DILEPTONS AND HADRON SPECTRAL FUNCTIONS IN HEAVY-ION COLLISIONS[1]

W. CASSING[2]

Institut für Theoretische Physik, Universität Giessen
D-35392 Giessen, Germany

Abstract

We briefly review the dilepton experiments at BEVALAC/SIS and SPS energies for $p + p$, $p + A$ and $A + A$ collisions as well as our present understanding of the data within transport theoretical simulations. Since dileptons from $p + A$ and $A + A$ collisions in particular probe the in-medium spectral functions of vector mesons, a novel semiclassical off-shell transport approach is introduced on the basis of the Kadanoff-Baym equations that describes the dynamical evolution of broad hadron spectral functions. The implications of the off-shell dynamics – relative to the conventional on-shell transport dynamics – are discussed for proton spectra, high energy γ, pion, kaon and antikaon production from GANIL to AGS energies in comparison to experimental data.

1 Introduction

The properties of hadrons in hot and/or dense nuclear matter are of central interest for the nuclear physics community as one expects to learn about precurser effects for chiral symmetry restoration or to explore the vicinity of a quark-gluon plasma (QGP) phase transition. Whereas the early 'big bang' most likely evolved through equilibrium configurations from the QGP to a hadronic phase, this is not the case for the hot/dense systems produced in collisions of heavy ions at relativistic energies.

Nowadays, the dynamical description of strongly interacting systems out of equilibrium is dominantly based on transport theories and efficient numerical recipies have been set up for the solution of the coupled channel transport equations [1, 2] (and Refs. therein). These transport approaches have been derived either from the Kadanoff-Baym equations [3] or from the hierarchy of connected equal-time Green functions [4, 5] by applying a Wigner transformation and restricting to first order in the derivatives of the phase-space

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variables (X, P). Whereas theoretical formulations of off-shell quantum transport have been limited to the formal level for a couple of years [6, 7] only recently a tractable semiclassical form has been derived for testparticles in the eight dimensional phase-space of a particle [8, 9].

In this contribution a brief review is given in Section 2 on the present understanding of 'low mass' dilepton data from pp to AA collisions and the necessity for a quantum transport description is pointed out. A short reminder of the steps for a derivation of such transport theories is presented in Section 3 as well as generalized testparticle equations of motion that are extracted in the semiclassical limit. Section 4 is devoted to a presentation of the most important off-shell effects in nucleus-nucleus collisions from GANIL to AGS energies.

2   Low mass dileptons

Since the first dilepton studies from nucleus-nucleus collisions at the BE-VALAC by the DLS Collaboration [10] the field of dilepton measurements has rapidly expanded at CERN/SPS (see Refs. [2, 11]); the new detector HADES will complement the experimental programm at the SIS [12]. The data taken by the DLS Collaboration [13] on the elementary pp reaction can reasonably be well described by the production of intermediate baryonic resonances as well as $\pi^0, \eta, \omega, \rho^0$ and ϕ mesons and their Dalitz or direct decays to e^+e^- pairs. This has been demonstrated in detail in Refs. [14, 15]. A similar statement holds true for the $p + Be$ and $p + W$ reactions at 450 GeV and 200 GeV [2], respectively, which are fully described by the meson Dalitz or direct decays to dileptons since baryon resonance decays play no longer a substantial role at SPS energies. Whereas the e^+e^- and $\mu^+\mu^-$ differential spectra of the CERES and HELIOS-3 Collaborations – that show an excess of dileptons in the invariant mass regime 0.3 GeV $\leq M \leq$ 0.6 GeV – can be described within the 'dropping mass' scenario [16] or the 'melting' ρ-meson scenario [11, 17] – that involves a strong broadening of the ρ spectral function in the medium due to the coupling to dressed pions and resonance-hole loops – the latter concepts seem to work no longer for the DLS data at 1 A GeV as worked out by Bratkovskaya et al. [18, 19]. Here, the e^+e^- invariant mass spectra for $Ca+Ca$ are underestimated in the regime 0.2 GeV $\leq M \leq$ 0.6 GeV by a factor of 6–7 when involving 'vacuum' spectral functions for the ρ and ω mesons [18] and by a factor of 2–3 within the 'dropping mass' [18] and 'melting' ρ scenarios [15, 19] (DLS-puzzle). In part this discrepancy might be attributed to an improper 'use' of the vector-dominance-model (VDM) in elementary reactions or

unknown isospin dependencies in reactions involving neutrons as pointed out by Mosel at this workshop and should be examined experimentally first in γp or $\pi^- p$ and $\pi^- d$ reactions [20]. These elementary reactions also are expected to provide valuable insight into ρ/ω mixing and their individual production processes [21].

A general survey on low mass dilepton production in $Au + Au$ collisions from SIS to RHIC energies has been given in Refs. [2, 22]; here the energy regime from 2 – 10 A GeV is found to be most promising for studies of the vector meson spectral functions at high baryon density since the systems show high density regimes above $2 - 3 \, \rho_0$ for more than 10 fm/c which are large compared to the ρ and ω life times in the medium. Furthermore, at a couple of A GeV the scalar quark condensate $< \bar{q}q >$ is expected to approach zero, i.e. a chirally restored phase, for substantial space-time volumes [22, 23]. At SPS energies and above most vector mesons are produced dominantly by meson-meson channels in the longitudinally expanding fireball at rather low baryon density (but high temperature) [22]. This lowers the perspectives of low mass dilepton measurements at RHIC energies; however, intermediate mass dileptons from 1.2 – 2.5 GeV of invariant mass might show a signal from the QGP phase provided that open charm is strongly suppressed when propagating through a colored QGP medium [24].

In view of these more general considerations an accurate understanding of the vector meson properties in the medium – or the imaginary part of the current-current correlation functions [25] – can only be achieved if the full dynamical evolution of the hadron spectral functions – also for nonequilibrium phase-space configurations – can be followed throughout all stages of a nucleus-nucleus collision. The next Section is devoted to a formulation of such type of off-shell transport theory following Refs. [8, 9].

3 Extended semiclassical transport equations

The general starting point for the derivation of a transport equation for particles with a finite and dynamical width are the Dyson-Schwinger equations for the retarded and advanced Green functions S^{ret}, S^{adv} and for the non-ordered Green functions $S^<$ and $S^>$ [8]. In the case of scalar bosons – which is considered in the following for simplicity – these Green functions are defined by

$$i \, S^<_{xy} := < \Phi^\dagger(y) \, \Phi(x) > , \qquad i \, S^>_{xy} := < \Phi(x) \, \Phi^\dagger(y) > , \qquad (1)$$

$$i S^{ret}_{xy} := \Theta(x_0 - y_0) \, < [\Phi(x), \Phi^\dagger(y)] > , \; i \, S^{adv}_{xy} := -\Theta(y_0 - x_0) \, < [\Phi(x), \Phi^\dagger(y)] > .$$

They depend on the space-time coordinates x, y as indicated by the indices $\cdot_{xy}$. The Green functions are determined via Dyson-Schwinger equations by the retarded and advanced self energies $\Sigma^{ret}, \Sigma^{adv}$ and the collisional self energy $\Sigma^<$:

$$\hat{S}_{0x}^{-1} \, S_{xy}^{ret} \; = \; \delta_{xy} \; + \; \Sigma_{xz}^{ret} \odot S_{zy}^{ret} \; , \quad \hat{S}_{0x}^{-1} \, S_{xy}^{adv} \; = \; \delta_{xy} \; + \; \Sigma_{xz}^{adv} \odot S_{zy}^{adv} , \qquad (2)$$

$$\hat{S}_{0x}^{-1} \, S_{xy}^< \; = \; \Sigma_{xz}^{ret} \odot S_{zy}^< \; + \; \Sigma_{xz}^< \odot S_{zy}^{adv} \; , \qquad (3)$$

where Eq. (3) is the well-known Kadanoff-Baym equation. Here $\hat{S}_{0x}^{-1}$ denotes the (negative) Klein-Gordon differential operator which is given for bosonic field quanta of (bare) mass M_0 by $\hat{S}_{0x}^{-1} = -(\partial_x^\mu \partial_\mu^x + M_0^2)$; δ_{xy} represents the four-dimensional δ-distribution $\delta_{xy} \equiv \delta^{(4)}(x - y)$ and the symbol $\odot$ indicates an integration (from $-\infty$ to ∞) over all common intermediate variables (cf. [8]).

For the derivation of a semiclassical transport equation one now changes from a pure space-time formulation into the Wigner-representation with variable $X = (x+y)/2$ and the four-momentum P, which is introduced by Fourier-transformation with respect to the relative space-time coordinate $(x - y)$. In any semiclassical transport theory one, furthermore, keeps only contributions up to the first order in the space-time gradients. To simplify notation the operator $\diamond$ is introduced as [7, 8]

$$\diamond \, \{ \, F_1 \, \} \, \{ \, F_2 \, \} \; := \; \frac{1}{2} \left(\frac{\partial F_1}{\partial X^\mu} \frac{\partial F_2}{\partial P_\mu} - \frac{\partial F_1}{\partial P_\mu} \frac{\partial F_2}{\partial X^\mu} \right) , \qquad (4)$$

which is a four-dimensional generalization of the well-known Poisson-bracket. In first order gradient expansion one then obtains algebraic relations between the real and the imaginary part of the retarded Green functions. On the other hand Eq. (3) leads to a 'transport equation' for the Green function $S^<$. In order to obtain real quantities one separates all retarded and advanced functions – Green functions and self energies – into real and imaginary parts,

$$S_{XP}^{ret,adv} \; = \; Re S_{XP}^{ret} \mp \frac{i}{2} A_{XP} \, , \qquad \Sigma_{XP}^{ret,adv} \; = \; Re \Sigma_{XP}^{ret} \mp \frac{i}{2} \Gamma_{XP} \, . \qquad (5)$$

The imaginary part of the retarded propagator is given (up to a factor 2) by the normalized spectral function

$$A_{XP} \; = \; i \left[\, S_{XP}^{ret} - S_{XP}^{adv} \, \right] \; = \; -2 \, Im \, S_{XP}^{ret} \, , \qquad \int \frac{dP_0^2}{4\pi} \, A_{XP} \; = \; 1 \, , \qquad (6)$$

while the imaginary part of the self energy corresponds to half the width Γ_{XP}. By separating the complex equations into their real and imaginary contributions one obtains an algebraic equation between the real and the imaginary part of S^{ret} (Eq. (12) of [8]), an algebraic solution for the spectral function (in first order gradient expansion) as

$$A_{XP} = \frac{\Gamma_{XP}}{(P^2 - M_0^2 - Re\Sigma_{XP}^{ret})^2 + \Gamma_{XP}^2/4} , \qquad (7)$$

as well as for the real part of the retarded propagator [8].

The (Wigner-transformed) Kadanoff-Baym equation (3) allows for the construction of a transport equation for the Green function $S^<$. When separating the real and the imaginary contribution of this equation one obtains i) a generalized transport equation (Eq. (15) in [8]) and ii) a generalized mass-shell constraint (Eq. (16) in [8]). Furthermore, according to Botermans and Malfliet [6], in the transport equation the collisional self energy $\Sigma^<$ has to be replaced by $S^< \cdot \Gamma/A$ to gain a consistent first order gradient expansion scheme. Finally, the general transport equation (in first order gradient expansion) reads [8]

$$A_{XP}\,\Gamma_{XP}\,[\,\diamond\,\{\,P^2 - M_0^2 - Re\Sigma_{XP}^{ret}\,\}\,\{\,S_{XP}^<\,\}$$

$$-\frac{1}{\Gamma_{XP}}\,\diamond\,\{\Gamma_{XP}\}\,\{\,(\,P^2 - M_0^2 - Re\Sigma_{XP}^{ret}\,)\,S_{XP}^<\,\}\,] = \ i\,[\,\Sigma_{XP}^>\,S_{XP}^< \ - \ \Sigma_{XP}^<\,S_{XP}^>\,].$$
$$(8)$$

It has also been independently derived by Ivanov et al. [26] and Leupold [27]. Its formal structure is fixed by the approximations applied. Note, however, that the dynamics is fully determined by the different self energies, i.e. $Re\Sigma_{XP}^{ret}, \Gamma_{XP}, \Sigma_{XP}^<$ and $\Sigma_{XP}^>$ that have to be specified for the physical systems of interest.

Besides the drift term (i.e. $\diamond\{P^2 - M_0^2\}\{S^<\} = -P^\mu\partial_\mu^X S^<$) and the Vlasov term (i.e. $-\diamond\{Re\Sigma^{ret}\}\{S^<\}$) a third contribution appears on the l.h.s. of (8) (i.e. $\sim \diamond\{\Gamma_{XP}\}\{\cdots S_{XP}^<\}$), which vanishes in the quasiparticle limit and incorporates – as shown in [8, 9, 27] – the off-shell behaviour in the particle propagation that has been neglected so far in transport studies[3]. The r.h.s. of (8) consists of a collision term with its characteristic gain ($\sim \Sigma^< S^>$) and loss ($\sim \Sigma^> S^<$) structure, where scattering processes of particles into and out of a given phase-space cell are described.

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[3] This also holds true for the recent numerical studies in Ref. [28]

3.1 Testparticle approximation

In order to obtain an approximate solution to the transport equation (8) a testparticle ansatz is used for the Green function $S^<$, more specifically for the real and positive semidefinite quantity $F_{XP} = A_{XP} N_{XP} = i S_{XP}^<$,

$$F_{XP} \sim \sum_{i=1}^N \delta^{(3)}(\vec{X} - \vec{X}_i(t)) \; \delta^{(3)}(\vec{P} - \vec{P}_i(t)) \; \delta(P_0 - \epsilon_i(t)) \,. \qquad (9)$$

In the most general case (where the self energies depend on four-momentum P, time t and the spatial coordinates $\vec{X}$) the equations of motion for the testparticles read [9]

$$\frac{d\vec{X}_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[2\,\vec{P}_i + \vec{\nabla}_{P_i} Re\Sigma_{(i)}^{ret} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma_{(i)}^{ret}}{\Gamma_{(i)}} \vec{\nabla}_{P_i} \Gamma_{(i)} \right],$$
$$\qquad (10)$$

$$\frac{d\vec{P}_i}{dt} = -\frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[\vec{\nabla}_{X_i} Re\Sigma_i^{ret} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma_{(i)}^{ret}}{\Gamma_{(i)}} \vec{\nabla}_{X_i} \Gamma_{(i)} \right], \quad (11)$$

$$\frac{d\epsilon_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[\frac{\partial Re\Sigma_{(i)}^{ret}}{\partial t} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma_{(i)}^{ret}}{\Gamma_{(i)}} \frac{\partial \Gamma_{(i)}}{\partial t} \right], \qquad (12)$$

where the notation $F_{(i)}$ implies that the function is taken at the coordinates of the testparticle, i.e. $F_{(i)} \equiv F(t, \vec{X}_i(t), \vec{P}_i(t), \epsilon_i(t))$. These equations also have been derived by Leupold in the nonrelativistic limit recently [27].

In (10-12) a common multiplication factor $(1 - C_{(i)})^{-1}$ appears, which contains the energy derivatives of the retarded self energy

$$C_{(i)} = \frac{1}{2\epsilon_i} \left[\frac{\partial}{\partial \epsilon_i} Re\Sigma_{(i)}^{ret} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma_{(i)}^{ret}}{\Gamma_{(i)}} \frac{\partial}{\partial \epsilon_i} \Gamma_{(i)} \right] \qquad (13)$$

and yields a shift of the system time t to the 'eigentime' of particle i defined by $\tilde{t}_i = t/(1 - C_{(i)})$ [9]. As the reader immediately verifies, the derivatives with respect to the 'eigentime', i.e. $d\vec{X}_i/d\tilde{t}_i$, $d\vec{P}_i/d\tilde{t}_i$ and $d\epsilon_i/d\tilde{t}_i$ then emerge without this renormalization factor for each testparticle i when neglecting higher order time derivatives in line with the semiclassical approximation scheme.

For momentum-independent self energies one regains the transport equations as derived in [8]. Furthermore, in the limiting case of particles with vanishing gradients of the width Γ_{XP} these equations of motion reduce to the well-known transport equations of the quasiparticle picture.

Furthermore, the variable $M^2 = P^2 - Re\Sigma^{ret}$ is taken as an independent variable instead of P_0. Eq. (12) then turns to

$$\frac{dM_i^2}{dt} = \frac{M_i^2 - M_0^2}{\Gamma_{(i)}} \frac{d\Gamma_{(i)}}{dt} \qquad (14)$$

for the time evolution of the testparticle i in the invariant mass squared [8, 9].

3.2 Collision terms

The collision term of the Kadanoff-Baym equation can only be worked out by giving explicit approximations for $\Sigma^<$ and $\Sigma^>$. As in the conventional transport theory the formulation of collision terms is based on Dirac-Brueckner theory for the transition amplitudes. The formal structure resembles somewhat the on-shell formulations [2], however, includes additionally the spectral functions of all hadrons in the initial and final channels and four-momentum integrations instead of three-momentum integrals [9]. Furthermore, now all off-shell transition amplitudes enter that so far are scarcely known for strong interaction processes. The collisional width $\Gamma_{coll}(X, \vec{P}, M^2)$ for each hadron then is defined via the loss-term of the corresponding collision integral [9].

For the numerical studies mentioned below the off-shell transition amplitudes squared have been taken as the on-shell values employed in the conventional HSD approach [2], however, corrected by the final phase-space for the particles with off-shell masses. For a detailed description of the numerical recipies and approximations employed the reader is refered to Refs. [8, 9].

4 Off-shell effects in nucleus-nucleus collisions

The off-shell propagation of hadrons in nucleus-nucleus collisions has been examined from GANIL to AGS energies in Refs. [8, 9] and for infinite nuclear matter problems in Ref. [29]. The main results are briefly summarized:

- The numerical implementations of off-shell dynamics can be shown to lead to the proper equilibrium mass distributions (for $t \to \infty$) in line with the quantum statistical limit when employing the same equilibrium spectral functions for all hadrons [29].

- Numerical studies of systems in a finite box with periodic boundary conditions show that the off-shell dynamics lead to practically the same equilibration times as the on-shell dynamics [29].

- At GANIL energies (~ 95 A GeV) the off-shell effects (OSE) are most pronounced since the excitation of Δ-resonances is strongly suppressed. OSE show up especially in the high momentum tails of the collisional $\sqrt{s}$ distribution, in high momentum proton spectra and most pronounced in high energy photon production as demonstrated in comparison to the γ spectra from the TAPS collaboration for $Ar + Au$ at 95 A GeV [8].

- At SIS energies (1–2 A GeV) the OSE lead only to a slight enhancement of the high transverse momentum spectra since the high mass tail of the nucleon spectral function remains small compared to the Δ mass distribution [9]. The latter again is only marginally enhanced at high masses relative to the on-shell dynamics which leads to almost the same pion spectra in this energy domain. It's worth noting that pions in the high density phase become somewhat harder, which leads to an enhanced production of kaons and antikaons. The enhancement of K^- for $Ni + Ni$ reactions at 1.8 A GeV amounts to a factor of about 2 such that in-medium antikaon potentials should be substantially less attractive than suggested in [2].

- The 'DLS puzzle' mentioned in Section 2 might be solved within the off-shell dynamics once there is a strong coupling of low-mass ρ^0 mesons to baryons at high baryon density, which approximately 'thermalizes' the ρ mass distribution. Since this is presently a *speculation*, no definite conclusions can be drawn so far.

- At AGS energies of 11 A GeV the particle production is dominated by 'string' continuum excitations, which themselves are similar to very broad hadron spectral functions. Since all productions thresholds for $K, \bar{K}, \rho, \omega, \phi$ are by far exceeded in the initial collisions, no substantial effects from the off-shell dynamics could be established within the numerical accuracy on rapidity distributions and transverse mass spectra.

In closing it is necessary to point out that although the general equations for off-shell transport are available by now and efficient numerical solution schemes exist, the final understanding of this approach requires the knowledge of *all off-shell (in-medium) transition amplitudes*! The present recipe of using final state phase-space corrections is only a first step on this way.

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