Effects of Trilinear Term in Softly Broken $N = 1$ Supersymmetric QCD

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Softly broken dual magnetic theory of $N = 1$ supersymmetric $SU(N_c)$ QCD with N_f flavours is investigated with the inclusion of trilinear coupling term of scalar fields in the case of $N_f > N_c + 1$. It is found that the the trilinear soft supersymmetric breaking term greatly change the phase and the vacuum structure.

There has been a big progress in understanding strongly coupled $N = 1$ supersymmetric Yang-Mills theory in the last few years [1,2]. A complete phase diagram of the theory, the particle spectrum and the dynamical phenomenon in each phase have been quantitively or qualitatively figured out. In particular, it was found that $N = 1$ supersymmetric QCD with gauge group $SU(N_c)$ and N_f flavour quarks possesses a "conformal window" $3N_c/2 < N_f < 3N_c$ in the infrared region of the theory when $N_f > N_c + 1$, where the theory can be described by a physically equivalent $N = 1$ supersymmetric $SU(N_f - N_c)$ theory with N_f flavours and N_f^2 singlets but with the strong and weak coupling be exchanged and vice versa. This is exactly the realization of the old Montonen-Olive non-Abelian electric-magnetic duality conjecture [3] in $N = 1$ supersymmetric gauge theory, which exists only in $N = 4$ [4] and has an analogue in low-energy $N = 2$ [5] supersymmetric gauge theories.

It is natural to extend these non-perturbative analysis to the ordinary QCD since its non-perturbative aspect is not clear yet. To do this, the first step is breaking supersusymmetry. The most convenient breaking method is the the introduction of superparticle mass terms such as squarks and gaugino [6–8]. However, when $N_f > N_c + 1$ the dual magnetic theory has an additional flavour interaction superpotential. A trilinear soft supersymmetry breaking term composed of the scalar fields is thus allowed. In this talk we shall emphasize the effects of this trilinear term in determining the phase structure and the vacuum structure of the soft broken $N = 1$ dual QCD [9].

$N = 1$ supersymmetric QCD with gauge group $SU(N_c)$ and N_f flavour quark chiral supermultiples Q_{ir}, $\widetilde{Q}_{ir}$ $i = 1, \cdots, N_f$, $r = 1, \cdots, N_c$, has an anomaly free global symmetry

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R \tag{1}$$

At the low-energy the quark supermultiplets are confined, the dynamical degrees of freedom are the chiral supermultiplets, meson M_{ij} and baryons $B^{[i_1 \cdots i_{N_c}]}$ and $\widetilde{B}^{[i_1 \cdots i_{N_c}]}$. The low-energy effective theory still possesses the global symmetry (1). When $N_f > N_c + 1$, the dynamics of M, B and $\widetilde{B}$ can be described $N = 1$ supersymmetric $SU(N_f - N_c)$ gauge theory with N_f flavours of quark chiral supermultiplets $q_{i\tilde{r}}$, $\widetilde{q}_{i\tilde{r}}$ and colour singet chiral fields $\mathcal{M}_{ij}$, $i, j = 1, \cdots, N_f$, $\tilde{r} = 1, \cdots, N_f - N_c$, and an additional flavour interaction superpotential

$$W = \widetilde{q}^{i\tilde{r}} \mathcal{M}_{ij} q^{jr} \tag{2}$$

This conjecture is supported by 't Hooft anomaly matching.

The most important property in supersymmetric field theory is non-renormalization theorem, which determine the superpotential must be a holomorphic function of the chiral superfield. The holomorphy of superpotential and global symmetry (1) as well as the instanton calculation can give a series of exact result of low-energy supersymmetric QCD. Since supersymmetric QCD is very sensitive to the relative numbers of colour and flavours, so we list the non-perturbative dynamical phenomena according to the different ranges of the colour and flavour numbers.

When $N_f < N_c$ there will dynamically generate a superpotential, which eliminates all the supersymmetry vacua. In the case $N_f = N_c$, the non-perturbative quantum correction modify the classical moduli space constrained by $\det M - B\widetilde{B} = 0$ as the quantum one, $\det M - B\widetilde{B} = \Lambda^{2N_c}$. Depending on the vacuum choice in the moduli space, the theory can present various dynamical patterns. For examples, the vacuum $M^i_j = \Lambda^2 \delta^i_j$, $B = \widetilde{B} = 0$ leads to the chiral symmetry breaking and confinement; while the other vacuum choice $M^i_j = 0$, $B = -\widetilde{B} = \Lambda^{N_c}$ makes chiral symmetry unbroken and the baryon number violation. In the case $N_f = N_c + 1$, the quantum moduli space is the same as classical moduli space. Consequently, the low-energy theory present confinement but no chiral symmetry breaking. When $N_f > N_c + 1$, if $N_f > 3N_c$, the theory at high energy level is not asymptotically free and hence the low-energy theory in a free electric phase, the coupling constant behaves as $\alpha(R) \sim 1/\ln(R\Lambda)$; The more interesting is the range $3N_c/2 < N_f < 3N_c$, here the theory can have an non-trivial IR fixed point, at which the low-energy theory becomes an interacting conformal field theory. Thus it is called Seiberg's conformal window. The $SU(N_f - N_c)$ theory describes the same physics as the high energy $SU(N_c)$ QCD, but with the strong and weak coupling exchanged and vice versa This is called the non-Abelian electro-magnetic duality..

When $N_f < N_c$ the quadratic soft supersymmetry breaking term at low-energy can be written out near the origin of the moduli space [6]

$$\mathcal{L}_{\rm sb} = \int d^4\theta \left[B_T M_Q \mathrm{Tr}(M^\dagger M) + B_B M_Q \left(B^\dagger B + \widetilde{B}^\dagger \widetilde{B} \right) \right] - \left[\int d^2\theta M_g \langle W^\alpha W_\alpha \rangle + h.c. \right] \quad (3)$$

In the case $N_f \geq N_c + 1$, the composite superfields are equivalently replaced by the dual magnetic quarks and the soft breaking Lagrangian is [6,7]

$$\mathcal{L}_{\rm sb} = B_M m_M^2 \mathrm{Tr}(\phi_M^\dagger \phi_M) + B_q m_q^2 (\phi_q^\dagger \phi_q + \phi_{\widetilde{q}}^\dagger \phi_{\widetilde{q}}) \quad (4)$$

In the decoupling limit, $m_g, m_Q \to \infty$, the features of the ordinary QCD are expected.

The non-perturbative dynamical features in this soft broken supersymmetric QCD had been analyzed [6]. The results show that when $N_f < N_c$, the standard $SU_V(N_f) \times U(1)_B$ QCD vacuum can arise, while in the case $N_f = N_c$, there emerges an exotic vacua with chiral symmetry and spontaneously breaking of the baryon number symmetry, i.e. the vacuum is invariant $SU_L(N_f) \times SU_R(N_f) \times U(1)_R$. When $N_f > N_c$, There is a vacuum state with unbroken chiral symmetry, but it is interesting that the non-Abelian electric-magnetic duality persists in the presence of soft supersymmetry breaking.

When $N_f \geq N_c + 1$, in addition to (4), the trilinear term

$$\mathcal{L}'_{SB} = h \phi_{qi} \phi_{Mj}^i \phi_{\widetilde{q}}^j \quad (5)$$

can also make the supersymmetry softly broken due to the superpotential (2), where h is the trilinear coupling constant. With the inclusion of (5) the scalar potential becomes [9]

$$\begin{aligned} V(\phi_q, \phi_{\widetilde{q}}, \phi_M) &= \frac{1}{k_T} \mathrm{Tr} \left(\phi_q \phi_q^\dagger \phi_{\widetilde{q}}^\dagger \phi_{\widetilde{q}} \right) \\ &+ \frac{1}{k_T} \mathrm{Tr} \left(\phi_q \phi_M \phi_M^\dagger \phi_q^\dagger + \phi_{\widetilde{q}}^\dagger \phi_M^\dagger \phi_M \phi_{\widetilde{q}} \right) \\ &+ \frac{\widetilde{g}^2}{2} \left(\mathrm{Tr} \phi_q^\dagger \widetilde{T}^a \phi_q - \mathrm{Tr} \phi_{\widetilde{q}} \widetilde{T}^a \phi_{\widetilde{q}}^\dagger \right)^2 \\ &+ m_q^2 \mathrm{Tr}(\phi_q^\dagger \phi_q) + m_{\widetilde{q}}^2 \mathrm{Tr}(\phi_{\widetilde{q}}^\dagger \phi_{\widetilde{q}}) + m_M^2 \mathrm{Tr}(\phi_M^\dagger \phi_M) \\ &- \left(h \mathrm{Tr} \phi_{qi} \phi_{Mj}^i \phi_{\widetilde{q}}^j + h.c. \right) \end{aligned} \quad (6)$$

where $\widetilde{T}^a$ are the generators of magnetic gauge group $SU(N_f - N_c)$, $\widetilde{g}$ is the magnetic gauge coupling constant. The phase (or vacuum) structure can be revealed by analyzing the minima of (6). With assumption that h is real, the minimum of potential can be obtained along diagonal direction

$$\phi_{qi}^r = \begin{cases} \phi_{q(i)} \delta^r{}_i & i = 1, \cdots, N_f - N_c \\ 0 & \text{otherwise} \end{cases}$$

$$\phi_{\widetilde{q}i}^r = \begin{cases} \phi_{\widetilde{q(i)}} \delta^r{}_i & i = 1, \cdots, N_f - N_c \\ 0 & \text{otherwise} \end{cases}$$

$$\phi_{Mj}^i = \begin{cases} \phi_{M(i)} \delta^i{}_j, & i, j = 1, \cdots, N_f - N_c \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

The analysis shows that in the direction $\phi_{q(i)} = q$ and $\phi_{\widetilde{q(i)}} = 0$ (or $\phi_{q(i)} = 0$ and $\phi_{\widetilde{q(i)}} = q$), the vacuum expectation value $\langle \phi_{Mj}^i \rangle = 0$. If $m_q^2 > 0$ (or $m_{\widetilde{q}}^2 > 0$), the scalar potential has the minimum $V = 0$ at $q = 0$, thus theory is in chiral symmetric phase. However, if $m_q^2 < 0$ (or $m_{\widetilde{q}}^2 < 0$), the scalar potential unbounded from below and the theory becomes unphysical. In the flat direction of D-term, $\phi_{q(i)} = \phi_{\widetilde{q(i)}} = X_i$, if soft breaking parameters satisfy $h^2 \geq 2/k_q \left(m_q^2 + m_{\widetilde{q}}^2 \right)$, then we find

$$\frac{h - \sqrt{h^2 - 2(m_q^2 + m_{\widetilde{q}}^2)/k_q}}{2/k_q} \leq \langle \phi_{M(i)} \rangle \leq \frac{h + \sqrt{h^2 - 2(m_q^2 + m_{\widetilde{q}}^2)/k_q}}{2/k_q} \quad (8)$$

Thus chiral symmetry broken phase arises and the baryon number violation occurs. Furthermore, depending on the ratio $\rho \equiv 2 m_M^2 / (m_q^2 + m_{\widetilde{q}}^2) k_q / k_M$, the phase structure in D-flat directions presents various patterns. For examples, in the phase diagram labeled by $(h^2, (m_q^2 + m_{\widetilde{q}}^2)/2)$, when $\rho = 1$, the theory only has one chiral symmetry broken phase and one unbroken phase iff all m_q^2, $m_{\widetilde{q}}^2$ and m_M^2 are positive, whereas when $\rho = 20$ theory has two unbroken phases and two chiral symmetry broken phases. If m_q^2, $m_{\widetilde{q}}^2$ and m_M^2 are negative, scalar potential is unbounded from below and becomes unphysical. Furthermore, in chiral symmetric phase, the $SU(N_f)^3$ and $SU(N_f)^2 U(1)_B$ 't Hooft anomalies match. In the broken phase, the two softly broken dual theories also present same anomaly structure. This seems to suggest Seiberg's duality remains after SUSY breaking, even in the chiral symmetry broken phase.

In the case $N_f = N_c + 1$, the flavour symmetry (1) allows a trilinear soft breaking term $h' \phi_{Bi} \phi_{Mj}^i \phi_{\widetilde{B}}^j$. With the combination of the quadratic term and the terms from the effective potential $\mathcal{W} = (B_i M_j^i \widetilde{B}^j - \det M)/\Lambda^{2N_c - 1}$, the whole scalar potential is [9]

$$\begin{aligned} V &= \lambda_M^2 \sum_{i,j} |\phi_{Bi} \phi_{\widetilde{B}}^j - (\det{}' \phi_M)_i^j|^2 \\ &+ \lambda_B^2 \mathrm{Tr}(|\phi_{\widetilde{B}} \phi_M|^2 + |\phi_B \phi_M|^2) \\ &+ m_B^2 \mathrm{Tr}|\phi_B|^2 + m_{\widetilde{B}}^2 \mathrm{Tr}|\phi_{\widetilde{B}}|^2 m_M^2 \mathrm{Tr}|\phi_M|^2 \\ &- (h' \phi_{Bi} \phi_{Mj}^i \phi_{\widetilde{B}}^j + h.c.) \end{aligned} \quad (9)$$

where $\lambda_M = 1/(k_M \Lambda^{2N_c - 1})$, $\lambda_B = 1/(k_B \Lambda^{2N_c - 1})$, and $(\det' M)_i^j \equiv \partial \det M / \partial M_j^i$. The minimum of superpotential has been analyzed along the diagonal direction, $M^i_j = M_i \delta^i_j$. Choosing a special direction with $B_i \widetilde{B}^j - (\det' M)^j_i = 0$, considering a simple case where $B_i = \widetilde{B}^i = B$, $M_{(i)} = M$, and assuming B and M are real, we write the scalar potential as

$$\frac{V}{N_c+1} = 2\lambda_B^2 \phi_M^{N_c+2} - 2h' \phi_M^{N_c+1} + (m_B^2 + m_{\widetilde{B}}^2)\phi_M^{N_c} + m_M^2 \phi_M^2 \tag{10}$$

This scalar potential yields that if h' is sufficiently large compared with soft scalar masses, there will arise vacua with $\langle M \rangle \neq 0$, $\langle B \rangle \neq 0$,$\langle \widetilde{B} \rangle \neq 0$, thus we get the vacua with chiral symmetry breaking but baryon number symmetry also spontaneously breaking.

When $N_f \leq N_c$, there is no possible trilinear term, but a new quadratic SUSY breaking term, $h_B B \widetilde{B} + h.c.$ can be introduced. However, it produces no new effects compared with the case with only quadratic terms.

In summary, we studied the softly broken $N = 1$ supersymmetric QCD with the inclusion of the trilinear terms and find that in comparison with only the quadratic soft breaking term, some remarkable effects on the phase structure can be produced. First, In case $N_f > N_c + 1$, depending on trilinear coupling constant and other soft breaking parameters, we get vacua with both the chiral symmetry breaking and baryon number violation. Furthermore, with various choices of soft breaking parameters, we find different phase structures. Whereas in case with only quadratic breaking term, only exotic vacua with unbroken chiral symmetry arise. In addition, Seiberg's duality seems to persist in both chiral symmetry broken and unbroken phases. Second when $N_f = N_c + 1$, if the trilinear coupling is strong enough, there will arise vacua with spontaneously chiral symmetry breaking but baryon number violation. In the case with only quadratic terms, the origin of the moduli space is the only vacuum and is hence $SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R$ i nvariant. The significance of this investigation is providing an enlightenment that there is a long way to go for us to understand the non-perturbative QCD starting from the soft breaking of supersymmetric QCD.

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