

# Symmetry Breaking by Proton-Neutron Pairing

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The symmetries of the  $t = 1$  and  $t = 0$  pair-fields are different. The consequences for rotational spectra are discussed. For  $t = 1$ , the concept of spontaneous breaking and subsequent restoration of the isospin symmetry turns out to be important. It permits us to describe the proton-neutron pair-correlation within the conventional frame of pairing between like particles. The experimental data are consistent with the presence of a  $t = 1$  field at low spin in  $N \approx Z$  nuclei. For a substantial  $t = 0$  field, the spectra of even-even and odd-odd  $N \approx Z$  nuclei become similar. The possibility of a rotationally induced  $J = 1$  pair-field at high spin is considered.

## I. INTRODUCTION

The understanding of the proton-neutron pair-correlations is one of the major goals of the spectroscopy of nuclei with  $N \approx Z$ . The progress in sensitivity achieved with the large  $\gamma$ -ray detector arrays combined with mass separators permits us to do detailed spectroscopy in the mass 80 and 50 regions. The advent of radioactive beams will hopefully allow us to study even heavier  $N \approx Z$  nuclei. These experimental opportunities revived the theoretical activities devoted to the study of the proton-neutron pairing.<sup>1</sup> What are the consequences of proton-neutron pairing for the excitation spectra? This is an important question, because the excitation spectra and in particular rotational bands are the information that come from  $\gamma$  spectroscopy. In this lecture we shall address a more specific question: What are the consequences of the **proton-neutron pair-field** for the rotational spectra? The restriction to the pair-field has the advantage that its symmetries show up directly in the spectra, which are qualitatively different for the various symmetry types.

## II. THE PAIR-FIELD

The pair-field appears when the mean-field approximation is applied, which results in the Hartree-Fock-

Bogolubov (HFB) equations

$$\mathcal{H}' \begin{pmatrix} U \\ V \end{pmatrix} = e'_i \begin{pmatrix} U \\ V \end{pmatrix}, \quad (1)$$

where

$$\mathcal{H}' = \begin{pmatrix} h'_{ij} + \Gamma_{ij} & -(\lambda + \lambda_\tau \tau_i) \delta_{ij} & \Delta_{ij} \\ -\Delta_{ij}^* & -h'_{ij} - \Gamma_{ij} & +(\lambda + \lambda_\tau \tau_i) \delta_{ij} \end{pmatrix}, \quad (2)$$

$$\Gamma_{ij} = \sum_{kl} \langle ik | v_a | jl \rangle \rho_{lk}, \quad (3)$$

$$\Delta_{ij} = \frac{1}{2} \sum_{kl} \langle ij | v_a | kl \rangle \kappa_{kl}, \quad (4)$$

$$\rho = V^* V^T, \quad (5)$$

$$\kappa = V^* U^T. \quad (6)$$

The quantities in the brackets  $\langle v_a \rangle$  in (3) and (4) are the antisymmetric uncoupled matrix elements of the interaction. In Eq. (2), we have introduced the isospin label  $\tau = 1, -1$  for neutrons and protons, respectively and rearranged the chemical potentials  $\lambda_n$  and  $\lambda_p$  which constrain  $N$  and  $Z$  into  $\lambda = (\lambda_n + \lambda_p)/2$  and  $\lambda_\tau = \lambda_n - \lambda_p$  which fix mass  $A$  and the isospin projection  $T_z$ , respectively. The HFB solutions are obtained by solving the equations (1) -(6) self-consistently.

The pairs of states  $\{ij\}$  that define the pair-field (4) can be rewritten in a coupled representation as  $\{t, t_z, i, j\}$ , which explicitly indicates the isospin  $t$  and  $t_z$  and  $i, j$  denote all quantum numbers except the isospin. If  $t = 0$ , the pair-field is an isoscalar and for  $(t = 1, t_z)$  it is an isovector. The proton-proton (pp) pair-field has  $(t = 1, t_z = -1)$  and the neutron-neutron (nn) has  $(t = 1, t_z = 1)$ . There are two proton-neutron (pn) pair-fields with  $(t = 1, t_z = 0)$  and  $(t = 0, t_z = 0)$ . We use the lower case letters  $t$  and  $t_z$  for the isospin of the pair-field in order to avoid confusion with the isospin of the states, which we denote by  $T$  and  $T_z$ . Let us restrict to the simple case of one  $j$ -shell, for which the symmetries become most obvious. The generalization to many  $j$ -shells is straight forward. Then the field is specified by  $i, j = J, M$ , where  $J, M$  label the angular momentum of the pair. Anti-symmetry implies that for  $t = 1$  the angular momentum  $J$  is even and for  $t = 0$  it is odd. The monopole  $t = 1, J = 0$  and the dipole  $t = 0, J = 1$  are the most important pair-fields because they have the largest matrix elements for a short range interaction as well as a sufficient number of states available to built up

<sup>1</sup>See A. Goodman's contribution to this meeting and [1], which give an overview of the relevant literature.

correlations.<sup>2</sup>

In the following we assume that the pair-field is either  $t = 1$ ,  $J = 0$  or  $t = 0$ ,  $J = 1$ . This is the most common solution of the HFB equations. But coexistence of  $t = 1$  and  $t = 0$  fields is possible, as demonstrated by A. Goodman [1].

### III. THE $T = 1$ PAIR-FIELD

We shall proceed with the assumption that for  $40 < A < 80$  the  $N \approx Z$  nuclei have a  $t = 1$  pair-field at low spin. This assumption is supported by the experiments with two-particle transfer reactions on these nuclei. As reviewed by Bes, Broglia, Hansen and Nathan [2], the observed cross section can be very convincingly be interpreted in terms of a  $t = 1$  pair-field. A dynamic picture of the field is used and multi-boson excitations are considered. We shall stay within the static HFB approach and consider the consequences of the  $t = 1$  pair-field for the energy spectra. The following discussion is based on the material in ref. [3].

The pair-potential  $\Delta$  has the same symmetries as the pairing-tensor  $\kappa$ , because the interaction is an invariant with respect to the symmetry operations. For  $t = 1$  monopole pairing we have

$$\vec{\kappa} = \sum_m \langle j, m, j, -m | 00 \rangle \times \begin{pmatrix} \frac{1}{\sqrt{2}} \langle c_{jm,n}^+ c_{j-m,p}^+ + c_{jm,p}^+ c_{j-m,n}^+ \rangle & t_z = 1 \\ \langle c_{jm,n}^+ c_{j-m,p}^+ \rangle & t_z = 0 \\ \langle c_{jm,p}^+ c_{j-m,p}^+ \rangle & t_z = -1 \end{pmatrix}. \quad (7)$$

It is a vector in isospace in spherical representation. The pair-potential  $\vec{\Delta}$  can be written in an analogous way. The original two-body Hamiltonian conserves the isospin (we neglect the Coulomb interaction). It is a scalar with respect to rotations in isospace. We consider the case  $N = Z$ . Then  $\lambda_\tau = 0$ . The mean field Routhian  $\mathcal{H}'$  (cf. (2)) consists of isoscalar terms, except the pair-potential, which is an isovector. Therefore, it breaks the isospin symmetry spontaneously<sup>3</sup>

#### A. Spontaneous breaking of the isospin symmetry by the $t = 1$ pair-field

Before discussing the breaking of the symmetry by the isovector pair-field, it is useful to state the familiar case of spontaneous breaking of the spatial isotropy by a mean-field solution with a deformed density distribution (c.f. ref. [4,5]). Since the two-body Hamiltonian is isotropic, this symmetry is broken spontaneously. There is a family of mean-field solutions with the same energy which correspond to different orientations of the density distribution. All represent one and the same intrinsic quasi-particle configuration, which is not an eigenfunction of the total angular momentum. Any of these solutions can be chosen as the intrinsic state. The principal axes of its density distribution define the body-fixed coordinate system. The states of good angular-momentum are superpositions of these states of different orientation, the weight being given by the Wigner  $D$ -functions. Thus, the relative importance of the different orientations is fixed by restoring the angular-momentum symmetry. At the simplest level of the cranking model, which is valid for sufficiently strong symmetry breaking, the energy of the good angular-momentum state is given by the mean-field value.

Let us now consider a  $t = 1$  HFB solution found for the  $N = Z$  system. The  $t = 1$  pair-field  $\vec{\Delta}$  is a vector that points in a certain direction in isospace, breaking the isospin symmetry. Since the two-body Hamiltonian is isospin invariant, the symmetry is a spontaneously broken and all orientations of the isovector pair-field :

$$\begin{aligned} \Delta_{J,M,t=1,t_z=\pm 1} &= \mp \Delta_{J,M,t=1} \sin \theta \exp \mp i\phi / \sqrt{2} \\ \Delta_{J,M,t=1,t_z=0} &= \Delta_{J,M,t=1} \cos \theta \end{aligned} \quad (8)$$

are equivalent. Fig. 1 illustrates this family of HFB solutions, the energy of which does not depend on the orientation of the pair-field. In particular, the cases of a pure pn- field (z-axis) and pure pp- and nn-pair-fields (y-axis) represent the same intrinsic state. Hence, at the mean-field level the ratio between the strengths of pp-, nn- and pn- pair-fields is given by the orientation of the pair-field, which is not determined by the HFB procedure. The relative strengths of the three types of pair-correlations becomes only definite when the isospin symmetry is restored. The symmetry breaking by the isovector pair-field has been discussed before in [6,7], where references to earlier work can be found.

The most simple way to restore the symmetry is the above discussed rotor limit, which assumes that the rotation is slow as compared with the intrinsic motion. For the isospace it was discussed in [8]. In essence it amounts to pick one of the orientations of  $\vec{\Delta}$  and call this the intrinsic state. The direction of the y-axis, corresponding to  $\Delta_{np} = 0$ , is particular useful, as will be discussed below. The symmetry conserving wavefunction is a product of this intrinsic state and Wigner  $D$ -function, which is the probability amplitude of the different orientations

<sup>2</sup>The  $t = 0$ ,  $J = 2j$  field has also a large matrix element but there is only one such state in the  $j$ -shell. It may built up correlations in the space of many  $j$ -shells. Since its symmetries are the same as for the  $J = 1$  field, we shall not consider it further.

<sup>3</sup>A discussion of the concept of spontaneous symmetry breaking is given in [4].

of the intrinsic state in isospin. For  $T = 0$  states it is a constant. All orientations of  $\vec{\Delta}$  are equally probable, corresponding to an equal amount of pn-, pp- and nn-correlation energy. In this way the pn- pair-field reappears via restoration of the isospin symmetry, although the intrinsic state has only the pp- and nn- pair-fields. For states  $T_z = T$ , the  $D$ -function becomes more and more peaked in the  $y$ -direction, i. e. with increasing  $T$  the orientations with  $\Delta_{pn} \neq 0$  become less and less probable.

This simple pair-rotational scheme is completely analogous to the familiar rotational bands. The intrinsic excitations represent the  $T = 0$  states. The energies of states with larger isospin are

$$E(T) = E(T=0) + \frac{T(T+1)}{2\mathcal{J}_T}. \quad (9)$$

The moment of inertia for the isorotation can be calculated by means of the cranking procedure. One solves the HFB eqs. for a finite "frequency"  $\lambda_\tau$  and calculates

$$\mathcal{J}_T = \frac{\langle T_z \rangle}{\lambda_\tau}. \quad (10)$$

The moment of inertia is approximately proportional to the level density at the Fermi surface. Realistic interactions or shell model potentials are tuned such that the experimental level density is well reproduced. Hence, the pair-rotational energy, which is a combination of symmetry and the Wigner terms of the binding energy, is expected to be reproduced well.

### B. Intrinsic excitations

Like in the case of spatial rotation, the intrinsic excitations are constructed from the quasi particles (qps) belonging to *one* of the orientations of pair-field. We choose the  $y$ -direction,  $\Delta_{nn} = \Delta_{pp}, \Delta_{np} = 0$ . This is a particularly convenient choice because it permits to reduce the construction of the qp excitation spectrum to the familiar case with no pn-pairing [12]. The choice of the qp operators is not unique [7]. We choose them to be pure quasi neutrons or quasi protons and denote their creation operators by  $\beta_{t_z, k}^+$ , where  $t_z$  indicates the isospin projection. They are pairwise degenerate, i.e. the qp Routhians  $e'(\omega)_{\frac{1}{2}, k} = e'(\omega)_{-\frac{1}{2}, k}$  are equal. Our choice of the orientation of the intrinsic state has the advantage that its symmetries become obvious. Since

$$[e^{-i\pi Z}, \mathcal{H}'] = [e^{-i\pi N}, \mathcal{H}'] = 0, \quad (11)$$

proton and neutron number parities are conserved. That is states with even or odd  $N$  are different quasi neutron configurations and states with even or odd  $Z$  are different quasi proton configurations. The HFB vacuum state has  $N$  and  $Z$  even: Configurations with an odd or even number of quasi neutrons belong to the odd or even  $N$ ,

respectively, and the same holds for the protons. In particular, the lowest  $T = 0$  state in odd-odd  $N = Z$  nuclei is a two-qp excitation and different from the ground state of its even-even neighbor, which is the vacuum.

However, not all qp configurations are permitted. If  $\lambda_\tau = 0$ , the qp Routhian commutes with  $T_y$

$$[T_y, \mathcal{H}'] = 0 \quad (12)$$

This implies that the qp configurations have  $T_y$  as a good quantum number. Since  $T \geq T_y$ , only configurations with  $T_y = 0$  are permitted. The detailed discussion of this restriction can be found in [3].

### C. Comparison with the exact shell model solutions

As a study case, we used the deformed shell model Hamiltonian which consists of a cranked deformed one-body term,  $h'$  and a scalar two-body delta-interaction [9–11]. The one-body term is the familiar cranked-Nilsson mean-field potential which takes into account of the long-range part of the nucleon-nucleon interaction. The residual short-range interaction is specified by the delta-interaction,

$$H' = h' - g\delta(\hat{r}_1 - \hat{r}_2) \quad (13)$$

where,

$$h' = -4\kappa\sqrt{\frac{4\pi}{5}}Y_{20} - \omega J_x \quad (14)$$

We use  $G = g \int R_{nl} r^2 dr$  as our energy unit and the deformation energy  $\kappa$  and is related to the deformation parameter  $\beta$ . We have diagonalized the Hamiltonian (13) exactly for neutrons and protons in the  $f_{7/2}$  shell, for which  $\kappa=1.75$  approximately corresponds to  $\beta = 0.16$ . In addition to its invariance with respect to rotations in isospace (1) is invariant with respect to  $\mathcal{R}_x(\pi)$ , a spatial rotation about the  $x$ -axis by an angle of  $\pi$ . As a consequence, the signature  $\alpha$  is a good quantum number [4,12], which implies that the shell model solutions represent states with the angular momentum  $I = \alpha + 2n$ ,  $n$  integer.

The exact energies obtained by diagonalizing the shell model Hamiltonian (13) for  $(Z + N = 3 + 3)$  particles in the  $f_{7/2}$  shell are shown in the upper panel of figs. 2. The states are classified with respect to the isospin and the signature.

We have solved the HFB equations self-consistently for  $(4+4)$  at  $\omega = 0$ . The solution is a  $t = 1$  pair-field. In order to construct mean field solutions for finite frequency we adopted the Cranked Shell Model (CSM) approach [12]. The fields  $\Gamma$  and  $\Delta_{nn} = \Delta_{pp}$  determined for  $\omega = 0$  are kept fixed for all other values of  $\omega$ . They are also used to describe the  $(3+3)$  and  $(3+4)$  systems, for which only  $\lambda$  and  $\lambda_\tau$  are adjusted to have  $\langle N \rangle = N$  and  $\langle Z \rangle = Z$  at  $\omega = 0$ . Earlier studies of a small number of particles

in a  $j$ -shell showed that the CSM gives better agreement with the exact shell model than than demanding full self-consistency for all  $\omega$  [9].

Fig. 3 shows the quasiparticle Routhians  $e'_i(\omega)$ . All are two-fold degenerated. They correspond to a quasi proton and a quasi neutron, which are labeled, respectively, by  $a, b, c, \dots$  and  $A, B, C, \dots$ , adopting the popular CSM letter convention. The configurations are constructed by the standard qp occupation scheme, as described in ref. [12]. The vacuum  $[0]$  corresponds to all negative qp orbitals filled. It has signature  $\alpha = 0$ , even  $N$  and  $Z$  and  $T_y = 0$ . It represents the even-spin  $T = 0$  yrast band of the  $(N = Z = 4)$  system. The AB-crossing at  $\omega = 0.6$  corresponds to the *simultaneous* alignment of a proton- and a neutron- pair (because the Routhians are degenerate). Fig. 4 demonstrates that the CSM approximation describes the double alignment fairly well. It also shows a shell model calculation where we took off all the  $t = 0$  components of the delta interaction. The crossing appears at almost the same frequency as in the calculation with the full interaction. Hence the possible  $t = 0$  correlations cannot influence the crossing in an important way, as has been speculated [13].

The lowest two-qp excitation is generated by putting one quasi proton and one quasi neutron on the lowest Routhian. We denote this configuration by  $[A, a]_0$ . It has  $T_y = 0$  and thus correspond to a  $T = 0$  band. The subscript indicates the isospin  $T$  of the configuration. The total signature is  $\alpha = 1$  and corresponds to an odd-spin band. The particle numbers  $N$  and  $Z$  must be odd, because exciting one quasi neutron changes  $N$  from even to odd or from odd to even and the same holds for the quasi protons. Thus  $[A, a]$  is the lowest  $T = 0$  odd-spin band in the odd-odd  $N = Z$  system. Fig. 2 shows the CSM estimate for this band. The configuration  $[B, b]$  is the second odd-spin  $T = 0$  band and  $[A, b]$  the first even-spin  $T = 0$  band in the odd-odd system. The configuration  $[a, B]$  does not generate a new state, because only the superposition  $([A, b] - [a, B])/\sqrt{2}$  corresponds  $T_y = 0$ , the other must be discarded. To keep the notation simple, we label the configuration as  $[A, b]$ . But it is understood that the superposition is meant. The lower panel of fig. 2 shows these configurations, which represent the three lowest  $T = 0$  bands.

The lowest  $T = 1$  band is pair-rotational level based intrinsic vacuum state, which we denote by  $[0]_1$ . Its energy is given by (9) and the moment of inertia  $\mathcal{J}_T$  is found by “cranking in isospace” according to (10).

The comparison with the shell model calculation in fig. 2 demonstrates that the simple pair-rotational scheme reproduces well the position of the  $T = 1$  even-spin band relative to the three lowest  $T = 0$  bands, the relative position of which is also well reproduced by the CSM. The appearance of the  $T = 1$  even-spin band below the  $T = 0$  bands is a specific feature of the  $Z = N$  system. (In odd-odd nuclei with  $N \gg Z$  all bands start with an energy larger than  $2\Delta$ .) Its low energy for  $\omega = 0$  has the

consequence that the  $T = 1$  even-spin band is crossed by the aligned odd-spin  $T = 0$  band. This crossing has been observed in  $^{74}\text{Rb}$  [14]. The similar energy of the  $T = 1$  and  $T = 0$  states at  $\omega = 0$  appears as a cancellation between the pair-gap and the isorotational energy. The configuration  $[Aa]_0$  is shifted by  $2\Delta$  with respect to the qp vacuum  $[0]_0$ . The configuration  $[0]_1$  is shifted by  $T(T + 1)/2\mathcal{J}_T$ . Both quantities are nearly equal. This is not a special feature of the  $j$ -shell model, as discussed now.

#### D. Realistic nuclei

The energy difference between the lowest  $T = 0$  and  $T = 1$  states in odd-odd  $N = Z$  nuclei has recently by studied by P. Vogel [15] and A. Macchiavelli *et al.* [16]. It turns out to be few 100 keV for all nuclei with  $A > 22$ , except  $A = 42$  and 46. The small difference is an indication for the presence of the  $t = 1$  pair-field: The  $T = 0$  state lies at  $2\Delta$  because it is a two-qp excitation. The  $T = 1$  state lies at  $1/\mathcal{J}_T$  because it is the zero-qp state but the first excited state of the pair-rotational band. Since the two terms are about the same the two states have almost equal energy. One may derive experimental values for  $\Delta$  from the odd-even mass differences and for  $1/\mathcal{J}_T$  from the experimental energies  $E(T_z, A)$  within a isobaric chain. These independently determined values are consistent with the experimental energy differences  $E(T = 1) - E(T = 0)$  observed in the odd-odd nuclei [15,16].

For the even-even  $N = Z$  nuclei the  $T = 0$  state is the vacuum. Then lowest  $T = 1$  state must be a two-qp excitation, which lies at  $2\Delta + 1/\mathcal{J}_T$ , which is about 5 MeV. The experimental energy of the lowest  $T = 1$  state agrees well with this estimate [15].

The fact that  $2\Delta \approx 1/\mathcal{J}_T$  holds not only the experimental values but also for the single  $j$ -shell model points to a general feature, which remains to be understood.

Since we the orientation of  $\vec{\Delta}$  with  $\Delta_{pn} = 0$  is a legitimate choice for the intrinsic state, the analysis of rotational bands in realistic nuclei can be carried out along the familiar scheme without a pn- pair-field. One has only to take into account the possibility of low lying pair-rotational states and the exclusion of states due to the condition  $T_y = 0$ . This sheds light on the results of the recent analyzes of high spin data in nuclei with  $T_z = 1/2$  and 1 by means of this conventional approach [18,23], which find good agreement between theory and experiment. For  $T_z = 1/2$  the first excited pair-rotational state has  $T = \frac{3}{2}$ . It lies at least  $\frac{3}{2}(\frac{3}{2} + 1)/1(1 + 1) \approx 1.8$  times higher than in the nuclei with  $T_z = 0$ , where it has  $T = 1$ . In the  $T_z = 1$  nuclei it lies even higher. Thus the lowest bands are only the intrinsic excitations, which can be described as qp excitations with  $\Delta_{pn} = 0$ . These results support our suggestion that in the investigated nuclei with  $70 < A < 80$  there is strong  $t = 1$  pairing. In

this connection we want to point out once more that the fact that the mean field theory without an explicit pn-pair-field works well does by no means imply that there is no  $t = 1$  pn- field. On the contrary, it must have a strength comparable with the pp- and nn- fields in order to restore the isospin symmetry.

Fig. 5 displays the spectrum of  $^{74}\text{Rb}_{37}$ . The upper panel also shows the  $T_z = 1$  bands measured in  $^{74}\text{Kr}_{38}$ . They are isobaric analog to the  $T = 1$  bands in  $^{74}\text{Rb}$  and should give a good estimates of these bands. Since the  $T = 1$  states belong to an isobaric triplets, we set the ground state energy of  $^{74}\text{Kr}$  equal to the energy of the  $I = 0$  state in  $^{74}\text{Rb}$ .

Fig. 6 shows the quasi neutron Routhians for  $N = 36$ , which are nearly identical with the quasi proton ones. The standard letter coding is used to label the qp Routhians. The use of A, a, ... indicates that the diagram is relevant for both neutrons and protons. In the lower panel of fig. 5 the rotational spectrum of  $^{74}\text{Rb}_{37}$  is constructed from the qp Routhians. The lowest  $T = 0$  configurations are generated by exciting a quasi proton and a quasi neutron. The first is the positive parity odd spin band  $[Aa]_0$ . Next,  $[Ab]_0$  and  $[Ae]_0$  are expected. As discussed in section III B, the condition  $T_y = 0$  permits only one linear combination of the two excitations, obtained by exchanging the quasi proton with the quasi neutron, which we arbitrary label by only one of the terms in order to keep the notation simple. The lowest  $T = 1$  bands are generated by cranking in isospace, using the realistic deformed Nilsson potential (for details see [3]). The lowest band is the vacuum  $[0]_1$ . It is crossed by the  $T = 0$  band  $[Aa]_0$ , which has a large alignment. The crossing frequency is fairly well reproduced. Thus it seems, that this crossing is a phenomenon belonging to the realm of  $t = 1$  pair-correlations.

The CSM assumptions of fixed deformation and pairing are too inaccurate for the high frequency region. Of course one may combine the concept isorotation with a more sophisticated mean-field calculations. Fig. 7 presents the spectrum of  $^{74}\text{Rb}$  as an example. Only pp- and nn- pairing is considered, but in addition to the monopole a quadrupole pair-field is taken into account. For each configuration and frequency  $\omega$ , the deformation parameters are individually optimized. The calculations of [18] for the yrast sequence in  $^{74}\text{Kr}$  are used for the configuration  $[0]_1$  and the results of an analogous TRS calculation [19] for  $^{74}\text{Rb}$  are used for the configurations  $[Aa]_0$  and  $[Ae]_0$ . The relative energy of the  $T = 0$  and  $T = 1$  bands is calculated by setting at  $\omega = 0$  the energy difference between the configurations  $[0]$  in  $N = 38$ ,  $Z = 36$  and  $N = 37$ ,  $Z = 37$  equal to the experimental value for the isorotational energy. The same Harris reference as used for the experimental Routhians is subtracted from the calculated ones. The calculated spectrum now agrees rather well with the data at high  $\omega$ .

#### IV. THE $T = 0$ PAIR-FIELD

Since the  $t = 0$  pair-field is an isoscalar

$$[\vec{T}, \kappa_M] = 0, \quad (15)$$

and the qp Routhian conserves the isospin. The qp operators have  $t = 1/2$  and either  $t_z = 1/2$  (neutron + proton hole) or  $-1/2$  (proton + neutron hole). The qp vacuum has  $T = 0$ . The one-qp excitations have  $T = \pm 1/2$ . The two-qp excitations can be combined into  $T = 1, T_z = -1, 0, 1$  states, analogous to eq. (7), and into  $T = 0$  states, which correspond to the odd linear combination of the  $T_z = 0$  pairs.

The field conserves the parity of the total number of particles

$$e^{-i(N+Z)\pi} \kappa_M e^{i(N+Z)\pi} = \kappa_M. \quad (16)$$

This means that even- $A$  nuclei and odd- $A$  nuclei correspond to different qp configurations (even or odd number of quasi particles) and have different excitation spectra. However, it does not conserve the neutron or proton number parity separately

$$e^{-iN\pi} \kappa_M e^{iN\pi} = e^{-iZ\pi} \kappa_M e^{iZ\pi} = -\kappa_M. \quad (17)$$

This means that the same qp configuration with zero or an even number of excited qps represents both even-even and odd-odd nuclei. The wave function is a linear composition of states of even-even and odd-odd particle numbers. Adjacent even-even and odd-odd  $N = Z$  nuclei should have similar excitation spectra.

In order to understand this statement better, let us briefly return to the familiar case of the pure neutron pair-field. For a configuration with an even number of qps the wave function is a linear combination of states with even particle number. It may be viewed as the intrinsic state of a pair-rotational band. The different members of the band correspond to  $N$ ,  $N \pm 2$ ,  $N \pm 4$ , ... In the limit of very strong breaking of the particle number  $N$  conservation, all the members of the band have the same excitation spectrum, which is given by the different intrinsic states. This situation is reached for a very strong pair-field, when  $\Delta$  is much larger than the distance between the single-particle levels. In reality,  $\Delta$  is only somewhat larger than the level distance (about 3 times in the heavy nuclei) and the pair-correlations do not completely smear out the region near the Fermi surface. Still one observes quite a remarkable similarity between the spectra of the adjacent even-even nuclei. For example, the first excited two-qp state appears always at about  $2\Delta$ .

For the  $t = 0$  pair-field the situation is analogous. However, adding proton-neutron pairs, brings us from an even-even nucleus to an odd-odd one and the again to an even-even one, etc. This means, if  $\Delta$  is larger than the distance between the single-particle levels, the spectra of neighboring even-even nucleus and odd-odd  $N = Z$

nuclei must be similar. This represents a clear evidence, which can be checked experimentally.

The  $t = 0$  field consists of  $J = 1$  pairs, which have the three angular momentum projections  $M = -1, 0$ , and  $1$ . Let us assume that  $M$  is the projection on the  $x$ -axis, which is the axis of rotation. The components have different signature

$$\mathcal{R}_x(\pi)\kappa_M\mathcal{R}_x^{-1}(\pi) = (-)^M\kappa_M. \quad (18)$$

Let us consider the important case that the deformed potential  $\Gamma$  (cf. eq. (3)) conserves the signature. If there is a  $M = 0$  field the qp Routhian conserves the signature if there are  $M = \pm 1$  fields the signature is not conserved.

### A. The $M = 0$ field

Let us first consider the case  $M = 0$ . It was first discussed by Goswami and Kisslinger [20]. It corresponds to the ( $\alpha = m, \bar{\alpha} = -m, t = 0$ ) pairs in Goodman's classification (cf. his lecture and [1]). He finds for the even-even  $N = Z$  nuclei with  $76 \leq A < 90$  a  $t = 1$  pair-field at low spin. This is in accordance with the our discussion of the spectra in section IV. The observed spectra of adjacent even-even and odd-odd nuclei up to mass 80 are distinctly different. This excludes a  $t = 0$  pair-field with  $\Delta$  larger than the single particle level distance. As discussed before, the different spectra of the even-even and odd-odd  $N = Z$  nuclei can easily be understood in assuming a  $t = 1$  pair-field.

Goodman finds a  $t = 0$  pair-field of the  $(\alpha, \bar{\alpha})$ -type for  $^{92}\text{Pd}$  [1]. The potential  $\Gamma$  is near spherical. The chemical potential is situated in the  $g_{9/2}$  shell, which is almost degenerate. In such a situation one can expect that the spectra of the even-even and odd-odd neighbors are similar. It would be interesting to see if the experiment confirms this.

Goswami and Kisslinger [20] considered the possibility that  $\Delta$  is much smaller than the level distance  $d$ . In such a case, the gap between the ground state and the first excited state is much smaller in the odd-odd nucleus ( $2\Delta$ ) than in the even-even ( $2\sqrt{\Delta^2 + (d/2)^2}$ ). It is difficult to derive information about such a weak pair-field from the spectra. Similarly, it seems hard to extract from the spectra clearcut evidence for a mixed pair-field composed of  $t = 1$  and  $t = 0$  components, which Goodman obtains for one set of input parameters [1].

### B. The $M = 1$ field

This type of pair-field is favored by rotation, because it carries angular momentum. One may speculate that such a field appears at high spin, where the rotation has destroyed the  $t = 1$  pairing and made the phase space available for the new type of correlation.

The  $M = 1$  pair-field has a special symmetry. It is odd under  $e^{-iN\pi}$  (cf. (17)) and under  $\mathcal{R}_x(\pi)$  (cf. (18)). Therefore it is even under the combination of both and the qp Routhian is invariant,

$$\mathcal{S}_N\mathcal{H}'\mathcal{S}_N^{-1} = \mathcal{H}', \quad \mathcal{S}_N = e^{-iN\pi}\mathcal{R}_x(\pi). \quad (19)$$

There is an analogy to reflection asymmetric axial nuclei [21,22]. Although both  $\mathcal{R}_x(\pi)$  and the space inversion  $\mathcal{P}$  do not leave the qp Routhian invariant, the combination  $\mathcal{S} = \mathcal{P}\mathcal{R}_x(\pi)$  does, which implies the quantum number simplex. The bands are  $\Delta I = 1$  sequences of alternating parity. The simplex determines which parity belongs to  $I$ . In the same way  $\mathcal{S}_N$  implies the quantum number  $\gamma$

$$\mathcal{S}_N|\rangle = e^{-i\gamma\pi}|\rangle, \quad (20)$$

which takes the values 0 and 1 for even  $A$  and  $\pm 1/2$  for odd  $A$ . Referring to the analogy with the simplex we suggest the name gauge-simplex or shorter gaugeplex. It relates the parity of the neutron number (or proton number) with the signature  $\alpha$ ,

$$e^{-i(I+N)\pi} = e^{-i\gamma\pi}. \quad (21)$$

Here we used  $e^{-i\alpha\pi} = e^{-iI\pi}$ . A strong  $M = 1$  pair-field implies that adjacent even-even and odd-odd nuclei join into a pair-rotational band of fixed gaugeplex. This means that the even- $I$  states become similar to the odd- $I$  states of the neighbor and vice versa.

In order to investigate this interesting structure we carried out shell model calculations for our study case of a  $f_{7/2}$ -shell. In order to enhance the  $t = 0$  correlations we modified the interaction. Only the odd- $J$  multipoles of the  $\delta$ -interaction, which have  $t = 0$ , were taken into account ( $t = 0$   $\delta$ -interaction). Figs. 8 and 9 show the results. It is seen that for  $\omega/G > 1$  the spectra of the systems  $Z = N = 4$  and  $Z = N = 3$  become very similar if the states of opposite signature are compared. The similarity may be ascribed to a gradual built up of the  $t = 0$ ,  $M = 1$  correlations caused by the increasing frequency. This interpretation is supported by the development of a gap between the yrast and yrare state. At  $\omega/G = 2$ , the distances between the lowest Routhians are in units of  $G$ : 2.4, 1.0, 1.2 for  $Z = N = 3$  and 2.2, 0.9, 0.9 for  $Z = N = 4$ . So far we have not been able to find a self-consistent pair-field in this frequency region. It is not clear at this point what this means. The work is very recent. It may be that we just did not use the right initial wavefunction for the iterative solution of the HFB equations. It is possible that the pair-correlations are of vibrational type and there is no static HFB solution. But it could also be that there is a completely different explanation for the similarity.

## V. CONCLUSIONS

The spectra of  $N \approx Z$  nuclei in the region  $40 < A < 80$  are consistent with a  $t = 1$  pair-field at low and moderate

spin (cf. S. Lenzi's lecture at this meeting). This conclusion is in accordance with the analysis of two-particle transfer data, which has already provided strong independent evidence for this type of pair-field [2].

The  $t = 1$  pair-field breaks the isotropy with respect to rotations in isospace. Therefore, the mean field solutions must be interpreted as intrinsic states of pair-rotational bands. This has two consequences.

As intrinsic state, one may use the orientation in isospace with a zero proton-neutron pair-field. The intrinsic spectrum looks as if there was no such field. Only for  $N = Z$  certain excitations ( $T_y \neq 0$ ) are forbidden. The proton-neutron pair-correlations appear via the restoration of the isospin in the total wavefunction.

On top of each intrinsic state, there is a pair-rotational band. However, these additional states with  $T > T_z$  lie high in energy. Only in the odd-odd  $N = Z$  nuclei the first excited pair-rotational state based on the intrinsic ground state has a similar energy as the first excited intrinsic state. Hence the appearance of this low lying  $T = 1$  rotational band is a consequence of the  $t = 1$  pair-correlations.

The  $t = 0$  pair-field, which has other symmetries than the  $t = 1$  field, leads to a different pattern of excited states. Since the pairs carry finite angular momentum ( $J = 1$  is expected to be most important) one must distinguish between the fields with signature 0 and 1.

A substantial pair-field with signature 0 would show up as similar spectra in adjacent even-even and odd-odd  $N = Z$  nuclei. Such a similarity is not seen in the experimental spectra. It should appear around  $A = 92$ , where Goodman [1] predicts this type of pair-field at low spin.

The pair-field with signature 1 is favored by rotation. It may appear at high spin. A substantial field of this type would show up as similar spectra in adjacent  $N = Z$  nuclei, provided the odd/even spins in even-even nucleus are compared with the even/odd spins in the odd-odd nucleus.

## VI. ACKNOWLEDGMENT

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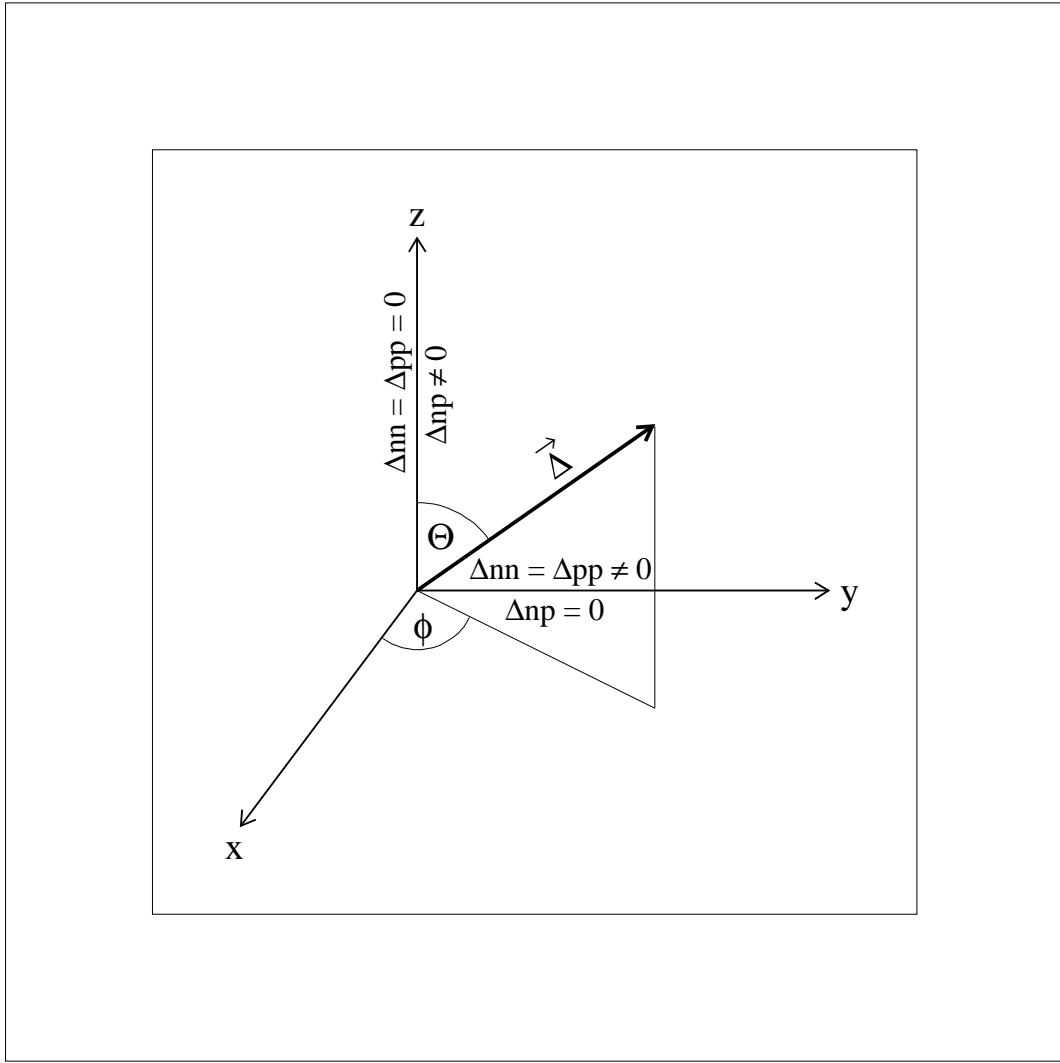


FIG. 1. The isovector pair-field  $\vec{\Delta}$ .



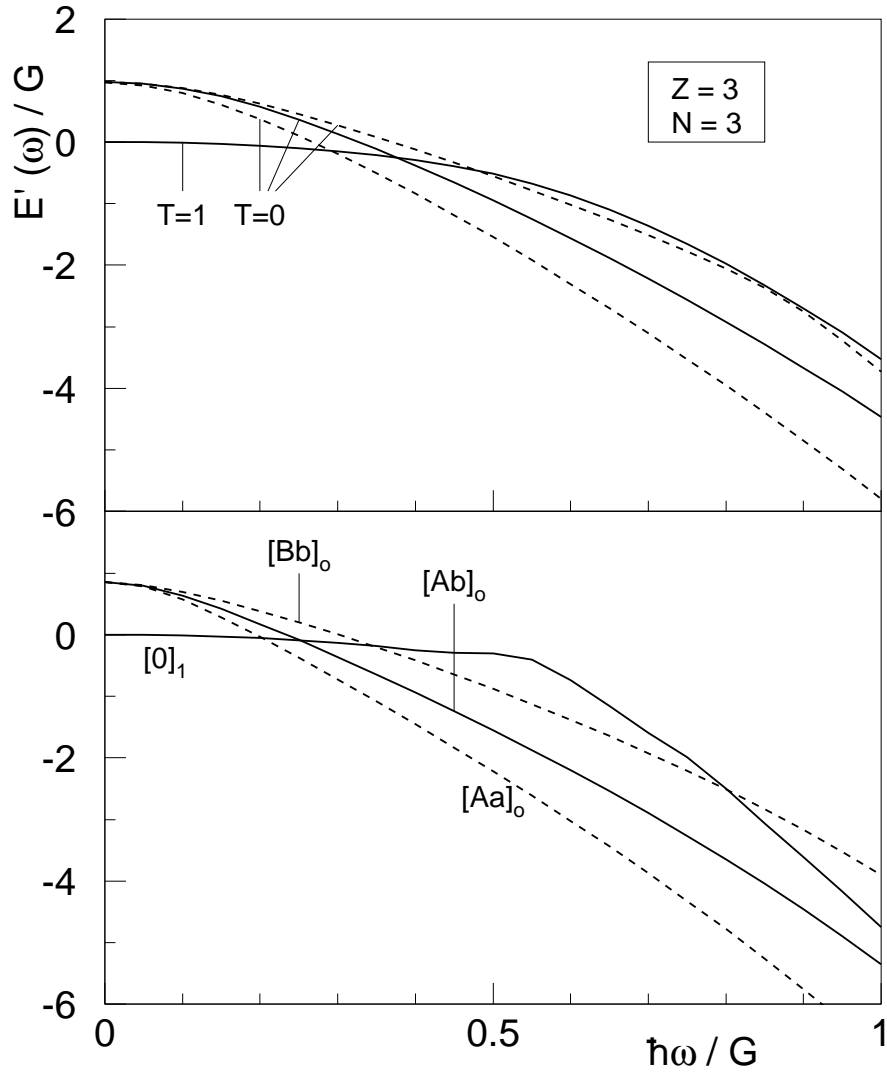


FIG. 2. Total routhians for the ( $Z = N = 3$ ) system. The upper panel shows the shell model results and the lower the CSM approximation. Full lines correspond to even spins and dashed ones to odd spins. The labeling of the quasiparticle configurations is explained in the text.

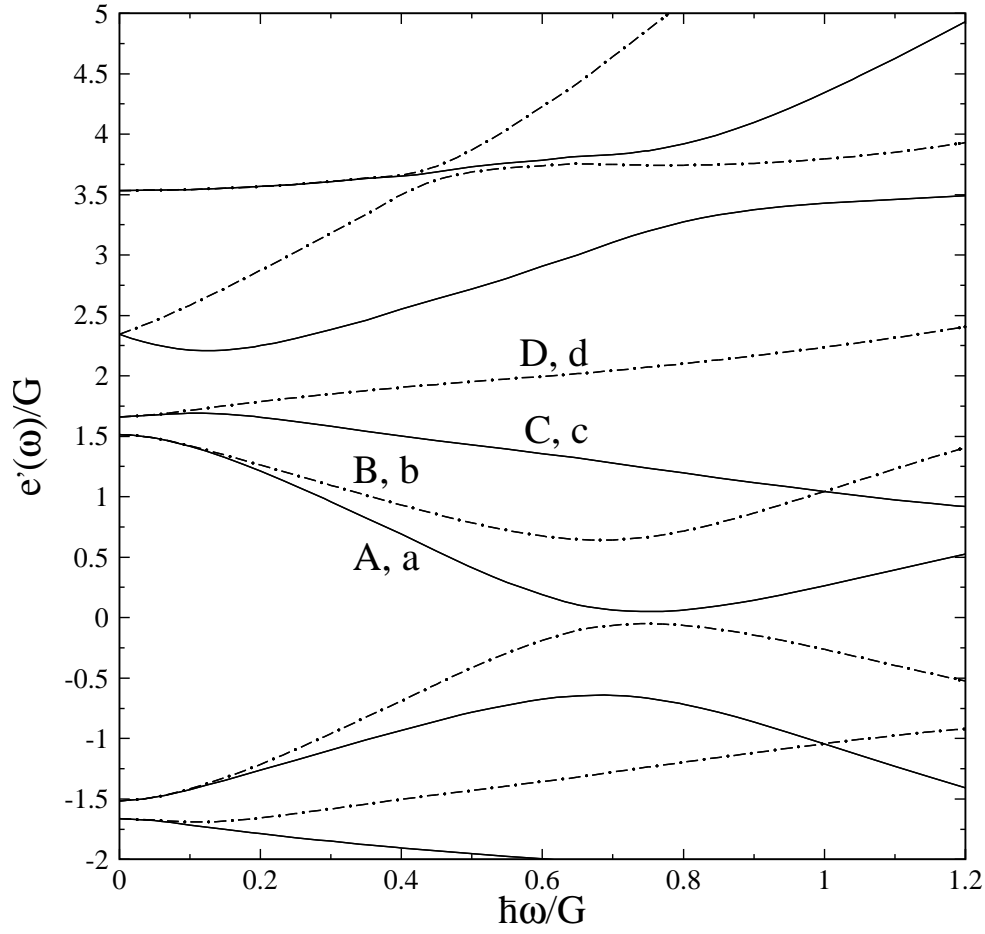


FIG. 3. Quasiparticles in the  $f_{7/2}$  shell as function of the rotational frequency  $\omega$ . The chemical potential corresponds to a half filled shell  $\langle Z \rangle = \langle N \rangle = 4$ . The mean-field is kept fixed to the values calculated by solving the HFB equations (1) for  $\omega = 0$ . Full drawn and dashed dotted lines denote the favored and unfavored signature ( $\alpha = -1/2$  and  $1/2$  for  $f_{7/2}$ ), respectively.

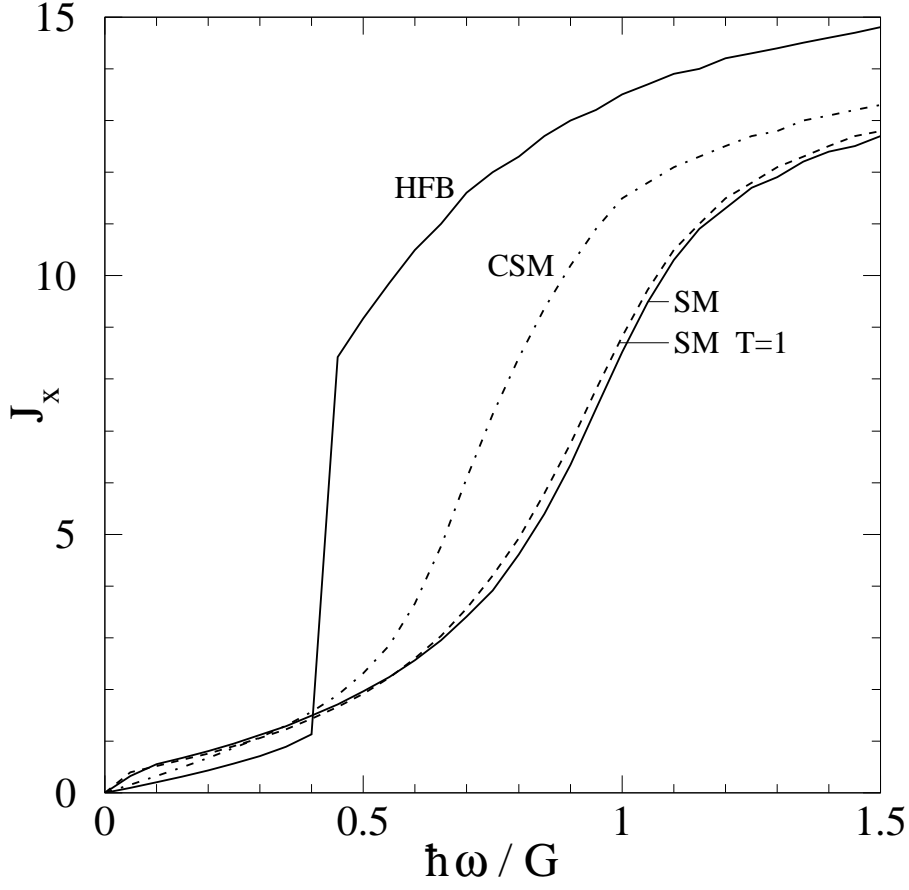


FIG. 4. Angular momentum expectation value  $\langle J_x \rangle$  for the yrast-band in the  $(Z = N = 4)$  system. The full shell model result is denoted by SM, the shell model result with a modified two-body interaction leaving out the  $T = 0$  components of the  $\delta$ -force by SM T=1, the fully selfconsistent HFB calculation by HFB and the CSM approximation by CSM.

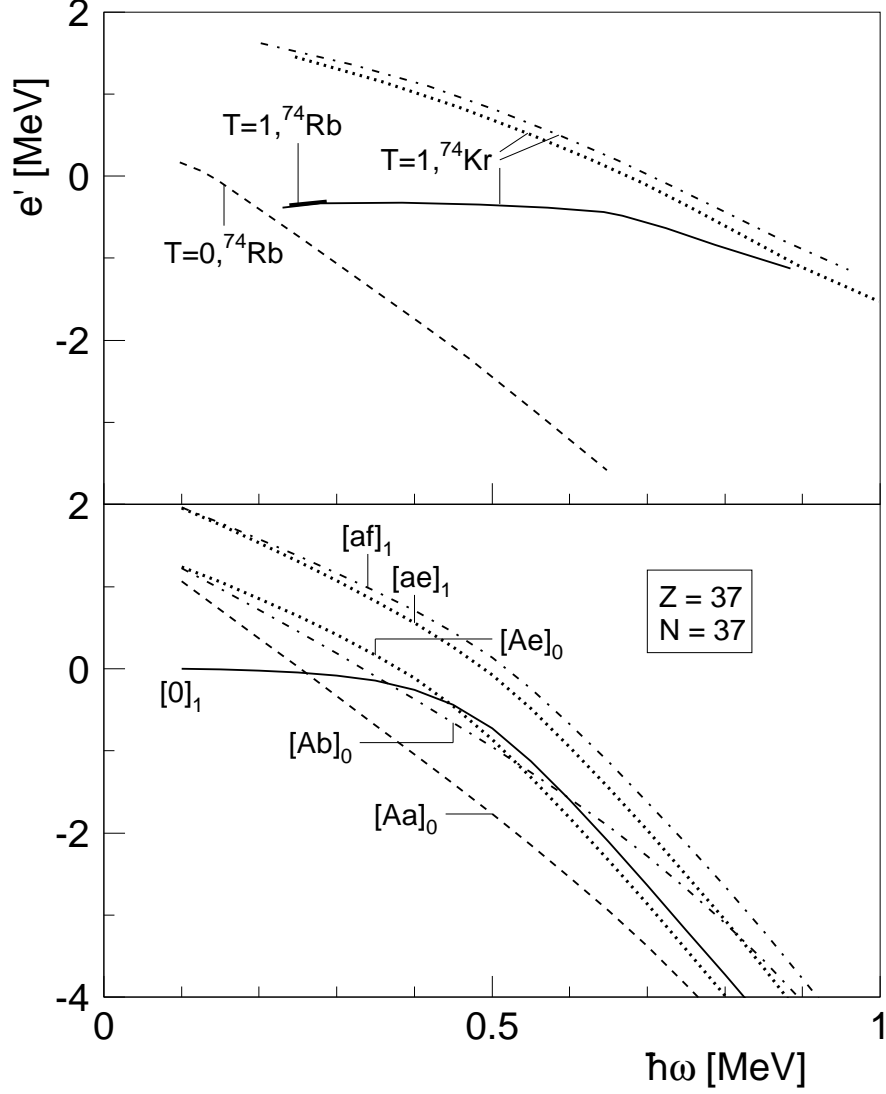


FIG. 5. Total routhians for  $^{74}_{37}\text{Rb}_{37}$ . The upper panel shows the experimental routhians [14] and the lower the CSM approximation. For  $T = 1$  also the isobaric analog  $T_z = 1$  bands in  $^{74}_{36}\text{Kr}_{38}$  [18] are shown. The parity and signature assignments  $(\pi, \alpha)$  are: Full lines  $(+,0)$ , dashed  $(+,1)$ , dashed dotted  $(-,0)$  and dotted  $(-,1)$ . A Harris reference is subtracted.

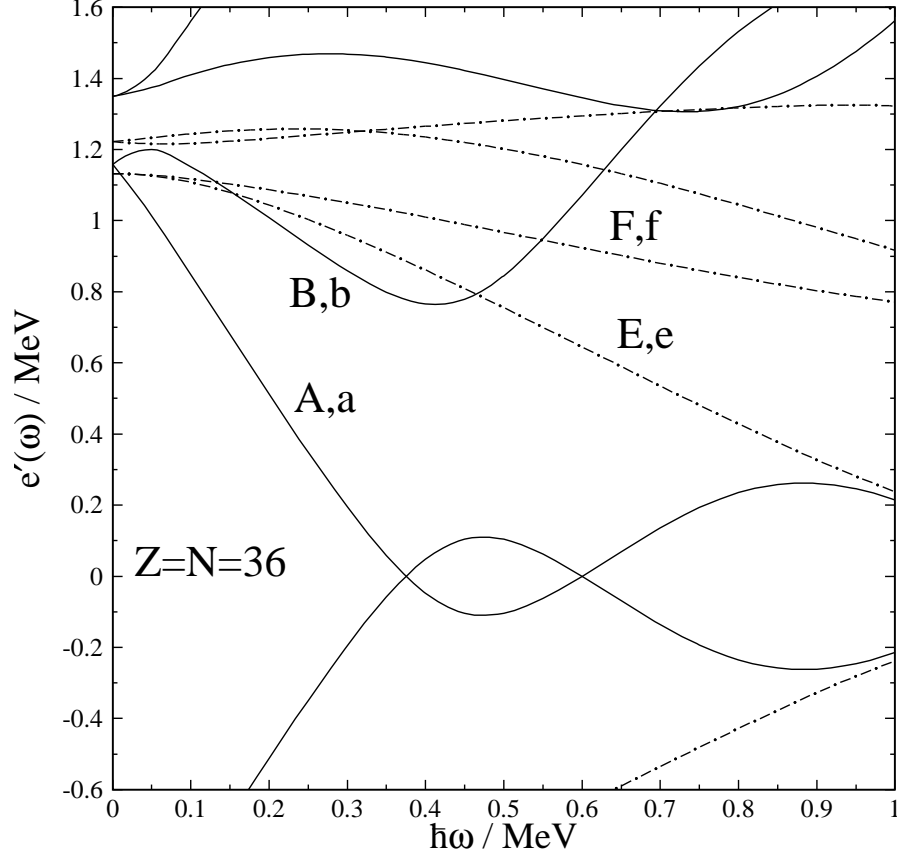


FIG. 6. Quasiparticles for  $(N = Z = 36)$  as function of the rotational frequency  $\omega$ . The mean-field is the modified oscillator with the deformations  $\varepsilon = 0.3$ ,  $\varepsilon_4 = 0$  and  $\gamma = 0$  and  $\Delta_n = \Delta_p = 1.1 \text{ MeV}$ . The diagram is relevant for both protons and neutrons. Full drawn and dashed dotted lines denote positive and negative parity, respectively. The signature is indicated by the letters:  $\alpha = 1/2$  for A,E and  $\alpha = -1/2$  for B,F.

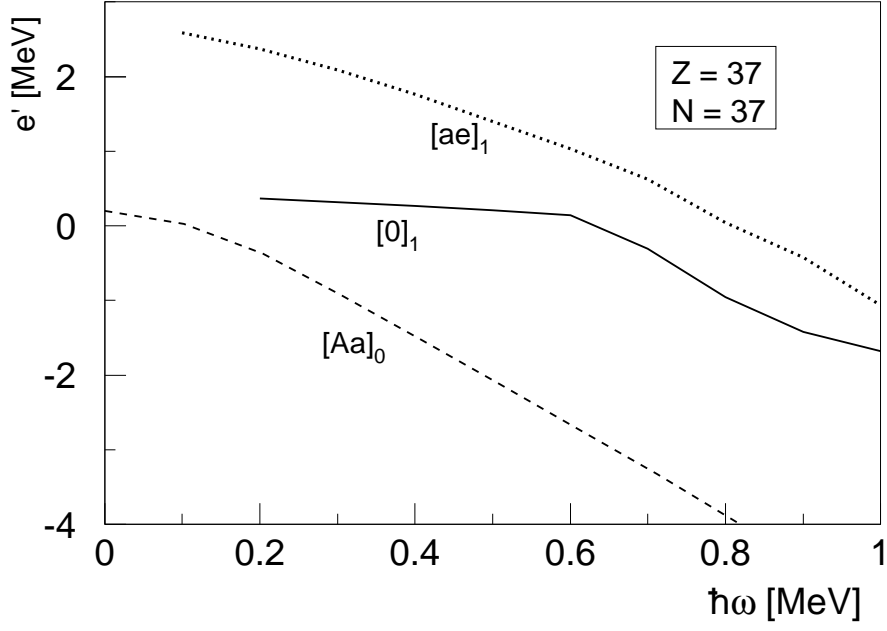


FIG. 7. Total routhians for  $^{74}_{37}\text{Rb}_{37}$  calculated by means of the deformation optimized Woods Saxon Strutinsky method. The text explains how the energy of the  $T = 1$  bands relative to the energy of the  $T = 0$  ground state is fixed. The parity and signature assignments  $(\pi, \alpha)$  are: Full lines  $(+, 0)$ , dashed  $(+, 1)$  and dotted  $(-, 1)$ . A Harris reference is subtracted.

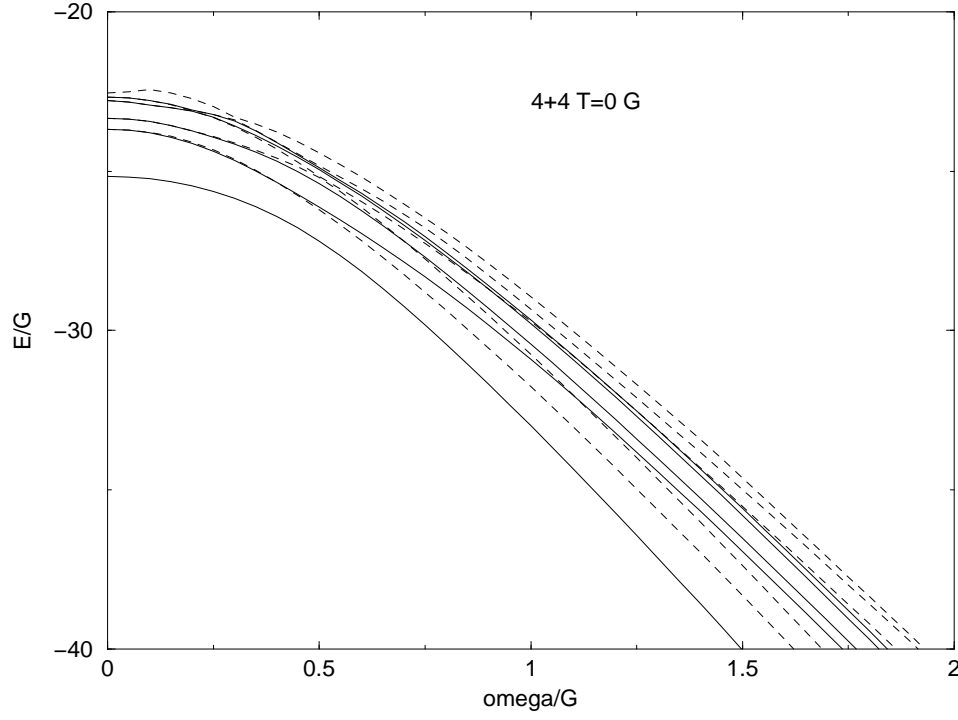


FIG. 8. Total routhians of the ( $Z = N = 4$ ) system as obtained by the shell model for a  $t = 0$   $\delta$  -interaction. Full lines correspond to even spins and dashed ones to odd spins.

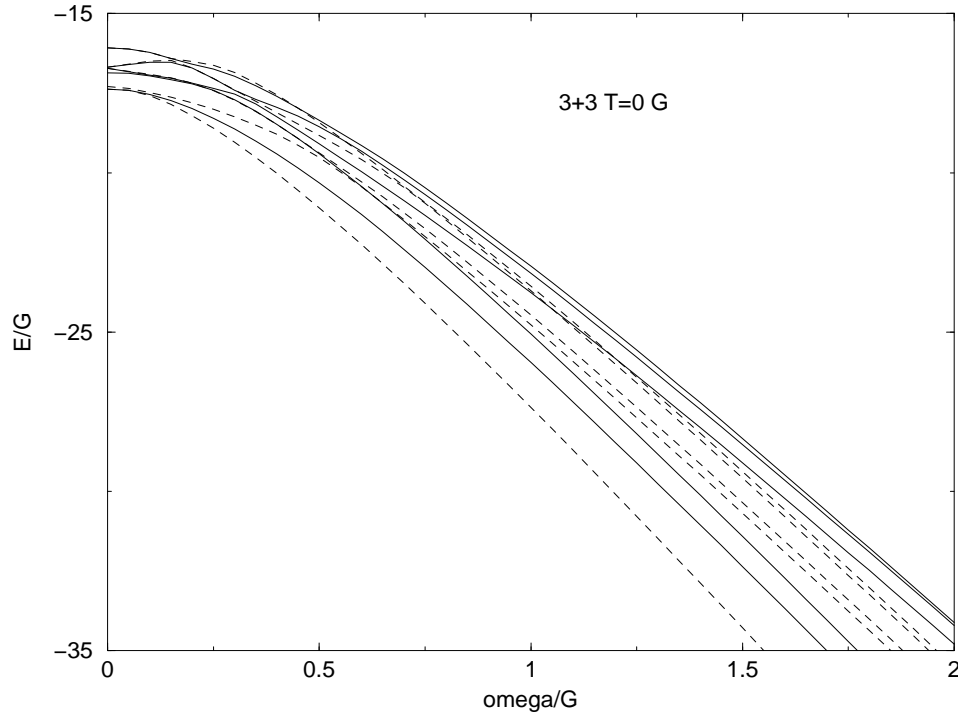


FIG. 9. Total Routhians of the ( $Z = N = 3$ ) system obtained by the shell model for a  $t = 0$   $\delta$  -interaction. Full lines correspond to even spins and dashed ones to odd spins.