Cooling of Particle Beams in Storage Rings

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Abstract

Old and new cooling methods are discussed in reference to $e^{\pm}$, ion and $\mu^{\pm}$ beams.

Contents

1 Introduction

Different cooling methods were suggested to decrease the emittances of charged particle beams in storage rings. Among them methods based on synchrotron radiation damping [1]-[4], electron cooling [5], laser cooling in traps [6] and in storage rings [7], [8, 9], ionization cooling [3], [10]-[13], cooing of ions through the inelastic intrabeam scattering [15], [16], [17], and stochastic cooling [14]. The majority of cooling methods are based on a friction of particles in external electromagnetic fields or in media when the Liouville's theorem does not work. Only stochastic method of cooling is not based on a friction. It consists in the individual observation of particles and the action of external control fields introduced in the storage ring for the correction of particle trajectories.

The friction and corresponding energy losses are determined by the next processes:

1) the spontaneous incoherent emission of the electromagnetic radiation in external fields produced by bending magnets, undulators/wigglers, laser beams etc.

2) ionization and excitation of atoms of a target at rest installed on the orbit of the storage ring,

3) the transfer of the kinetic energy from particles of a being cooled beam to particles of a co-propagating cold beam of e^-, e^+ or ions in the process of the elastic scattering [13],

4) excitation of being cooled ions and emission of photons by these ions through the inelastic intrabeam scattering [15], [16], [17],

5) the $e^{\pm}$ pair production by photons of a laser beam in fields of being cooled ions [18].

A friction originating from a media or in the process of emission (scattering) of photons by charged particles in external fields leads, under definite conditions, to a damping of amplitudes of both betatron and phase oscillations of these particles when they are captured in buckets of storage rings, i.e. there occurs a three-dimensional cooling of particle beams[1]. In this case particles of a beam loose theirs momentum. At that the friction force is parallel to the particle velocity, and therefore the momentum losses include both the transverse and longitudinal ones. Longitudinal momentum losses are compensated by a radio frequency accelerating system of the storage ring. Meanwhile the longitudinal momentum of a particle tends to a certain equilibrium. This is because the rate of the momentum loss of the particle is higher/less than the equilibrium value when this momentum is higher/less than the equilibrium value. The transverse vertical momentum of particles disappears irreversibly. Such a way the compression of phase-space density for a given ensemble of particles takes place.

The transverse radial and longitudinal particle oscillations are dispersion coupled through energy losses [4][2]. Their damping rates can be corrected or redistributed by insertion devices[3] placed at straight sections of storage rings to introduce an additional friction of particles, and by a correction of the ring lattice parameters. The damping rates of both transverse and longitudinal oscillations can be redistributed by coupling transverse and longitudinal particle oscillations near betatron and synchro-betatron resonances [3].

[1]This conclusion is valid for all possible types of friction processes and was emphasized in some pioneer papers on synchrotron radiation and ionizing damping/cooling (see, e.g. [3, 11]).

[2]Owing to this coupling radial betatron oscillations can have negative damping rate.

[3]Undulators, laser beams, wedge shaped material targets having in general case non-homogeneous field strengths or density in the radial direction at the position of being cooled particle beam .

In the case of emission of particles in a laser wave the radiation friction force appears together with the laser wave pressure force. Friction force is determined by self-fields of the accelerated particles and the pressure force is determined by the electromagnetic fields of the laser wave (average value of the vector product of the particle velocity and the magnetic field strength of the laser wave which is not equal to zero when the self-force is taken into account) [19]. The total averaged force can be directed both in the same direction, or in the transverse and the opposite directions relative to the average particle velocity. It depends on the direction of the particle velocity relative to the wave propagation. We will consider the relativistic case $\gamma \gg 1$ and conditions corresponding to the interaction angle between vectors of the particle velocity and the direction of the laser beam propagation $\theta_{int} \gg 1/\gamma$, where $\gamma = \varepsilon/Mc^2$ is the relativistic factor of the particle, M particle rest mass, and ε the energy of the particle[4]. In this case both the radiation friction force and the pressure force are directed opposite to the particle velocity.

The process of cooling of particle beams based on friction forces has a classical nature and can be described in the framework of classical electrodynamics. However, such data as atomic and nuclear levels, oscillator strengths, degeneracy parameter, and so on, which determine the corresponding laser wavelengths, cross-sections of particle interactions and friction forces should be used from quantum mechanics. At the same time, the excitation of longitudinal and transverse oscillations of particles in storage rings has a quantum nature. In this case the quasi-classical description of excitation of these oscillations can be used [4].

In the ordinary three-dimensional cooling, the particle beams are cooled under conditions when all particles interact with the external fields or media independent of their energy and amplitudes of betatron oscillations. Insertion devices/targets in this case (wigglers, laser beams, material media) overlap the particle beam and are motionless. In this case the difference in the rates of energy losses of particles of the beam having maximum and minimum energies is not high. That is why the cooling time of the particle beam determined by this difference is high and equal to the time interval, at which particle energy losses in the target are equal to about the two-fold initial energy of the particle[5]. At that the particle energy must be recovered by the RF system of the storage ring.

Cooling of particle beams can be enhanced. In this case the damping time of the particle beams in the longitudinal phase space can be reduced essentially (in the energy of particle over the energy spread of the particle beam ratio) if we will use selective interactions of particles of the beam and targets. For this purpose we have to choose such targets which interact with particles having definite energies or amplitudes of betatron oscillations and do not interact with another particles of the same beam. For example, target can interact with particles of the energy higher then minimum energy of the beam and do not interact with particles of minimum and lesser energy. In this case the rate of the energy loss of particles is not increased but the difference in the rates of losses of particles of the beam having maximum and minimum energies will be increased essentially and all particles will be gathered at the minimum energy in a short time (equal to the time interval, at which a particle looses the energy equal to the initial energy spread of the particle beam).

In this paper some peculiarities of ordinary and enhanced cooling methods based on

[4]In the case $\theta_{int} < 1/\gamma$ the particles will be accelerated by the light pressure force.

[5] This conclusion is valid for damping times both in the longitudinal and transverse planes.

a friction are discussed in reference to electron, ion and muon storage rings. The main characteristics of cooling methods (damping time, equilibrium emittance of the being cooled beam) will be presented. Cooling of electron and ion beams in linear accelerators will be presented as well.

2 Three-dimensional radiative cooling of particle beams in storage rings by laser beams

In the method of the ordinary three-dimensional radiative laser cooling of particle beams in buckets of storage rings, a laser beam overlaps a particle beam at a part of its orbit. We shall consider a cooling configuration where the laser beam is colliding head-on with a particle beam in a dispersion-free straight section of a storage ring. Both the laser beam and the particle beam are focused to a waist at the center of the straight section of the storage ring. In limits of this region the particle beam is affected by a friction force through scattering of laser photons. Particles lose their energy mainly in the process of backward Compton or Rayleigh scattering of laser photons. The accelerating fields of the radiofrequency systems of storage rings compensate the radiative losses of the particle energy. We assume that the incident laser beam has a uniform spectral intensity $I_\omega = dI_L/d\omega = I_L/\Delta\omega_L$ in the frequency interval $\Delta\omega_L$ centered around ω_L, where I_L is the total intensity (power per unit area).

In the case of an ion cooling, the electronic transitions of the not fully stripped ions or nuclear transitions and broadband laser beam have to be used. If the ion beam has an angular spread $\Delta\psi$, around $\psi = 0$, and a relative energy spread $\Delta\gamma$, around average $\overline{\gamma}$, the full bandwidth required for the incoming laser to interact with all ions simultaneously (to shorten the damping time of radial betatron oscillations) is $(\Delta\omega/\omega)_L = (\Delta\psi)^2/4 + \Delta\gamma/\overline{\gamma}$, where $\omega_L = \omega_{tr}/\overline{\gamma}(1+\overline{\beta_z})$; ω_{tr}, the resonant transition energy in the rest frame of the ion; $\beta_z = v_z/c$; and v_z, the longitudinal component of the vector of the particle velocity. In the case of radiative cooling of electron or fully stripped ion beams (Compton scattering) a monochromatic laser beam can be used.

The physics of damping in this method is similar to a synchrotron radiation damping originating from a particle emission in bending magnets of storage rings. The difference is in the appearance of other regions, where the emission of photons takes place, lattice parameters of these regions, and in spectral distributions of the emitted (scattered) photons.

Equilibrium emittances of particle beams are determined by a product of damping times and the rates of excitation of longitudinal or transverse oscillations of particles in the storage rings. The rate of excitation of particle oscillations is determined by hardness and power of the emitted radiation and by the lattice features of the storage ring such as its global parameter "momentum compaction factor α" and its local parameter "dispersion function" in the regions where the particle emits the radiation [4]. By analogy with synchrotron radiation damping, in order to shorten a bunch length in a storage ring, one should reduce $\alpha \to 0$ by manipulating with the ring optics [20, 21]. To shorten the transverse radial emittance one should use long dispersion-free straight sections filled with strong wigglers or laser beams (to produce fast damping without additional excitation of betatron oscillations). In the case of cooling of electron beams by lasers, the lattice of the storage ring must have large-radius arcs with strongly focused FODO to produce low quanta excitation by synchrotron radiation in bending magnets of the ring [22]-[24].

2.1 Three-dimensional radiative cooling of ion beams in storage rings by broadband lasers

In the three-dimensional laser cooling, the ion beams are cooled under conditions of Rayleigh scattering of laser photons when all ions interact with the homogeneous laser beam independent of theirs energy and position [25]-[28]. In this case, the average cross-section of the photo-ion interactions, $\overline{\sigma} = \pi f_{12} r_e \lambda_{tr} (\omega/\Delta\omega)_L$, is larger than the Compton (Thompson) cross-section, $\sigma_T \simeq 8\pi r_e^2/3 \simeq 6.65 \cdot 10^{-25}$ cm^2, by about a factor of $(\lambda_{tr}/r_e)(\omega/\Delta\omega)_L$, which is large, about $10^6 - 10^9$ for many cases of the practical interest. In the previous expressions, value f_{12} is the oscillator strength, $r_e = e^2/mc^2$ the classical electron radius, $\lambda_{tr} = 2\pi c/\omega_{tr}$.

Assuming that photo-ion interaction takes place in dispersion-free straight section, the damping time of horizontal betatron oscillation τ_x is the same as the vertical oscillation τ_y, because a variation in the radiated energy due to a variation in the orbit vanishes. The damping time of amplitudes of betatron and phase (τ_ϵ) oscillations of ions is

$$\tau_x = \tau_y = \frac{\tau_\epsilon}{(1+D)} = \frac{2\varepsilon}{P}, \tag{2.1}$$

where $P = f \Delta N_{int} \varepsilon_{loss}$ is the average power of the electromagnetic radiation emitted (scattered) by the ion; f, the frequency of the ion beam revolution in the storage ring, $\Delta N_{int} = (1+\beta) I_{sat} l_{int} \overline{\sigma} D / c\hbar\omega_L (1+D)$, the number of ion interactions with the laser beam photons per one ion-laser beam collision, l_{int} the length of the interaction region of the laser and ion beams; $D = I_L/I_{sat}$, the saturation parameter; $I_{sat} = [g_1/4(g_1+g_2)](\hbar\omega_{tr}^4/\pi^2 c^2 \gamma\overline{\gamma})(\Delta\omega/\omega)_L$, the saturation intensity, $g_1(g_2)$, the degeneracy factor of the state 1(2); $\varepsilon_{loss} = \hbar\omega_{tr}\gamma = (1+\beta)\hbar\omega_L\gamma\overline{\gamma}$ the average loss of the ion energy per one event of ion-photon interaction.

The expression τ_ϵ in Eq(2.1) is specific to the assumption that the spectral intensity of the laser beam $I_\omega(\omega, x, y)$ is constant inside its bandwidth and inside the area of the laser beam occupied by the being cooled ion beam[6]. Moreover, we assume that the length of the ion decay $l_{dec} = c^2 g_2 \beta\gamma / 2 g_1 f_{12} r_e \omega_{tr}^2 = g_2 \beta\gamma \lambda_{tr}^2 / 8\pi^2 g_1 f_{12} r_e$ is much less than the length of the dispersion-free straight section [25]-[28]. The length of the ion decay is determined by the spontaneous decay time $\tau_{sp} = 1/\Gamma_{21}$, where $\Gamma_{21} = 2 g_1 f_{12} r_e \omega_{tr}^2 / g_2 c$ is the probability of the spontaneous photon emission of the excited ion or the natural linewidth $\Delta\omega_{nat}$. Usually the relative natural linewidth $(\Delta\omega/\omega)_{nat} = 4\pi f_{12}(g_1/g_2)(r_e/\lambda_{tr})$ is less than the line width of a laser beam $(\Delta\omega/\omega)_L$ necessary for a three-dimensional ion cooling by broadband laser beams, and is determined by the energy and angular spreads of ion beams. Otherwise the monochromatic laser beams can be effectively used for the same purpose.

[6]The presence of a derivative $\partial I_\omega/\partial\omega$ will lead to another value τ_ϵ [27]. The longitudinal-radial coupling arising in non-zero dispersion straight sections of the storage rings leads to a redistribution of the longitudinal and radial damping times when the radial gradient of the laser beam intensity $\partial I_\omega/\partial x$ is introduced [25, c]. Experiments [17] confirm this observation. The same idea is in the three-dimensional scheme of the ionizing cooling of muon beams through dispersion coupling in the wedge-shaped material target. It was discussed in the paper of D.V.Neuffer [29].

The quantum nature of the laser-photon scattering provides excitation of betatron and phase oscillations of ions. The calculation of equilibrium amplitudes is similar to the case of ordinary electron storage rings, except that the spectral-angular probability distribution of the scattered photons here is given by that of the undulator radiation. We have found that the relative r.m.s. energy spread of the ion beam at equilibrium is given by [27]

$$\frac{\sigma_\varepsilon}{\varepsilon} = \sqrt{1.4(1+D)\hbar\omega_{tr}/Mc^2}. \tag{2.2}$$

In the present case, where the interaction takes place in a dispersion-free straight section, the excitation of both the horizontal and vertical betatron motions is due to the fact that the propagation direction of emitted photons is not exactly parallel to the vector of the ion momentum. The r.m.s. equilibrium horizontal ion beam emittance is found to be

$$\epsilon_x = \frac{3}{20}\frac{\hbar\omega_{tr}}{\gamma^2 Mc^2} < \beta_x > . \tag{2.3}$$

In (2.3), $< \beta_x >$ is the average horizontal beta function in the interaction region. The equilibrium vertical emittance is obtained by replacing $< \beta_x >$ by $< \beta_y >$. The r.m.s. transverse size of the ion beam at the waist $\sigma_x = \sqrt{\beta_x \cdot \epsilon_x}$.

In practice the three-dimensional method of laser cooling of ion beams have to realize after one-dimensional one (see Section 3). This will permit to cool the ion beam in the longitudinal plane for a short time using low power monochromatic laser beam in the first case and then to shorten the bandwidth and power of the laser beam in the second one. After that the one dimensional cooling can be used again to decrease the longitudinal emittance of the being cooled ion beam[7].

The three-dimensional laser cooling of ion beams can be realized by a monochromatic laser beam with accelerating fields of radio frequency cavities as well. The nature of the transverse cooling by the monochromatic laser beam in the three-dimensional method of cooling does not differ from the case of the broadband laser cooling. A difference is in conditions of interaction of ion and laser beams. In the case of a broadband laser beam all ions interact with the laser beam independent of their energies and amplitudes of betatron oscillations. In the case of a monochromatic laser beam every ion interacts with the laser beam only a part of time, when it passes the ion resonance energy in the process of phase oscillations in a bucket of a storage ring. That is why it has greater damping time at the same saturation parameter as in the case the broadband laser and the same cooling configuration (at that the power of the broadband laser is higher). Smaller value of the transverse damping time is the advantage of the broadband laser cooling in the case of a three-dimensional laser cooling of ion beams.

Experimentally, the version of longitudinal cooling of a bunched non-relativistic beam of $^{24}Mg^+$ ions (kinetic energy $\sim$ 100 keV) was observed first in the storage ring ASTRID [30]. The monochromatic laser beam co-propagated with the ion beam (conditions of the ion acceleration) at scanning of its frequency and using the accelerating system of the storage ring. At such cooling, some degree of the radiative transverse cooling could be observed.

[7] The energy spread of the being cooled beam is minimum in the case of one-dimensional laser cooling.

2.2 Three-dimensional radiative cooling of electron beams in storage rings by lasers

A three-dimensional cooling of electron and proton beams based on the backward Compton scattering of laser photons in the dispersion-free straight sections of the storage rings can be used [25, 27, 31, 32]. In this case we can use the expressions (2.1) - (2.3) if we replace the values $\overline{\sigma} \to \sigma_T$, $I_{sat}D = I_L$ and accept $D = 1$[8]. The method is identical to that suggested in papers [22]-[24] where magnetic wigglers were used instead of laser beams[9]. In this case the excitation of radial betatron oscillations will take place only through the emission of photons of synchrotron radiation from bending magnets of the ring where the dispersion function differ from zero. At the same time, the rate of the betatron oscillation damping will be determined by total power emitted both in the form of synchrotron radiation and Compton scattering of laser photons. The electromagnetic radiation emitted by electrons in the process of Compton scattering can lead to a significant shortening of damping time and hence equilibrium emittance of stored beams if the power of scattered radiation will be much higher than the power of the synchrotron radiation.

The method of the radiative cooling considered in this section is not optimal. The damping time and the emittance of particle beams can be shortened significantly by using a selective interaction of particles and laser beams. Below we shall consider one- and two-dimensional enhanced cooling schemes based on selective interactions of particle beams and targets.

3 Enhanced cooling of particle beams

In the case of ordinary longitudinal cooling all particles of a beam loose their energy in storage rings such a way that the ratio of a difference in rates of the energy loss of particles having maximum and minimum energies is small. The small difference in rates in comparison with the rates of the energy loss of particles leads to a small relative velocity of bringing closer of their energies. That is why the damping time of longitudinal emittance of the particle beam is high. It is determined by about two-fold loss of the initial particle energy in the external fields or targets under conditions of recovering of the energy in the radio frequency system of the storage ring.

Introduction of special damping magnets (wigglers) or wedge-shaped targets can lead to decreasing of damping time of particles in the longitudinal phase space. But such insertion devices simultaneously, according to Robinson's damping criterion, lead to increase of damping time of the particles in the transverse radial phase space or to untidamping [4]. This is

[8]The difference in the physics of cooling of electron and ion beams is in the dependence of the average hardness ε_{loss} and power P of scattered radiation on the energy γ. The damping time of the longitudinal oscillations and equilibrium energy spread of a particle beam are determined by the derivative $\partial P/\partial\gamma$. The powers emitted by electron and ion are: $P_e \sim \gamma^2$, $P_{ion} \sim \hbar\omega\gamma$ ($\omega_{tr} = const$, ion of higher energy scatter laser photons of lesser energy!). Because of this difference we will have correct result for the case of cooling of electron beams if we will choose $D = 1$ in (2.1), (2.2) [27].

[9] Electromagnetic waves can be considered as objects which belong to the type of undulators/wigglers [33]. Wigglers with high deflecting parameters can change the lattice parameters of the storage ring.

because of the Robinson's theorem is valid for the case of linear systems, when target overlap the particle beam completely and when longitudinal and transverse betatron oscillations are dispersion coupled.

The damping time of the particle beam in the longitudinal phase space can be reduced essentially (in the energy over the energy spread ratio) if we will use selective interactions of a particle beam and a target when only a part of particles of the beam interact with the target. In this case we have to choose such targets and conditions of interaction, when targets interact with particles having definite energies or amplitudes of betatron oscillation and do not interact with another particles of the same beam. For example, we can switch on interaction between target and particles of the beam having maximum energy first, extend in time continuously the interaction with particles of lesser energy and switch off the interaction at the energy equal to minimum energy of particles in the beam[10]. Then we can repeat this manipulation and gather all particles of the beam at minimal energy. Such a way we will realize the cooling of a particle beam in the longitudinal plane. In this case the rate of the energy loss by a particle must depend mainly on the position of its instantaneous orbit and must not depend on the deviation of the particle from this orbit because of the betatron oscillations. The damping time of the particle beam will be determined by the condition in which a particle having maximum energy will loose the energy equal to the energy spread of the being cooled beam.

The selectivity can be achieved in different ways. Among them note a method based on a resonant and energy threshold interaction ($e^{\pm}$ pair production, interaction of ions with a broadband laser beam having sharp frequency edges), a time dependent degree of overlapping in the radial direction of a being cooled particle beam and a moving target.

We can see a similarity between enhanced longitudinal and stochastic cooling of particle beams in the individual manipulations in turns with parts of particle beams which are differ by definite parameters (energies, amplitudes of betatron oscillations) in the first case and individual particles in the second case. However this is only similarity. The reason of the enhanced cooling is in a friction and the selective interaction of particles and transversely moving target (the degree of overlapping of the target and particle beam depends on time, particles of the energy lesser then minimum one does not interact with the target, the duration of interaction is proportional to the energy deviation of the particle from the minimum initial energy of particles in the beam). The enhancing effect is possible only in the case when the cooled beam occupies a small lauer in the phase space. It is proportional to the ratio of the minimum energy of the beam to its energy spread. The enhanced cooling would be absent when the particles of the beam where distributed in the range of energies from zero to maximum one. Below we will discuss some possible enhanced schemes of cooling.

3.1 One-dimensional laser cooling of ion beams in storage rings

A typical version of one-dimensional laser cooling of ion beams is based on the resonant interaction of unbunched ion beam and homogeneous monochromatic laser beam overlapping the ion one in the transverse direction. The initial frequency of the laser beam (photon

[10]At that the velocity of extending of the interaction with particles must be higher then the maximum velocity of shrinking of the instantaneous orbits of particles under interaction with the target.

energy) in this version of cooling is chosen so that photons interact first with the most high energy ions. Then the frequency is scanning (frequency chirp) in the high frequency direction, and ions of a lower energy begin to interact with the laser beam and decrease their energy. The scanning of the laser frequency is stopped when all ions are gathered at the minimum energy of ions in the beam.

The resonance ion energy $\varepsilon_r = Mc^2\gamma_r$, where $\gamma_r = [1 + (\omega_{tr}/\omega_L)^2]/2(\omega_{tr}/\omega_L)$ is the resonance relativistic factor. The initial resonance energy is above the equilibrium energy of the storage ring, and is higher than that corresponding to the maximum ion energy in the beam. Minimum ion energy must be higher than the equilibrium one as well (the accelerating RF electric field strength will be switched on after cooling). The rate of scanning must correspond to the condition $\dot{\varepsilon}_r = d\varepsilon_r/dt < P$, where P corresponds to the power of radiation scattered by an ion at resonance conditions.

The cooling time is equal to the time interval, at which the particle with maximum initial energy emits the electromagnetic radiation energy equal to the initial energy spread $\sigma_{\varepsilon\, in}$ of the ion beam, and the energy spread of the cooled beam is determined by either the average energy of the scattered photons ε_{loss} or the natural line width of the laser beam:

$$\tau_\epsilon = \frac{\sigma_{\varepsilon\, in}}{P}, \qquad\qquad \frac{\sigma_\varepsilon}{\varepsilon} = max\left(\frac{\varepsilon_{loss}}{Mc^2\gamma} = \frac{\hbar\omega_{tr}}{Mc^2}, \quad (\frac{\Delta\omega}{\omega})_{nat}\right). \tag{3.1}$$

This time is $\sim \varepsilon/\sigma_{\varepsilon\, in} \sim 10^3 \div 10^4$ times lower than that in the case of the three-dimensional cooling. This is the consequence of the selective resonance interaction of photon and ion beams in the one-dimensional method when ions of the energy higher than minimum initial energy of the beam interact with the laser beam.

The considered method is one of the possible one-dimensional ion cooling methods[11]. The first similar method was used for cooling of non-relativistic ion beams [8], [9], [34][12]. Relativistic version of such a method was developed in [36].

One-dimensional laser cooling of bunched ion beams by monochromatic laser beam is possible with accelerating fields of radiofrequency cavities (see section 2.1) [30]. The broad-band laser beam with a sharp low frequency edge can be used as well. In this case the edge frequency must have such a value that only ions with energies above the equilibrium one can be excited [37]. To gather the cooled ion beam into short bunches the radiofrequency accelerating cavity should be switched on adiabatically.

3.2 Two-dimensional cooling of particle beams in storage rings

One-dimensional laser cooling is highly efficient in the longitudinal direction, but rather difficult in the transverse direction unless a special longitudinal-radial coupling mechanism

[11]In another version of cooling, the laser frequency can be constant, and the acceleration of ions in the direction of given resonance energy can be produced by eddy electric fields of a linear induction accelerator or by phase displacement mechanism. The use of an induction accelerator is not efficient in high energy ($\varepsilon > 1$ Gev/nucleon) storage rings.

[12]Two laser beams of different frequencies, co- and counter-propagating with the ion beam, can be used in the non-relativistic and moderate relativistic case. In the coordinate system connected with the ion beam the frequencies of laser waves can be equal and form a standing wave at the resonance energy [35].

is applied (synchro-betatron resonance [3, 38, 25], dispersion coupling [10, 17]). Moreover this is the resonance method of cooling. It can be applied to cooling of only complicated ions. In papers [25]-[28] a three-dimensional radiative ion cooling method is proposed and considered above. Nevertheless, the quest for new more efficient enhanced three-dimensional cooling methods remains vital for cooling of electrons, protons, muons and both not fully stripped and fully stripped high current ion beams. Below we will discuss a two-dimensional method of cooling when alternative targets and selective interactions are used[13].

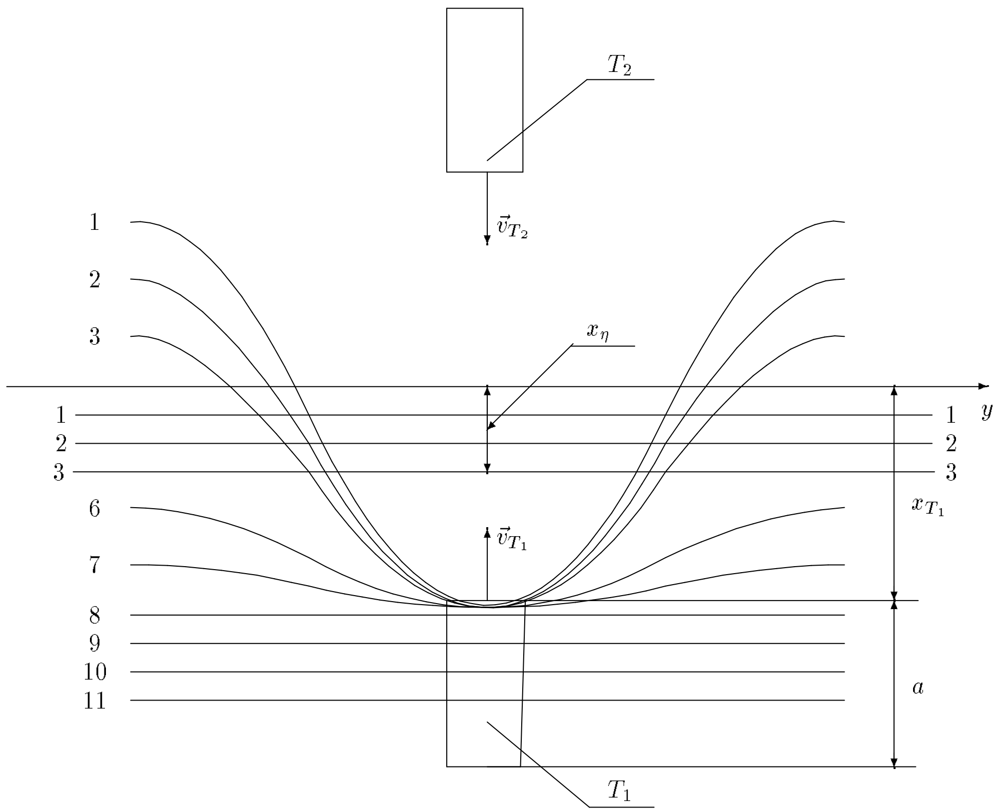

Figure 1: *The scheme of the two-dimensional ion cooling.The axis y is the equilibrium orbit of the storage ring, 1-1, 2-2 ... the location of the instantaneous ion orbit after 0,1,2 ... events of the ion energy loss, T_1 and T_2 are the targets moving in turn with the velocity $\vec{v}_{T_{1,2}}$ from outside to an equilibrium orbit.*

We consider the process of change of amplitudes of betatron oscillations A and positions of instantaneous orbits x_η in the process of the energy loss of particles in targets when the RF accelerating system of the storage ring is switched off [39]. The internal target T_1 at the first stage and external target T_2 of constant thickness at the second one will be moved in the transverse directions in turn (see Fig.1). They can be located in different positions along the way of the particle beam in the storage ring. Every stage of cooling can be used in combination with another schemes of enhanced cooling or with schemes of the emittance

[13]Coupling resonance of betatron oscillations permits cooling of a particle beam in three dimensions.

exchange in the longitudinal and transverse planes.

The velocity of a particle instantaneous orbit $\dot{x}_\eta$ depends on the distance $x_{T_{1,2}} - x_\eta$ between the target and the instantaneous orbit, and on the amplitude of particle oscillations. When the instantaneous orbit of a particle enters the target at the depth higher than the amplitude of the particle oscillations then its velocity $\dot{x}_{\eta\, in}$ is maximum one by the value and negative. The velocity $x_{\eta\, in}$ is given by the material target thickness or the intensity and the length of the interaction region of the laser target.

In the general case particles do not interact with the target every turn. That is why the velocity $\dot{x}_\eta = \dot{x}_{\eta\, in} \cdot W$, where $W < 1$ is the probability of a particle to cross the target. The probability W is determined by the ratio of the period of betatron oscillations of a particle to a part of the period which is determined by conditions: 1) the deviation of the particle from the instantaneous orbit is greater than the distance between the orbit and the target ($|x_{T_{1\,2}} - x_\eta| \leq |x_0^{'}| \leq A$), 2) the deviation is directed to the target. It can be presented in the form $W = \Delta\varphi_{1,\,2}/\pi$, where $\Delta\varphi_1 = \pi - arccos\xi_1$, $\Delta\varphi_2 = arccos\xi_2$, $\xi_{1,\,2} = (x_{T_{1,\,2}} - x_\eta)/A$, labels 1, 2 correspond to first and second targets used at first and second stages of cooling accordingly.

In the approximation when random processes of excitation of betatron oscillations of particles in a target of a storage ring can be neglected the behavior of the amplitudes of betatron oscillations of these particles is determined by the equation $\partial A^2/\partial x_\eta = -2 < x_0^{'} >$ or $\partial A/\partial x_\eta = - < x_0^{'} > /A$ (see Appendix A), where $< x_0^{'} >$ is the particle deviation from the instantaneous orbit averaged through the range of phases $2\Delta\varphi_{1,\,2}$ of betatron oscillations where the particle cross the target[14]. The value $< x_0^{'} >= \pm A sinc\,\Delta\varphi_{1\,2}$, where $sinc\Delta\varphi_{1\,2} = sin\Delta\varphi_{1\,2}/\Delta\varphi_{1\,2}$, signs + and − are related to the first and second stages of cooling. Thus the cooling processes at the first and second stages are determined by the system of equations

$$\frac{\partial A}{\partial x_\eta} = \pm sinc\Delta\varphi_{1,2}, \qquad \frac{\partial x_\eta}{\partial t} = \frac{\dot{x}_{\eta\, in}}{\pi}\Delta\varphi_{1,2}. \tag{3.2}$$

where $\Delta\varphi_1 = \pi - arccos\xi_1$, $\Delta\varphi_2 = arccos\xi_2$, $\xi_{1,2} = (x_{T_{1,\,2}} - x_\eta)/A$.

These equations are valid both for one and for a set of thin foils of the same total thickness, located at different azimuths and at definite depths of storage rings. This system of equations permits to investigate the main processes connected with cooling of particle beams.

From the Eqs. (3.2) and the expression $\partial A/\partial x_\eta = [\partial A/\partial t]/[\partial x_\eta/\partial t]$ it follows:

$$\frac{\partial A}{\partial t} = \frac{\dot{x}_{\eta\, in}}{\pi}\sin\Delta\varphi_2 = \frac{\dot{x}_{\eta\, in}}{\pi}\sqrt{1 - \xi_{1,\,2}^2}. \tag{3.3}$$

Let the initial instantaneous particle orbits be distributed in a region $\overline{x_\eta} \pm \sigma_{x,\varepsilon,0}$ and initial amplitudes of particle radial betatron oscillations A_0 be distributed in a region $\sigma_{x,b,0}$ relative to their instantaneous orbits, where $\overline{x_\eta}$ is the location of the middle instantaneous orbit of particles of the beam; $\sigma_{x,\varepsilon,0}$, the mean-root square deviation of instantaneous orbits from the middle one. The value $\sigma_{x,\varepsilon,0}$ is determined by the initial energy spread $\sigma_{\varepsilon,0}$. Suppose

[14]We suppose that particles loose their energy in the target. When a complicated ion is excited in the target, then we suppose that the length of decay of the excited ion is much less then the length of betatron oscillations of the particle.

that the initial spread of amplitudes of betatron oscillations of particles $\sigma_{x,b,0}$ is identical for all instantaneous orbits of the beam. The velocity of the instantaneous orbit in a target $\dot{x}_{\eta,in} < 0$, the velocity of the first target $v_{T_1} > 0$ and the velocity of the second target $v_{T_2} < 0$. Below we will use the relative velocities of targets $k_{1,2} = v_{T_{1,2}}/\dot{x}_{\eta,in}$. In our case $k_1 < 0$ and $k_2 > 0$.

First stage of cooling

At the first stage of the two-dimensional cooling, an internal target T_1 (laser beam or a material medium target) is located at the orbit region $(x_{T_1},\, x_{T_1} - a)$, where a is the target width (see Fig. 1). The internal edge of the target has the form of a flat sharp boundary. At the initial step the target overlaps only a part of the particle beam so that particles with largest initial amplitudes of betatron oscillations interact with the target. The interaction takes place only at the moment when the particle deviation caused by betatron oscillations is directed toward the target and has approximately the maximum initial amplitude $A_0 \simeq \sigma_{x,b,0}$.

In this case immediately after the interaction and loss of energy the position of a particle and the direction of its momentum remain the same, but the instantaneous orbit is displaced inward in the direction of the target[15]. The radial coordinate of the instantaneous orbit and the amplitude of betatron oscillations are decreased to the same value owing to the dispersion coupling. After every interaction the position of the instantaneous orbit will approach the target more and more and the amplitude of betatron oscillations is coming smaller until it will reach some small value when the instantaneous orbit will reach the edge of the target. Up to this moment the instantaneous orbit go in the direction to the target, but the particle depth of dipping do not move forward deeper into the target.

At the moment when the instantaneous orbit of the particle enter the target the amplitude of the particle betatron oscillations is much decreased. The instantaneous orbit will continue its movement in the target with constant velocity $\dot{x}_{\eta\, in}$ if the target is homogeneous one, has constant thickness and when its depth of dipping in the target is greater then the amplitude of particle betatron oscillations[16].

The particle beam has a set of amplitudes of betatron oscillations and instantaneous orbits. To cool the beam we must move the target in the direction to the particle beam or move instantaneous orbits of the particle beam in the direction to the target[17] at some velocity v_{T_1} until the target will reach the instantaneous orbit having maximum energies. After all particles of the beam will interact with the target, the latter must be removed or the particle beam must be returned to the initial position for a short time. After this, all particles will have small amplitudes of betatron oscillations and increased energy spread.

Here we will estimate the possibilities of the transverse cooling of particle beams at the first stage of cooling. Let the internal target be displaced in the direction of the being cooled

[15] We suppose that the friction leads to a decrease in the momentum by the value only and neglect scattering of the particle in the target for the time of the enhanced cooling

[16] The amplitude will be increased in proportion to the r.m.s. number of particle interactions with the target when random processes of deviation of energy losses from an average value exist. The amplitude increase here is slower than the amplitude reduction when it changed proportionally to the number of interactions. We will neglect this increase here.

[17] A kick, decreasing of the value of the magnetic field in bending magnets of the storage ring, a phase displacement or eddy electric fields can be used for this purpose.

beam step by step with a pitch $\Delta x_{T_1} = \sigma_{x,b,0}/M$, where $M \geq 2$ is the whole number. In this case at the first step of the first stage of cooling the target overlaps a pitch Δx_{T_1} of a being cooled beam. The instantaneous orbits of particles having maximum amplitudes of betatron oscillations after the moment of their entry into the target will be displaced in the direction of the target with a velocity $\dot{x}_\eta$ and the amplitudes of betatron oscillations will be reduced. When $x_\eta - x_{T_1} \simeq A_f \gg \Delta x_{T_1}$ then $\Delta\varphi_1 \simeq \sqrt{2\Delta x_{T_1}/A_f} \ll 1$, $sinc\,\Delta\varphi_1 \simeq 1 - \Delta x_{T_1}/3A_f \simeq 1$ and according to (3.2) the value $\partial A/\partial x_\eta \simeq 1$, $A(\xi_1) - (x_\eta - x_{T_1}) \simeq \Delta x_{T_1}$ up to the time when the instantaneous orbits approach the target to the distance $x_\eta \simeq x_{T_1}$ and when final amplitude of betatron oscillations $A_f \sim \Delta x_{T_1}$. When the orbits enter the target ($\Delta\varphi_1 = \pi/2$) the value dA/dx_η is decreased by $\pi/2$ times only. Then the orbits of particles will be deepened into the homogeneous target at the depth greater than their final amplitudes of betatron oscillations A_f, i.e. when $\Delta\varphi_1 = 0$, $\partial A/\partial x_\eta = 0$. After that the particles will interact with the target every turn without change of their amplitudes. This means that amplitudes of betatron oscillations in the first approximation will not tend to zero, but to some finite value $A_f \sim \Delta x_{T_1} \sim \sigma_{x\,b\,0}/M$.

More accurate numerical calculations of the dependencies A_f/A_0 and $A_f/\Delta x_{T_1}$ based on the equation (B.5) for the motionless target ($k_1 = 0$, $\xi_0 = -1 + \Delta x_{T_1}/A_0$.) are presented in the Table 1.

Table 1

$\Delta x_{T_1}/A_0$	0.000	0.010	0.050	0.100	0.500	1.00
A_f/A_0	0.000	0.024	0.082	0.137	0.442	0.707
$A_f/\Delta x_{T_1}$	—	2.40	1.640	1.37	0.884	0.707

The regime of the target movement step by step was considered for illustration of the cooling mechanism. In practice we can move the target uniformly with the velocity v_{T_1}. Detailed numerical calculations of the first stage of the transverse cooling in this case are presented in the Appendix B. According to these calculations $A_f/A_0 = \sqrt{|k_1|/(|k_1|+1)}$.

The time of the target movement in the first stage of cooling is $\Delta t_1 \simeq \sigma_{x,0}/v_{T_1}$, where $\sigma_{x,0} = \sigma_{x,b,0} + \sigma_{x,\varepsilon,0}$ is the total initial radial dimension of the particle beam. For this time the instantaneous orbits of particles of a beam having minimum energy and maximum amplitudes of betatron oscillations will pass the distance $\simeq |\dot{x}_{\eta\,in}|\Delta t_1$. Hence the final radial dimension of the beam determined by the final energy spread of the beam and the total radial dimension of the beam will be increased to the value

$$\sigma_{x,f} \simeq \sigma_{x,\varepsilon,f} = \sigma_{x,0}\frac{1}{|k_1|}. \tag{3.4}$$

Thus, at the first stage we have a high degree enhanced cooling of particle beams in the transverse plane (3.4) and more high degree of heating in the longitudinal one (B.6).

At the first stage of the two-dimensional transverse cooling of particle beams it is desirable to use the straight section with low-beta and high dispersion functions. In this case, less events of the photon emission are required to cool the beam in the transverse direction. This is because the change of amplitudes of betatron oscillations is the same as the change of positions of instantaneous orbits of the particle. Meanwhile, the spread of amplitudes of betatron oscillations is small and the step between positions of instantaneous orbits is high.

Second stage of cooling

At the second stage of cooling an external target T_2 is moving with a velocity $v_{T_2} < 0$ from outside of the working region of the storage ring in the direction of a being cooled particle beam (or instantaneous orbits of the particle beam are moved in the direction of the target). The instantaneous orbits of particles will go in the same direction with a velocity $\dot{x}_\eta \leq \dot{x}_{\eta,in}$ after the moment of their first interaction with the target. In this case, the target will start to interact first with particles having the largest amplitudes of betatron oscillations and the highest energies at some moment t_0. Then it will interact with particles of lesser amplitudes and energies. When the target will pass through the instantaneous orbit of particles having zero amplitudes and minimum initial energies then it must be removed to the initial position.

We will start from the estimation of the value of the transverse heating. According to (3.2) and (3.3) the value $\partial A/\partial t = (\dot{x}_{\eta\, in}/\pi)\sqrt{1-\xi_{1,2}^2} \leq |\dot{x}_{\eta\, in}|/\pi$ for the arbitrary time and the value $|\partial x_\eta/\partial t| \leq |\dot{x}_{\eta\, in}|/2$ when $x_{T_2} - x_\eta \geq 0$ that is up to the time $t_{1/2} \simeq t_0 + A_0/|v_{T_2} - \dot{x}_{\eta\, in}/2|$ when the target will reach the instantaneous orbit. For the time $t_{1/2} - t_0$ the target will pass the way $\Delta x_{T_2} < 2A_0k_2/(2k_2-1)$, the instantaneous orbit will pass the way $\Delta x_\eta < A_0/(2k_2-1)$ and the increase of the amplitude of betatron oscillations will be $\Delta A < 2A_0k_2/\pi(k_2-1)$. Specifically, when $v_{T_2} = 1.5\dot{x}_{\eta\, in}$ the values $\Delta x_{T_2} = 1.5A_0$, $\Delta x_\eta < A_0/2$, $\Delta A_1 < A_0/\pi$.

After the target passed the instantaneous orbit at a moment t_c then in a time interval $t_c - t_{1/2}$ the value $\dot{x}_\eta$ is increased to the value $\dot{x}_{\eta\, in}$, and $\partial A/\partial t$ is decreased to zero. At that the value $x_\eta < x_{T_2} + A$ and the amplitude of betatron oscillations will reach its final value A_f. Specifically, when $v_{T_2} = 1.5\dot{x}_{\eta\, in}$ then $t_c - t_0 \simeq 2(t_{1/2} - t_0)$, and at this step the increase of the amplitude of betatron oscillations will be $\Delta A_2 < A_0(1+1/\pi)(1/\pi) \simeq 0.42A_0$. Finally we will have the amplitude $A_f \simeq A_0 + \Delta A_1 + \Delta A_2 \simeq 1.74A_0$.

Now we will estimate the behavior of the energy spread of the beam in the second stage of cooling. Let, for the simplicity, that the initial spread of positions of instantaneous orbits $\sigma_{x,\varepsilon,0}$ is much greater then the spread of the amplitudes of betatron oscillations $\sigma_{x,b,0}$ of the beam. In this case high energy particles first and then particles with smaller energies will interact with the moving target until the target will reach the instantaneous orbit with the least energy. Then the target must be removed or the particle beam instantaneous orbits must be returned to the initial position for a short time.

The particles of the beam having maximum energy and zero amplitudes of betatron oscillations will interact with the target during the time $\Delta t_2^{'} \simeq \sigma_{x,\varepsilon,0}/v_{T_2}$. For this time the instantaneous orbits of particles will pass the distance $|\dot{x}_{\eta\, in}|\Delta t_2{}' = k_2^{-1}\sigma_{\varepsilon,b,0}$. At that particles having minimum energy and zero amplitudes of betatron oscillations will stay at rest. Hence it follows that the spread of instantaneous orbits of these particles will be compressed to the value $\sigma_{x,\varepsilon,f} \simeq \sigma_{x,\varepsilon,0}(1-k_2^{-1})$. If we take into account the fact that the behavior of the instantaneous orbit depends on the initial amplitude of particle betatron oscillations then the total radial dimension of the beam can be presented in the form $\sigma_{x,\varepsilon,f} =$

$\sigma_{x,\varepsilon,0}(k_2-1)/k_2+\Delta\sigma_{x,\varepsilon}$, which, according to the numerical calculations produced in the Appendix B, can be represented in the form ($t>t_0^{'}$)

$$\sigma_{x,\varepsilon,f}\le\begin{cases}[\frac{k_2-1}{k_2}\frac{\sigma_{x,0}-A_{T_2}}{\sigma_{x,b,0}}-\xi_{2,st}\cdot D_{2,tr}+0.28]\sigma_{x,b,0}, & \sigma_{x,0}\le A_{T_2}<l_c;\\ [\frac{k_2-1}{k_2}\frac{\sigma_{x,0}}{\sigma_{x,b,0}}+\sqrt{\frac{k_2}{k_2-1}}-\pi(k_2-1)\psi(k_2,\xi_{2,c})+0.28]\sigma_{x,b,0}, & A_{T_2}>l_c,\sigma_{x,0},\end{cases}\qquad(3.5)$$

where A_{T_2} is the amplitude of displacement of the second target; $\xi_{2,st}$, the parameter ξ_2 at the moment of the second target stop; $D_{2,tr}=A_f/A_0$, the coefficient of the decompression of the amplitudes of betatron oscillations of particles at the second stage of cooling; $l_c=\pi k_2\psi(k_2,\xi_{2,c})A_0$, the way of movement of the second target, for which the instantaneous orbits of particles having maximum initial amplitudes of betatron oscillations will be deepened into the target on the depth higher then their final amplitudes of betatron oscillations; the symbol $\le$ is because of we have got the maximum value 0.28 in (3.5) for the value $\Delta\sigma_{x,\varepsilon,1}$.

Notice that the Eq. (3.2) does not take into account that the target pass of a finite distance per one turn $\delta x_{T_2}=|v_{T_2}|\cdot T$, where $T=1/f$ is the period of the particle revolution around its orbit in the storage ring. When

$$\delta x_{T_2}>\sigma_{x,b,0},\qquad(3.6)$$

$|v_{T_2}|\ge|\dot{x}_{\eta\,in}|$ then all instantaneous orbits of the particle beam can enter the target at the distance $x_\eta-x_T>\sigma_{x,b,0}$, that is, all at once under conditions $\partial A/\partial t=0$ ($\varphi_2=\pi$). In this case there will not be any heating process in the transverse plane. This case can be realized easier if we do a high-degree cooling of the particle beam for the first stage of cooling, and when the target is installed at the straight section with low β - function and high dispersion function at the second stage of cooling.

We considered the example of the transverse heating and longitudinal cooling at the second stage for illustration. Detailed numerical calculations of the amplitude of betatron oscillations increase and damping of the energy spread of the beam at the second stage of cooling are presented in the Appendix B.

The energy spread of the particle beam being cooled in the storage ring

$$\frac{\sigma_\varepsilon}{\varepsilon}>max\left(\frac{\varepsilon_{loss}}{Mc^2\gamma},\quad\frac{\delta r}{\overline{R}},\quad(\frac{\Delta\omega}{\omega})_{nat}\right),\qquad(3.7)$$

where δr is the length of the slope of the target edge; $\overline{R}=cT$ the average radius of the ring; ε_{loss}, the average loss of the particle energy per one event of particle-photon interaction. The value $\delta r/\overline{R}$ influence on the energy spread mainly at the second stage of cooling.

The damping times of the particle beam at the first and second stages of cooling in the transverse and longitudinal planes are

$$\tau_x=\frac{\sigma_{eq}}{k_1P},\qquad\tau_s=\frac{\sigma_{\varepsilon\,in}}{P},\qquad(3.8)$$

where in the smooth approximation $\sigma_{eq}=\varepsilon\sigma_{x,b,0}/\alpha\overline{R}$ the energy interval corresponding to the energy spread of the particle beam whose instantaneous orbits are distributed through the interval of radii $\sigma_{x,b,0}$; $k_1 \sim 0.1 \div 0.2$; α, the momentum compaction function, P is the power of the particle energy loss.

The transverse emittances of beams are proportional to theirs damping times. It means that the emittance of a beam in the plane "i" in the two-dimensional method of cooling $\epsilon_i^{(2)}$ is equal to the emittance corresponding to a three-dimensional one $\epsilon_i^{(3)}$ multiplied by the ratio of their damping times

$$\epsilon_i^{(2)} = \epsilon_i^{(3)} \frac{\tau_i^{(2)}}{\tau_i^{(3)}}. \tag{3.9}$$

The described process of transverse cooling is based on particle interactions with external and internal targets. Similar interactions were described in 1956 by O'Neil [10]. However, the targets in that case were motionless and could not lead to cooling[18]. They could be used for injection and capture of only one portion of particles. For the purpose of the multi-cycle injection and storage of heavy particles O'Neil, in addition to targets, suggested the ordinary three-dimensional ionization cooling based on a thin hydrogen target jet situated in the working region of the storage ring.

Discussion

The dynamics of positions of instantaneous orbits and amplitudes of betatron oscillations of particles interacting with a target strongly depends on the target velocity when the instantaneous orbits are deepened into the target on the depth less then the amplitude of particle betatron oscillations (see (3.2)). Moreover, the moving target begin interact with particles of the beam located at different instantaneous orbits at different moments of time and that is why can compress or decompress the spread of these orbits. These features of interaction of moving target can be used for improving existing and adopting new schemes of enhanced three-dimensional cooling.

We have found (see (B.6), (3.4)), that at the first stage of cooling there is the significant decrease of amplitudes of betatron oscillations (transverse cooling) and, at the same time, a greater increase of the spread of instantaneous orbits (longitudinal heating). If the degree of transverse cooling (B.6) is defined by the coefficient of compression $C_{1,tr} = A_0/A_f = \sqrt{(1+|k_1|)/|k_1|}$ then, according to (3.4), the increase of the spread of the instantaneous orbits of the beam (decompression) will be $D_{1,l} \simeq C_{1,tr}^2$ times.

At the second stage of cooling there is a significant decrease of the spread of instantaneous orbits of particles defined by the compression coefficient $C_{2,l} = \sigma_{x,\varepsilon,0}/\sigma_{x,\varepsilon,f}$ and, at the same time, lesser value of increase of amplitudes of betatron oscillations (see (3.5), (B.8), Tables 8, 9). If the condition (3.6) is not fulfilled then the degree of the transverse heating (decompression) can be about the square root of the degree of the longitudinal cooling ($D_{2,tr} \simeq \sqrt{C_{2,l}}$), when $\xi_{2,st} \simeq 0$ (see Table 8,9) or when $\xi_{2,st} = -1$, $A_{T_2} \simeq \sigma_{x,\varepsilon,0} \gg \sigma_{x,b,0}$ (see Table 9 at $\xi_{2,st} = -1$, $A_{T_2} = 101\sigma_{x.b.0}$). In the last case the degree of selectivity of interaction of a target with instantaneous orbits is greater. It can be realized easier if we will locate

[18] Internal target could be rotated out of the medium plane only to prevent the particle beam losses.

the target in the straight section of the storage ring with a low-beta and high dispersion function. When the condition (3.6) is fulfilled then heating process can be neglected at all.

The successive application of two stages of the two-dimensional cooling in tern will lead to cooling of the particle beam in both degrees of freedom only in the case when the condition (3.6) is fulfilled.

When the interaction of the particle beam and the target has the resonance or threshold nature then we can use transverse cooling of particles at the first stage of the two dimensional method of cooling and then to use one dimensional method of longitudinal cooling at the second one (see section (3.2.1)).

At the second stage, contrary to the first one, the degree of longitudinal cooling can be much greater then the degree of heating in the transverse plane. That is why we can use the emittance exchange between longitudinal and transverse phase spaces (say using the synchro-betatron resonance) and such a way to have enhanced two-dimensional cooling of the particle beam based on the second stage of cooling only. In this case the first stage of cooling can be omitted and the second one can be repeated many times (see sections (3.2.2), (3.2.3)).

3.2.1 Enhanced laser cooling of ion beams in storage rings

In the two-dimensional method of ion cooling based on the selective resonant interaction, as the targets one can use two different laser beams. The first broadband laser beam must have sharp geometrical internal boundary. Metal screens can be used at the exit of the laser beams from an optical resonators to produce the extracted laser beams with sharp edges[19]. The second laser beam must overlap the ion beam as a whole, have sharp frequency edge or can be monochromatic one and must have scanning central frequency. In this case the ordinary one-dimensional resonance laser cooling will take place at the second stage (see Section 3.1). At that we can start from the second stage. Then the first stage and the second one must be repeated.

Example 1. The two-dimensional cooling of a hydrogen-like beam of ${}^{207}_{82}Pb^{+81}$ in the CERN LHC through the backward Rayleigh scattering of photons of two laser beam targets. The broadband laser beams overlap the ion beam, have sharp frequency edges, and scanning central frequencies.

The relevant parameters of LHC and the beam in LHC are: $2\pi\overline{R} = 27$ km, $f = 1.11 \cdot 10^4$ Hz, $\alpha = 2.94 \cdot 10^{-4}$, $< \beta_x >= 0.5m$, $\gamma = 3000$, $Mc^2\gamma = 575$ TeV, $\sigma_{\varepsilon,0}/\varepsilon = 2 \cdot 10^{-4}$ ($\sigma_{\varepsilon,0} = 1.15 \cdot 10^{11}$ eV), the value $\sigma_{x,b,,0} = \sigma_{x,\varepsilon,0} = 1.2 \cdot 10^{-2}$ cm.

The relevant characteristics of the hydrogen-like ($f_{12} = 0.416$, $g_1 = 1$, $g_2 = 3$) lead ions are: the transition between the $1S$ ground state and the $2P$ excited state of the particle corresponds to the value of the resonant transition energy $\hbar\omega_{tr} = 68.7$ keV, $\lambda_{tr} = 1.8 \cdot 10^{-9}$ cm, $(\Delta\omega/\omega)_{nat} = 2.72 \cdot 10^{-4}$.

The relevant parameters of a laser: the laser wavelength $\lambda_L = 4\pi c\gamma/\omega_{tr} = 1080\mathring{A}$, $\hbar\omega_L = 11.49$ eV, the bandwidth of the laser beam $(\Delta\omega/\omega)_L = 5 \cdot 10^{-4}$, the r.m.s. transverse laser beam size at its waist $\sigma_L = 1.52 \cdot 10^{-2}$ cm, the Rayleigh length $z_R = \pi\sigma_L^2/\lambda_L = 67.2$ cm, $l_{int} = 2z_R = 135$ cm, the power of the laser beam is $P_L = 400$ W.

[19] Production of laser beams with sharp edges in resonators is another problem which would be solved.

Under this condition the average energy of the scattered photons $< \hbar\omega^s >= \varepsilon_{loss} = \gamma\hbar\omega_{tr} = 206$ MeV, $I_{sat} = 4.94 \cdot 10^{12}$ W/cm^2, $I_L = 5.5 \cdot 10^5$ W/cm^2, $D = 1.1 \cdot 10^{-7} \ll 1$, $\overline{\sigma} = 1.32 \cdot 10^{-18}$ cm^2, $\Delta N_{int} = 3.52 \cdot 10^{-3}$, $P = 8.04 \cdot 10^9$ eV/s. According to (3.1), (3.7) (3.9) the longitudinal damping time $\tau_s \simeq 14.3$ sec., the transverse damping time $\tau_x|_{k1=0.1} \simeq 143$ sec., the total damping time $\tau = 157.3$ sec., the limiting relative energy spread $\sigma_\varepsilon/\varepsilon \simeq (\Delta\omega/\omega)_{nat} \simeq \sigma_{\varepsilon,0}/\varepsilon$, $l_{dec} \simeq 4.2 \cdot 10^{-3}$ cm. According to (2.1), (2.3), the damping time and the emittance of the ion beam in the case of three-dimensional ion cooling method are: $\tau_x = 1.43 \cdot 10^5 sec.$, $\varepsilon_x = 3 \cdot 10^{-15}$ m·rad[20]. It means that in the two-dimensional method of ion cooling, according to (3.9), $\varepsilon_x = 2.86 \cdot 10^{-16}$ m·rad.

Notice, that at the first stage of the resonance ion cooling the width of the frequency band can be small. It can correspond to the spread of the instantaneous orbits $\Delta x_\eta \simeq A_f$. In this case the amplitude of betatron oscillations of particles will be in time to be cooled before their instantaneous orbits will go out of the spread and will be stopped without interaction with the laser target. Heating of the particle beam in the transverse plane at the another side of the laser target will be absent.

The two-dimensional method of laser cooling of ion beams will work in the case when the RF system of the storage ring is switched on as well. In this case the scanning of the frequency of the accelerating fields can be used instead of moving targets. At that the laser targets T_1 and T_2 have to be switched on and off in turn.

3.2.2 Enhanced laser cooling of electron and proton beams

To produce the enhanced laser cooling of electron and proton beams based on the backward Compton scattering we are forced to use laser beams with sharp geometrical boundaries. We can not neglect synchrotron radiation of electron beams in the guiding magnetic fields of lattices of storage rings. That is why we are forced to coll such beams in the radio frequency buckets. Cooling of the particle beams in buckets is another problem which can be considered in a separate paper. Obviously this problem can be solved when only second stage of cooling is used under conditions of synchro-betatron resonance.

Here we would like to notice that when the spread of amplitudes of betatron oscillations is near to zero then we can take the parameter $k_2 = 1$. In this case the cooled beam according to (3.5) will be monochromatic. It means that we can add the second stage of cooling to the case of the three dimensional laser cooling of electron beams considered in [32] one time and then extract the beam with low both transverse and longitudinal emittances.

3.2.3 Enhanced ionization cooling of muon beams in storage rings

Muons have rather small lifetime ($\sim 2.2\mu sec$) in their rest frame. They can do about $3 \cdot 10^3$ turns only in the strong magnetic field ($\sim$ 10 T). That is why the muon beams require enhanced cooling. Muons have no nuclear interactions with the material medium of the target. That is why they have no problems with the inelastic scattering in the target.

The enhanced method of the two-dimensional cooling of particle beams in storage rings can be used in the case of muon cooling as well. Two material medium targets are to be

[20] The damping time of bunched ion beams can be done less at the same average power P [27].

placed in the inner and external sides of the working region of the storage ring. The use of thick targets can lead to high rate of the ionization energy losses of muons in the targets. The kick/bump can be used for the displacement of the instantaneous orbits forth and back with high velocity ($\sim \dot{x}_{\eta\, in}$) instead of the displacement of targets.

If the velocity of movement of the instantaneous orbit $\dot{x}_{T_2}$ by a kick in the direction of the external target is slightly greater then the velocity of movement of the orbit in the target ($k_2 = \dot{x}_{T_2}/\dot{x}_{\eta\, in} \simeq 1$) then during the second stage of longitudinal cooling the instantaneous orbits will be gathered at the inner orbit position in a damping time (3.1), (3.8), where P is the average power of the ionization energy loss. At this moment the kick must be switched off for a short time. In this case, the muon beams will be cooled down to high degree in the longitudinal plane. To avoid the heating of the beam in the transverse plane we must work under condition (3.6), that is $\dot{x}_{T_2} \cdot T > \sigma_{x,\, b,\, 0}$[21].

The enhanced transverse muon cooling will take place when the instantaneous orbit will be moved to the muon beam with the velocity $\dot{x}_{T_1} \simeq 0.1 \cdot x_{\eta\, in}$ (see section 3.1).

The two-stage cooling process can be repeated several times to increase the degree of cooling. To keep the position of instantaneous orbits of particles at central part of the working region of the storage ring after cooling stages the magnetic field of the storage ring can be decreased in time.

Another version of the two-dimensional muon cooling is based on using of the second stage of cooling only and the coupling of the longitudinal and transverse planes through the synchro-betatron resonance. In this case the first stage of cooling can be omitted.

3.2.4 Enhanced cooling of ion beams through the $e^{\pm}$ pair production

The three-dimensional method of laser cooling of ion beams based on the nonselective interaction of counterpropagating ion and photon beams through the $e^{\pm}$ pair production were considered in [18]. More effective two-dimensional method of cooling based on the threshold phenomena of the $e^{\pm}$ pair production in Coulomb fields of ions was considered in [39]. Below we will consider the latter case.

The cross-section of the electron-positron pair production has the form

$$\sigma|_{\hbar\omega - 2m_ec^2 \leq m_ec^2} \sim \frac{\pi}{12} Z^2 \alpha r_e^2 \left(\frac{\hbar\omega - 2m_ec^2}{m_ec^2}\right)^3,$$

$$\sigma|_{\hbar\omega \gg 2m_ec^2} \simeq \frac{28}{9} Z^2 \alpha r_e^2 [\ln \frac{2\hbar\omega}{m_ec^2} - \frac{109}{42} - f(\alpha Z)], \tag{3.10}$$

where Z is the atomic number of the ion, $f(\nu)/\nu^2 \simeq 1.203 - \nu^2$ [40].

The cross-section (3.10) has a threshold photon energy $\hbar\omega'_{thr} = 2m_ec^2$ in the ion rest frame or $\hbar\omega_{thr} = 2m_ec^2/(1+\beta_{thr})\gamma_{thr}$ in the laboratory reference frame, where the ion threshold relativistic factor $\gamma_{thr} = [1 + (\omega'_{thr}/\omega_L)^2]/2(\omega'_{thr}/\omega_L)$.

Below we will consider an example of the two-dimensional laser cooling of the lead ion beam through the $e^{\pm}$ pair production.

[21] We suppose that after acceleration the transverse and longitudinal emittances of muon beams are much less then the corresponding acceptances of the storage ring.

Example 2. The two-dimensional cooling of the fully stripped ion beam of lead ${}^{207}_{82}Pb^{+82}$ in the CERN LHC through the $e^{\pm}$ pair production.

The relevant parameters of LHC and the beam in LHC are the same as in the previous example: $2\pi\overline{R} = 27$ km, $f = 1.11 \cdot 10^4$ Hz, $\alpha = 2.94 \cdot 10^{-4}$, $\gamma = 3000$, $Mc^2\gamma = 575$ TeV, $\sigma_{\varepsilon\, in}/\varepsilon = 2 \cdot 10^{-4}$ ($\sigma_{\varepsilon\, in} = 1.15 \cdot 10^{11}$ eV), the value $\sigma_{\varepsilon b} = \sigma_{\varepsilon\, in}$, $\sigma_{x\, b} = \sigma_{x\, \varepsilon} = 1.2 \cdot 10^{-2}$ cm.

In this method of cooling $\hbar\omega'_{thr} = 2m_ec^2 = 1.02MeV$, $\lambda'_{thr} = 0.01217\mathring{A}$, $\hbar\omega_{thr} = 2m_ec^2/(1+\beta)\gamma = 170$ eV, $\lambda_{thr} = 73\mathring{A}$.

The relevant parameters of a laser: wavelength $\lambda_L = 36.5\mathring{A}$, $\hbar\omega_L = 340$ eV, the bandwidth $(\Delta\omega/\omega)_L < 10^{-4}$, the r.m.s. transverse size at a beam waist $\sigma_L = 1.52 \cdot 10^{-2}$ cm, the Rayleigh length $z_R = \pi\sigma_L^2/\lambda_L = 1989$ cm, $l_{int} = 2z_R = 3978$ cm; the power $P_L = 400$ W.

In this case the average energy loss per one event of pair production $\varepsilon_{loss} \simeq 2\gamma m_ec^2 = 3.06$ GeV, $I_L = 5.48 \cdot 10^5$ W/cm^2, $\sigma = 8.1 \cdot 10^{-24}$ cm^2, $\Delta N_{int} = 2.69 \cdot 10^{-9}$, $P = 6.43 \cdot 10^{10}$ eV/s., the longitudinal damping time $\tau_s \simeq 1.58 \cdot 10^5$ s., the damping time for the betatron oscillations $\sim$ 10 times greater. According to (3.7) the limiting relative energy spread is $\sigma_\varepsilon/\varepsilon \simeq 5.3 \cdot 10^{-6}$.

The average power of the X-ray laser in the considered example is rather high but it can be realized in future FELs [41]. The X-ray FELs are proposed to operate in the pulsed regime. In this case the damping time or/and the power of the laser can be decreased essentially (two-four orders) if we use ion beam gathered in short bunches separated by a long distances (duty cycle $\sim 10^2 \div 10^4$), and use interaction regions with smaller diameters of ion and photon beams.

4 Cooling of electron and ion beams in linear accelerators

Electron beams can be cooled in linear accelerators if external fields (undulators, electromagnetic waves) producing radiation friction forces will be distributed along the axes of these accelerators. The physics of cooling of electron beams under conditions of linear acceleration is similar to one in storage rings where external fields are created in the dispersion-free straight sections of the rings, and the synchrotron radiation in bending magnets of the rings can be neglected (see Section 2). The element of the irreversibility takes place in this case too. The electron momentum losses are parallel to the particle velocity and therefore include transverse and longitudinal momentum losses. The reacceleration of the electron beams in accelerating structures of linear accelerators restores the longitudinal momentum. The transverse emittance is reduced by $1/e$ with as little as 2ε of the total energy exchange. The expected emittances of cooled beams are small in both transverse directions.

First, the effect of undulator/wiggler radiation damping on the transverse beam emittance was studied by A.Ting and P.Sprangle for linear accelerators based on inverse free-electron lasers [42]. In [43], the same effect applied to the case of the radio frequency linear accelerators is considered. In [44] and later in [45], the case of linear acceleration was investigated, where a laser beam was used instead of a wiggler. General formulas in this cases are similar. Some peculiarities are in the hardness of the emitted radiation which determines the energy spread and transverse emittance of being cooled beams. The hardness of the backward scattered laser radiation is more high then undulator/wiggler radiation. That is why laser beams for damping can be used at small ($\sim 10^3 \div 10^4$ MeV) electron energies. A strong focusing of electron and laser beams at the interaction point is necessary in this case. Damping wigglers can be used at high energies ($10 \div 100$ GeV) and in the limits of more long distances along the axis of the accelerator (about some kilometers).

The analogies with the enhanced particle cooling in linear accelerators are possible as well when bending magnets will be used for dispersion separation of particles and selective cooling. Monochromatization of ion beams can be realized by broadband lasers with sharp frequency edge located at the exit of the linear accelerator.

5 Conclusion

The fundamental ideas of cooling of particle beams based on a friction were invented during the past five decades. First the synchrotron radiation damping/cooling was developed theoretically. The theory of the synchrotron radiation damping is used during designing of the $e^{\pm}$ synchrotrons and storage rings for the $e^{\pm}$ circular colliders and next generations of the synchrotron and undulator radiation sources. Activity in this field have led to the development of storage ring lattices (straight sections with high- and low-beta functions and zero momentum compaction factors, using of magnets with low magnetic fields and large bending radii). The necessity in increasing of luminosity of the $p, \overline{p}$ colliders have led to a development of electron and stochastic methods of cooling of heavy particles. The development of the idea of the inertial confinement fusion have led to a necessity of ion cooling of non-fully stripped ion beams through the intrabeam scattering [15, a]. The development of μ colliders stimulated development of schemes based on well-known methods of ionization friction. Laser cooling in gaps was naturally extended to the enhanced one-dimensional laser cooling in storage rings, to the enhanced three-dimensional cooling through a synchro-betatron resonance or through the longitudinal-transverse dispersion in storage rings.

In this review we have presented different methods of cooling of relativistic particle beams in a single particle approximation. We hope that the development and adoption of these methods will lead to the next generation of storage rings for colliders of different particles, new light sources in optical to X-ray and γ -ray regions [28], [32], ion fusion [15, a], sources of gravitational radiation in IR and more hard regions [46], and so on.

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Appendix A

In a smooth approximation, the movement of a particle relative to its instantaneous orbit position x_η is described by

$$x' = A\cos(\Omega t + \varphi). \tag{A.1}$$

where $x' = x - x_\eta$.

The amplitude of betatron oscillations of the particle $A_0 = \sqrt{x_0'^2 + \dot{x}_0'^2/\Omega^2}$, where x_0' is the particle deviation from the instantaneous orbit at the moment t_0 of change of the particle energy in a target (laser beam, material medium), $\dot{x}_0' = -A\Omega\sin(\Omega t + \varphi)$ the transverse velocity of the particle. After the interaction, the position of the instantaneous orbit will be changed at a value δx_η. The particle will continue its movement relative to a new orbit. Its deviation relative to the new orbit will be $x_0' - \delta x_\eta$, and the angle will not be changed. The new amplitude will be $A_1 = \sqrt{(x_0' - \delta x_\eta)^2 + \dot{x}_0'^2/\Omega^2}$. The change of the square of the amplitude

$$\delta(A)^2 = A_1^2 - A_0^2 = -2x_0'\delta x_\eta + (\delta x_\eta)^2. \tag{A.2}$$

When $|\delta x_\eta| \ll |x_0'| < A_0$, the second term in (A.2) can be neglected[22]. In this case the value $\delta(A)^2 \simeq -2x_0'\delta x_\eta$ and $\delta A = -(x_0'/A)\delta x_\eta$. The amplitude of betatron oscillations of the particle will be changed by the low $|\delta A| \simeq |\delta x_\eta|$ when $|x_0'| \simeq A$. It means that the particle will change its amplitude of oscillations proportional to the number of passages N of the particle through the target when the instantaneous orbit of the particle is at a distance of about their amplitude ($\sim A$) away from the target. This is the highest rate of increase in the amplitude of the particle oscillations.

When the target is located at the external side of the working region of the storage ring $x_{T_2} > 0$, the instantaneous orbit position $x_\eta < x_{T_2}$, the particle enter the target under conditions of deviations $x_0' > 0$, and when the energy loss of the particle leads to the decrease of its instantaneous orbit position ($\partial x_\eta/\partial\varepsilon > 0$) then the amplitude of radial betatron oscillations of the particle will be increased (heating conditions). In the opposite case when the target is located at the inner side of the working region of the storage ring $x_{T_1} < 0$, $x_\eta > x_{T_1}$, and $x_0' < 0$ the amplitude of radial betatron oscillations of the particle will be decreased (cooling conditions).

The term $-2x_0'\delta x_\eta$ in (A.2) determines the classical damping (antidamping) processes in particle beams of storage rings. The value $\delta x_\eta = D_x\Delta p/p$, where D_x is the local dispersion function [4], $p = Mc\beta\gamma$ the momentum of the particle. This means that the scheme works when the dispersion function $D_x \neq 0$. The greater D_x the greater the rate of cooling.

[22] In the framework of classical electrodynamics particles loose theirs energy continuously. That is why the value $(\delta x_\eta)^2$ in (A.2) in this framework can be neglected. The averaged value $< 2P(x')x' > \neq 0$ when the gradient of the energy loss $\partial P(x')/\partial x' \neq 0$ (see Appendix B). Both damping and antidamping of betatron oscillations of particles in storage rings can be effective in this case [4]. The term $< P(x')(\delta x_\eta)^2 >$ leads to the excitation of betatron oscillations only in the case of a intermittent random energy loss.

Appendix B

From the definition of $\xi_{1,\,2}$ we have a relation $x_\eta = x_{T_{1,\,2}} - \xi_{1,\,2}A(\xi_{1,\,2})$. The time derivative $\partial x_\eta/\partial t = v_{T_{1,2}} - [A + \xi_{1,\,2}(\partial A/\partial \xi_{1,\,2})]\partial \xi_{1,\,2}/\partial t$, where $v_{T_{1,2}} = dx_{T_{1,2}}/dt$ is the velocity of the target. Equating this value to the second term in (3.2) we will receive the time derivative

$$\frac{\partial \xi_{1,\,2}}{\partial t} = \frac{\dot{x}_{\eta\, in}}{\pi}\frac{\pi k_{1,2} - \Delta\varphi_{1,2}}{A(\xi_{1,\,2}) + \xi_{1,\,2}(\partial A/\partial \xi_{1,\,2})}. \tag{B.1}$$

Using this equation we can transform the first value in (3.2) to the form

$$\pm sinc\Delta\varphi_{1,2}(\xi_{1,\,2}) = \frac{\partial A}{\partial \xi_{1,\,2}}\frac{\partial \xi_{1,\,2}}{\partial t}/\frac{\partial x_\eta}{\partial t} = \frac{\pi k_{1,2} - \Delta\varphi_{1,2}}{[A + \xi_{1,\,2}(\partial A/\partial \xi_{1,\,2})]\Delta\varphi_{1,2}}\partial A/\partial \xi_{1,\,2}.$$

which can be transformed to

$$\frac{\partial \ln A}{\partial \xi_{1,\,2}} = \frac{\pm \sin\Delta\varphi_{1,2}}{\pi k_{1,2} - (\Delta\varphi_{1,2} \pm \xi_{1,\,2}\sin\Delta\varphi_{1,2})}.$$

When velocities of targets $v_{T_{1\,2}}$ and velocities of instantaneous orbits in the targets $\dot{x}_{\eta\, in}$ are constant (targets have constant thickness) then the received equation leads to the law of change of the amplitudes of betatron oscillations of particles in the storage ring

$$A_f = A(\xi_{1,\,2,\,f}) = A_0 \exp\int_{\xi_{1,2,0}}^{\xi_{1,2,f}} \frac{\pm \sin\Delta\varphi_{1,2} d\xi_{1,\,2,\,f}}{\pi k_{1,2} - (\Delta\varphi_{1,2} \pm \xi_{1,\,2}\sin\Delta\varphi_{1,2})}, \tag{B.2}$$

where the labels 0, f correspond to the initial time t_0 and observation time t_f accordingly.

The time dependence of the amplitudes and positions of the instantaneous orbits of particles are determined by (3.2) through the parameter $\xi(t)$. This parameter is determined by (B.1) and (B.2). Substituting the values A and $\partial A/\partial \xi_{1,\,2,\,f}$, which are determined by (B.2), in (B.1) we can find the connection between time of observation and parameter $\xi_{1,\,2}$

$$t - t_0 = \frac{\pi A_0}{|\dot{x}_{\eta\, in}|}\psi(k_{1,2}, \xi_{1,\,2,\,f}), \tag{B.3}$$

where

$$\psi(k_{1,2}, \xi_{1,\,2,\,f}) = \int_{\xi_0}^{\xi_{1,\,2,\,f}} \frac{-[A(\xi_{1,\,2,\,f})/A_0]d\xi_{1,\,2,\,f}}{\pi k_{1,\,2} - (\Delta\varphi_{1,\,2} \pm \xi_{1,\,2,\,f}\sin\Delta\varphi_{1,\,2})}.$$

The equations (B.3) determine the time dependence of the functions $\xi_{1,\,2}(t_f - t_0)$. The time dependence of the amplitudes $A[\xi_{1,\,2}(t - t_0)]$ is determined by the equation (B.2) through the functions $\xi_{1,\,2}(t - t_0)$ in a parametric form. The time dependence of the position of the instantaneous orbit follow from the definition of $\xi_{1,\,2}$

$$x_\eta(t - t_0) = x_{T_{1,\,2}0} + v_{T_{1,\,2}}(t_f - t_0) - A[(\xi_{1,\,2}(t_f - t_0)] \cdot \xi(t_f - t_0). \tag{B.4}$$

First stage of cooling

The law of change of the amplitudes of particle betatron oscillations is determined by the equation (B.2), which in the first stage of cooling can be presented in the form

$$A_f = A_0 \exp \int_{\xi_0}^{\xi_{1,\,f}} \frac{-\sqrt{1-\xi_1^2}\,d\xi_1}{-\pi k_1 + \pi - \arccos \xi_1 + \xi_1 \sqrt{1-\xi_1^2}}. \tag{B.5}$$

The dependence of the ratio A_f/A_0 of the final amplitude of betatron oscillations of particles A_f to the initial one A_0 on the relative velocity $k_1 < 0$ of the target T_1 is described by the equation (B.7). The numerical calculations of this dependence are presented at the Fig.2 and in the Table 2 for the case $\xi_0 = -1$, $\xi_f = 1$.

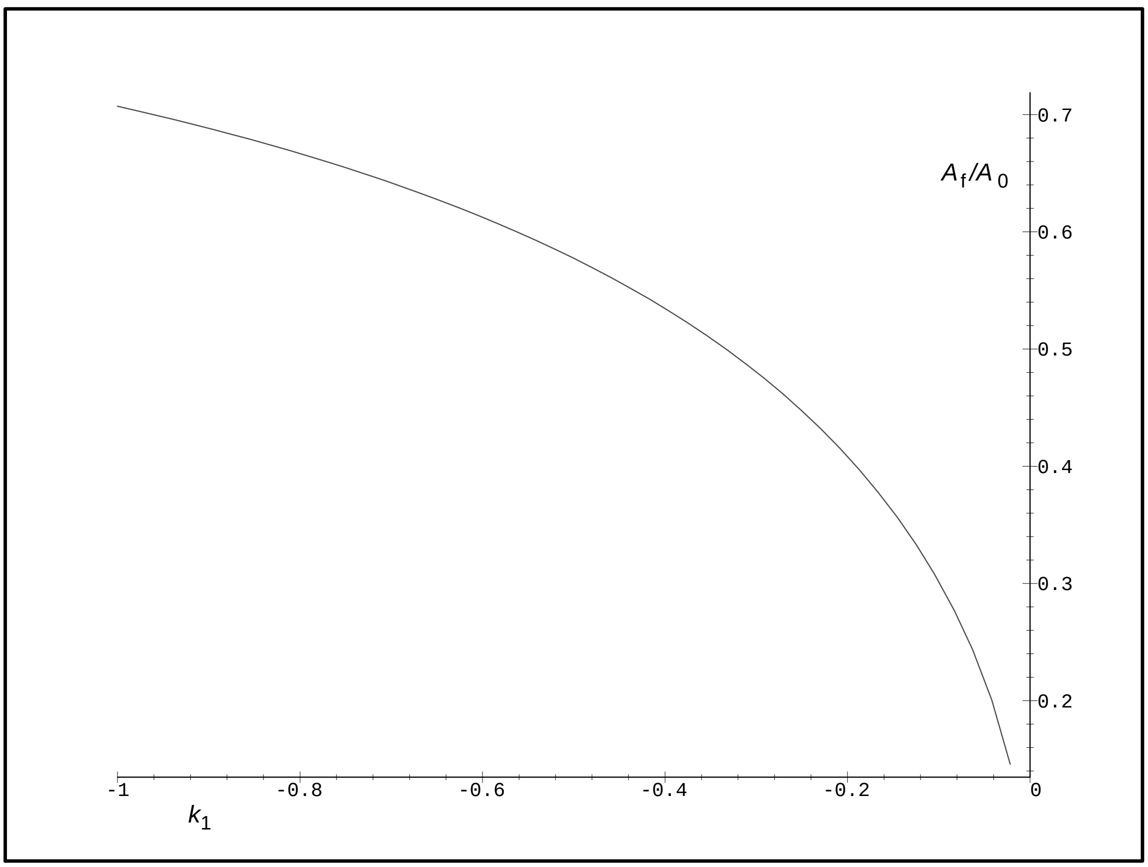

Figure 2: *The dependence of the ratio A_f/A_0 on k_1.*

Table 2

$\lvert k_1 \rvert$	0	0.2	0.4	0.6	0.8	1.0
$C_1^{-1} = A_f/A_0$	0	0.408	0.535	0.612	0.667	0.707
$\sqrt{\lvert k_1 \rvert/(\lvert k_1 \rvert+1)}$	0	0.408	0.535	0.612	0.667	0.707

This dependence can be presented by the next approximate expression (possibly this expression is the exact solution: precision of an estimate 10^{-10}).

$$A_f \simeq A_0 \sqrt{\frac{|k_1|}{|k_1|+1}}. \tag{B.6}$$

Second stage of cooling

1) The law of change of the amplitudes of particle betatron oscillations is determined by the equation (B.2), which in the second stage of cooling can be presented in the form

$$A_f = A_0 \exp \int_{\xi_0}^{\xi_{2,f}} \frac{-\sqrt{1-\xi_2^2}d\xi_2}{\pi k_2 - \arccos\xi_2 + \xi_2\sqrt{1-\xi_2^2}}. \tag{B.7}$$

where the parameter $\xi_{2,f}$ is determined by a moment $t_f = min\{t_{st}, t_A\}$, where $t_{st} = t_0 + A_{T_2}/|v_{T_2}|$ is the moment of the target T_2 stop, $A_{T_2}/|v_{T_2}|$ is the duration of movement of the target T_2 through its amplitude of displacement A_{T_2} and t_A, corresponds to the moment when the instantaneous orbit of a particle will reach in the target the depth equal its amplitude of betatron oscillations A, i.e. when $\xi_{2,A} = \xi_{2,f}(t_A) = -1$. After this moment the amplitude of the particle oscillations is not changed. We will consider here the cooling of particle beams under conditions $\xi_0 = 1$. When $A_{T_2} < l_A$ then, according to (B.7), the value A_f has to be calculated in the limits $(\xi_{2,st}, 1)$, where $\xi_{2,st} > -1$ corresponds to the moment t_{st}.

The moment t_A can be realized only when the relative velocity of the second target $k_2 > 1$ and $t_{st} > t_A$. This moment, according to (B.3), is determined by a moment $t_A = t_0 + \pi A_0 \psi(k_2, \xi_{2,c})/|x_{\eta,in}|$. During the time interval $t_c - t_0$ the target T_2 will pass a way $l_{A_0} = |v_{T_2}|(t_A - t_0) = \pi k_2 A_0 \ \psi(k_2, \xi_{2,c})$. The dependence $\psi(k_2, \xi_{2,c})$ determined by (B.9) is presented in the Table 3. The value l_A depends on initial amplitude of a particle betatron oscillations and tend to maximum one $l_c = \pi k_2 \psi(k_2, \xi_{2,c}) \sigma_{x,b,0}$.

Table 3

k_2	1.0	1.02	1.03	1.05	1.1	1.2	1.3	1.4	1.5	1.7	2.0
$\psi(k_2, \xi_{2,c})$	∞	13.80	9.90	6.52	3.71	2.10	1.51	1.18	0.980	0.735	0.538

The ratio of a maximum amplitude of betatron oscillations of a particle to the initial one $D_{2,tr,c} = A_{f,c}/A_0$ ($A_{f,c} = A_f(\xi_{2,c})$ corresponding to the case $t_{st} > t_c$ on the relative velocity k_2 of the second target is presented at the Fig.3 and in the Table 4. According to calculations this ratio can be presented by the next approximate expression (possibly this expression is the exact solution: precision of an estimate 10^{-10}).

$$A_{f,c} \simeq A_0 \sqrt{\frac{k_2}{k_2 - 1}}. \tag{B.8}$$

2) The function $\psi(k_2, \xi_{2,f})$ according to (B.3) can be presented in the form

$$\psi(k_2,\xi_{2,f}) = \int_{\xi_{2,f}}^{1} \frac{\exp\int_x^1 [\sqrt{1-t^2}/(\pi k_2 - \arccos t + t\sqrt{1-t^2})]dt}{\pi k_2 - \arccos x + x\sqrt{1-x^2}} dx. \tag{B.9}$$

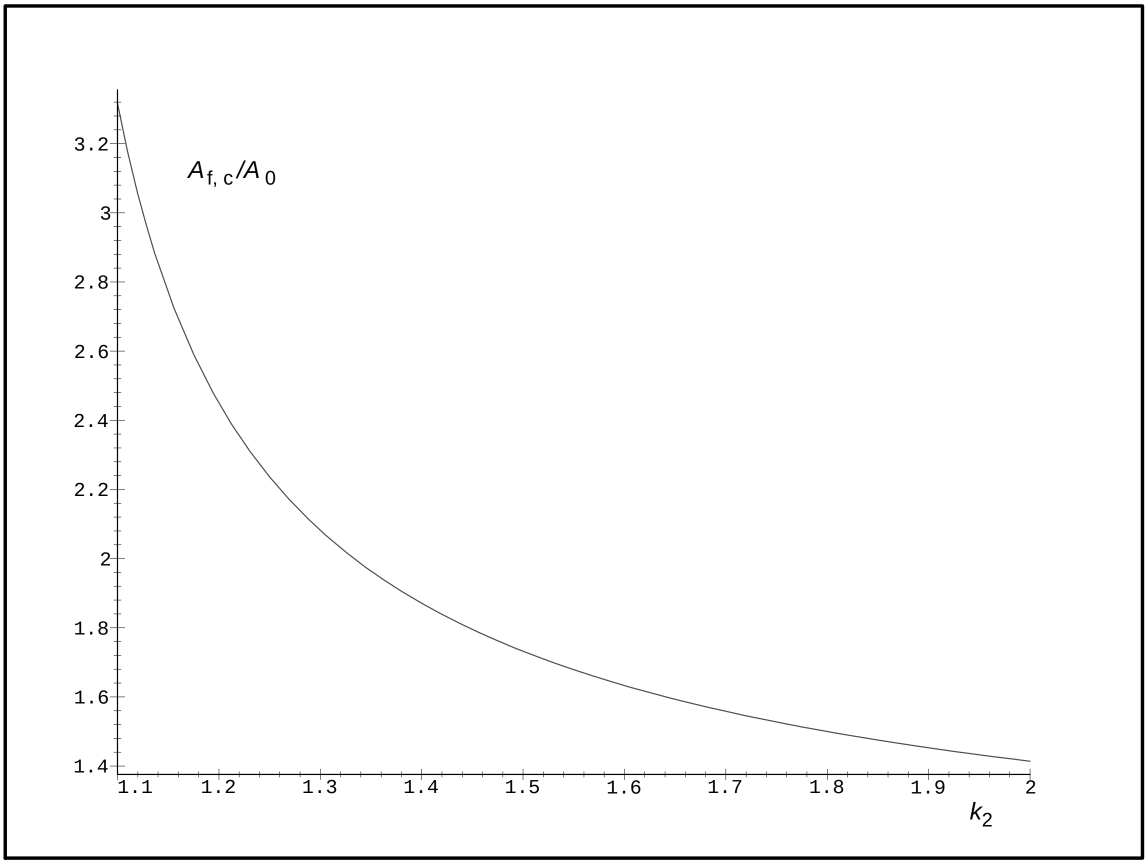

Figure 3: *The dependence of the ratio* $D_{2,tr,c} = A_{f,c}/A_0$ *on* k_2.

Table 4

k_2	1.0001	1.0010	1.0100	1.1000	1.5000	2.0000
$D_{2,tr,c} = A_{f,c}/A_0$	100.005	31.64	10.04	3.32	1.73	1.414
$\sqrt{k_2/(k_2-1)}$	100.005	31.64	10.04	3.32	1.73	1.414

Numerical calculations of this function for the cases $k_2 = 1.0$, $k_2 = 1.1$ and $k_2 = 1.5$ are presented in the Table 5, Table 6 and Table 7 accordingly. It can be presented in the next approximate form

$$\psi(k_2,\xi_{2,f}) \simeq C_3(k_2)\psi(\frac{1-\xi_{2,f}}{k_2+\xi_{2,f}}), \tag{B.10}$$

where $C_3(k_2) \simeq 0.492 - 0.680(k_2-1) + 0.484(k_2-1)^2 + ...$, $\psi[(1-\xi_{2,f})/(k_2+\xi_{2,f})|_{k_2=1} \simeq (1-\xi_{2,f})/(1+\xi_{2,f}))$.

Table 5 $(k_2 = 1.0)$

ξ_2	1.0	0.5	0.2	0	-0.2	-0.5	-0.8	-0.9	-1.0
$\psi(k_2, \xi_2)$	0	0.182	0.341	0.492	0.716	1.393	4.388	10.187	-∞

Table 6 $(k_2 = 1.1)$

ξ_2	1.0	0.5	0.2	0	-0.2	-0.5	-0.8	-0.9	-1.0
$\psi(k_2, \xi_2)$	0	0.163	0.300	0.423	0.595	1.033	2.076	2.759	3.710

Table 7 $(k_2 = 1.5)$

ξ_2	1.0	0.5	0.2	0	-0.2	-0.4	-0.6	-0.8	-1.0
$\psi(k_2, \xi_2)$	0	0.116	0.202	0.273	0.359	0.466	0.602	0.772	0.980

3) The evolution of instantaneous orbits and amplitudes of betatron oscillations under the influence of the target pass the next moments.

a) First, the target T_2 will interact with particles having the largest initial amplitudes of betatron oscillations $A_0 = \sigma_{x,b,0}$ and the highest energies. The instantaneous orbit of these particles, according to the definition of the function ξ_2 will be changed by the low $x^{'}_{\eta} = x_{T_2} - \xi_{2,f}\sigma_{x,b}(\xi_{2,f})$, where $x_{T_2} = x_{T_2,0} + v_{T_2}(t - t_0)$ up to the time $t = t_c$. At the same time instantaneous orbits $x^{''}_{\eta}$ of particles having the same maximum energy but zero amplitudes of betatron oscillations will be at rest up to the moment $t^{'}_0 = t_0 + \sigma_{x,b,0}/|v_{T_2}|$. The orbit $x^{'}_{\eta}$ at the moment $t^{'}_0$ will be displaced relative to the orbit $x^{''}_{\eta}$ by the distance $|\Delta x_{\eta}|$, where the difference between the positions of orbits $\Delta x_{\eta}|_{t_0 \leq t \leq t^{'}_0} = x^{'}_{\eta} - x^{''}_{\eta} < 0$. At the moment $t^{'}_0$ the distance $|\Delta x_{\eta}|$ will be maximum. It will be determined by the value

$$\Delta x_{\eta,1} = \Delta x_{\eta}|_{t=t^{'}_0} = -\xi_2(t^{'}_0) \cdot \sigma_{x,b}(t^{'}_0) < 0, \tag{B.11}$$

where the parameter $\xi_2(t^{'}_0)$, according to (B.3) and the condition $|v_{T_2}|(t^{'}_0 - t_0) = \sigma_{x,b,0}$, will be determined by the equation $\psi[k_2, \xi_{2,f}(t^{'}_0)] = 1/\pi k_2$.

The value $\psi[k_2, \xi_{2,f}|_{k_2 \simeq 1} \simeq 1/\pi$, $\xi_{2,f}(k_2, t^{'}_0)|_{k_2 \simeq 1} \simeq 0.22$ (see Tables 5-7), $\sigma_{x,b}(t^{'}_0) = 1.26\sigma_{x,b,0}$ and the distance $|\Delta x_{\eta,1}| \simeq 0.28\sigma_{x,b,0}$. This distance will be decreased with increasing k_2.

b) The instantaneous orbit of particles $x^{''}_{\eta}$ inside the time interval $t^{'}_0 < t \leq t_c$ will be changed by the low $x^{''}_{\eta} = x_{T_2,0} - \sigma_{x,b,0} + \dot{x}_{\eta,\,in}(t - t_0 - \sigma_{x,b,0}/|v_{T_2}|)$. The difference between the positions of two instantaneous orbits of particles having maximum and zero initial amplitudes of betatron oscillations and equal initial energies will be equal to $\Delta x_{\eta,2} = [(k_2 - 1)/k_2][\sigma_{x,b,0} + v_{T_2}(t - t_0)] - \xi_{2,f}\sigma_{x,b}(\xi_{2,st})$. At the moment t_{st}, i.e. at the position of maximum displacement of the second target, when $t_{st} - t_0 = -A_{T_2}/v_{T_2}$ the distance

$$\Delta x_{\eta,2}|_{t_0 \leq t_{st} \leq t_c} = -\left[\frac{k_2 - 1}{k_2}\left(\frac{A_{T_2}}{\sigma_{x,b,0}} - 1\right) + \xi_{2,st}D_{2,tr})\right]\sigma_{x,b,0}, \tag{B.12}$$

where $D_{2,tr} = D_2(k_2, \xi_{2,st}) = \sigma_{x,b,st}/\sigma_{x,b,0}$, $\sigma_{x,b,st} = \sigma_{x,b,f}|_{t=t_{st}}$, $A_{T_2} = \pi k_2 \psi(k_2, \xi_{2,st})\sigma_{x,b,0} \leq l_c$. The typical dependence $D_{2,tr}$ defined by (B.7) is presented on the Fig.4.

c) After the moment t_c ($t_c < t < t_{st}$) the value (B.12) have a maximum corresponding to $\xi_{2,st} = \xi_{2,c} = -1,\ A_{T_2} = l_c,\ D_{2,tr,c} = \sqrt{k_2/(k_2-1)}$:

$$\Delta x_{\eta,2}|_{t_c \leq t_{st} \leq \infty} = \left[\frac{k_2 - 1}{k_2} + \sqrt{\frac{k_2}{k_2 - 1}} - \pi(k_2 - 1)\psi(k_2, \xi 2, c)\right]\sigma_{x,b,0}. \qquad \text{(B.13)}$$

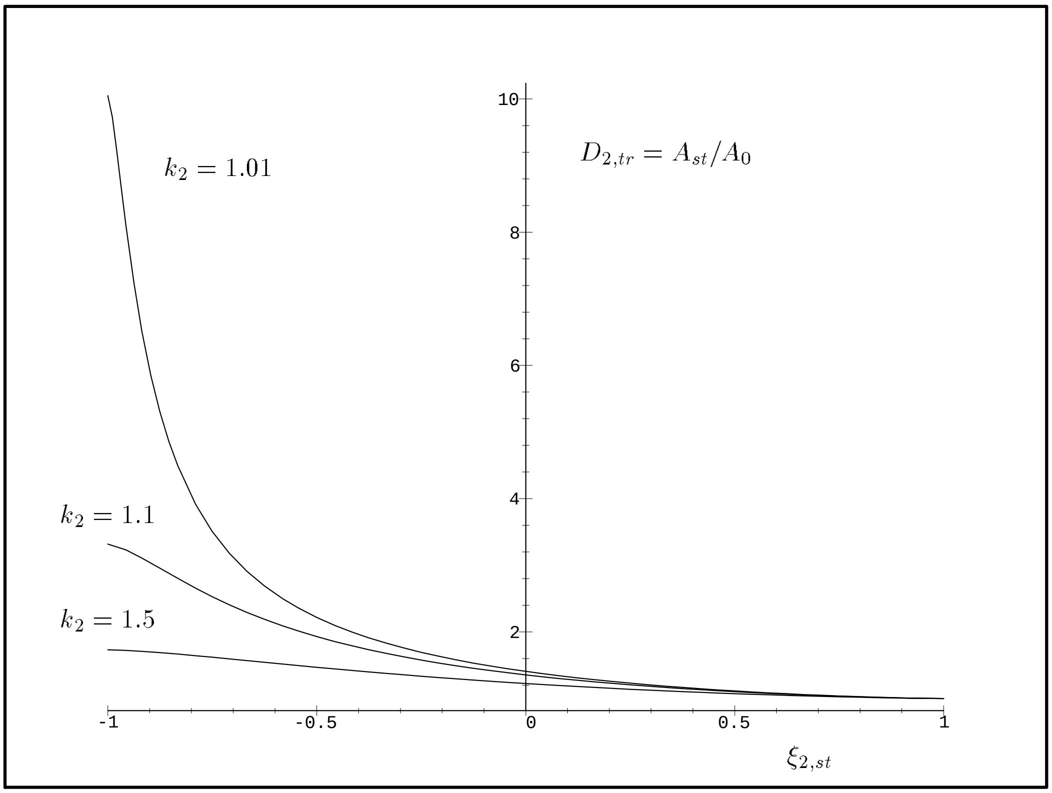

Figure 4: *The dependence of the ratio* $D_{2,tr} = A_{st}/A_0 = \sigma_{x,b,f}/\sigma_{x,b,0}$ *on* $\xi_{2,st}$.

First the target T_2 interacts with instantaneous orbits of particles $x_\eta^{'}$ having maximum energy and amplitudes of betatron oscillations. The energy of particles is decreased and amplitudes of betatron oscillations are increased. The instantaneous orbits of particles $x_\eta^{"}$ having zero amplitudes and the same maximum energy will be at rest up to a moment $t_0^{'}$. The difference between positions of these orbits $\Delta x_{\eta,1}$ at the moment $t_0^{'}$ will be maximum by the value and negative. Instantaneous orbits of particles having maximum initial energies and total set of amplitudes of betatron oscillations at this moment will be mixed with instantaneous orbits of particles having lesser initial amplitudes and energies. The maximum value $|\Delta x_{\eta,1}|_{max} \leq 0.28\sigma_{x,b,0}$. After the moment $t_0^{'}$ the distance $|\Delta x_{\eta,2}(t)|$ between the orbits $x_\eta^{'}$ and $x_\eta^{"}$ will be decreased, at some moment $t_0^{"}$ will be equal zero $\Delta x_\eta^{"} = 0$, and then it will be increased up to the moment t_c ($A_{T_2} = l_c$), where it will be maximum. After the moment

t_c both the amplitude of betatron oscillations and the spread of instantaneous orbits $\Delta x_{\eta,2}$ will be constant. This assertion is valid for the instantaneous orbits of the arbitrary energy.

When the value $\Delta x_{\eta,1}$ is negative then the instantaneous orbits of particles having minimum initial energies and maximum amplitudes of betatron oscillations will leave behind the instantaneous orbits of particles having the same initial energies and zero amplitudes of betatron oscillations. When the value $\Delta x_{\eta,2}$ is positive then the instantaneous orbits of particles having maximum initial energies and amplitudes of betatron oscillations will fall behind from the instantaneous orbits of particles having the same initial energies and zero amplitudes of betatron oscillations. That is why the energy spread of the beam (3.5) will have an addition $\Delta\sigma_{x,\varepsilon} = |\Delta x_{\eta,1}| + 0.5[\Delta x_{\eta,2} + |\Delta x_{\eta,2}|]$. This addition is proportional to the initial spread of amplitudes of betatron oscillations of particles in the beam.

Examples

In the Tables 8, 9 the examples are presented for the longitudinal cooling of ion beams at $k_2 = 1.0$ and $k_2 = 1.1$. The next notations were used: $A_{T_2}/\sigma_{x,b,0} = \pi\psi(k_2, \xi_{2,st})$; $\sigma_{x,\varepsilon,0} = A_{T_2} - \sigma_{x,b,0}$, $D_{2,tr} = \sigma_{x,b,f}/\sigma_{x,b,0}$, $C_{2,l} = \sigma_{x,\varepsilon,0}/\sigma_{x,\varepsilon,f}$; $C_{x,b} = \sigma_{x,0}/\sigma_{x,f}$.

Table 8 $\quad k_2 = 1.0, \quad l_c = \infty \quad A_{T_2} = \sigma_{x,0} = \sigma_{x,\varepsilon,0} + \sigma_{x,b,0}$

$\xi_{2,st}$	$\psi(1.0, \xi_{2,st})$	$A_{T_2}/\sigma_{x,b,0}$	$\sigma_{x,\varepsilon,0}/\sigma_{x,b,0}$	$D_{2,tr}$	$\sigma_{x,\varepsilon,f}/\sigma_{x,b,0}$	$C_{2,l}$	$C_{x,b}$
0.0	0.492	1.545	0.545	1.414	0.278	1.96	0.92
−0.2	0.716	2.25	1.25	1.636	0.61	2.05	1.00
−0.5	1.392	4.38	3.38	2.261	1.41	2.40	1.19
−0.9	10.187	32.00	31.00	7.314	6.86	4.52	2.26

Table 9 $\quad k_2 = 1.1, \quad l_c = 12.82\sigma_{x,b,0}, \quad A_{T_2} = \sigma_{x,0} = \sigma_{x,\varepsilon,0} + \sigma_{x,b,0}$

$\xi_{2,st}$	$\psi(1.1, \xi_{2,st})$	$A_{T_2}/\sigma_{x,b,0}$	$\sigma_{x,\varepsilon,0}/\sigma_{x,b,0}$	$D_{2,tr}$	$\sigma_{x,\varepsilon,f}/\sigma_{x,\varepsilon,0}$	$C_{2,l}$	$C_{x,b}$
0.0	0.423	1.33	0.33	1.35	0.28	1.17	0.82
−0.2	0.595	1.87	0.87	1.52	0.584	1.49	0.89
−0.5	1.033	3.25	2.25	1.93	1.245	1.80	1.02
−0.9	2.759	9.54	8.54	3.04	3.02	2.82	1.70
−1.0	3.71	12.82	11.82	3.32	3.60	3.06	1.85
−1.0	3.71	101	100	3.32	11.61	8.61	6.76

The high degree enhanced cooling of a particle beam in the longitudinal plane and much lesser degree of heating in the transverse one takes place at the second stage of cooling.

The MAPLE V computer program was used to calculate the dependencies presented in the Tables and Figures of this Appendix.