Quantum computation with mesoscopic superposition states

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We present a strategy to engineer a simple cavity-QED two-bit universal quantum gate using mesoscopic distinct quantum superposition states. The dissipative effect on decoherence and amplitude damping of the quantum bits are analyzed and the critical parameters are presented.

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I. INTRODUCTION

Quantum Mechanics is now fundamental to the modern world we live and interact with, not being just the abstract realm of theoretical physics. Many new areas of emerging technology depend on the principles contained within it [1]. One of the most striking features of quantum systems are superposition states. They have given rise to a large amount of discussion in the literature [2] and now play a central role for the recent developments made in the area of quantum information. This is due to their possibility to encode information in a way impossible to be attained by any classical system. Quantum computation has become a significant subject within quantum information theory, due to the powerful property of superposition states to execute large parallel processing. Quantum information research has also improved significantly the understanding about the quantum systems involved on the factual realisation of a quantum computer and has raised many interesting problems such as in the encoding of information [3], entanglement of states [4] and quantum cryptography [5].

A number of core technologies are currently under investigation for constructing a quantum computer which is necessary to fully implement quantum algorithms. These include ion-traps [6,7], cavity QED [8–10], solid state [11,12] and liquid state NMR [13] to name but a few. The proposals to engineer a quantum computer or as a first step a single logic gate in the realm of quantum optics are generally based on discrete atomic states and cavity field number states of zero and one photons. A central proposal which has gained much attention in recent years is the Cirac and Zoller [14] trapped ions scheme to encode a n-conditional gate. We also cite the proposals of Sleator and Weinfurter [8] and of Domokos *et al.* [9] based on cavity-QED (quantum electrodynamics) technology and dealing with two-bit universal gates. Experimentally, there are few initiatives for logical operations in ion traps [7] and in NMR [13], which allow for a scalable implementation. These proposals require a technological domain, which to date has not been attained [6,15,16]. In cavity-QED technology, for optical frequencies, a conditional interaction between two-modes, the idler and pump, have been proposed to encode a phase gate (P-gate) [16] due to the high non-linearity that can be presented by single atoms. At microwave frequencies, logical elements have been demonstrated experimentally as a means of encoding a quantum memory with a single photon [17].

In this article we are going to focus on cavity QED and the technology associated with it. Cavity QED has had a very rich past and has been instrumental in a huge amount of fundamental quantum and atom optical research [18]. Such a system has been used for photon number quantum non demolition measurements [10,19], generation of single Fock state [10,20] and generation and measurement of the time of decoherence of Schrödinger cats states [19,21,22]. With such a rich history recent attention in cavity QED has been focused on quantum information. With the non demolition measurement of a single photon number in the cavity [10,20], the technology became available to encode qubits and realise a quantum gate [10]. The quantum information proposals based on cavity QED technology [8,9] makes use of only zero and one field number states. More recently there is the significant evidence of generation of trapped states of more than one photon [23] which could be used in an encoding scheme.

With a CNOT gate based on an encoding scheme using zero and one Fock states, spontaneous errors have a disastrous effect. Quantum information is irreversibly lost. It is possible to protect the system against such errors. In fact to protect the qubit against general one qubit errors it is necessary to encode the original state by distributing its quantum information over at least five qubits. Basically the 5-qubit quantum circuit takes the initial state with four extra qubits in the state $|0\rangle$ to an encoded state. This state is then protected versus all single qubit errors. Decoding this state and then applying a simple unitary transformation yields the original state. Implementing a five qubit error

correcting code is quite expensive in terms of quantum resources. Other encoding schemes may allow simpler error correction circuits.

There is no fundamental reason to restrict oneself to physical systems with two dimensional Hilbert spaces for the encoding. It may be more natural in some contexts to encode logical states as a superposition over a large number of basis states. Significant advances can be achieved. For instance in the protection against errors incoming due to the coupling of the qubit system to a dissipative environment. Recent work by Cochrane *et. al.* [24] have proposed how macroscopically distinct quantum superposition states (Schrödinger cat states) may be used as logical qubit encoding. Spontaneous emission causes a bit-flip error in these superposition state qubit encoding, which is easily corrected by a standard 3-qubit error correction circuit (compared to five qubits for Fock states). This is particularly relevant, as the bit-flip error is much easier to fix than spontaneous emission errors in Fock state systems. Another good reason for using superposition of coherent states to encode qubits, is that they are naturally generated in any cavity system, while number states of more than one photon require a large amount of control [23].

In this paper we propose how even and odd mesoscopic coherent superposition of states can be used to implement and encode a CNOT quantum gate in a realistic superconducting cavity-QED system, where those states were already generated [18,21]. We define the even cat state as the 0 qubit and the odd cat state as the 1 qubit. This encoding can be represented as

$$|0\rangle_L \equiv \frac{1}{N_+} \left(|\alpha\rangle + |-\alpha\rangle \right), \tag{1}$$

$$|1\rangle_L \equiv \frac{1}{N_-} \left(|\alpha\rangle - |-\alpha\rangle \right). \tag{2}$$

where $N_\pm = \sqrt{2(1 \pm e^{-2|\alpha|^2})}$. This normalisation is important and will be retained throughout the paper.

Given the generation of the two logic qubits how does one implement a quantum gate in cavity QED. Essentially any two-bit quantum gate is universal [8,25]. One of these universal quantum gates is the control not gate (CNOT) and consist of a conditional gate - here if the control bit is 0 the target bit will be maintained, but if the control bit is 1 the target bit will suffer a flip transform to 0. The CNOT gate can be engineered by two Hadamard transforms [26] plus a phase (P) transform [24,26]. The Hadamard transform is a single qubit operation that leads to a rotation in the state while the P-transform is a conditional two-bits transform necessary to identify the state of the control bit. The question posed here is how to identify these Hadamard and P transforms in a realizable physical cavity QED system when the encoding for the qubits is in terms of odd and even cat states.

To begin this paper we show how the apparatus similar to the one used to generate Schrödinger cat field states [19,22,27] can be generalised to perform a CNOT gate conditional transform involving two levels of a Rydberg atom and the field mesoscopic superposition state. Here the two levels of a Rydberg atom are considered to encode the controlled (or target) bit and the field cat state will be the control bit. Since the generation of Schrödinger even and odd cat field states in cavity QED experiments is dependent of a conditional measurement [21,22], giving a random outcome, we propose in Section III a strategy based on resonant atomic feedback [28] which allow us to definitely prepare the state of the control bit. The essence of this proposal involves using a feedback scheme based on the injection of appropriately prepared atoms. Basically the state of the cavity is monitored indirectly via the detection of atoms that have interacted dispersively with it. If the cavity field state is not in the required state, a photon is injected into the cavity. Finally in the last section of this paper we present a reasonable detailed discussion of dissipation and their effect on the CNOT gate. We explicitly discuss the advantages of encoding with superposition states over zero one photon number states used in previous proposals [8,9]. Attention is focused on the decoherence phenomenon, as this is one of the main difficulties for quantum computation.

II. SUPERPOSITION STATE ENCODING

In the last few years a great amount of experimental progress in cavity QED has enabled work at the level of single atoms and single photons, where only two electronic energy states of Rydberg atoms participate in the exchange of a photon with the cavity [18]. This has enabled cavity QED technology to be responsible for a large number of interesting experiments showing, the generation of mesoscopic coherent superposition field states, called Schrödinger cat states [19], the decoherence phenomenon [21] and non-local entanglement of quantum systems [17]. These systems have gained much attention due to the quantum non demolition (QND) property of measurement on the field photon number by atomic interferometry [19].

Our experimental proposal is based on the cavity-QED scheme [15,19,22,27] to generate the field superposition states and is depicted schematically in Fig.(1). It consists of a Rydberg atom beam crossing three cavities, R_1^ϕ, C and R_2^θ. Here R_1^ϕ and R_2^θ are Ramsey zones and C is a superconducting Fabry-Perot cavity of high quality factor [29]. To achieve our desired encoding the atoms are initially prepared at B in circular states of quantum principal number of the order of 50. Such atoms are well suited for this scheme since their lifetime is over 3×10^{-2}s [15,19,27].

The R_1^ϕ and R_2^θ cavities, where classical fields resonant with an atomic $|g\rangle \rightarrow |e\rangle$ transition (51.099 GHz) [20] are injected during the time of interaction with the atoms, constitute the usual setup for Ramsey interferometry [15]. There, for a selected atomic velocity, the state of the atom will suffer a rotation in the vector space spanned by $\{|e\rangle, |g\rangle\}$.

The experiment is started when one selects the initial state of an atom prepared in the $|g\rangle$ or $|e\rangle$ by the laser field L. This atom has a resonant interaction with the field in R_1^ϕ given by [15,30]

$$H_I = \hbar\Omega \left(a_r \sigma^+ + a_r^\dagger \sigma^- \right) \tag{3}$$

where $\sigma^+ \equiv |e\rangle \langle g|$ and $\sigma^- \equiv |g\rangle \langle e|$ are the atomic pseudo-spin Pauli operators, $a_r^\dagger$ (a_r) are the creation (annihilation) operator for the mode of the field in R_1^ϕ and Ω is the one photon Rabi frequency. With a proper choice of the field phase ϕ in R_1^ϕ the atomic states $|g\rangle$ and $|e\rangle$ are rotated to

$$|g\rangle \rightarrow \frac{1}{\sqrt{2}} \left(|g\rangle + e^{-i\phi} |e\rangle \right) \tag{4}$$

$$|e\rangle \rightarrow \frac{1}{\sqrt{2}} \left(|e\rangle - e^{i\phi} |g\rangle \right) \tag{5}$$

The cavity C is tuned near the resonance of the transitions between the atomic states $|e\rangle$ and $|i\rangle$, a reference state corresponding to the higher level from $|e\rangle$. The frequency of the transition $|e\rangle \rightarrow |i\rangle$ is 48.18 GHz [19] and is distinct of any transition involving the level $|g\rangle$. The mode geometry inside the cavity is configured in such a way that the intensity of the field rises and decreases smoothly through with the atomic trajectory inside C. For sufficiently slow atoms and for sufficiently large cavity mode detuning from the $|e\rangle \rightarrow |i\rangle$ frequency transition, the atom-field evolution is adiabatic and no photonic absorption or emission occurs [15]. On the other hand, dispersive effects emerge - an atom in the state $|e\rangle$ crossing C induces a phase shift in the cavity field which can be adjusted by a proper selection of the atomic velocity (~ 100 m/s) [19]. For a π phase shift the coherent field $|\alpha\rangle$ in C transforms to $|-\alpha\rangle$. On the other hand, the phase shift caused by an atom in the $|g\rangle$ state is null. The atom field interaction can be written effectively as [31]

$$H_{off} = \hbar\Omega_2 a^\dagger a \, \sigma^+ \sigma^- \tag{6}$$

where Ω_2 is the effective Rabi frequency for the interaction of the atom with the field and $a^\dagger$ (a) is the creation (annihilation) operator for the field in C. After the atomic interaction with the field in C, the atom crosses the second Ramsey zone R_2^θ which introduces a new rotation in the atomic vector space, analogously to Eq. (4) and (5), but for the phase θ. The atomic state is detected in D by an ionization zone detector, instantaneously giving the atomic state and the field state in C. This is due to the entanglement of their states. The important point we emphasise here is that the resonant interaction of the Ramsey zones can be used as Hadamard transform since they induce rotations in the vector space of the target bit (atomic state) and the off-resonant interaction between atom and field in C can be used for the P-transform [24].

We begin the description of the implementation of the CNOT gate by specifying that the coherent field state will be responsible for the encoding of the control bit and the atomic states $|g\rangle$ and $|e\rangle$ will be the target bits $|0\rangle_T$ and $|1\rangle_T$, respectively. The procedures to implement the CNOT gate is described as follows. The laser field L prepares the target bit in $|g\rangle$ or $|e\rangle$; a one bit Hadamard transform is applied to the target qubit by the first Ramsey zone R_1^ϕ; then the two-bit P-gate is realized by the off-resonant atom-field interaction in C and the second Hadamard transform is realized by R_2^θ. Finally the atom is detected simultaneously specifying the atomic and field states. The effective unitary operator related to the evolution of the atom-field in cavity C entangled state, due to the sequential interaction of the atom with the field in R_1^ϕ, C and R_2^θ is given by

$$U(\phi, \theta) = U_2^\theta \exp\left[i\mu a^\dagger a \sigma^+ \sigma^- \right] U_1^\phi \tag{7}$$

where U_1^ϕ and U_2^θ are the unitary operators related to the evolution of the joint state in R_1^ϕ and R_2^θ, respectively. In eqn. (7), $\mu = \Omega_2 t$, where t is the time interval for the off-resonant interaction. Proceeding through the immediate

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states generated by the atomic passing through each of the cavities it is easy to show, for $\phi = \pi$ and $\theta = 0$, the following table

Input	R_1^ϕ	C	R_2^θ	Output
$\lvert g \rangle \otimes \lvert 0 \rangle_L$ $\rightarrow$	$\frac{1}{\sqrt{2}} (\lvert g \rangle - \lvert e \rangle) \otimes \lvert 0 \rangle_L$ $\rightarrow$	$\frac{1}{\sqrt{2}} (\lvert g \rangle - \lvert e \rangle) \otimes \lvert 0 \rangle_L$ $\rightarrow$	$\lvert g \rangle \otimes \lvert 0 \rangle_L$	
$\lvert e \rangle \otimes \lvert 0 \rangle_L$ $\rightarrow$	$\frac{1}{\sqrt{2}} (\lvert e \rangle + \lvert g \rangle) \otimes \lvert 0 \rangle_L$ $\rightarrow$	$\frac{1}{\sqrt{2}} (\lvert e \rangle + \lvert g \rangle) \otimes \lvert 0 \rangle_L$ $\rightarrow$	$\lvert e \rangle \otimes \lvert 0 \rangle_L$	
$\lvert g \rangle \otimes \lvert 1 \rangle_L$ $\rightarrow$	$\frac{1}{\sqrt{2}} (\lvert g \rangle - \lvert e \rangle) \otimes \lvert 1 \rangle_L$ $\rightarrow$	$\frac{1}{\sqrt{2}} (\lvert e \rangle + \lvert g \rangle) \otimes \lvert 1 \rangle_L$ $\rightarrow$	$\lvert e \rangle \otimes \lvert 1 \rangle_L$	
$\lvert e \rangle \otimes \lvert 1 \rangle_L$ $\rightarrow$	$\frac{1}{\sqrt{2}} (\lvert e \rangle + \lvert g \rangle) \otimes \lvert 1 \rangle_L$ $\rightarrow$	$\frac{1}{\sqrt{2}} (\lvert g \rangle - \lvert e \rangle) \otimes \lvert 1 \rangle_L$ $\rightarrow$	$\lvert g \rangle \otimes \lvert 1 \rangle_L$	

which verifies the standard CNOT truth-table.

Above we have discussed a setup where the atoms encode the target qubit and the cavity field mode encodes the control qubit. Nevertheless, it is also possible to proceed with atoms responsible by both the control and target qubit. In this second case, the state of the control atom must be transferred to the cavity C and with a proper selection of the cavity state (to what we address to the next section) the procedure for implementing the CNOT gate follows as above. After the second atom, which encodes the target qubit interaction in the process described above, a third atom is sent across the system to read the cavity state in a process similar to the scheme already proposed by Sleator and Weinfurter [8]. To envisage a quantum network, *i.e.*, the interconnection of quantum gates, the carriers of qubits between gates can be achieved by atoms transferring the state of one cavity to another [8], or even by the coupling of these cavities by superconducting wave-guides which can be responsible by an exchange of states [32] between two gates.

It is important for this proposal to include a brief discussion of the realistic parameters. We first note that an atom crosses the cavity in a time of order of 10^{-4} s, which is well below the relaxation time of the field inside C (typically of the order of 10^{-3}-10^{-2}s for Niobium superconducting cavities [29]) and below the atomic spontaneous emission time of $(3 \times 10^{-2}$s) [15,19]. Therefore, the limits considered in that proposal must be far away from the problematic limits found in those experiments.

Our entire proposal for encoding a CNOT gate discussed here is reliant on being able to generate the zero ($\lvert 0 \rangle_L$) and one ($\lvert 1 \rangle_L$) logical states. For this reason we address in Section (III) a strategy for guaranteeing the exact choice of the initial cavity field state. Without such a strategy, the logical states can only theoretically be generated with a 50% probability. More explicitly there is a 50% probability that the $\lvert 0 \rangle_L$ state actually contains only even photon number states and a 50% probability that it contains only odd photon number states.

III. INITIAL CONDITIONS FOR THE CONTROL BIT

Our generation of the CNOT gate outlined in the previous section relies on our ability to be able to generate the coherent logical state encoding with a high degree of certainty. The initial state of the control bit (the field state of the cavity) has to be prepared with a probability greater than 50% as usually occurs in the preparation of superposition field states by Rydberg atoms. The state of the field in the cavity is $\lvert 0 \rangle_L$ or $\lvert 1 \rangle_L$ conditioned by the measurement of the atomic $\lvert g \rangle$ or $\lvert e \rangle$ state in the process of generation of superposition states. Such a scheme is analogous to the depicted in fig.(1), however here we have $\theta = \pi$ in the second Ramsey zone and for the initial cavity state a coherent one, considering that the atom was prepared in the $\lvert e \rangle$ state. Let us suppose we are interested in preparing the state $\lvert 0 \rangle_L$ for the control bit. If the atomic state $\lvert e \rangle$ was detected, then our scheme would have failed. For it to succeed we have to apply a process conditioned to the measurement of the atomic $\lvert e \rangle$ state to guarantee the flip of the cavity field state from $\lvert 1 \rangle_L$ to $\lvert 0 \rangle_L$. Analogously we have to apply a process conditioned to the measurement of the atomic $\lvert g \rangle$ state to guarantee the flip of the cavity field state from $\lvert 0 \rangle_L$ to $\lvert 1 \rangle_L$ if we are interested in prepare the control bit in the $\lvert 1 \rangle_L$ state.

First noting the fact that an atom interacting resonantly with the field in C, with a controlled velocity, can exchange a single photon and regarding that a single photon emission by the cavity field causes

$$a \lvert 0 \rangle_L = \alpha \frac{N_-}{N_+} \lvert 1 \rangle_L \approx \alpha \lvert 1 \rangle_L \quad (\alpha \text{ large}) \tag{8}$$

$$a \lvert 1 \rangle_L \approx \alpha \lvert 0 \rangle_L . \tag{9}$$

We can now formulate an atomic feedback scheme that operates whenever the atomic detector clicks, if we are interested in the control qubit $\lvert 0 \rangle_L$ or $\lvert 1 \rangle_L$. In fact this process is very similar to the stroboscopic feedback proposed

by Vitali *et al.* [28] for the suppression of decoherence of superposition field states. Of course we do not need a stroboscopic action, but just one event conditioned to the atomic state measurement.

The scheme proposed is depicted schematically in fig.(2), where B_2 is a source of atoms which are tuned in resonance with the field in C by the Stark shift conditioned to the atomic state measurement made in the ionization zones D_e or D_g. The resonant atom-field interaction is given by the Hamiltonian

$$H_I = \hbar \Gamma \left(a \sigma_f^+ + a^\dagger \sigma_f^- \right) \tag{10}$$

where Γ is the coupling constant between the field and atomic variables. Here σ_f^+ and σ_f^- are rising and lowering operator for the feedback atom. If the feedback atom is prepared in the state $|e\rangle$ then the field state is given by

$$\rho_f^{\binom{g}{e}} = \cos(\Gamma \tau \sqrt{a^\dagger a + 1}) \, \rho_C^{\binom{g}{e}} \, \cos(\Gamma \tau \sqrt{a^\dagger a + 1}) + a^\dagger \, \frac{\sin(\Gamma \tau \sqrt{a^\dagger a + 1})}{\sqrt{a^\dagger a + 1}} \, \rho_C^{\binom{g}{e}} \, \frac{\sin(\Gamma \tau \sqrt{a^\dagger a + 1})}{\sqrt{a^\dagger a + 1}} \, a. \tag{11}$$

where $\rho_C^{\binom{g}{e}}$ is the density operator associated with the field state in C before the feedback action. Here ρ_f^g (ρ_f^e) explicitly is the ground (excited) state density operator. τ is the time of interaction of the feedback atom with the field.

As a measure of the field state in the cavity a second atom is sent through the setup and again measured in D_g or D_e [27]. The conditional probability $P^{\binom{g}{e}}(T)$ that the second atom will be detected in the $|g\rangle$ or $|e\rangle$ state, at the time T after detection of the first atom follows

$$P^{\binom{g}{e}}(T) = \frac{1}{2} \left\{ 1 \pm \frac{1}{1 + \cos\varphi e^{-2|\alpha|^2}} \left[e^{-2|\alpha|^2 e^{-\gamma T}} + \cos\varphi e^{-2|\alpha|^2 \left(1 - e^{-\gamma T} \right)} \right] \right\}, \tag{12}$$

conditioned to $\varphi = 0$ $[\pi]$ if the first atom is detected in the $|g\rangle_1$ $[|e\rangle_1]$ state and to the signal + [-] for the second atom be detected in the $|g\rangle_2$ $[|e\rangle_2]$ state. For the computation of Eq. (12) at time T we have included the relaxation of the field state due to dissipation. Considering a reservoir at zero temperature, this state is now given by

$$\rho_C^{\binom{g}{e}}(T) = \frac{1}{N_\pm^2} \left\{ |\alpha e^{-\gamma T/2} \rangle\langle \alpha e^{-\gamma T/2}| + |-\alpha e^{-\gamma T/2} \rangle\langle -\alpha e^{-\gamma T/2}| \right.$$
$$\left. \pm \, e^{-2|\alpha|^2(1 - e^{-\gamma T})} \left[|-\alpha e^{-\gamma T/2} \rangle\langle \alpha e^{-\gamma T/2}| + |\alpha e^{-\gamma T/2} \rangle\langle -\alpha e^{-\gamma T/2}| \right] \right\}. \tag{13}$$

where γ is the relaxation constant of the field. By analyzing Eq.(12) we observe that if the second atom is detected instantaneously after the first one ($\gamma T \ll 1$) then

$$P^{\binom{g}{e}}(T) = \frac{1}{2} \left[1 \pm \cos\varphi \right], \tag{14}$$

again with $\varphi = 0$ (π). This gives the conditional probability of detection of the first atom in $|e\rangle_1$ and the second atom in $|e\rangle_2$ as $P(e,e) \equiv P^{\binom{g}{e}}|^{\pi,-} = 1$ and the probability of detection of the first in $|g\rangle_1$ and the second in $|e\rangle_2$ as $P(g,e) \equiv P^{\binom{g}{e}}|^{0,-} = 0$. Analogously $P(g,g) \equiv P^{\binom{g}{e}}|^{0,+} = 1$ and $P(e,g) \equiv P^{\binom{g}{e}}|^{\pi,+} = 0$. This is a signature of the measurement for the state in which the cavity field was prepared. It presents our undesired results $P(g,e)$ and $P(e,g)$ equal to zero, that is no probability of them occurring. However if the feedback loop is taken in to account in the calculation of the probabilities $P(g,e)$ and $P(e,g)$ then, instead of using $\rho_C^{\binom{g}{e}}$ in Eq. (12) we must use $\rho_f^{\binom{g}{e}}(T+\tau)$ from Eq. (11), where $T' = T + \tau$. Substituting Eq. (13) for the field relaxation into Eq. (11) it follows

$$P_f^{\binom{g}{e}}(T+\tau) = \frac{1}{2} \left\{ 1 \pm \frac{2}{N^2} e^{-|\alpha|^2} \sum_m \frac{\left(-|\alpha|^2 e^{-\gamma T} \right)^m}{m!} \left[1 + (-1)^m \cos\varphi \, e^{-2|\alpha|^2 \left(1 - e^{-\gamma T} \right)} \right] \cos\left(2\Gamma\tau\sqrt{m+1} \right) \right\}, \tag{15}$$

which accounts for the conditional probability of detection of the second atom in the state $|g\rangle_2$ $[|e\rangle_2]$ if the first atom was detected in the state $|e\rangle_1$ $[|g\rangle_1]$. In Fig. (3) we show the respective four conditional probability of atomic detection, $P(e,e)$ and $P(g,g)$ without feedback and $P_f(e,g)$ and $P_f(g,e)$ considering the feedback loop for some values of $\Gamma\tau$. This shows the feasibility of the feedback to control the initial state of the cavity. The figure is plotted until $\gamma T = 1$ since there is no reason to consider times longer than this once the decoherence of the state has already taken place. In fact the scale of time to be taken into account in figures 3(c) and 3(d) is $T' = T + \tau$, the time interval after the

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detection of the first atom plus the time interval of the feedback atom. In these figures the continuous solid line represents the absence of feedback. As can be seen there is an optimum value for the feedback process at $\Gamma\tau = \pi/6$ which gives a 93% of chance for the cavity field qubit to be prepared in the right state. It also must be noted that an optimal value is possible only when the feedback atom is sent instantaneously after the click of the respective detector. The performance of the setup decreases considerably when a time delay exists, as can be observed in Figs. 3(c) and 3(d) for $\gamma T' \geq 0.1$. The limit of those curves around 0.5 means that the field state already decohered, and so there is 50% of chance again for generation of the $|0\rangle_L$ or $|1\rangle_L$ states. For $\gamma T' > 1.0$ (not shown in the figures) the effect of dissipation implies amplitude damping. The field asymptotically tends to be in a vacuum state, and when this occurs is easily shown through Eq. (15) that the second atom will always be detected in the $|g\rangle$ state. With the feedback it tends to always be detected in $|e\rangle$ state.

IV. EFFICIENCY AND SOURCES OF ERROR

This section discusses in detail the advantages and disadvantages of encoding qubits in superposition states instead of number states of only one and zero photon and the effect of dissipation on these. As is already well known for cavity QED experiments the dominant source of error that will affect the implementation of quantum logic elements is cavity damping. Since the cavities are not isolated, when the states $|0\rangle_L$ or $|1\rangle_L$ are constructed, the presence of dissipative effects will alter the free evolution of the cavity field state introducing amplitude damping as well as coherence loss. The zero temperature master equation describing the bosonic damping is simply

$$\frac{d\rho}{dt} = \frac{\gamma}{2}\left(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a\right), \tag{16}$$

and its solution for any initial state can be written as [28]

$$\rho(t) = \sum_{k=0}^{\infty} \Upsilon_k(t)\rho(0)\Upsilon_k^\dagger(t), \tag{17}$$

where

$$\Upsilon_k(t) = \sum_{n=k}^{\infty} \sqrt{\binom{n}{k}} \left(e^{-\gamma t}\right)^{(n-k)/2} \left(1 - e^{-\gamma t}\right)^{k/2} |n - k\rangle\langle n|. \tag{18}$$

We are interested in the effect of dissipation on the information encoded in the qubits. For that we will consider first a superposition of Schrödinger cats qubits and thereafter a superposition of one and zero photon number states qubit encoding.

The action of a single decay event Υ_1 on the state

$$|\psi_1\rangle = E_1 |0\rangle_L + E_2 |1\rangle_L, \tag{19}$$

leads to

$$\Upsilon_1 |\psi_1\rangle = \alpha \left(1 - e^{-\gamma t}\right)^{1/2} e^{-|\alpha|^2\left(1 - e^{-\gamma t}\right)/2} \left(E_1 \frac{N_-}{N_+} |\tilde{1}\rangle_L + E_2 \frac{N_+}{N_-} |\tilde{0}\rangle_L\right), \tag{20}$$

that is, a simple bit-flip occurs. Here $|\tilde{0}\rangle_L \equiv \frac{1}{N_+}\left(\left|e^{-\gamma t/2}\alpha\right\rangle + \left|-e^{-\gamma t/2}\alpha\right\rangle\right)$ and $|\tilde{1}\rangle_L \equiv \frac{1}{N_-}\left(\left|e^{-\gamma t/2}\alpha\right\rangle - \left|-e^{-\gamma t/2}\alpha\right\rangle\right)$, account for the amplitude damping. A simple unitary process will transform Eq. (20) back to Eq. (19), meaning the reversibility of the process. Under a double decay event Υ_2,

$$\Upsilon_2 |\psi_1\rangle = \alpha^2 \left(1 - e^{-\gamma t}\right) e^{-|\alpha|^2\left(1 - e^{-\gamma t}\right)/2} \left(E_1 |\tilde{0}\rangle_L + E_2 |\tilde{1}\rangle_L\right), \tag{21}$$

which is exactly our initial state but with amplitude damping. This special superposition is invariant under even number of decay events. This fact brings one important information about these states. However a single decay event, Υ_1 on the Fock superposition state

$$|\psi_2\rangle = F_1 |0\rangle + F_2 |1\rangle, \tag{22}$$

leads to

$$\Upsilon_1 \left| \psi_2 \right\rangle = F_2 \left(1 - e^{-\gamma t} \right)^{1/2} \left| 0 \right\rangle . \tag{23}$$

No unitary operation can recover (22) indicating the irreversibility of the process. This means that in one photon state information processing schemes, one single photon decay is fatal, since there is no way in which the resulting error can be corrected once it occurs. However for qubits consisting of superpositions of odd and even number states, one decay event cause a bit-flip, which could be, in principle be corrected. So here, we classified two different kind of error arising from dissipation, one impossible to be corrected (called irreversible error) and the other a bit-flip which can be corrected (reversible error) by unitary processes [24,33].

There is a number of error correction schemes that protect the quantum information against single errors. As we have mentioned previously a spontaneous emission error for the Schrödinger cat encoding results in a bit-flip. It is well known that such errors can easily be prevented by a 3-qubit error correction circuit [34] (schematically depicted in Fig (4a)). This circuit is reasonably simple and the superposition state it produces is relatively simple. In fact for an arbitrary qubit $\left| \psi \right\rangle = E_1 \left| 0 \right\rangle_L + E_2 \left| 1 \right\rangle_L$ the correction circuit generates the encoded superposition state

$$\left| \psi \right\rangle = E_1 \left| 000 \right\rangle + E_2 \left| 111 \right\rangle . \tag{24}$$

To protect against arbitrary error requires normally a 5-qubit error correction circuit [35] (schematically depicted in Fig (4b)). For an arbitrary qubit $\left| \psi \right\rangle = F_1 \left| 0 \right\rangle + F_2 \left| 1 \right\rangle$, the correction circuit generates the superposition state

$$\begin{aligned}
\left| \psi \right\rangle = {} & F_1 \left[\left| 00000 \right\rangle + \left| 00110 \right\rangle + \left| 01001 \right\rangle - \left| 01111 \right\rangle + \left| 10011 \right\rangle + \left| 10101 \right\rangle + \left| 11010 \right\rangle + \left| 11100 \right\rangle \right] \\
& + F_2 \left[\left| 00011 \right\rangle - \left| 00101 \right\rangle - \left| 01010 \right\rangle - \left| 01100 \right\rangle - \left| 10000 \right\rangle + \left| 10110 \right\rangle + \left| 11001 \right\rangle + \left| 11111 \right\rangle \right] .
\end{aligned} \tag{25}$$

This is quite a complicated superposition state to create (as can be seen from the quantum circuit). The 3-qubit correction circuit is much simpler and hence we see the advantage of the Schrödinger cat encoding. Also, while here we are only discussing a single gate, a reasonable quantum computer has to be constituted of many gates. Then, if the above 5-qubit protection circuit has to be implemented, this will become much more expensive in terms of qubits in comparison to the 3-qubit circuit for bit-flip protection. The bit-flip protection scheme saves 2 qubits at each needed qubit in comparison to the 5-qubit protection circuit described above. It does however only protect against a specific type of error. An unavoidable error incoming from dissipation over superposition states is decoherence. Let us consider the general effect of dissipation on the quantum coherent superposition state. At zero temperature the state of the cavity field is described by the density operator

$$\begin{aligned}
\rho_C^{\pm}(t) = {} & \frac{1}{N_{\pm}^2} \left\{ \left| \alpha e^{-\gamma t/2} \right\rangle \left\langle \alpha e^{-\gamma t/2} \right| + \left| - \alpha e^{-\gamma t/2} \right\rangle \left\langle -\alpha e^{-\gamma t/2} \right| \pm e^{-2|\alpha|^2 (1 - e^{-\gamma t})} \right. \\
& \left. \times \left[\left| -\alpha e^{-\gamma t/2} \right\rangle \left\langle \alpha e^{-\gamma t/2} \right| + \left| \alpha e^{-\gamma t/2} \right\rangle \left\langle -\alpha e^{-\gamma t/2} \right| \right] \right\} .
\end{aligned} \tag{26}$$

We see that two characteristic times are involved in this evolution. The first one, the *decoherence time* is the time in which the pure state given by Eq.(7) is turned into a statistic mixture

$$\rho_C(t) \approx \frac{1}{2} \left\{ \left| \alpha \right\rangle \left\langle \alpha \right| + \left| - \alpha \right\rangle \left\langle -\alpha \right| \right\} , \tag{27}$$

the second is the *damping time* or *relaxation time* of the field $t_c = \gamma^{-1}$, the time that the dissipative effect reduces the energy of the field leading it in to a vacuum state.

The decoherence of the field state is characterized by the $\exp \left[-2 \left| \alpha \right|^2 \left(1 - e^{-\gamma t} \right) \right]$ factor, that for short times, $\gamma t \ll 1$, turns to be $\exp \left[-2 \left| \alpha \right|^2 \gamma t \right]$ and the coherence decays with the time $t_d = \left(2\gamma \left| \alpha \right|^2 \right)^{-1}$. Unfortunately the coherence time depends on inversely on $\left| \alpha \right|^2$ and hence the larger the $\left| \alpha \right|^2$ the smaller the coherence time. Decoherence constitutes the main obstacle to quantum computation [1], since the encoding is completely based in the purity of the field state.

The relaxation time of microwave fields in superconducting cavities is of the order of 10^{-2} s [15], what means $t_d \approx 10^{-2} \left| \alpha \right|^{-2}$ s. So all the interactions involved in this proposal must consider this time and more specifically the number of photons as critical quantities. Moreover, the initial information encoded in the superpositions given by Eqs. (19) and (22) also suffer the effect of decoherence, which for $\left| \alpha \right| \approx 1$ occur at the same time for both encoding schemes. Again, decoherence prevention schemes play a crucial role for any quantum information encoding.

One favorable point for the superposition state encoding is that proposals for sustaining the coherence of these field states have already been considered which could be well adapted for our case. For example the stroboscopic feedback proposal of Vitali *et al.* [28]. This proposal is particularly appropriate here since it guarantee that at each single decay event a feedback atom is sent through the setup restituting the coherence and the state parity. In fact in [28] the authors claim that the coherence is restored but for a slightly different state. For the proposal presented here what is important is not the original superposition of states, but the original parity of the state, if it was originally a superposition of even or odd photon number states. It is important to emphasize the experimentally critical values for the physical elements involved. The time of flight of the atom across the setup (10^{-4} s), the relaxation time of the field (10^{-3}-10^{-2}s for Niobium superconducting cavities) and the atomic spontaneous emission time (3×10^{-2}s) [15,29].

V. CONCLUSION

In conclusion we have presented a feasible scheme to encode the CNOT quantum gate, based on a field superposition of states. These states have been already generated in superconducting microwave cavities which constitute a system almost dominated by the current technology [15]. The proposal here to encode the CNOT gate based on a superposition of states is less susceptible to irreversible errors due to dissipative effect imposed by the environment than number states [8,9]. The generation of these kind of states is dependent of a conditional measurement giving a random assignment of initial control bits, which would be useless if no further process is considered. Hence we propose a conditional feedback scheme, which guarantees that the initial control bit is prepared in the required state. Once the amplitude damping of a coherent state (at zero temperature) still constitutes a coherent state the method proposed works until the inevitable effect of decoherence takes place. For that a reset of the qubits must be done after a time of the order of the time of the decoherence or a coherence control scheme [28] must be applied. The reset process is done repeating the process here described.

The state of a logic unit can be transferred to another logic unit (if the time of decoherence is respected), constituting a sort of quantum memory circuit [17]. That can be attained by the proper choice of atomic interactions between atoms and the field in the microwave cavity or even by the direct photonic process of coupling two cavities by a superconducting wave-guide, which permits an exchange of information (exchange of states) between the coupled units [32,36]. This problem is of fundamental importance for the engineering of quantum networks [37]. The major sources of error here are the loss of coherence of the field state and the control bit-flip due to the dissipative effect. Analysis of these kind of errors on quantum networks constituted by the basic element here described is left for further investigation.

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FIG. 1. Experimental setup for a CNOT gate. Here B is a source of atoms, L is a laser field which prepares the atomic state, C is a superconducting microwave cavity, R_1^ϕ and R_2^θ are Ramsey zones and D is an ionization zone atomic detector, while Ss are classical microwave sources. The state of the field in C encodes the control bit and the atomic state, the target bit.

FIG. 2. Experimental feedback setup for the control of the initial state of the field in C. Here B_1 and B_2 are atomic sources of atoms, L is a laser field that prepares the atomic state, C is a superconducting microwave cavity, R_1^ϕ and R_2^θ are Ramsey zones, D_e, D_g are ionization zone atomic detectors and Ss are classical microwave sources. Once the undesired state is measured by D_e or D_g the B_2 source of atoms is turned on and a resonant interacting atom flips the state of the field in C.

FIG. 3. Conditional probability of detection for the second atom being in a particular state given the result of a measurement on the first atom. Figure a) shows the probability of obtaining the second atom in the excited state $|e\rangle_2$ given the first atom was measured in the $|e\rangle_1$ state. Figure b) shows the probability of obtaining the second atom in the ground state $|g\rangle_2$ given the first atom was measured in the $|g\rangle_1$ state. Figure c) gives the conditional probability of detecting the second atom in the $|g\rangle_2$ state given the first atom was measured in the $|e\rangle_1$ when the feedback action is taken into account. Here $T' = T + \tau$. Figure d) gives the conditional probability of detecting the second atom in the $|e\rangle_2$ state given the first atom was measured in the $|g\rangle_1$ when the feedback action is taken into account. Again $T' = T + \tau$. In Figures c) & d) the solid line represents the absence of feedback.

FIG. 4. Three (a) and Five (b) qubit error correction circuits. The 3-qubit circuit correct bit-flip errors while the 5-qubit circuit correct arbitrary errors.
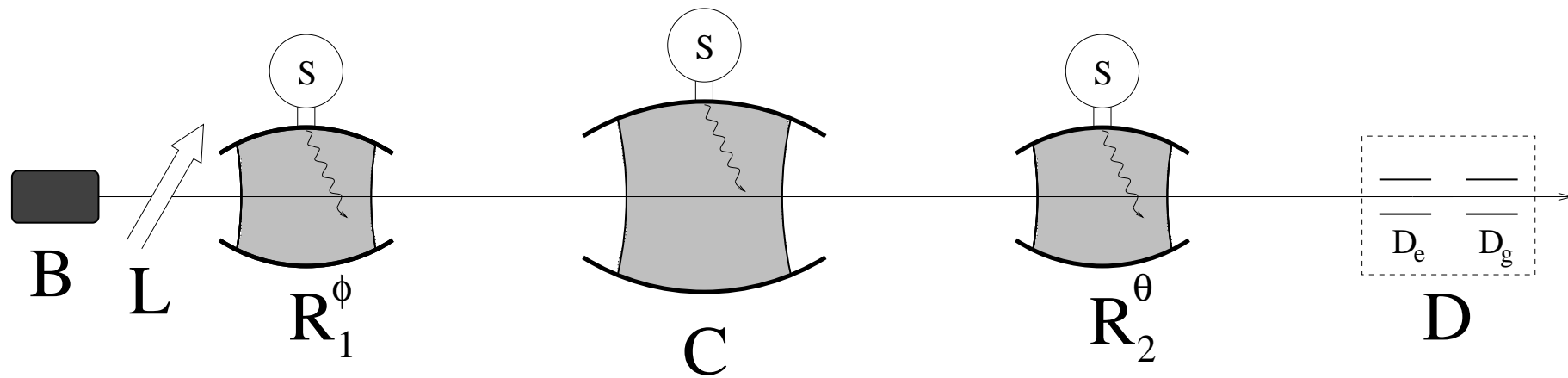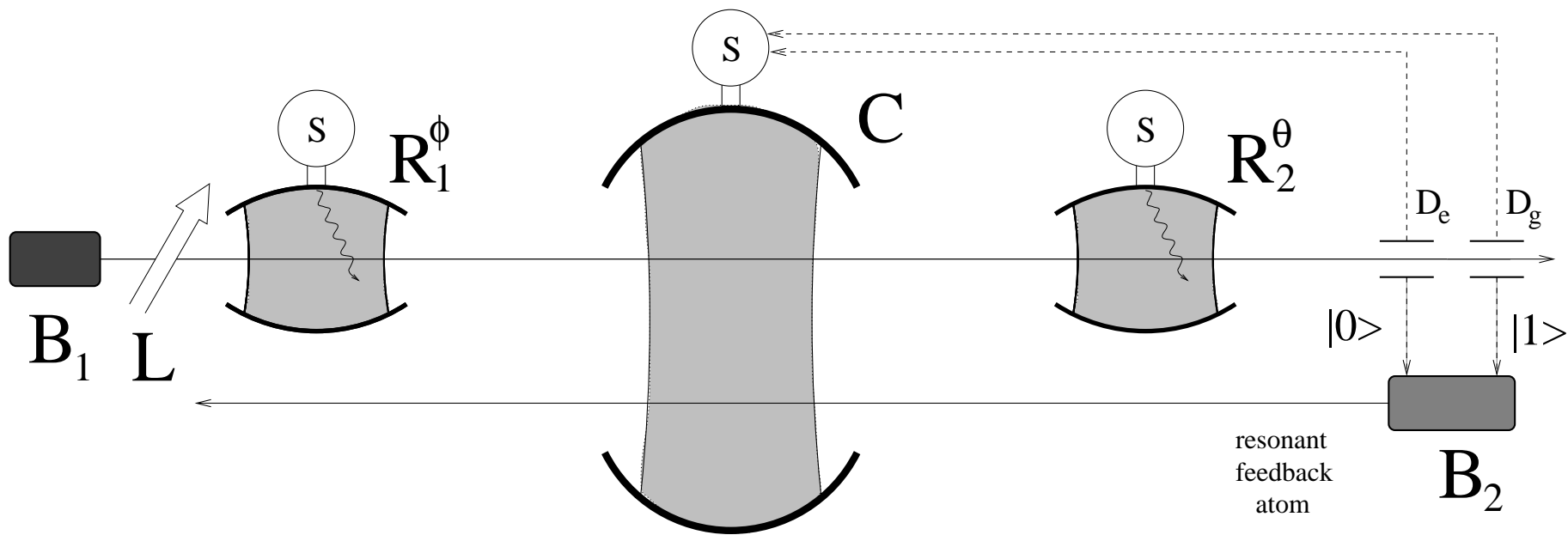
Fig. (1) - M.C. de Oliveira et al.

Fig. (2) - M.C. de Oliveira et al.
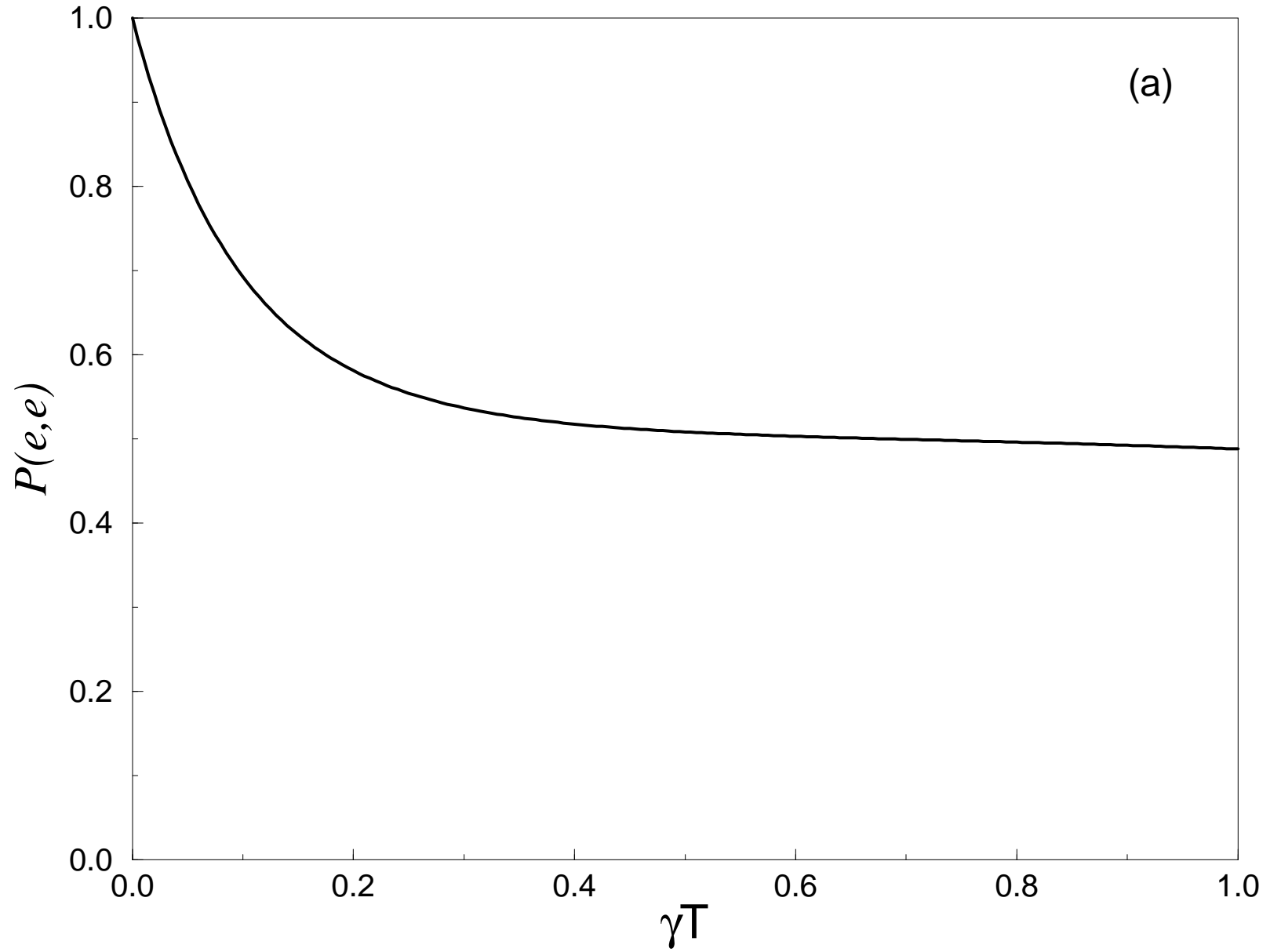
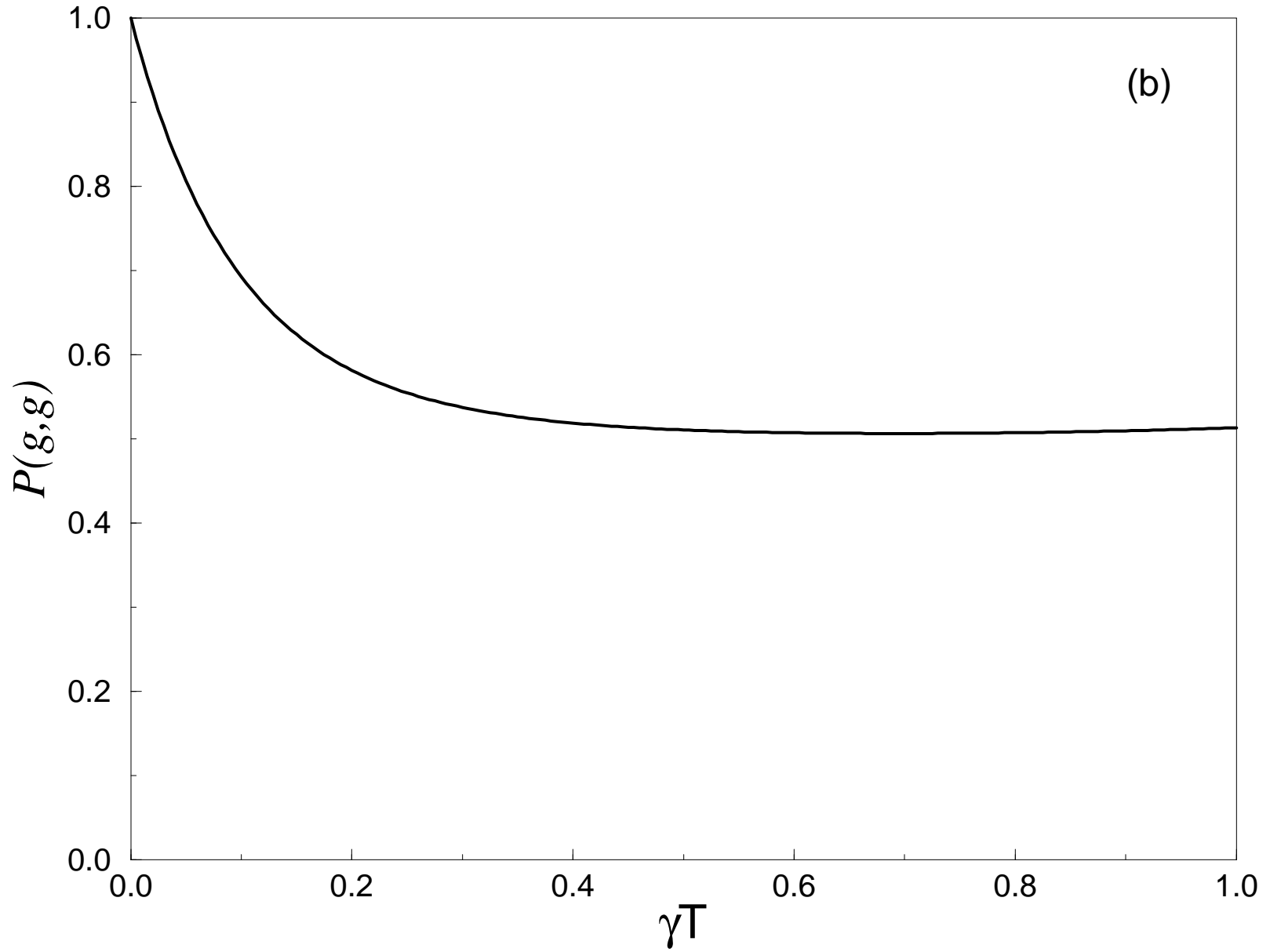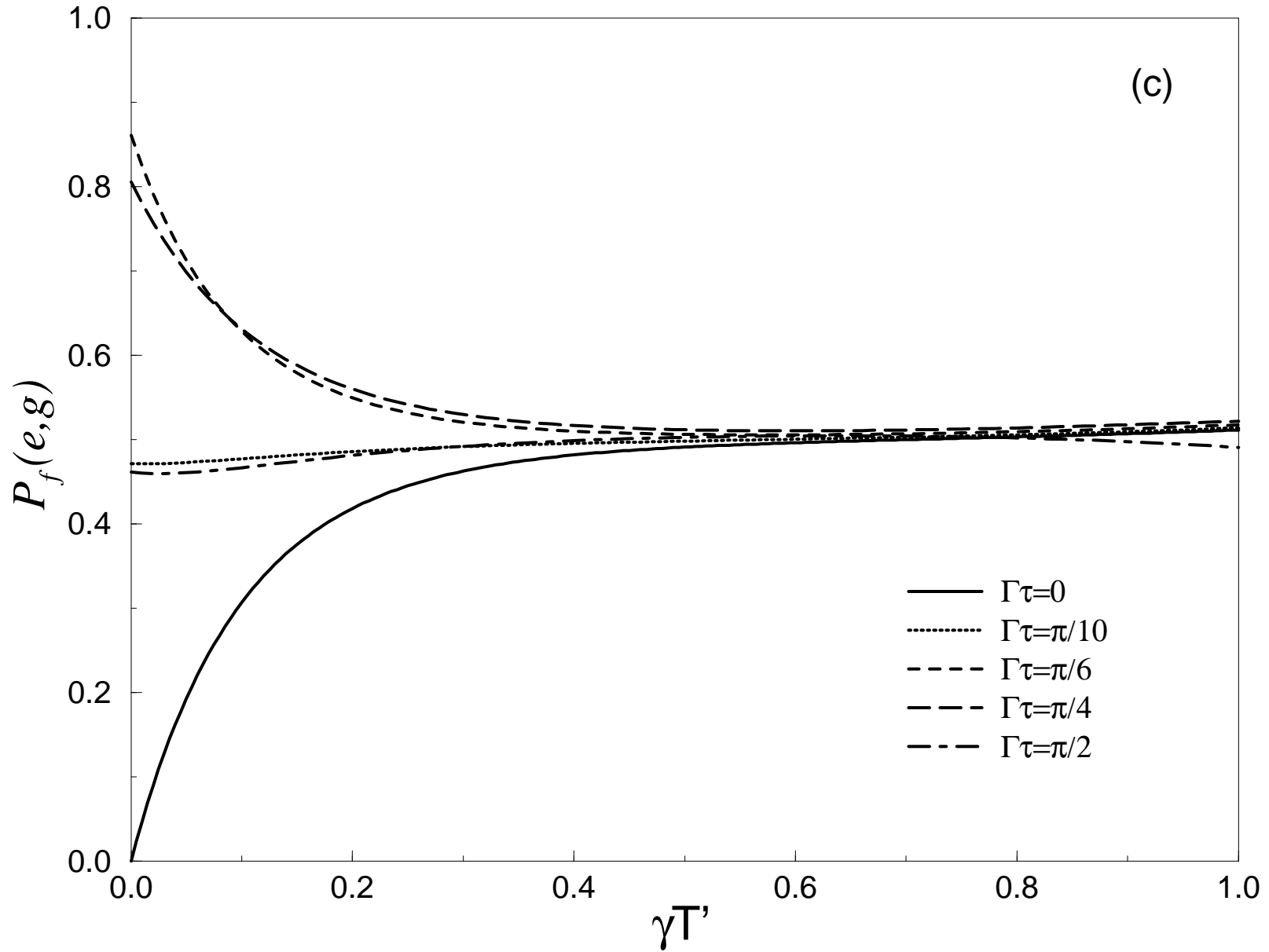Fig. (3a) - M.C. de Oliveira et al.

Fig. (3b) - M.C. de Oliveira et al.

Fig. (3c) - M.C. de Oliveira et al.

Fig. (3d) - M.C. de Oliveira et al.