The $\Delta I = 1/2$ rule in non-mesonic weak decay of Λ-hypernuclei

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Abstract

By employing recent data on non-mesonic decay of s-shell Λ-hypernuclei we study, within the phenomenological model of Block and Dalitz, the validity of the $\Delta I = 1/2$ rule in the $\Lambda N \to NN$ process. Due to the low experimental precision, a possible violation of this rule can be neither proved nor excluded at present with sufficient accuracy: a pure $\Delta I = 1/2$ transition amplitude is excluded at 40% confidence level.

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The $\Delta I = 1/2$ rule has been first established from the experimental observation of the free Λ decay rates into the two pionic channels:

$$\Lambda \rightarrow \begin{array}{ll} \pi^- p & (\Gamma^{\rm free}_{\pi^-}/\Gamma^{\rm free}_{\Lambda} = 0.639) \\ \pi^0 n & (\Gamma^{\rm free}_{\pi^0}/\Gamma^{\rm free}_{\Lambda} = 0.358), \end{array} \tag{1}$$

the total lifetime being $\tau^{\rm free}_{\Lambda} \equiv \hbar/\Gamma^{\rm free}_{\Lambda} = 2.632 \cdot 10^{-10}$ sec. The experimental ratio of the above widths, $(\Gamma^{\rm free}_{\pi^-}/\Gamma^{\rm free}_{\pi^0})_{\rm Exp} \simeq 1.78$, as well as the Λ polarization observables are suggestive of a change in isospin of $\Delta I = 1/2$, which also holds true for other non–leptonic strangeness changing processes, like the decay of the Σ hyperon and pionic kaon decays.

Actually this rule is slightly violated in the Λ decay, since (taking the same phase space for both channels and neglecting the final state interactions) it predicts $\Gamma^{\rm free}_{\pi^-}/\Gamma^{\rm free}_{\pi^0} = 2$, a value which is slightly larger than the experimental one. Nevertheless, the ratio $A_{1/2}/A_{3/2}$ between the $\Delta I = 1/2$ and the $\Delta I = 3/2$ transition amplitudes for the decay $\Lambda \rightarrow \pi N$ is very large (of the order of 30).

This isospin rule is based on experimental observations, but its dynamical origin is not yet understood on theoretical grounds: indeed the free Λ decay in the *Standard Model* can occur through both $\Delta I = 1/2$ and $\Delta I = 3/2$ transitions, with comparable strengths. Moreover, the effective 4-quark weak interaction derived from the *Standard Model* including perturbative QCD corrections gives too small $A_{1/2}/A_{3/2}$ ratios ($\simeq 3 \div 4$, as calculated at the hadronic scale of about 1 GeV by using renormalization group techniques). One can guess that non–perturbative QCD effects at low energy (such as hadron structure and reaction mechanism) and/or final state interactions could be responsible for the enhancement of the $\Delta I = 1/2$ amplitude and/or for the suppression of the $\Delta I = 3/2$ one. Unfortunately these low energy effects are more difficult to handle: for example chiral perturbation theory, which is frequently employed for describing hadronic phenomena in the low energy regime, even if used together with perturbative QCD corrections, is not able to reproduce the hyperon non–leptonic weak decay rates.

Let us turn now to the weak decay of hypernuclei: here the mesonic decay is disfavoured by the Pauli principle, particularly in heavy systems, as the momentum of the final nucleon in (1) is only about 100 MeV/c (much smaller than the average Fermi momentum). Yet, it is the dominant decay channel in s-shell hypernuclei. From theoretical calculations and measurements there is evidence [1] that in nuclei the ratio $\Gamma_{\pi^-}/\Gamma_{\pi^0}$ strongly oscillates around the value 2 predicted by the $\Delta I = 1/2$ rule in a symmetric nucleus ($N = Z$). However, these oscillations are essentially due to nuclear shell effects and might not be directly related to the weak process itself.

In all but the lightest hypernuclei, the dominant weak decay mode is the non–mesonic one, induced by the interaction of the hyperon with one or more nucleons, e.g.

$$\Lambda N \rightarrow NN \quad (\Gamma_1), \tag{2}$$

$$\Lambda NN \rightarrow NNN \quad (\Gamma_2). \tag{3}$$

In this letter we wish to explore the validity of the $\Delta I = 1/2$ rule in the non–mesonic (one–nucleon induced) Λ–decay: since this process can only occur in nuclei, its analysis is complicated by nuclear structure effects, which can alter the simple balance of the isospin change based on angular momentum couplings.

The one–nucleon induced mechanism concerns the following isospin channels:

$$\begin{aligned} \Lambda n &\to nn \quad (\Gamma_n), \\ \Lambda p &\to np \quad (\Gamma_p), \end{aligned} \tag{4}$$

where, in the average, the final nucleons have large momenta (about 420 MeV). Although the two–nucleon stimulated decay (which has not been detected yet) is non–negligible (for p–shell to heavy hypernuclei Γ_2 is about 15% of the total rate, while for s–shell hypernuclei its contribution is $\simeq 5\%$ [2, 3]), in the following we shall consider the process $\Lambda N \to NN$ only.

Direct measurements of the $\Lambda N \to NN$ process are very difficult to perform, due to the short lifetime of the hyperons, which gives flight paths limited to less than 10 cm. The inverse reaction $pn \to p\Lambda$ in free space is now under investigation at COSY and KEK. Nevertheless, only the precise measurement of the non–mesonic width $\Gamma_{NM} = \Gamma_n + \Gamma_p$ in s-shell hypernuclei is nowadays, at least potentially, a good tool to study the spin and isospin dependence and the validity of the $\Delta I = 1/2$ rule in the $\Lambda N \to NN$ weak interaction. In s–shell hypernuclei all the nucleons are confined (as the hyperon) into the s–level (hence, only the relative orbital angular momentum $L = 0$ is present in the ΛN initial state), while complications arise with increasing mass number, due to the appearance of more ΛN states and of the nucleons' rescattering inside the nucleus, which complicates the kinematic of the measured nucleons.

Nowadays, the main problem concerning the weak decay of Λ–hypernuclei is to reproduce the experimental values for the ratio Γ_n/Γ_p between the neutron– and the proton–induced widths (on the contrary, the total non–mesonic widths are well explained by the available models [2–6]). The theoretical calculations underestimate the central data points for all considered hypernuclei, although the large experimental error bars do not permit any definitive conclusion. The data are quite limited and not precise since it is difficult to detect the products of the non–mesonic decays, especially for the neutron–induced one. Moreover, the present experimental energy resolution for the detection of the outgoing nucleons does not allow to identify the final state of the residual nucleus in the processes ${}^A_\Lambda\mathrm{Z} \to {}^{A-2}\mathrm{Z} + nn$ and ${}^A_\Lambda\mathrm{Z} \to {}^{A-2}(\mathrm{Z}-1) + np$. As a consequence, the measurements supply decay rates averaged over several nuclear final states.

In order to solve the Γ_n/Γ_p puzzle, many attempts have been made up to now, but without success. Among these we recall the introduction of mesons heavier than the pion in the $\Lambda N \to NN$ transition potential [4, 5, 7, 8], the inclusion of interaction terms that explicitly violate the $\Delta I = 1/2$ rule [9, 10] and the description of the short range baryon–baryon interaction in terms of quark degrees of freedom (by using a hybrid quark model in [11] and a direct quark mechanism in [6]), which automatically introduce $\Delta I = 3/2$ contributions.

As we shall see in the following, the analysis of the non–mesonic decays in s–shell hypernuclei is important both for the solution of the Γ_n/Γ_p puzzle and for testing the validity of the related $\Delta I = 1/2$ rule. Let us start by considering the possible $\Lambda N \to NN$ transitions in s–shell hypernuclei: since the ΛN pair is in a state with relative orbital angular momentum $L = 0$, the only possibilities are the following ones (we use the spectroscopic notation ${}^{2S+1}L_J$):

$$\begin{aligned} {}^1S_0 &\to {}^1S_0 \quad (I_f = 1) \\ &\to {}^3P_0 \quad (I_f = 1) \\ {}^3S_1 &\to {}^3S_1 \quad (I_f = 0) \\ &\to {}^1P_1 \quad (I_f = 0) \\ &\to {}^3P_1 \quad (I_f = 1) \\ &\to {}^3D_1 \quad (I_f = 0). \end{aligned} \tag{5}$$

The $\Lambda n \to nn$ process has final states with isospin $I_f = 1$ only, while for $\Lambda p \to np$ both $I_f = 1$ and $I_f = 0$ are allowed.

Classically, the interaction probability of a particle which crosses an infinite homogeneous system of thickness ds is $dP = ds/\lambda$, where $\lambda = 1/(\sigma\rho)$ is the mean free path of the particle, σ is the relevant cross section and ρ is the density of the system. The width $\Gamma_{NM} = dP/dt$ for the process $\Lambda N \to NN$, is then given by:

$$\Gamma_{NM} = v\sigma\rho, \tag{6}$$

v being the Λ velocity in the rest frame of the homogeneous system. For a finite nucleus, one can weight its density $\rho(\vec{r})$, within the semi-classical approximation, by the Λ wave function in the hypernucleus, $\psi_\Lambda(\vec{r})$:

$$\Gamma_{NM} = \langle v\sigma \rangle \int d\vec{r} \rho(\vec{r}) \mid \psi_\Lambda(\vec{r}) \mid^2, \tag{7}$$

where $\langle\,\rangle$ denotes an average over spin and isospin states. In the above equation the nuclear density is normalized to the mass number $A = N + Z$, hence the integral gives the average nucleon density ρ_{A+1} at the position of the Λ particle. In this scheme, the non–mesonic width of the hypernucleus ${}^{A+1}_{\Lambda}Z$ is then:

$$\Gamma_{NM}({}^{A+1}_{\Lambda}Z) = \frac{N\overline{R}_n + Z\overline{R}_p}{A}\rho_{A+1} \equiv \Gamma_n({}^{A+1}_{\Lambda}Z) + \Gamma_p({}^{A+1}_{\Lambda}Z), \tag{8}$$

where $\overline{R}_n$ ($\overline{R}_p$) denotes the spin–averaged rate for the neutron–induced (proton–induced) process appropriate for the considered hypernucleus.

Then, by introducing the rates R_{NJ} for spin–singlet (R_{n0}, R_{p0}) and spin–triplet (R_{n1}, R_{p1}) interactions, the non–mesonic decay widths of s–shell hypernuclei are [12]:

$$\begin{aligned} \Gamma_{NM}({}^3_\Lambda\mathrm{H}) &= (3R_{n0} + R_{n1} + 3R_{p0} + R_{p1})\,\frac{\rho_3}{8}, \\ \Gamma_{NM}({}^4_\Lambda\mathrm{H}) &= (R_{n0} + 3R_{n1} + 2R_{p0})\,\frac{\rho_4}{6}, \\ \Gamma_{NM}({}^4_\Lambda\mathrm{He}) &= (2R_{n0} + R_{p0} + 3R_{p1})\,\frac{\rho_4}{6}, \\ \Gamma_{NM}({}^5_\Lambda\mathrm{He}) &= (R_{n0} + 3R_{n1} + R_{p0} + 3R_{p1})\,\frac{\rho_5}{8}, \end{aligned} \tag{9}$$

where we have taken into account that the hypernuclear total angular momentum is 0 for ${}^4_\Lambda$H and ${}^4_\Lambda$He and 1/2 for ${}^3_\Lambda$H and ${}^5_\Lambda$He. In terms of the rates associated to the partial–wave transitions (5), the R_{NJ}'s of eq. (9) are:

$$\begin{aligned} R_{n0} &= R_n(^1S_0) + R_n(^3P_0), \\ R_{p0} &= R_p(^1S_0) + R_p(^3P_0), \\ R_{n1} &= R_n(^3P_1), \\ R_{p1} &= R_p(^3S_1) + R_p(^1P_1) + R_p(^3P_1) + R_p(^3D_1), \end{aligned} \tag{10}$$

the quantum numbers of the NN final state being reported in brackets.

If we assume that the $\Lambda N \to NN$ weak interaction occurs with a change $\Delta I = 1/2$ of the isospin, the following relations (simply derived by angular momentum coupling coefficients) hold among the rates for transitions to isospin 1 final states:

$$R_n(^1S_0) = 2R_p(^1S_0), \;\; R_n(^3P_0) = 2R_p(^3P_0), \;\; R_n(^3P_1) = 2R_p(^3P_1), \tag{11}$$

then:

$$\frac{R_{n1}}{R_{p1}} \leq \frac{R_{n0}}{R_{p0}} = 2. \tag{12}$$

For pure $\Delta I = 3/2$ transitions, the factor 2 in the right hand side of eqs. (11), (12) should be replaced by 1/2.

Let us now introduce the ratios:

$$r = \frac{\langle I_f = 1 || A_{1/2} || I_i = 1/2 \rangle}{\langle I_f = 1 || A_{3/2} || I_i = 1/2 \rangle} \tag{13}$$

between the $\Delta I = 1/2$ and $\Delta I = 3/2$ $\Lambda N \to NN$ transition amplitudes for isospin 1 final states (r being real, as required by time reversal invariance) and:

$$\lambda = \frac{\langle I_f = 0 || A_{1/2} || I_i = 1/2 \rangle}{\langle I_f = 1 || A_{3/2} || I_i = 1/2 \rangle}. \tag{14}$$

Then, for a general $\Delta I = 1/2$–$\Delta I = 3/2$ mixture, we have:

$$\frac{R_{n1}}{R_{p1}} = \frac{4r^2 - 4r + 1}{2r^2 + 4r + 2 + 6\lambda^2} \leq \frac{R_{n0}}{R_{p0}} = \frac{4r^2 - 4r + 1}{2r^2 + 4r + 2}\, . \tag{15}$$

By using equations (9) and (15) together with the available experimental data, it is possible to extract the spin and isospin behavior of the $\Lambda N \to NN$ interaction without resorting to a detailed knowledge of the interaction mechanism. It is worth pointing out that, according to the above relations, the value of Γ_n/Γ_p is not a universal function of the ratios R_{nJ}/R_{pJ}: it rather depends upon the spin, isospin and structure of the considered hypernucleus. A ratio $\Gamma_n/\Gamma_p < 2$ does not necessarily imply that the $\Delta I = 1/2$ rule is valid; on the contrary the presence of $\Delta I = 3/2$ transitions can even lower this ratio. The opposite situation, $\Gamma_n/\Gamma_p > 2$, cannot be simply explained through a violation of the $\Delta I = 1/2$ rule, and

TABLE I. Experimental data (in units of $\Gamma_\Lambda^{\rm free}$) for s-shell hypernuclei.

	Γ_n	Γ_p	Γ_{NM}	Γ_n/Γ_p	Ref.
${}^4_\Lambda$H			0.22 ± 0.09		reference value (average)
			0.17 ± 0.11		KEK [16]
			0.29 ± 0.14		[12]
${}^4_\Lambda$He	0.04 ± 0.02	0.16 ± 0.02	0.20 ± 0.03	0.25 ± 0.13	BNL [17]
${}^5_\Lambda$He	0.20 ± 0.11	0.21 ± 0.07	0.41 ± 0.14	0.93 ± 0.55	BNL [18]

more complicated reaction mechanisms, including the two–nucleon induced decay and the nucleons final state interactions, are likely to play a role.

We must notice that this analysis makes use of several assumptions. For example, the decay is treated incoherently on the stimulating nucleons, within a simple 4–barions point interaction model, and the nucleon final state interactions are neglected. Moreover, the calculation requires the knowledge of the nuclear density at the hyperon position (here, in particular, the same density is employed for ${}^4_\Lambda$H and ${}^4_\Lambda$He). Since the available data have large error bars, the above approximations can be considered as satisfactory. This phenomenological model was introduced by Block and Dalitz [12]. More recently, the analysis has been updated by other authors [13–15]. These works suggested a sizeable violation of the $\Delta I = 1/2$ rule, although the authors pointed out the need for more precise data to draw definitive conclusions.

Here we shall use more recent data (which are summarized in table I) and we shall employ a different analysis. Unfortunately, no data are available on the non–mesonic decay of hypertriton. We use the BNL data [17, 18] for ${}^4_\Lambda$He and ${}^5_\Lambda$He together with the *reference value* of table I for ${}^4_\Lambda$H. This last number is the average of the previous estimates of refs. [12, 16], which have not been obtained from direct measurements but rather by using theoretical constraints.

We have then 5 independent data which allow to fix, from eq. (9), the 4 rates $R_{N,J}$ and ρ_4. Instead, the density ρ_5 also entering into eq. (9), has been *estimated* to be $\rho_5 = 0.045$ fm^{-3} by employing the Λ wave function of ref. [19] (obtained through a quark model description of the ΛN interaction) and the Gaussian density for ^{4}He that reproduces the experimental mean square radius of the nucleus. For ${}^4_\Lambda$H and ${}^4_\Lambda$He no realistic hyperon wave function is available and we can obtain the value of ρ_4 from the data of table I, by imposing that [see eq. (9)]:

$$\frac{\Gamma_p({}^5_\Lambda{\rm He})}{\Gamma_p({}^4_\Lambda{\rm He})} = \frac{3}{4}\frac{\rho_5}{\rho_4}. \tag{16}$$

This yields $\rho_4 = 0.026$ fm^{-3}. Furthermore, the best determination of the rates $R_{N,J}$ is obtained from the relations for the observables:

$$\Gamma_{NM}({}^4_\Lambda{\rm H}), \quad \Gamma_{NM}({}^4_\Lambda{\rm He}), \quad \Gamma_{NM}({}^5_\Lambda{\rm He}), \quad \frac{\Gamma_n}{\Gamma_p}({}^4_\Lambda{\rm He}), \tag{17}$$

which have the smallest experimental uncertainties. Solving these equations we extracted the following partial rates (the decay rates of eq. (9) are considered in units of the free Λ decay width):

$$R_{n0} = (4.7 \pm 2.1)\ \mathrm{fm}^3, \tag{18}$$

$$R_{p0} = (7.9^{+16.5}_{-7.9})\ \mathrm{fm}^3, \tag{19}$$

$$R_{n1} = (10.3 \pm 8.6)\ \mathrm{fm}^3, \tag{20}$$

$$R_{p1} = (9.8 \pm 5.5)\ \mathrm{fm}^3, \tag{21}$$

The errors have been obtained with the standard formula:

$$\delta[O(r_1, .., r_N)] = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial O}{\partial r_i} \delta r_i \right)^2}, \tag{22}$$

namely by treating the data as independent and uncorrelated ones. Due to the large relative errors implied in the extraction of the above rates, the Gaussian propagation of the uncertainties has to be regarded as a poor approximation.

For the ratios of eq. (15) we have then:

$$\frac{R_{n0}}{R_{p0}} = 0.6^{+1.3}_{-0.6}, \tag{23}$$

$$\frac{R_{n1}}{R_{p1}} = 1.0^{+1.1}_{-1.0}. \tag{24}$$

while the ratios of the spin–triplet to the spin–singlet interaction rates are:

$$\frac{R_{n1}}{R_{n0}} = 2.2 \pm 2.1, \tag{25}$$

$$\frac{R_{p1}}{R_{p0}} = 1.2^{+2.7}_{-1.2}. \tag{26}$$

The large uncertainties do not allow to drawn definite conclusions about the possible violation of the $\Delta I = 1/2$ rule and the spin–dependence of the transition rates. Eqs. (23) and (24) are still compatible with eq. (12), namely with the $\Delta I = 1/2$ rule, although the central value in eq. (23) is more in agreement with a pure $\Delta I = 3/2$ transition ($r \simeq 0$) or with $r \simeq 2$ [see eq. (15)]. Actually, eq. (23) is compatible with r in the range $-1/4 \div 40$, while the ratio λ of eq. (15) is completely undetermined.

By using the results of eqs. (18)-(21) into eq. (9) we can predict the neutron to proton ratio for hyper-triton, which turns out to be:

$$\frac{\Gamma_n}{\Gamma_p}({}^3_\Lambda\mathrm{H}) = 0.7^{+1.1}_{-0.7}, \tag{27}$$

and, by using $\rho_3 = 0.001\ \mathrm{fm}^{-3}$ [12],

$$\Gamma_{NM}({}^{3}_{\Lambda}\mathrm{H}) = 0.007 \pm 0.006. \tag{28}$$

The latter is of the same order of magnitude of the detailed 3-body calculation of ref. [20], which provides a non-mesonic width equal to 1.7% of the free Λ width.

The compatibility of the data with the $\Delta I = 1/2$ rule can be exploited in a different way. By *assuming* this rule, $R_{n0}/R_{p0} = 2$; then one can use the three observables (instead of the four in eq. (17)):

$$\Gamma_{NM}({}^{4}_{\Lambda}\mathrm{He}), \quad \Gamma_{NM}({}^{5}_{\Lambda}\mathrm{He}), \quad \frac{\Gamma_n}{\Gamma_p}({}^{4}_{\Lambda}\mathrm{He}), \tag{29}$$

to extract the following partial rates:

$$R_{n0} = (4.7 \pm 2.1)\ \mathrm{fm}^3, \tag{30}$$

$$R_{p0} \equiv R_{n0}/2 = (2.3 \pm 1.0)\ \mathrm{fm}^3, \tag{31}$$

$$R_{n1} = (10.3 \pm 8.6)\ \mathrm{fm}^3, \tag{32}$$

$$R_{p1} = (11.7 \pm 2.4)\ \mathrm{fm}^3. \tag{33}$$

These values are compatible with the ones in eqs. (18)-(21) (actually R_{n0} and R_{n1} are unchanged with respect to the above derivation). For pure $\Delta I = 1/2$ transitions the spin–triplet interactions seem to dominate over the spin–singlet ones:

$$\frac{R_{n1}}{R_{n0}} = 2.2 \pm 2.1, \tag{34}$$

$$\frac{R_{p1}}{R_{p0}} = 5.0 \pm 2.4. \tag{35}$$

Moreover, since:

$$\frac{R_{n1}}{R_{p1}} = 0.9 \pm 0.8, \tag{36}$$

from eq. (15) one obtains the following estimate for the ratio between the $\Delta I = 1/2$ amplitudes:

$$\left|\frac{\lambda}{r}\right| \equiv \left|\frac{\langle I_f = 0||A_{1/2}||I_i = 1/2\rangle}{\langle I_f = 1||A_{1/2}||I_i = 1/2\rangle}\right| \simeq \frac{1}{3.7} \div 2.3. \tag{37}$$

Finally, the other independent observable (which has not been utilized here) is predicted to be:

$$\Gamma_{NM}({}^{4}_{\Lambda}\mathrm{H}) = 0.17 \pm 0.11, \tag{38}$$

in good agreement with the experimental data of table I, within $\simeq 0.6\,\sigma$ deviation. This implies that the data are consistent with the $\Delta I = 1/2$ rule at the level of 60%. Or, in other words, the $\Delta I = 1/2$ rule is excluded at the 40% confidence level.

TABLE II. Experimental data for ${}^{12}_{\Lambda}$C.

Γ_{NM}	Γ_n/Γ_p	Ref.
1.14 ± 0.20	$1.33^{+1.12}_{-0.81}$	BNL [18]
0.89 ± 0.18	$1.87^{+0.67}_{-1.16}$	KEK [21]
1.01 ± 0.13	$1.61^{+0.57}_{-0.66}$	average

The phenomenological model of Block and Dalitz can be extended to hypernuclei beyond the s–shell; for the sake of illustration we consider here ${}^{12}_{\Lambda}$C. In table II the data on the non–mesonic decay of this hypernucleus are quoted. Within the present framework, the relevant decay rate can be written in the following form:

$$\Gamma_{NM}({}^{12}_{\Lambda}\mathrm{C}) = \frac{\rho^s_{12}}{\rho_5}\Gamma_{NM}({}^{5}_{\Lambda}\mathrm{He}) + \frac{\rho^p_{12}}{7}\left[3\overline{R}_n(P) + 4\overline{R}_p(P)\right], \tag{39}$$

where ρ^s_{12} (ρ^p_{12}) is the average s–shell (p–shell) nucleon density at the hyperon position, while $\overline{R}_n(P)$ [$\overline{R}_p(P)$] is the spin–averaged P–wave neutron–induced (proton–induced) rate. By using the previous results from s–shell hypernuclei and the average values in tab. II, we obtain:

$$\overline{R}_n(P) = (18.3 \pm 10.7)\ \mathrm{fm}^3, \tag{40}$$

$$\overline{R}_p(P) = (3.6^{+12.6}_{-3.6})\ \mathrm{fm}^3. \tag{41}$$

The densities ρ^s_{12} ($= 0.064\ \mathrm{fm}^{-3}$) and ρ^p_{12} ($= 0.043\ \mathrm{fm}^{-3}$) have been calculated from the appropriate s– and p–shell Woods–Saxon nucleon wave functions. The s– and p–shell contributions in eq. (39) are 0.58 ± 0.20 and 0.43 ± 0.24, respectively. The former is calculated starting from the experimental non–mesonic decay width for ${}^{5}_{\Lambda}$He, the latter by subtracting the s–shell contribution from the average value of table II for Γ_{NM}. The central values quoted above are in disagreement with the detailed calculation of refs. [4, 5], where the contribution of the P partial wave to Γ_{NM} is estimated to be only $5 \div 10\%$ in p–shell hypernuclei. However, due to the large uncertainties, at the 2σ level our result is compatible with a negligible P–wave contribution. Moreover, we must notice that in eq. (39) the contribution of S–wave ΛN relative states originating from the interaction with p–shell nucleons is neglected. It is possible to include this contribution by changing $\rho^s_{12} \to \rho^s_{12} + \alpha\rho^p_{12}$ in the first term in the right hand side of eq. (39), α being the fraction of $1p$ nucleons which interact with the Λ in S relative wave. In order to reproduce a 10% contribution of the P-wave interaction to $\Gamma_{NM}({}^{12}_{\Lambda}\mathrm{C})$ [second term in the right hand side of eq. (39)], a large α is required: $\alpha \simeq 0.8$.

Other applications can be considered in heavier hypernuclei, providing one neglects ΛN interactions in D, F, etc waves; then, by using the description and results of eqs. (39)-(41), we can easily predict, e.g., the non–mesonic rate for ${}^{56}_{\Lambda}$Fe, with the result:

$$\Gamma_{NM}({}^{56}_{\Lambda}\mathrm{Fe}) = \frac{\rho^s_{56}}{\rho_5}\Gamma_{NM}({}^{5}_{\Lambda}\mathrm{He}) + \frac{\rho^p_{56}}{2}\left[\overline{R}_n(P) + \overline{R}_p(P)\right] = 1.48^{+0.59}_{-0.45}, \tag{42}$$

where $\rho^s_{56} = 0.087$ fm^{-3} ($\rho^p_{56} = 0.063$ fm^{-3}) now embodies the contributions from both the $1s$ and $2s$ ($1p$ and $2p$) nucleon levels. The central value of eq. (42) overestimates the KEK result [22], which measured a total width $\Gamma_T = 1.22 \pm 0.08$ (note that for iron the mesonic rate is negligible at this level of accuracy), but the two numbers are compatible at the 1σ level. Notice that, should we use the prescription (suggested above for ^{12}C) $\rho^s_{56} \to \rho^s_{56} + 0.8\rho^p_{56}$, eq. (42) would yield about the same result for $\Gamma_{NM}(^{56}_{\Lambda}\mathrm{Fe})$ (1.39), but with a different balance of the S– and P–wave terms and a smaller Γ_n/Γ_p ratio (1.09 instead of 1.85).

In concluding this letter we wish to stress that the phenomenological model of Block and Dalitz employed here, in spite of its relative simplicity, allows to set rather precise constraints on the partial contributions to the non–mesonic decay width of s–shell hypernuclei. From the available experimental data we have found some limits on the ratio between $\Delta I = 1/2$ and $\Delta I = 3/2$ transition amplitudes. A violation of the $\Delta I = 1/2$ rule in the one–nucleon induced non–mesonic decay seems to be present at 40% confidence level. Unfortunately the experimental uncertainties are still too large to allow any definitive conclusion. Similar considerations hold valid in heavier systems, in which however the present analysis requires more severe approximations and must be applied with some caution.

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