)} \mathcal{P}^1_{\mu\nu}(p)\ , \tag{30}$$

where

$$\mathcal{P}^1_{\mu\nu}(p) = g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\ , \tag{31}$$

is the spin-1 projection operator and $\Pi_\rho(p^2)$ is the self-energy which has the same structure as for a scalar particle.

V. CHANGING REPRESENTATION

It is known that interacting Green's functions depend on the representation of fields in the Lagrangian. There exist a wide class of field transfomations which do not affect the asymptotic behaviour of the fields and hence leave the S-matrix (and thus all observables) invariant [15,16]. The invariance of the S-matrix under field transformations is known as the equivalence theorem [14,16].

In this Section we will take advantage of this irrelevance of representation and transform the effect of the dressing of the nucleon propagator into new πNN vertices. The representation constructed in this Section is an example of the "physical representation" discussed in Ref. [24].

We introduce the notation where the subscript Σ labels the representation where the propagator S contains a non-trivial self-energy. The new representation is defined by the two requirements: 1) the nucleon propagator must be equal to the free propagator $S^0(p)$, 2) it must be possible to construct the K-matrix as in Fig. (1), i. e. solely in terms of 2- and 3-point Green's functions. The new πNN vertex Γ must thus be a solution of the equation

$$\Gamma(p)\, S^0(p)\, \overline{\Gamma}(p) = \Gamma_\Sigma(p)\, S(p)\, \overline{\Gamma}_\Sigma(p)\ , \tag{32}$$

where only the dependence on the internal, off-shell, 4-momentum, p, is indicated. All effects of the dressing are now contained in the difference between the new dressed and the bare vertex, $\Gamma - \Gamma^0$.

The solution of Eq. (32) can be written as

$$\left[ReG(p^2) \pm \frac{W}{2M} ReG_V(p^2)\right] = \left[ReG_\Sigma(p^2) \pm \frac{W}{2M} ReG_{\Sigma,V}(p^2)\right] \sqrt{\frac{S(\pm W)}{S^0(\pm W)}} \tag{33}$$

where $G = G_S + G_V/2$ and $W = \sqrt{p^2} \geq 0$ is the invariant mass of the virtual nucleon. Also, $S^0(\pm W) = -1/(M \mp W)$ and

$$S(\pm W) = \frac{-1}{Z_2(M \mp W) - Z_2 \delta M \pm ReA(p^2)W + ReB(p^2)M} \tag{34}$$

are positive- and negative-energy parts of the free and dressed nucleon propagators, respectively.

In the present work we consider only those solutions for the dressed propagator S that do not have real poles in addition to the nucleon pole $W = M$. With this qualification, the solution Eq. (33) for the πNN vertex in the new representation is well defined, since, due to the renormalization procedure, the ratio $S(\pm W)/S^0(\pm W)$ is positive.

An extra pole at positive W would correspond to an additional asymptotic state, different to the free nucleon. To take it into account properly, certain modifications would be necessary of the standard renormalization of the nucleon field Eq. (21): an additional field renormalization constant would probably be required to account for the fact that a new particle species occured as a result of the dressing [25]. Such a study lies outside the scope of the present work.

VI. DISCUSSION

The masses of the particles included in the model are given in Table I [9] and kept fixed in the calculation of pion-nucleon phase shifts.

An important characteristic of the bare vertex is its half-width Λ_N^2, see Eq. (10). To investigate the dependence of the dressing on Λ_N^2, calculations have been done for two values, $\Lambda_N^2 = 2$ GeV^2 and $\Lambda_N^2 = 3$ GeV^2, referred to as calculations (I) and (II), respectively. The requirement that a converged solution of the dressing procedure can be obtained, without developing additional poles of the propagator (see Section V), puts an upper bound on Λ_N^2. While the exact value of this limit depends also on other parameters of the model, it is certain that the bare form factor cannot be arbitrarily hard. Note however that the scale introduced by the bare form factor, which is of the order of $M^2 + \Lambda_N^2$, is larger than the scale due to the degrees of freedom explicitly included in the dressing. The values of the bare coupling constant f_N, introduced in Eq. (9), are given in Table II, where also the values of the field and mass renormalization constants are listed.

We find that a sizable pseudoscalar admixture in the bare vertex (with $|\chi| > 0.1$ in Eq. (9)) leads to a poor description of low energy phase shifts. This is intimately related to the smallness of explicit chiral-symmetry breaking. Besides, even without resorting to phenomenology, the range of variation of χ is severely constrained by the requirement of convergence. Both calculations presented in this work were done with $\chi = 0.055$.

The values of the parameters in the vertices for the Δ-resonance and ρ- and σ-mesons, Eqs. (12 - 17), are summarized in Table III. The constants $g_{\pi N\Delta}$, $g_{\rho\pi\pi}$ and $g_{\sigma\pi\pi}$ were fixed from the decay widths of the Δ, ρ and σ [26]. The value of half-width Λ^2, see Eq. (14), was kept fixed and had to be sufficiently soft to provide convergence of the dressing procedure.

The coupling constants $g_{\rho NN}$, κ_ρ, $g_{\sigma NN}$, as well as the parameter $\widetilde{m}_\Delta^2$, were chosen from a comparison of the calculated πN phase shifts with the data, taken from [27]. Together with χ, discussed above, the five adjustable parameters are given in the last five columns in Table III. It should be stressed that only for a rather restricted range of these constants a convergent solution of the dressing procedure could be found.

The phase shifts in pion-nucleon scattering are shown in Figs. (3) and (5) as function of the pion kinetic energy in the laboratory system, corresponding to calculations (I) and (II), respectively. The solid lines are the phase shifts calculated with the dressed K-matrix as shown in Fig. (1). The dashed lines are obtained in the approximation where K is set equal to the potential V, hence without taking the dressing into account.

The effect of the dressing on the πNN vertex, can be seen more clearly from Figs. (4) (calculation(I)) and (6) (calculation (II)). The form factors are shown as functions of p^2, the invariant mass squared of the virtual nucleon. The representation constructed in Section V is particularly useful because in it the nucleon propagator is free and the effects of nucleon dressing are encapsulated solely in the difference $G_{V,S}(p^2) - G^0_{V,S}(p^2)$ between the dressed and bare πNN form factors. For this reason we do not present results for the self-energy functions. It should be stressed that in virtue of the equivalence theorem [14,16], either of the two representations described in Section V lead to identical results for the phase shifts. The upper and lower panels contain pseudovector and pseudoscalar form factors, respectively (please note that Fig. (4) and Fig. (6) have different vertical scales). The dotted lines are the bare form factors, see Eq. (9), with the constants f_N and χ given in Tables II and III. The dashed lines are the form factors obtained after the first iteration step (essentially, a one-loop correction to the bare vertex) and the solid lines are the fully dressed form factors. A comparison of the solid and dashed lines exhibits a non-perturbative aspect of the dressing in the sense that it goes beyond an inclusion of few loop corrections. It can be seen that the ratio of pseudoscalar and pseudovector form factors remains small if the nucleon is not far off the mass-shell. The dash-dotted curves correspond to the form factors in the representation where the nucleon self-energy has not been eliminated. We see that the dressed πNN vertex may depend significantly on the representation chosen.

Comparing the form factors in Fig. (4) with those in Fig. (6), we conclude that the converged solution depends strongly on the width of the bare form factor. However, independent of this width, the dressing causes considerable softening of the form factor at higher invariant masses. The results shown in Figs. (3) and (5) suggest that it is possible to obtain a reasonable description of phase shifts up to pion laboratory energies of about 400 MeV starting from bare form factors with rather different widths.

In this model only one resonance of the πN scattering, the Δ, was included. The lack of other resonances becomes especially conspicuous at higher energies. In fact, in the calculations of phase shifts we also included the Roper resonance, though it is not taken into account in the dressing. This improved the calculated P11 phase shift at energies of about 300 MeV and higher, with a negligible effect on the other phase shifts. In principle, the Roper can be easily included in the dressing, as well as other important degrees of freedom (for example, the S11-resonance). In the present version of the model we have forgone doing so, limiting ourselves to the lowest lying Δ-resonance only.

VII. CONSLUSIONS

We have presented a model in which considerations of unitarity and anlyticity (causality) are implemented in the K-matrix approach to pion-nucleon scattering. The principal ingredient of the model is the dressing procedure, formulated in terms of half-off-shell vertices and propagators, the building blocks of the K-matrix. Analyticity properties are exploited through the use of dispersion relations to obtain the principal value parts of loop integrals required for the unitarization of the scattering amplitude.

The five parameters of the model are constrained rather much by the requirement of convergence of the dressing procedure. By the same token, there is an implicit interdependence of the parameters. This means that a comparison with experiment is an important test for this approach. We showed that a good overall description of phase shifts in pion-nucleon scattering can be achieved at the energies exceeding the scale due to the degrees of freedom explicit in the model. This suggests that the developed dressing procedure provides a physically reasonable method for studying higher-order correction to the (unitarized) Born approximation traditionally adopted in the K-matrix approach.

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TABLES

TABLE I. Physical masses of particles included in the model. All the masses were fixed in the calculations of the πN scattering phase phifts.

Particle	N	π	Δ	ρ	σ
Mass, GeV	0.938	0.138	1.232	0.770	0.760

TABLE II. Bare πNN coupling constant and field and mass renormalization constants obtained in calculations (I) and (II).

Calculation	f_N	Z_2	Z_2^Δ	Z_ρ	Z_σ	δM (GeV)	δM_Δ (GeV)	δm_ρ^2 (GeV^2)	δm_σ^2 (GeV^2)
(I)	11.04	0.77	1.10	1.17	1.05	-0.11	-0.07	-0.09	-0.65
(II)	10.80	0.60	1.09	1.17	1.05	-0.28	-0.08	-0.09	-0.65

TABLE III. Parameters of the model used in calculations (I) and (II). Parameters in the last five columns only were varied in the calculations of the πN scattering phase phifts.

Calculation	Λ_N^2 (GeV^2)	Λ^2 (GeV^2)	$g_{\pi N\Delta}$	$g_{\rho\pi\pi}$	$g_{\sigma\pi\pi}$	χ	$g_{\rho NN}$	κ_ρ	$g_{\sigma NN}$	$\widetilde{m}_\Delta^2$ (GeV^2)
(I)	2	1	0.248	6.07	1.88	0.055	7.83	0.54	19.0	1.04
(II)	3	1	0.248	6.07	1.88	0.055	8.03	-1.43	18.5	1.06

FIGURES

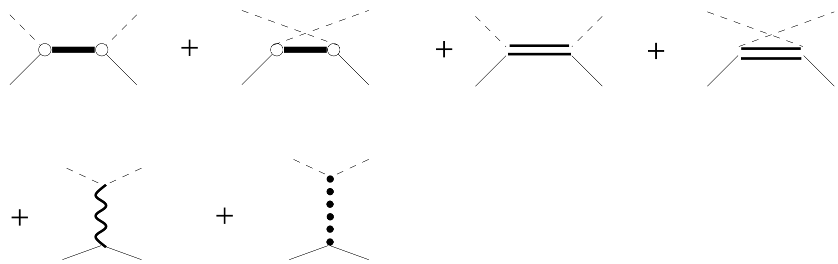

FIG. 1. Diagrams included in the calculation of the K-matrix. The solid lines are nucleons, the dashed lines pions, the solid double-lines Δs; the wavy and dotted lines represent the ρ- and σ-mesons, respectively. The intermediate propagators are dressed, as indicated by the thicker lines. The circle represents the dressed πNN vertex.

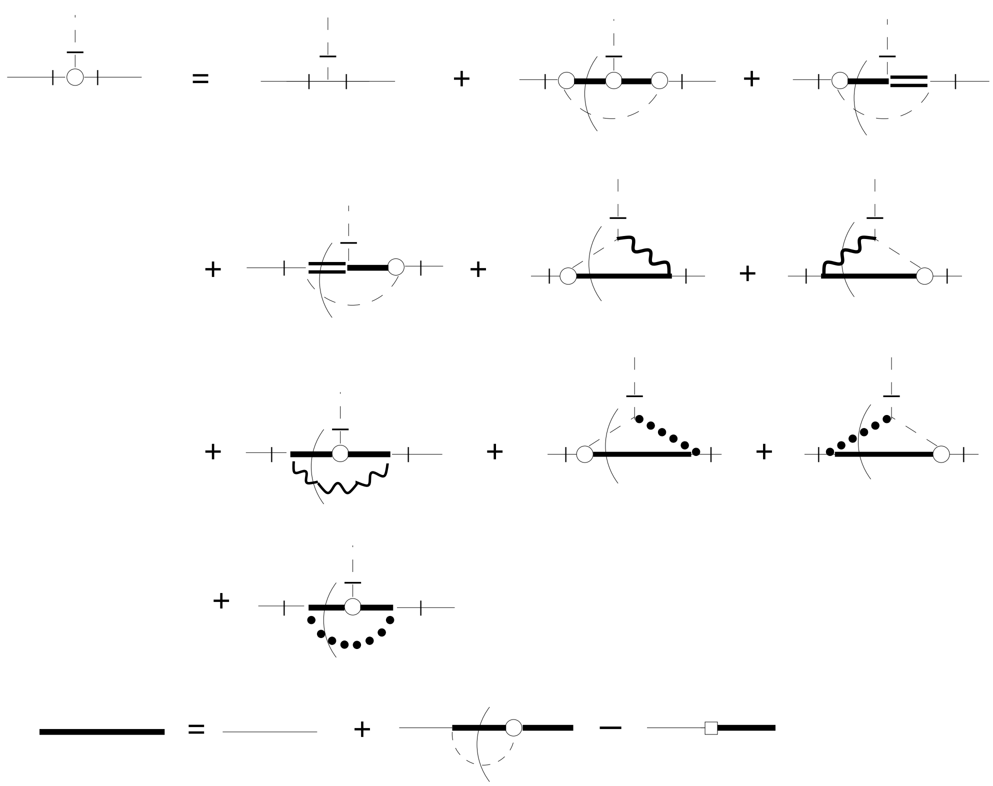

FIG. 2. Diagrammatic presentation of the dressing procedure for the half-off-shell πNN vertex and nucleon propagator. The notation is the same as in Fig. (1). In addition, the curved lines represent the cuts applied to calculate the pole contributions of the diagrams. The external lines are stripped away, as indicated by the dashes. The square in the second equation is the counter-term contribution to the nucleon self-energy.

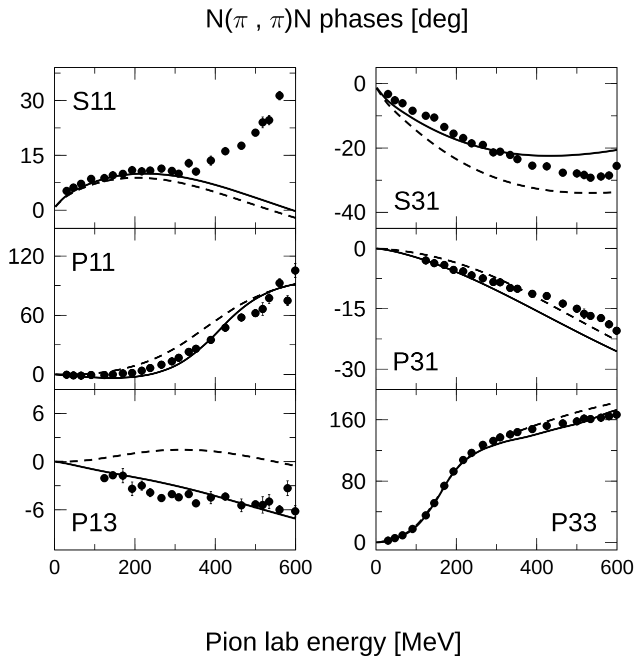

FIG. 3. Pion-nucleon phase shifts from calculation (I). The drawn curves are obtained in the full calculation. The dashed curves represent the calculation with the bare form factors and free propagators. The data are from [27].

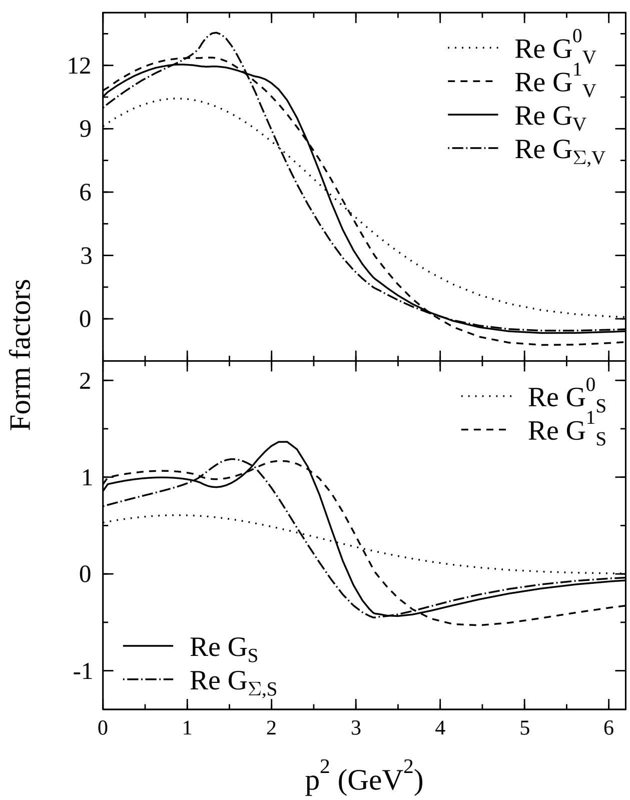

FIG. 4. Pion-nucleon form factors from calculation (I). The different curves are explained in the text.

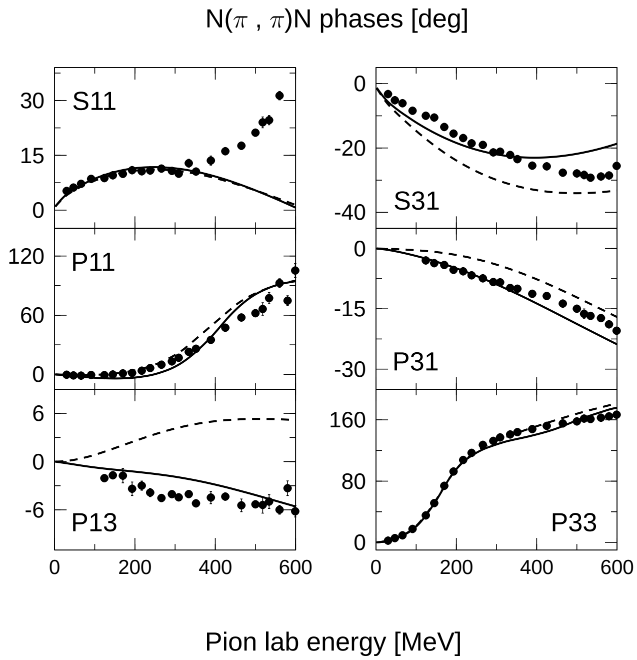

FIG. 5. Same as in Fig. (3), but for calculation (II).

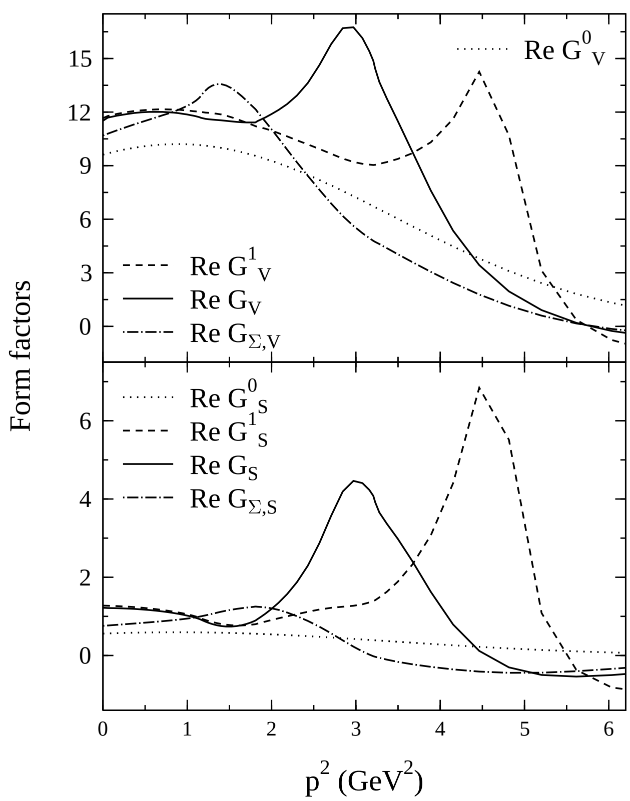

FIG. 6. Same as in Fig. (4), but for calculation (II).