

Quantum Decoherence from Adiabatic Entanglement

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Abstract

In order to understand quantum decoherence of a quantum system due to its interaction with a large system behaving classically, we introduce the concept of adiabatic quantum entanglement based on the Born-Oppenheimer approximation. In the adiabatic limit, it is shown that the wave function of the total system formed by the quantum system plus the large system can be factorized as an entangled state with correlation between adiabatic quantum states and quasi-classical motion configurations of the large system. In association with a novel viewpoint about quantum measurement, which has been directly verified by most recent experiments [e.g, S. Durr et.al, Nature 33, 359 (1998)], it is shown that the adiabatic entanglement is indeed responsible for the quantum decoherence and thus it can be regarded as a “clean” quantum measurement when the large system behaves as a classical object. The large system being taken respectively to be a macroscopically distinguishable spatial variable, a high spin system and a harmonic oscillator with a coherent initial state, three illustrations are present with their explicit solutions in this paper.

I. INTRODUCTION

In quantum measurement process, wave packet collapse (WPC, also called von Neumann's projection or wave function reduction) physically resembles the disappearance of interference pattern for Young's two-slit experiment in the presence of a "which-way" detector. Associated with the wave-particle duality, this phenomenon of losing quantum coherence is referred to as the so-called quantum decoherence. In fact, before a measurement to observe "which-way" the particle actually takes, the quantum particle seems to move from a point to another along several different ways simultaneously. This just reflects the wave feature of a quantum particle. The detection of "which-way" means a probe for the particle feature, which leads to the disappearance of wave feature or quantum decoherence[2].

Further explanation for decoherence phenomena was made in the view point of complementarity by Niels Bohr based on Heisenberg's position-momentum uncertainty [3]: A particular interaction between a classical instrument (detector) and the measured quantum system can be regarded as a quantum measurement; but once enough data about the states of the quantum system is "read out" from the motion configuration of the detector, the interaction unavoidably destroys the interference pattern. According to Heisenberg's uncertainty principle, to locate the position of a particle to the uncertainty of order Δx along the direction orthogonal to its moving direction, the "which-way" measurement must kick the momentum of the particle to an uncertainty of order $1/\Delta x$ and thereby washes out the spatial interference pattern. (In this paper the Plank constant is taken to be unity). Bohr's argument sounds correct, but a recent experiment [4] on Bragg's reflection of cold atoms shows that Schrodinger's concept of entangled state, rather than the unavoidable measurement distribution, is crucial for the wave-particle duality in this "which-way" experiment. Another "which-way" experiment [5] , which uses the electron Aharonov-Bohm interference with a quantum point contact, also manifests the importance of quantum entanglement. Actually, similar *gedenken* experiments using photon and neutron have been considered before [6-7]

A quantum entangled state [8-10] such as

$$|\Psi\rangle = \sum_n C_n |S_n\rangle \otimes |D_n\rangle \quad (\neq |S\rangle \otimes |D\rangle) \quad (1.1)$$

for any $|S\rangle$ and $|D\rangle$ is a coherent superposition of states of a quantum system of many particles or of a single particle with many degrees of freedom. It involves a correlation

between the states $|S_n\rangle$ of the quantum system and the states $|D_n\rangle$ of the detector. Once the detector is found in a state $|D_n\rangle$, the total system must collapse into a certain component $|S_n\rangle \otimes |D_n\rangle$. Then one can infer the state $|S_n\rangle$ of the quantum system. The interference pattern can be obtained from the total wave function $|\Psi\rangle$ by “summing over” all possible states of the detector. Assuming the states of the detector are normalized, we have

$$\sum_m |\langle D_m| \otimes \langle x|\Psi\rangle|^2 = \sum_n |C_n|^2 |S_n(x)|^2 + \sum_{n \neq m} C_m^* C_n S_m^*(x) S_n(x) \langle D_m|D_n\rangle \quad (1.2)$$

where $S_n(x) = \langle x|S_n\rangle$ is the state of the quantum system in position representation. The second term on the r.h.s of the above equation is responsible for the interference pattern. It is easy to see the interference fringes completely vanish when the states of the detector are orthogonal to one another [10], i.e., when $\langle D_m|D_n\rangle = \delta_{m,n}$. In this situation, an ideal quantum measurement results from the ideal entanglement with the orthogonal correlated components $|D_n\rangle$, in which one can distinguish the state of detector very well. Mathematically, by using the reduced density matrix

$$\rho = \text{Tr}_D(|\Psi\rangle\langle\Psi|) = \sum_n |C_n|^2 |S_n\rangle\langle S_n| + \sum_{m \neq n} C_m^* C_n |S_n\rangle\langle S_m| \langle D_m|D_n\rangle \quad (1.3)$$

which is obtained by tracing out the variables of detector, the above-mentioned decoherence phenomenon can be equivalently expressed as a projection or reduction of the reduced density matrix from a pure state $\rho = \sum_{m,n} |S_n\rangle\langle S_m|$ to a mixed state $\hat{\rho} = \sum_n |S_n\rangle\langle S_n|$.

It is noticed that, so long as the “which-way” information already stored in the detector *could be read out*, the interference pattern has been destroyed *without any data read out in practice* [4,5]. In this sense the environment surrounding the quantum system behaves as a detector to realize a “measurement-like” process. This is because the environment *never needs to read out* the data. Thus, the above argument is also applicable to the analysis of decoherence problem of an interfering quantum system coupling to the environment [8-10]. In this kind of problems, the environment is imagined as an objective detector detecting the states of the quantum system and thereby the detector states $|D_n\rangle$ are thought to be the macroscopic quantum states of the environment. Provide the environment couples with the quantum system and produce an ideal entanglement, the quantum system must lose its coherence. It is worthy to point out that this simple entanglement conserves the energy of the quantum system while destroying the quantum coherence. The loss of energy of

the quantum system can be separately discussed in the quantum dissipation theory well developed in recent years [11-16]

In our previous works on quantum measurement theory[17-23] , we investigate how an ideal entanglement appears in the macroscopic limit that the number N of particles making up the detectors approaches infinity. It was found that the *factorization structure*

$$F_{m,n} = \langle D_m | D_n \rangle \equiv \prod_{j=1}^N \langle D_m^{[j]} | D_n^{[j]} \rangle \quad (1.4)$$

concerning the overlapping of detector-states plays a crucial part in quantum decoherence. Here, $|D_n^{[j]}\rangle$ are the single states of those blocks constituting the detector, and $F_{m,n}$ is called decoherence factor. Since each factor $\langle D_m^{[j]} | D_n^{[j]} \rangle$ in $F_{m,n}$ has a norm less than unity, the product of infinite such factors may approach zero. This investigation was developed based on the Hepp-Coleman mode and its generalizations[24-27]. In 1998, this theory was applied to the analysis of the universality [28] of the environment influences on quantum computing process [29-31]. Parallely, the classical limit that certain quantum numbers (such as angular momentum) are huge is also investigated in our previous works .

However we have not got a totally-satisfactory answer to the question why the large system entangling with the small system behaves so classically in such limit situations. In fact, concerning the transition of the detector from quantum status to classical status , there were only some vague presentations[17, 19, 21,] . In a general situation the classical feature of the large system can not simply be characterized by large quantum numbers, and thus what is responsible for the classical feature remains unclear yet. Besides, all of our previous discussions about quantum decoherence are based on interaction of particular forms, namely the non-demolition interaction [3]. In this paper, using Born-Oppenheimer (B-O) approximation [32], we universally consider the decoherence problem for a quantum system coupling to a large system through a general interaction. This basic approach can be applied to analyzing influences exerted by environment and detector as well. Our discussion is also involved with a fundamental problem that the physicist can not avoid completely : how does the time reversal symmetry implied by the Schrodinger equation on the microscopic scale turn into the time reversal asymmetry manifested by quantum decoherence or quantum dissipation on the macroscopic scale?

This paper is organized as follows. We describe in Sec.2 the adiabatic factorization of slow

and fast dynamic variables in terms of the B-O approximation and show how the interaction of the large object with a quantum system causes a quantum entanglement dynamically. In Sec.3, incorporating the semi-classical approach to the quasi-classical motion of slow variable in a smooth potential, we manifest that, driven by the adiabatically-effective Hamiltonians, the final states of the large object initially in an appropriate state are orthogonal to one another, and their entanglement with the quantum system leads to decoherence. In Sec.4, Sec.5 and Sec.6, the universal formalism in Secs.2 and 3 is illustrated by three explicit examples : *a.* A particle with spin $\frac{1}{2}$ moves slowly in an inhomogeneous magnetic field of varying direction; *b.* A two level quantum system interacting with a very large spin; *c.* A quantized cavity field is coupled with a simple harmonic oscillator. The first illustration is similar to the Stern-Gerlach experiment [33]. It stresses that the classical properties of the large system can be understood in terms of the macroscopic distinguishability of its quantum states. The second illustration reflects a simple presentation of quantum- classical transition when the quantum number is huge [34]. The third illustration has certain practical significance as it is relevant to the problem of detecting gravitational wave by intracavity dynamics [35,36]. It demonstrates the necessity of choosing a quasi-classical initial state of the large system to realize the quantum coherence of the quantum system. In Sec.7, based on the adiabatic approach for quantum decoherence, we discuss the spatial localization of the macroscopic object resulting from the adiabatic entanglement between its collective coordinate and the dynamic variables of particles constituting it. This study provides us with a possible solution to the Schroedinger cat paradox. Concluding remarks are given in the end. It includes a brief discussion about the development of quantum dynamic theory of decoherence. In connection with the coupled channel theory, one of whose concrete realization is the B-O approach, our discussion reveals the possibility of generalizing our present work to the case with a complicated and hence more practical interaction than the over-simplified interaction as shown in the two examples presented in Sec. 4 , 5 and 6.

II. QUANTUM ENTANGLEMENT VIA BORN-OPPENHEIMER APPROACH

In a very wide sense, any interaction between two quantum systems can cause an entanglement between them. In general, it then realizes a quantum measurement in a certain meaning. This is because one quantum system in different states can act on another with

different effects correspondingly. However, this entanglement and its relevant quantum measurement is generally not very ideal because the usual interaction can not produce a one-one correspondence between the states of the two systems. Indeed, only a very particular interaction or its effective reduction can lead to an ideal entanglement and thereby an ideal quantum measurement. Nevertheless, fortunately, so long as one of the two systems can be separated *adiabatically* and behaves *classically*, as we will prove in the following, any interaction can result in an ideal entanglement in the evolution of the total system through its adiabatic reduction based on Born-Oppenheimer (B-O) approximation.

From the view point of BO approximation, we consider a total quantum system (“molecular”) with two sets of variables, a fast (“electric”) one q and a slow (nuclear) one x . Resolving the dynamics of fast variables for a given motion of the slow subsystem, we obtain certain quantum states labeled by n for the fast part. To the first order approximation, the left effective Hamiltonian governing the slow variables involves an external scalar potential $V_n(x)$ and an magnetic-like vector potential $A_n(x)$ induced by the fast variables [37,38]. The latter is called the induced gauge potential or Berry’s connection. If we assume the motions of the slow subsystem are “classical”, we naturally observe that, due to the back-actions of the fast part, there are different induced forces

$$F_n = -\nabla_x V_n(x) + \frac{d}{dt}x \times (\nabla_x \times A_n) \quad (2.1)$$

exerting on the slow part. Their direct physical effects are that the information of the “fast” states labeled by n is recorded in the different motion configurations of the slow part. An entanglement just stems from this correlation between the quantum states of the fast subsystem and the classical motion configurations of the slow subsystem. In spirit of this physically-intuitive observation, we study the production of such quantum entanglement from the adiabatic separation of slow and fast variables based on the B-O approach.

Let us consider the interaction between a quantum system S with fast dynamic variable q and the large system E with slow variable x . The former with the Hamiltonian $H_s = H_s(q)$ can be regarded as a subsystem soaked in an environment or a measured system monitored by a detector, and the latter with the Hamiltonian $H_E = H_E(x)$ as the environment or the detector accordingly. In general the interaction Hamiltonian is written as $H_I = H_I(x, q)$. For a fixed value of slow variable x of E , the dynamics of the quantum system is determined

by the eigen-equation

$$[H_s(q) + H_I(x, q)]|n[x]\rangle = V_n(x)|n[x]\rangle \quad (2.2)$$

Both the eigen-values $V_n[x]$ and the eigen-state $|n[x]\rangle$ depend on the slow variable x as a given parameter.

Usually, the variation of the Hamiltonian $H_s(q) + H_I(x, q)$ with x can cause transition from an energy level $V_n(x)$ of the quantum system to another level $V_m(x)$. But within the spatial domain R to which the slow variable x belongs, if the variable x changes so slowly that the adiabatic conditions [39-42]

$$\left| \frac{\langle n[x] | \partial_x | m[x] \rangle \frac{d}{dt} x}{V_m(x) - V_n(x)} \right| = \left| \frac{\langle n[x] | \{ \partial_x H_I(x, q) \} | m[x] \rangle \frac{d}{dt} x}{\{ V_m(x) - V_n(x) \}^2} \right| \ll 1 \quad (2.3)$$

hold for any two of the different energy levels $\{V_n(x)\}$, this transition can be physically neglected and then the BO approximation works as an effective approach. Let $|\Phi_{n,\alpha}\rangle$ be the full eigen-function of the full Hamiltonian $H = H_E(x) + H_s(q) + H_I(x, q)$ for the total system formed by the large system plus the quantum system. The B-O approximation treats it as a partially factorized function

$$\langle x | \Phi_{n,\alpha} \rangle = \phi_{n,\alpha}(x) |n[x]\rangle \quad (2.4)$$

of the slow and fast variables x and q . Here, the set of slow components $\{\phi_{n,\alpha}(x) = \langle x | \Phi_{n,\alpha} \rangle\}$ and the corresponding eigen-values $\omega_{n,\alpha}$ are obtained by solving the effective eigen-equation

$$H_n(x) \phi_{n,\alpha}(x) = \omega_{n,\alpha} \phi_{n,\alpha}(x) \quad (2.5)$$

The effective Hamiltonian $H_n(x)$ is defined by

$$H_n(x) = H_{nE}(x) + V_n(x) \quad (2.6)$$

where $H_{nE}(x)$ is a gauge-covariant modification of $H_E(x)$. It was obtained by replacing the momentum operator $p = -i\hbar \nabla_x$ with its gauge-covariant form $p = -i\hbar \nabla_x - A_n(x)$. Here, $A_n(x) = i \langle n[x] | \nabla_x | n[x] \rangle$ is a $U(1)$ gauge potential induced by the motion of the quantum system. In the classical limit that the slow part behaves classically, an effective dynamics of interaction between quantum and classical objects naturally results from the effective Hamiltonians or its relevant Lagrangian[43].

The completeness relations $\sum_{n,\alpha} |\Phi_{n,\alpha}\rangle\langle\Phi_{n,\alpha}| = 1$ for the full eigen-functions $|\Phi_{n,\alpha}\rangle$ can be expressed in x - *representation* as

$$\sum_{n,\alpha} \int dx dx' \phi_{n,\alpha}(x') \phi_{n,\alpha}(x) |x\rangle\langle x'| \otimes |n[x]\rangle\langle n[x]| = 1 \quad (2.7)$$

which is equivalent to

$$\sum_n |x\rangle\langle x| \otimes |n[x]\rangle\langle n[x]| = |x\rangle\langle x|, \quad \sum_\alpha |\phi_{n,\alpha}\rangle\langle\phi_{n,\alpha}| = 1 \quad (2.8)$$

After obtaining the complete set $\{\phi_{n,\alpha}(x)|n[x]\rangle\}$ of eigenstates of the total system, we can now consider how the entanglement appears in the adiabatic dynamic evolution. Let the total system be initially in the state $|\Psi(t=0)\rangle$:

$$\langle x|\Psi(t=0)\rangle = \left\{ \sum_n c_n |n[x]\rangle \right\} \phi(x) \quad (2.9)$$

The first component of the initial state $|\Psi(t=0)\rangle$ is a superposition of the eigenstates of the quantum system while the second one a single pure state. Expanding $|\Psi(t=0)\rangle$ in terms of the complete set $\{\phi_{n,\alpha}(x)|n[x]\rangle\}$, we have the evolution wave function at time t

$$\langle x|\Psi(t)\rangle = \sum_{n,\alpha} c_n \langle\phi_{n,\alpha}|\phi\rangle \exp[-i\omega_{n,\alpha}t] |n[x]\rangle \phi_{n,\alpha}(x) \quad (2.10)$$

where we have used the completeness relation eq.(2.7). In terms of the effective Hamiltonian $H_n(x)$ related to each single adiabatic state $|n[x]\rangle$, the above wave function is rewritten in a concise form

$$\langle x|\Psi(t)\rangle = \sum_n c_n |n[x]\rangle \langle x|D_n(t)\rangle \quad (2.11)$$

with

$$|D_n(t)\rangle = \sum_\alpha \langle\phi_{n,\alpha}|\phi\rangle e^{-i\omega_{n,\alpha}t} |\phi_{n,\alpha}\rangle = \exp[-iH_n t] |\phi(x)\rangle \quad (2.12)$$

The full wave function $|\Psi(t)\rangle$ is obviously an entangled state. Starting from the same initial state $|\phi\rangle$ at $t=0$, the large system will be subject to different back-actions defined by (V_n, A_n) from the different adiabatic states $|n[x]\rangle$ of the quantum system. Then it evolves to a superposition of different final states $|D_n(t)\rangle$. This intuitive argument shows us that, there indeed exists an entanglement between two quantum systems with an quite general interaction, if one of them moves so slowly that their dynamic variables can be adiabatically factorised according to the B-O approximation. Roughly speaking, in the B-O approach, the slow subsystem is usually referred to as heavy particles (such as nucleons) while the fast one as light particles (such as the electrons). So it is reasonable to expect the slow subsystem to behave as a classical object.

III. DECOHERENCE: TRANSITION FROM QUANTUM TO CLASSICAL

In this section we will discuss under what conditions the large system, the environment or the detector, can behave classically so that the quantum system entangled with it could completely lose its coherence and approach the classical limit.

Consider the reduced density matrix of the quantum system

$$\begin{aligned} \rho_s(t) = \text{Tr}_D(|\Psi(t)\rangle\langle\Psi(t)|) &= \sum_n |C_n|^2 |n[x]\rangle\langle n[x]| + \\ &+ \sum_{n \neq m} C_m C_n^* |m[x]\rangle\langle n[x]| \langle D_n(t)|D_m(t)\rangle \end{aligned} \quad (3.1)$$

obtained by "summing over" the variables of the large system. The off-diagonal term responsible for interference is proportional to the overlapping $F_{n,m} = \langle D_n(t)|D_m(t)\rangle$ of the two large system states. Were there no large system interacting with it, the quantum system would be completely coherent for $\rho_s(t) = |\varphi(t)\rangle\langle\varphi(t)|$ is a pure state. Here $|\varphi(t)\rangle = \exp(-iH_s t)|\varphi\rangle$ is a free evolution state of the large system. Mathematically, the effect of the adiabatic effective interaction is to multiply the off-diagonal term of the reduced density matrix by the decoherence factor $F_{n,m}$. A complete decoherence is defined by $F_{n,m} = 0$ while a complete coherence by $F_{n,m} = 1 (m \neq n)$.

Before considering how the decoherence factor $F_{n,m}$ becomes zero for the large system, we need to review some known arguments about the meaning of the classical limit of the motion of the large system. According to a widely accepted viewpoint [44], in the classical limit, the expectation value of an observable for certain particular states should recover its classical value forms. These particular states can give definite classical trajectories of particle in this limit. Usually we call them quasi-classical states. A coherent state or its squeezed version is a typical example of such states. According to Landau and Lifshitz [44], in general, a quasi-classical state is a particular superposition $\sum_n c_n \phi_n$ with the non-zero coefficients c_n only distributing around a large quantum number \tilde{n} . Then the correspondence principle requires that $\tilde{n} \rightarrow \infty, \hbar \rightarrow 0$ and the product $\tilde{n}\hbar$ approaches a finite classical action. In such a limit, the expectation of an observable will take the Fourier series of its corresponding classical quantity; or strictly speaking, it takes the Fejér's arithmetic mean of the partial sums of the Fourier series [45]. In this sense the mean-square deviation of the observable is zero; and accordingly the mean of the position operator defines a classical path. Physically, the zero mean-square deviation of the position operator implies the zero width of each wave

packet $\langle x|D_m(t)\rangle$, and the overlapping $F_{n,m} = \langle D_n(t)|D_m(t)\rangle$ of zero width wave packets must vanish. From such a semi-classical picture, we will clearly see in the following how the decoherence factor $F_{n,m}$ approaches zero dynamically as the large system becomes classical.

In the semi-classical approach, for a heavy particle, the initial state $|\varphi\rangle$ can be regarded as a very narrow wave packet of width a . Since the heavy particle has a large mass M it hardly spreads in the evolution because without the environment induced quantum dissipation [15,16] the width of the wave packet at time t is

$$w(t) = a\sqrt{1 + \frac{t^2}{4M^2a^4}} \quad (3.2)$$

Then we describe the large system as an moving wave packet with the center along a classical path $x(t)$ on a manifold with local coordinates x . For a proper initial state $|\varphi\rangle$, we will see that the wave packet will split into several narrow peaks with the centers along different paths determined by different motion equations governed by the effective forces $F_n = -\nabla_x V_n(x) + \frac{d}{dt}x \times (\nabla_x \times A_n)$ with effective potentials $(V_n(x), A_n)$. Usually, the widths of these peaks are almost of the same order as that of the original wave packet and each peak is correlated to an adiabatic quantum state $|n[x]\rangle$ for a large mass. Except for some moments at which the centers of two or more peaks coincide, these narrow peaks hardly overlaps with one another. In this sense, the large system starting from a narrow initial state can reach a superposition of those states orthogonal to one another. Thus we approximately have $F_{n,m} = 0$ in the classical limit for $m \neq n$.

With reference to the useful analysis in ref.[10], we present an explicit but sketchy calculation to justify the above physically-intuitive observation about $F_{n,m} = 0$ in the classical limit. Assume the large system to be a heavy particle with very large mass M . In the duration τ of the adiabatic interaction with the quantum system, if the condition $v\tau \approx (\Delta p/M)$ $\tau \ll \Delta x$ holds, the momentum p_0 of the free heavy particle can not be changed notably. Thus the contributions of the kinetic term and the induced gauge potential can be ignored in the wave function evolution of the free heavy particle under this condition. From this consideration we can approximately write down

$$|D_n(t)\rangle = e^{-iH_n t}|\phi(x)\rangle \propto e^{-iV_n(x)t}|\phi(x)\rangle \quad (3.3)$$

The approximation requires that the effective potential $V_n(x)$ is satisfactorily smooth or the

interaction $H_I(x, q)$ is a smooth function of x . So we can use

$$V_n(x) \approx V_n(0) + F_n x; \quad F_n \approx \nabla V_n(0) \quad (3.4)$$

to re-express the decoherence factor

$$F_{n,m} = \langle \varphi | \exp \left(-it\delta F(m, n)x \right) | \varphi \rangle \quad (3.5)$$

Here, $\delta F(m, n) = F_m - F_n$ is the difference of two external forces exerted by two adiabatic potentials $V_m(x)$ and $V_n(x)$. Then the role of the back-action of the quantum system on the large system is summing up the momentum shift by a quantity $\delta F(m, n)t$ with respect to the initial state $|\varphi\rangle$. Obviously, when the width $\sigma = a^{-1}$ of the initial wave packet $\langle p | \varphi \rangle$ in the momentum space is much less than the momentum shift $\delta F(m, n)t$, the large system will adiabatically evolve into states orthogonal to one another. In fact, if the initial state is chosen to be a Gaussian wave packet $\langle x | \varphi \rangle = \frac{\sigma}{\sqrt{\pi}} \exp[-\frac{1}{2}\sigma^2 x^2]$ of width $\Delta x = \frac{1}{\sigma}$, the decoherence factor is a Gaussian decaying function of time t

$$F_{n,m} = \exp \left(-\frac{\delta F(m, n)^2}{4\sigma^2} t^2 \right) \quad (3.6)$$

As the evolution time t approaches infinity or if we have a very narrow width σ , $F_{n,m} \rightarrow 0$ and a quantum decoherence results from the dynamical evolution automatically.

Generally, we consider a system described by $H_n = p^2/2M + V_n(x)$ without the induced gauge field. Define $x_c = \langle \varphi | x | \varphi \rangle$ and $p_c = \langle \varphi | p | \varphi \rangle$ for an initial state $|\varphi\rangle$. In the classical regime one may expect that the variations $\xi = x - x_c$ and $p_\xi = p - p_c$ are small compared with x_c and p_c . Accordingly the potential can be expanded as

$$V_n(x) \simeq V_n(x_c) + V'_n(x_c)\xi + \frac{1}{2} V''_n(x_c)\xi^2. \quad (3.7)$$

So, approximately the Heisenberg equations of motion become

$$\frac{d}{dt}x = \frac{p}{M}, \quad \frac{d}{dt}p = -V'_n(x_c) - V''_n(x_c)\xi \quad (3.8)$$

Sandwiched by the initial state $|\varphi\rangle$, the above equations turn into the classical equations of motion

$$\frac{d}{dt}x_c = \frac{p_c}{M}, \quad \frac{d}{dt}p_c = -V'(x_c). \quad (3.9)$$

Now we turn to Schrödinger's picture. The evolution of the initial state is governed by $i\hbar\partial_t|\varphi(t)\rangle = H_n|\varphi(t)\rangle$. Introduce the following time-dependent translation

$$|\phi(t)\rangle = \exp \left\{ \frac{i}{\hbar} \left(\theta(t) + x_c p_\xi - p_c \xi \right) \right\} |\varphi(t)\rangle \equiv S(t) |\varphi(t)\rangle \quad (3.10)$$

where $\theta(t)$ is determined by $\dot{\theta}_t = p_c^2/2M + V(x_c)$. Then straightforward calculation gives

$$i\hbar\partial_t|\phi(t)\rangle = \left(\frac{p_c^2}{2M} + \frac{1}{2}M\omega_t^2\xi^2\right)|\phi(t)\rangle \quad (3.11)$$

where $M\omega_t^2 = V''(x_c)$. This exactly describes an oscillator with time-dependent frequency. The above direct derivation shows that in the non-inertial frame moving along the classical orbit, every quasi-classical system looks like a time-dependent oscillator whose frequency depends on the orbit. This fact is an established conclusion and illustrated in Fig 1. Actually, it is present in many textbooks about path integral. But our argument here is based on a clear physical picture and is applicable to the three dimensional case after a slight generalization.

Denote by $|0\rangle$ the vacuum state of the harmonic oscillator with frequency ω_0 which is equivalent to a Gaussian wave packet of width $\sigma_0^{-1} = \sqrt{2m\omega_0/\hbar}$. Suppose that initially the system is in the state $S^\dagger(0)|0\rangle$, a coherent state whose center lies at $(x_c(0), p_c(0))$. At time t , the center of the wave packet is obviously at $(x_c(t), p_c(t))$, and it is reasonable to expect that the width of the wave packet becomes $\sigma_t^{-1} = \sqrt{2m\omega_t/\hbar}$, since the frequency of the time-dependent oscillator changes very slowly. For two different potentials $V_1(x)$ and $V_2(x)$ the macroscopic distinguishability is ensured when the width sum of the two evolved packets is less than their orbital difference, that is, when

$$\sigma_{1t} + \sigma_{2t} \leq |x_{c1}(t) - x_{c2}(t)|$$

One cannot expect that this condition can always be fulfilled for all time t . The orbital difference is determined by something like $|V'_1 - V'_2|$ and the width is determined by the second derivative of the potential. But their relation is not very clear to us at present. What is clear is, to have $\langle D_1(t)|D_2(t)\rangle = 0$ one should require the points that fail the inequality form a zero measure set. On the other hand, the adiabatic approximation also imposes some restrictions on the potential. To clarify the situation further more sophisticated considerations are needed.

IV. FROM MACROSCOPIC DISTINGUISHABILITY TO DECOHERENCE

In the context of quantum measurement, a variant of the Stern-Gerlach (SG) experiment provides an illustration of the above formalism. Quantum measurement is mutationally an

observing process that “reads out” the system states from the “macroscopically distinguishable” states of the detector. As is shown in the above, if the large particle moves slowly enough, an adiabatic eigen-state of the quantum system will be correlated to one of the detector states in the B-O approximation. So the adiabatic correlation

$$|1[x]\rangle \rightarrow |D_1(t)\rangle, |2[x]\rangle \rightarrow |D_2(t)\rangle, \dots \dots |n[x]\rangle \rightarrow |D_n(t)\rangle \quad (4.1)$$

between the system states $|n[x]\rangle$ and the detector states $|D_n\rangle$ defines a quantum measurement. In the classical limit, this measurement is thought to be ideal for $|D_n\rangle (n = 1, 2, \dots)$ are orthogonal to one another, i.e., $|D_n\rangle$ are shown to be “classically- or macroscopically distinguishable”. Once the detector is found in the state $|D_n\rangle$, we can infer that the system is just in the state $|n\rangle$. In the following we will quantitatively analyze the dynamical realization of such an adiabatic measurement in a variant of the SG experiment.

The original Stern-Gerlach (SG) experiment can be considered as a quantum measurement process detecting the spin states of particles from their spatial distribution. The WPC or quantum decoherence can be described in an dynamical evolution governed by the interaction between the space- and spin- degrees of freedom. In its variant, a spin- $\frac{1}{2}$ particle initially in a certain superposition state enters an inhomogeneous magnetic field of amplitude $B(x)$ with *varying* direction $\mathbf{n}(x) = (\sin \theta \cos kx, \sin \theta \sin kx, \cos \theta)$ where θ is fixed. Its configuration is shown in Fig.2. A simple experiment though it is, it is among the candidates of experiments proposed to test the Berry’s phase or its corresponding induced gauge field for a neutron in a static heliacal magnetic field [46, 42]. In the usual S-G experiment, the direction of the magnetic field is along the fixed x -axis, but in our present model the *polarization direction varies* as the position x changes.

The spatial variable is considered to be the slow system while the spin -variable to be fast as a quantum system. Corresponding to the eigenvalues $V_{\pm}(x) = \pm\mu B(x)$, the adiabatic eigenstates of the spin-Hamiltonian $H_{spin} = \mu B(x) \mathbf{n}(x) \cdot \sigma$ are

$$|\chi_+[x]\rangle = \begin{bmatrix} \cos \frac{\theta}{2} e^{-ikx} \\ \sin \frac{\theta}{2} \end{bmatrix}, \quad |\chi_-[x]\rangle = \begin{bmatrix} \sin \frac{\theta}{2} e^{-ikx} \\ -\cos \frac{\theta}{2} \end{bmatrix}$$

Here $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ is the Pauli spin operator and μ the gyromagnetic ratio. Let the incoming beam be initially in a superposition of the adiabatic eigen-states $|\psi\rangle = c_+|\chi_+[x]\rangle + c_-|\chi_-[x]\rangle$ along a certain polarization direction depending on x . When the particle moves

so slowly that the adiabatic condition

$$|\frac{d}{dt}xk \sin \theta / \mu B(x)| \ll 1 \quad (4.2)$$

holds, to the lowest order of the B-O approximation, the total initial state $|\Psi(0)\rangle = \{c_+|\chi_+[x]\rangle + c_-|\chi_-[x]\rangle\} \otimes |\phi(x)\rangle$ will evolve into an entangled state

$$|\Psi(t)\rangle = c_+|\chi_+[x]\rangle \otimes |D_+(t)\rangle + c_-|\chi_-[x]\rangle \otimes |D_-(t)\rangle \quad (4.3)$$

Here, $|D_\pm(t)\rangle = \exp[-iH_\pm t]|\phi(x)\rangle$ are the spatial states governed by the effective Hamiltonians

$$H_\pm = \frac{1}{2M}(-i\partial_x - A_\pm)^2 + V_\pm(x) \quad (4.4)$$

The effective scalar potentials $V_\pm(x)$ and the induced vector potentials $A_\pm = \frac{1}{2}k(1 \pm \cos \theta)$ are determined from the adiabatic spin eigenstates $|\chi_+[x]\rangle$ and $|\chi_-[x]\rangle$. In the semi-classical picture, because the particles in the adiabatic spin states $|\chi_+[x]\rangle$ and $|\chi_-[x]\rangle$ separately suffer two forces $F_\pm = -\frac{\partial}{\partial x}V_\pm(x)$ of opposite directions along \mathbf{x} , they will finally form two macroscopically -distinguishable spots on the detecting screen, each of which is correlated to one of the spin states. This spin-space correlation process enables people to pick out different spin states according to the spatial distribution.

To analyze this measurement process in details we assume the spatial part $\phi(x)$ in the initial state is a Gaussian wave packet

$$|\phi(x)\rangle = \left(\frac{1}{2\pi a^2}\right)^{\frac{1}{4}} \int dx e^{-\frac{x^2}{4a^2}} |x\rangle \quad (4.5)$$

distributing along direction x with the center at the original point. Here a is the initial width of the atom beam. Adopting the semi-classical method, we have the linear approximation $B(x) \simeq [\partial_x B(x=0)]x$ and $f = \mu \partial_x B(x=0)$. Factorizing the evolution operator $U_\pm(t) = \exp[-iH_\pm t]$ by Wei-Norman method [47,48](see Appendix 1), we exactly obtain, in position representation, the following effective wave functions $|D_\pm(t)\rangle$ at time t

$$\langle x|D_\pm(t)\rangle = \left(\frac{a^2}{2\pi^3}\right)^{\frac{1}{4}} \left(\frac{\pi}{a^2 + \frac{it}{2M}}\right)^{\frac{1}{2}} e^{-i\Omega_\pm(t) \mp iftx} \exp\left[-\frac{(x - x_{\pm c}(t))^2}{4(a^2 + \frac{it}{2M})}\right] \quad (4.6)$$

where

$$\Omega_\pm(t) = \frac{f^2 t^3}{6M} + \frac{1}{2} f t^2 A_\pm$$

It is seen from Eq.(4.6) that the Gaussian wave packets $\langle x|D_{\pm}(t)\rangle$ center on the classical trajectories

$$x_{\pm c}(t) = \mp \frac{1}{2} \cdot \frac{f}{M} t^2 - \frac{A_{\pm}}{M} t \quad (4.7)$$

They have the different group speeds $v_{\pm} = \mp \frac{f}{M} t - \frac{A_{\pm}}{M}$ along the opposite directions, but have the same width $a(t) = a\sqrt{1 + t^2/(4M^2a^2)}$ spreading with time. It is obvious that the motions of the wave packet centers obey the classical motion law that a particle of mass M forced by $\mp f$ will move with the acceleration $\mp f/M$. The quantum character of this motion is mainly reflected in the spreading of the wave-packets. The induced gauge fields A_{\pm} are constant, but they change the initial value of $\frac{d}{dt}x$ according to the corresponding classical Hamilton equation.

$$M \frac{d^2}{dt^2}x = \mp f; \quad \frac{d}{dt}x = \frac{p}{M} - \frac{A_{\pm}}{M} \quad (4.8)$$

This means that the zero initial value of the canonical momentum $p = M \frac{d}{dt}x + A_{\pm}$ determines the initial velocity $\frac{d}{dt}x(0) = -A_{\pm}/M$. The quantum effects of A_{\pm} are to contribute the additional phases $-\frac{1}{2}ft^2A_{\pm}$ in the wave functions.

The macroscopic distinguishability of wave-packets in quantum measurement requires that the distance between the two wave-packets should be larger than the width of each wave packet, i.e.

$$ft^2 - k \cos \theta t \gg a \sqrt{M^2 + \frac{t^2}{4a^2}} \quad (4.9)$$

This condition is easily satisfied for a long time evolution.

To analyze the decoherence quantitatively, we compute the norm of the decoherence factor $F(t) = |\langle D_+(t)|D_-(t)\rangle|$. The extent of quantum coherence depends totally on this overlapping integral. We can explicitly integrate it

$$F(t) = \exp \left[-a^2 f^2 t^2 - \frac{1}{8a^2} \left(\frac{f}{M} t^2 - \frac{k \cos \theta}{M} t \right)^2 \right] \quad (4.10)$$

It is obvious that the decoherence process indeed happens as $t \rightarrow \infty$, but it does not obey the simple exponential law $e^{-\gamma t}$. In a long time scale, the temporal behavior of decoherence is described by $F(t) \approx \exp \left[-\frac{f^2 t^4}{8a^2 M^2} \right]$ and the characteristic time of the decoherence process can be defined by $F(\tau_d) = e^{-1}$, that is

$$\tau_d = \sqrt{\frac{2\sqrt{2}Ma}{f}} \quad (4.11)$$

This shows that the long time behavior of decoherence is independent of the spatial details of interaction, which is caused by the configuration of the external field.

V. DECOHERENCE RESULTING FROM LARGE SPIN

There is a second illustration to show the happening of decoherence owing to the adiabatic separation of two systems. Based on our previous investigation about quantum decoherence in the classical limit [17,21,22], we assign an arbitrary spin j to interact with a two-level system (such as a spin- $\frac{1}{2}$ system) through a coupling of particular form. Let $\mathbf{J} = (\hat{J}_x, \hat{J}_y, \hat{J}_z)$ be the angular momentum operator of the large system and $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ be the Pauli matrix describing the quasi-spin of the two-level quantum system with energy-level difference ω_s . The full Hamiltonian of this model is

$$H_I = \omega_s \sigma_z + \omega J_z + \mathbf{f}(\mathbf{J})\sigma_x, \quad (5.1)$$

The general interaction $\mathbf{f}(\mathbf{J})\sigma_x$ is linear with respect to the variable of the quantum system while it depends on the variable \mathbf{J} through a function $\mathbf{f}(\mathbf{J})$. Two free Hamiltonians $\omega_s \sigma_z$ and ωJ_z were introduced to consider the energy-exchange between the quantum system and the large system.

The interaction $\mathbf{f}(\mathbf{J})\sigma_x$ can not well distinguish the states $|\pm\frac{1}{2}\rangle$ of the quantum system for $|\pm\frac{1}{2}\rangle$ are not the eigenstates of the interaction Hamiltonian. So, in general, this model can not well describe a quantum measurement process and thus can not give a good description of quantum decoherence. However, if we think \mathbf{J} as the slowly-changing variable relative to the fast one σ , determined by the B-O approximation under the adiabatic condition, the effective potential $V_{\pm} = \pm\sqrt{\omega_s^2 + \mathbf{f}(\mathbf{J})^2}$ of the large system will clearly distinguish the adiabatic eigenstates $|u_+[\mathbf{J}]\rangle = (\cos\frac{\vartheta}{2}, \sin\frac{\vartheta}{2})^T$ and $|u_-[\mathbf{J}]\rangle = (\sin\frac{\vartheta}{2}, -\cos\frac{\vartheta}{2})^T$. Here, the angle parameter $\vartheta = \arctan(-\frac{\mathbf{f}(\mathbf{J})}{\omega_s})$ depends on the slow variable \mathbf{J} . Then, the adiabatic separation of the spin- $\frac{1}{2}$ and the large spin system will result in a quantum decoherence.

In fact, because of the introduction of the arbitrary spin j , which labels the $2j + 1$ -dimensional irreducible representation of the rotation group $\text{SO}(3)$, we are able to consider the behaviors of the quantum dynamics governed by this model Hamiltonian in the classical limit with infinite spin j . The reason why the limit with infinite j is called classical is that the mean square deviations of the components \hat{J}_x , and \hat{J}_y enjoy the following limit feature

$$\frac{\Delta \hat{J}_x}{j} = \frac{\Delta \hat{J}_y}{j} = \frac{1}{\sqrt{2}j} \rightarrow 0 \text{ as } j \rightarrow \infty [34,17,49].$$

To solve the dynamical evolution of the total system explicitly, we choose a particular form of interaction : $\mathbf{f}(\mathbf{J}) = \sqrt{g^2 J_x^2 - \omega_s^2}$. Taking this particular form is equivalent to making a linear approximation for the effective potential $V_{\pm}[\mathbf{J}]$. With this particular form the effective Hamiltonians $H_{\pm} = \omega J_z + V_{\pm}[\mathbf{J}]$ can be expressed as an rotation of the simple spin-Hamiltonian $H_o = \sqrt{g^2 + \omega^2} J_z$, i.e.,

$$H_{\pm} = \exp[i\hat{J}_y\phi_{\pm}]H_o\exp[-i\hat{J}_y\phi_{\pm}] \quad (5.2)$$

where the polar angle ϕ_{\pm} is defined by $\tan \phi_{\pm} = \pm \frac{g}{\omega}$.

According to the quantum angular momentum theory, the eigen-states of H_{\pm} can be constructed as

$$|j, m(\phi_{\pm})\rangle = \exp[i\hat{J}_y\phi_{\pm}]|j, m\rangle = \sum_{m'=-j}^j d_{m',m}^j(-\phi_{\pm})|j, m'\rangle \quad (5.3)$$

where $|j, m\rangle$ is a standard angular momentum state and $d_{m',m}^j(\phi) = \langle j, m' | \exp[i\hat{J}_y\phi] | j, m \rangle$ is the corresponding d -function ; the corresponding eigen-values are $E_m = m\sqrt{g^2 + \omega^2}$.

Here, we should remark that the exact solvability of the above model largely depends on the particular form of the function $\mathbf{f}(\mathbf{J})$. If this is not the case, the above method can not work well and then certain semi-classical approximation methods should be used to deal with the effective Hamiltonian in its classical limit with very large j . If the coupling function $\mathbf{f}(\mathbf{J})$ depends on \mathbf{J} quite slightly, we can generally linearize the above effective potential $V_{\pm}(\mathbf{J})$ to realize the particular form.

We are concerned with classical characters of the large-spin system. Let us suppose it is initially assigned the adiabatic ground state $|j, m = -j(\phi)\rangle$ with the lowest magnetic quantum number $m = -j$. In quantum measurement theory, the choice of ground state is required by a stable measurement. Starting with its initial state

$$|\psi(0)\rangle = (C_+|u_+[\mathbf{J}]\rangle + C_-|u_-[\mathbf{J}]\rangle) \otimes |j, -j(\phi)\rangle \quad (5.4)$$

the effective Hamiltonians (5.2) evolves the large spin system into an entanglement state

$$|\psi(t)\rangle = C_+|u_+[\mathbf{J}]\rangle \otimes |D_+(t)\rangle + C_-|u_-[\mathbf{J}]\rangle \otimes |D_-(t)\rangle, \quad (5.5)$$

with

$$|D_{\pm}(t)\rangle = \exp[\pm i\hat{J}_y\phi] \exp[-it \hat{\mathbf{J}}_z \sqrt{g^2 + \omega^2}] \exp[\mp i\hat{J}_y\phi] |j, -j(\phi)\rangle \quad (5.6)$$

Using the explicit expressions of the d -function $d_{m',m}^j(\phi_{\pm})$, we can calculate the overlapping $\langle D_-(t) | D_+(t) \rangle$, obtaining

$$F(j; t) = |\langle D_-(t) | D_+(t) \rangle| = \left| 1 - \sin^2 2\phi \sin^2 \frac{\sqrt{g^2 + \omega^2}}{2} t \right|^j. \quad (5.7)$$

The above formula directly manifests the happening of quantum decoherence in the classical limit $j \rightarrow \infty$. In fact, in a nontrivial case with $\phi \neq 0$, $|1 - \sin^2 \frac{t}{2} \sqrt{g^2 + \omega^2} \sin^2 2\phi|$ is usually a positive number less than 1. In the classical limit with $j \rightarrow \infty$, its j -th power $|\langle D_-(t) | D_+(t) \rangle|$ must approach for $t \neq t_n \equiv 2nt/\sqrt{g^2 + \omega^2}$, $n = 0, 1, 2, \dots$. At those instances t_n , quantum coherence revivals as so-called quantum jumps (see Fig.3). Then, as far as the present model is concerned, we reach the conclusion that, if the large spin system behaves classically, the decoherence can be dynamically realized for the entangled quantum system. In traditional quantum measurement, the detector was pre-required as a purely classical object to reduce the coherent superposition instantaneously. But now it is proved that the WPC occurs as the *quantum* detector moves slowly to approach the classical limit. This means in our treatment the detector is essentially still a quantum object. Thus it has the advantage of dealing with the problem of quantum measurement consistently within the framework of quantum theory.

VI. INTRACAVITY DYNAMICS WITH CLASSICAL SOURCE

Our third example about decoherence in quantum adiabatic process is the intracavity dynamics with a classical source, which is associated with the interferometric detection of the gravitational wave by a squeezed light [35,36].

We consider a cavity with two end mirrors (as in Fig.4), one of which is fixed while the other is treated as a simple harmonic oscillator of frequency Ω and mass M with the position and momentum x and p . The radiation pressure force of the cavity field on the moving mirrors is proportional to the intracavity photon density. Let a^\dagger and a be the creation and annihilation operators of the cavity with a single mode of frequency ω . The cavity-mirror coupling is described by an interaction Hamiltonian $H_I = gx a^\dagger a$ where g is the coupling constant depending on the electric dipole. In the radio frequency range the cavity field can be prescribed as a macroscopic current. From this consideration we describe the cavity field dynamics with the Hamiltonian $H_c = \omega a^\dagger a + f(a^\dagger + a)$. This cavity field -mirror coupling

system can also be used to detect the photon number in the cavity by the motion of mirror. Obviously, the motion of the mirror is slow with respect to the oscillation of the cavity field. Thus we can use the B-O approximation to approach the quantum decoherence problem in the measurement of the cavity field. Most recently, the special case of this model without classical source has been used as a scheme probing the decoherence of a macroscopic object [51].

Coupled with the mirror and the classical source, the adiabatic eigen-states

$$|n[x]\rangle = \frac{1}{\sqrt{n!}}[a^\dagger + \lambda(x)]^n|0\rangle \quad (6.1)$$

of the cavity field for displacement $\lambda(x) = \frac{f}{\omega+gx}$ are determined by

$$\{[\omega + gx]a^\dagger a + f(a^\dagger + a)\}|n[x]\rangle = v_n(x)|n[x]\rangle \quad (6.2)$$

with the corresponding eigen-values $v_n(x) = n(\omega + gx)$, $n = 0, 1, 2, \dots$. Under the B-O approximation, the effective Hamiltonians are also referred to as the forced harmonic oscillators in the same renormalization external potential (RNEP) $V_{rne} = \frac{f^2}{\omega+gx}$ [11,12]. Under the adiabatic condition

$$\left| \frac{\langle (n-1)[x] | \partial_x | n[x] \rangle \frac{d}{dt} x}{\omega + gx} \right| \sim \frac{|ngf \frac{d}{dt} x|}{\omega^3} \ll 1 \quad (6.3)$$

μ , the RNEP V_{rne} can be linearized as $\frac{f^2}{\omega}[1 - \frac{gx}{\omega}]$. Then the effective Hamiltonians can be rewritten as $H_n = \Omega b^\dagger b + g_n(b^\dagger + b)$ in terms of

$$b = \frac{M\Omega x + ip}{\sqrt{2M\Omega}}, \quad g_n = \frac{g(n - f^2/\omega^2)}{\sqrt{2M\Omega}} = \mu \left(n - \frac{f^2}{\omega^2} \right) \quad (6.4)$$

For each effective Hamiltonian H_n , the corresponding evolution is a displacement operator

$$D[\alpha_n(t)] = \exp(\alpha_n(t)b^\dagger - \alpha_n(t)^*b) \quad (6.5)$$

with $\alpha_n(t) = -g_n(\exp[i\Omega t] - 1)/\Omega$.

Let the initial state of the mirror be a well-defined quasi-classical state, a coherent state $|\alpha\rangle$ and the initial state of the cavity be a superposition $|c(0)\rangle = \sum_n c_n |n[x]\rangle$ of the adiabatic states. The evolution governed by the effective Hamiltonian H_n leads to an entangled state

$$|\psi_I(t)\rangle = \sum_n c_n |n[x]\rangle \otimes D[\alpha_n(t)]|\alpha\rangle \equiv \sum_n c_n |n[x]\rangle \otimes |D_n(t)\rangle \quad (6.6)$$

for the total system. The overlapping of the mirror states in this entanglement can be computed and its norm is

$$|\langle D_m(t)|D_n(t)\rangle| = \exp\left(-(n-m)^2 \frac{2\mu^2}{\Omega^2} \sin^2 \frac{\Omega t}{2}\right) \quad (6.7)$$

The changing rate $\frac{d}{dt}x$ (the velocity) of the slow variable x is proportional to Ω . In the adiabatic limit, Ω is very small. So we can rationally consider the limit $\Omega \rightarrow 0$ for a fixed μ . Then an ideal entanglement appears in this limit case for the overlapping becomes a non-linear exponential decaying factor

$$|\langle D_m(t)|D_n(t)\rangle| = \exp\left(-\frac{1}{2}(n-m)^2 \mu^2 t^2\right) \quad (6.8)$$

This result is quite similar to that of the Cini model in van Hove limit [34]. This decay phenomenon was first illustrated in ref.[21,23]. Mathematically, it results from the fact that in the strong coupling limit, the period of the oscillation is very large in comparison with the small frequency Ω .

Another interesting situation arises when the mirror is initially prepared in a Fock state $|n\rangle = \frac{1}{\sqrt{n!}} (a^\dagger)^n |0\rangle$. To show a macroscopic, but non-classical dynamic behavior, the Fock state should possess a very large occupation number n . The overlapping for the initial Fock state can be expressed as

$$\begin{aligned} F(t, n) &= \langle n|D[-\alpha_k(t)]D[\alpha_l(t)]|n\rangle \\ &= \exp\left[-\frac{1}{2}(l-k)^2 \frac{\mu^2}{\Omega^2} \sin^2 \frac{\Omega t}{2}\right] L_n\left((l-k)^2 \frac{\mu^2}{\Omega^2} \sin^2 \frac{\Omega t}{2}\right) \end{aligned} \quad (6.9)$$

in terms of the Laguerre polynomial $L_n(z)$. Fig.5 shows $F(t, n)$ as a function of time t for different j . In fact, according to the theory of special function, $L_n(z)$ approaches the zero-order Bessel function $J_0(\sqrt{n}z)$ when $n \rightarrow \infty$, hence [52],

$$F(t, n) \rightarrow e^{-\frac{1}{2}(l-k)^2 \mu^2 t^2} L_n((l-k)^2 \mu^2 t^2 / 4) \rightarrow e^{-\frac{1}{2}(l-k)^2 \mu^2 t^2} J_0(\sqrt{n}(l-k)^2 \mu^2 t^2) \quad (6.10)$$

The zero-order Bessel function of real variable $\zeta\sqrt{n}$ is a decaying-oscillating function and approaches zero as n tends to infinity. Therefore, when the cavity is occupied by a large number of photons, the macroscopic feature of the detector (the end mirror) dynamically decoheres the initial pure state of the cavity.

VII. LOCALIZATION OF MACROSCOPIC OBJECT THROUGH ADIABATIC ENTANGLEMENT

As another interesting application of the above adiabatic approach for decoherence, we will discuss how the adiabatic entanglement result in the spatial localization of a macroscopic object. This discussion is devoted to consider the quantum decoherence of the slow part rather than that of the fast part, which has been studied in previous sections.

The localization problem originated from the correspondence between Einstein and Born [53] and is closely related to the Schrodinger cat. They observed that, usually in a spatially-localized state, a macroscopic object can only be described by a time-dependent localized wave packet, which is a coherent superposition of the eigen-states of the center-of-mass Hamiltonian $H_0 = p^2/2M$. Though the spreading of an initially well localized wave packet can be reasonably ignored for the macroscopic object with very large mass, Einstein argued that the superposition of two narrow wave packets is no longer narrow with respect to the macro-coordinate, but it is still a possible state of the macroscopic object. So a contradiction to the superposition principle arises because of the requirement that the wave function of a macroscopic object must be narrow. To solve this problem, Wigner [54], Joos and Zeh [55] present the so called scattering -induced -decoherence mechanism (or WJZ mechanism): scattering of photons or atoms off a macroscopic object records the information of its position to form a quantum measurement about the position. In this mechanism the interference terms between different positions are destroyed by the generalized "which-way" detection. In spirit of Omnes 's observation [56], we argue that, mentioning macroscopicness implies the requirement that the macroscopic object must contain a large number of internal blocks. Then the macroscopic object, coupled to the internal variables, should be described by collective variables subject to the interaction similar to that concerning the external scattering in WJZ mechanism. In this section we will show that the spatial localization of a macroscopic object can be caused by an ideal entanglement between its collective position (or center-of-mass) and internal variables. This entanglement results from their adiabatic separation..

Let x and q be, respectively, the collective position and internal variables of a macroscopic object with the collective Hamiltonian $H_s = p^2/2M$ ($[x, p] = i$). To consider how different positions affect the quantum coherence of the internal motion of the macroscopic object, we

suppose that the total system is initially in a product state

$$|\Psi_x(t=0)\rangle = |x\rangle \otimes |\phi\rangle \quad (7.1)$$

where the first component $|x\rangle$ is the eigen-state of the collective position operator x while $|\phi\rangle$ is an arbitrary initial pure state of the internal degrees of freedom. Usually, the collective motion acts on the internal motion in certain ways and the back-action of the internal motion can not be neglected physically. So this generic interaction can not produce an ideal entanglement between the collective position and the internal states of the macroscopic object. By an argument similar to that by Joos and Zeh [54], who deal with quantum decoherence and its relevant localization by considering the scattering of external particles by the macroscopic object, we see only when the back-action is negligibly small, can the interaction between the collective and internal states realize a "measurement-like process":

$$|x\rangle \otimes |\phi\rangle \rightarrow U(t)|x\rangle \otimes |\phi\rangle = |x(t)\rangle \otimes S(x;t)|\phi\rangle \quad (7.2)$$

Here, $U(t)$ is the total evolution matrix and $|x(t)\rangle = U_0(t)|x\rangle$ represents the free evolution in the absence of the coupling to the internal variables; $S(x, t)$, acting on the internal states, denotes the effective S -matrix parametrized by the collective position x . If the collective motion is initially described by a wave packet $|\varphi\rangle = \int \varphi(x)|x\rangle dx$, then the reduced density matrix of the collective motion is

$$\rho(x, x', t) = \varphi(x, t)\varphi^*(x', t)\langle\phi|S^\dagger(x'; t)S(x; t)|\phi\rangle \quad (7.3)$$

Considering the translational invariance of the scattering process, Joos and Zeh showed that, in x -representation, the off-diagonal terms take the following form

$$\langle\phi|S^\dagger(x'; t)S(x; t)|\phi\rangle \sim \exp(-\Lambda t|x - x'|^2) \quad (7.4)$$

This means the decoherence factor is a damping function with the localization rate Λ , which depends on the total cross section.

Now, the question arises whether the negligibility of the back-action is the unique cause for the appearance of the above mentioned "measurement-like process". If not, what are the other causes beyond it? To resolve this problem, we assume the Hamiltonian $h(q, x) = H_i(q) + W(x, q)$ describes the motion of the internal variables q coupling to the collective variable x . For a fixed value of the slow variable x , the eigen-state $|n[x]\rangle$ and the corresponding

eigen-values $V_n[x]$ are determined by the eigen- equation $h(q, x)|n[x]\rangle = V_n(x)|n[x]\rangle$. Regarding x and q as the slow and fast variables respectively in the BO adiabatic approach , we approximately obtain the complete set $\{\phi_{n,\alpha}(x)|n[x]\rangle\}$ of eigenstates of the total system, where $\phi_{n,\alpha}(x)$ come from the eigen-equation $H_n\phi_{n,\alpha}(x) = E_{n,\alpha}\phi_{n,\alpha}(x)$ and $H_n = p^2/M + V_n[x]$ is the effective Hamiltonian correlated to the internal state $|n[x]\rangle$. Here, we do not consider the induce gauge potential. Then, we can see how the “measurement-like process” naturally appears as a result of the adiabatic dynamic evolution.

In fact, under the BO approximation, we can expand the factorized initial state $|\Psi(0)\rangle = |x\rangle \otimes |\phi\rangle$ in terms of the adiabatic basis $\{\langle x|n, \alpha\rangle \equiv \phi_{n,\alpha}(x)|n[x]\rangle\}$ and then we obtain the total wave function

$$\begin{aligned} |\Psi(t)\rangle &= \sum_{n,\alpha} \langle \phi_{n,\alpha}|x\rangle \langle n[x]|\phi\rangle e^{-iE_{n,\alpha}t} |n, \alpha\rangle \\ &= \sum_n \langle n[x]|\phi\rangle \int dx' \langle x'|e^{-iH_n t}|x\rangle |x'\rangle \otimes |n[x]\rangle \end{aligned} \quad (7.5)$$

where we have used the single-component completeness relation $\sum_\alpha |\phi_{n,\alpha}\rangle \langle \phi_{n,\alpha}| = 1$. Generally, the propagator $K(x', x, t) = \langle x'|e^{-iH_n t}|x\rangle$ is not diagonal for $|x\rangle$ is not an eigen-state of H_n . However, in the large mass limit , we can prove that, to the first order approximation , $K(x', x, t)$ takes a diagonal form proportional to a δ - *function*. In fact, in this case, the kinetic term $p^2/2M$ can be regarded as a perturbation in comparison with the effective potential $V_n(x)$. Using Dyson expansion to the first order of $\frac{1}{M}$, we have

$$\begin{aligned} e^{-iH_n t} &= e^{-iV_n t} \left(1 - i \int_0^t e^{iV_n t'} \frac{p^2}{2M} e^{-iV_n t'} dt' + \dots \right) \\ &= e^{-iV_n t} \left(1 - i \frac{p^2 t^2}{2M} + i \frac{t^2}{4M} (p \partial_x V_n + [\partial_x V_n] p) - \frac{it^3 \partial_x V_n^2}{6M} + \dots \right) \end{aligned} \quad (7.6)$$

Since $\int \langle x'|P^n|x\rangle f(x') dx = 0$ for $n=1,2,\dots$, we have

$$K(x', x, t) = e^{-iV_n[x]t} [\delta(x - x') + \frac{i}{2M} \int_0^t d\tau e^{-iV_n[x']\tau} \frac{\partial^2}{\partial x'^2} \delta(x - x') e^{iV_n(x)\tau}] \quad (7.7)$$

We notice this simple result has the following physical explanation: the evolution state of a heavy particle for very large M , which is almost steady, is approximately an eigenstate of the position operator if it is initially in a state with a fixed position. Then, it follows that, in the large-mass limit, the wave function $|\Psi(t)\rangle$ can be factorized approximately: $|\Psi(t)\rangle = |x\rangle \otimes S(x, t)|\phi\rangle$ where the entangling S - *matrices*

$$S(x, t) = \sum_{n,} e^{-ihV_n t} |n[x]\rangle \langle n[x]| \quad (7.10)$$

are defined in terms of the adiabatic projection $|n[x]\rangle\langle n[x]|$.

According to our previous argument about the factorized structure of S – *matrix* in the dynamic theory of quantum measurement [], if the internal degree of freedom has many components, e.g., if $q = (q_1, q_2, \dots, q_N)$, then in their normal non-interaction modes, $S(x; t)$ can be factorized as:

$$S(x; t) = \prod_{j=1}^N S_j(x; t) \quad (7.11)$$

with

$$S_j(x; t) = e^{-ih_j(q_j, x)t} \quad (7.12)$$

with $h(q, x) = \sum_j h_j(q_j, x)$. Of course in the derivation of the above factorized structure for the S – *matrix*, we have made some simplifications. Roughly speaking, we have assumed that the potential takes the form of direct sum and the eigenstate the form of direct product

$$V_n = \sum_j V_{nj}(q_j), \quad |n[x]\rangle = \prod_{j=1}^N \otimes |n_j[x]\rangle \quad (7.13)$$

neglecting the higher order terms $\approx O(\frac{1}{M})$.

For the initial state $|\phi\rangle = \prod_{j=1}^N \otimes |\phi_j\rangle$ factorized with respect to internal components, the reduced density matrix

$$\rho(x, x', t) = \varphi(x)\varphi^*(x')F_N(x', x, t) : \quad (7.14)$$

can be re-written in terms of the so called decoherence factor

$$F_N(x', x, t) = \prod_{j=1}^N F^{[j]}(x', x, t) \equiv \prod_{j=1}^N \langle \phi_j | S_{q_j}^\dagger(x'; t) S_{q_j}(x; t) | \phi_j \rangle. \quad (7.15)$$

This factor is expressed as an N -multiple product of the single decohering factors $F^j(x, x') = \langle \phi_j | S_{q_j}^\dagger(x'; t) S_{q_j}(x; t) | \phi_j \rangle$ with norms less than unity. Thus in the macroscopic limit $N \rightarrow \infty$, it is possible that $F_N(x', x, t) \rightarrow 0$, for $x' \neq x$. In fact, this factor reflects almost all the dynamic features of the influence of the fast part on the slow part. Physically, an infinite N means that the object is macroscopic since it is made of infinite number of particles in that case. On the other hand, the happening of decoherence at infinite N manifests a transition of the object from the quantum realm to the classical realm. Here, as expected, the physical picture is consistent.

As to the localization problem raised by Einstein and Born [53], we, based on the above argument, comment that one can formally write down the wave function of a macroscopic

object as an narrow pure state wave packet, but it is not the whole of a real story. Actually, the statement that an object is macroscopic should physically imply that it contains many particles. So a physically correct description of its state must concern its internal motions coupling to the collective coordinates (e.g., its center-of-mass) . Usually, one observe this collective coordinate to determine whether two spatially-localized wave packets can interfere with each other. If there does not exist such interference, one may say that, the superposition of two narrow wave packets for the macro-coordinate is no longer a possible pure state of the macroscopic object. Indeed, because the “which-way” information of the macro-coordinate is recorded by the internal motions of particles making up the macroscopic object, the induced decoherence must destruct the coherence in the original superposition so that the state of the macroscopic object is no longer pure.

The present argument also provides a possible solution for the Schroedinger cat paradox. If we consider the Schroedinger cat as a macroscopic object consisting of many internal particles, then we can never observe anything corresponding to the interference between the dead and the living cats because the macroscopically- dead and the macroscopically-living states of the cat are correlated to the corresponding internal states. In this sense, we conclude that the Schroedinger cat paradox is not a paradox at all in practice. Rather, it essentially arises from overlooking the internal motions of a macroscopic cat or the multi-particle scattering off it. Turning to the problem of quantum coherence of a subsystem within the total system, from the above argument we also conclude that the classical characters of both the quantum system and the large system entangled with it are “correlated” physically: when the large system transits from quantum to classical realms, the quantum system has to act in the same way. In other words, you can never see a coherent superposition of microscopic states entangled to a live or a dead cat’s states if the Schrodinger cat is classical. In the presence of a classical cat, the quantum system entangling with it should lose its own coherence. Actually, if , in a classical manner,one asks experimentally what a quantum system really does , then the quantum system would behave physically like a classical object . This is just the quantum mystery physicists have to face.

To make a deeper elucidation of the above general arguments about the localization of a macroscopic object of mass M , we model the macroscopic object as consisting of N two level particles, which are fixed at certain positions to form a whole without internal spatial motion. The collective position x is taken to be its mass-center or any reference position

on it while the internal variables are the quasi-spins associated with two level particles. Generally, if we assume the back-action of the internal variables on the collective position is relatively small, the model Hamiltonian can be written as

$$H = \frac{P^2}{2M} + \sum_{j=1}^N [f_j(x)|e_j\rangle\langle g_j| + f_j^*(x)|g_j\rangle\langle e_j|] + \sum_{j=1}^N \omega_j[|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|] \quad (7.16)$$

where $|g_j\rangle$ and $|e_j\rangle$ are the ground and the excited states of the j 'th particle and $f_j(x)$ denote the position-dependent couplings of the collective variable to the internal variables. Let l_j be the relative distance between the j 'th particle and the reference position x . We can further assume $f_j(x) = f(x + l_j)$. Physically, we may think that these couplings are induced by an inhomogeneous external field, e.g., they may be the electric dipole couplings of two-level atoms in an inhomogeneous electric field.

We remark that the above model enjoys some universality under certain conditions, compared with various environment models inducing both dissipation and decoherence of quantum processes. In fact, Caldeira and Leggett [25] have pointed out that any environment weakly coupling to a system may be approximated by a bath of oscillators under the condition that "each environmental degree of freedom is only weakly perturbed by its interaction with the system". We observe that any linear coupling only involves transitions between the lowest two levels (ground state and the first excitation state) of each harmonic oscillator in the perturbation approach though it has many energy levels. Therefore in such a case we can also describe the environment as a combination of many two level subsystems without losing generality [28]. To some extent, these arguments justify our choosing the two level subsystems to model the internal motion of the macroscopic object. We will soon see its advantage: the localization characters can be manifested naturally and clearly.

Now let us calculate the $S_j(x; t)$ for this concrete model. The single-particle Hamiltonian $h_j(x) = \omega_j(|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|) + (f_j(x)|e_j\rangle\langle g_j| + h.c)$ has the x -dependent eigenvalues

$$V_{jc} = n\Omega_j(x) \equiv \pm \sqrt{|f_j(x)|^2 + \omega_j^2} \quad (n = \pm) \quad (7.17)$$

and the corresponding eigen-vectors $|n_j[x]\rangle$ are

$$|+_j[x]\rangle = \cos \frac{\theta_j}{2} |e_j\rangle + \sin \frac{\theta_j}{2} |g_j\rangle, \quad (7.18)$$

$$|-_j[x]\rangle = \sin \frac{\theta_j}{2} |e_j\rangle - \cos \frac{\theta_j}{2} |g_j\rangle, \quad (7.19)$$

where $\tan \theta_j = \frac{f_j(x)}{\omega_j}$. Then we explicitly the corresponding single-particle S -matrix

$$S_j(x; t) = \begin{pmatrix} \cos(\Omega_j t) - i \sin(\Omega_j t) \cos \theta_j, & i \sin(\Omega_j t) \sin \theta_j \\ i \sin(\Omega_j t) \sin \theta_j, & \cos(\Omega_j t) + i \sin(\Omega_j t) \cos \theta_j \end{pmatrix} \quad (7.20)$$

Here in the derivation we have used the formula $\exp[i\vec{\sigma} \cdot \vec{A}] = \cos A + i\vec{\sigma} \cdot \vec{n}_A \sin A$ for a given vector \vec{A} of norm A along the direction \vec{n}_A . Having obtained the above analytic results about S -matrix, we can further calculate the single-particle decoherence factors $F^{[j]}(x', x, t) \equiv \langle g_j | S_j^\dagger(x'; t) S_j(x; t) | g_j \rangle$ for a given initial state $|\phi\rangle = \prod_{j=1}^N \otimes |g_j\rangle$. For simplicity we use the notation $f(x') = f'$. We have

$$\begin{aligned} F^{[j]}(x', x, t) = & \{ \sin(\Omega'_j t) \sin \theta'_j \sin(\Omega_j t) \sin \theta_j + \\ & \cos(\Omega'_j t) \cos(\Omega_j t) + \sin(\Omega'_j t) \cos \theta'_j \sin(\Omega_j t) \cos \theta_j \cos \theta_j \\ & + i \{ \cos(\Omega'_j t) \sin(\Omega_j t) \cos \theta_j - \sin(\Omega'_j t) \cos \theta'_j \cos(\Omega_j t) \} \} \end{aligned} \quad (7.22)$$

In the weakly coupling limit with $g_j \ll \omega_j$ and the coupling $f_j \simeq g_j x$, we have $\sin \theta_j \simeq \theta_j \simeq \frac{f_j}{\omega_j}$, $\cos \theta_j \simeq 1 - \frac{1}{2}\theta_j^2$ and $\Omega_j \simeq \omega_j$. Thus, the decohering factors can be simplified as

$$F^{[j]}(x', x, t) \simeq 1 - (x - x')^2 \frac{|g_j|^2}{2\omega_j^2} \sin^2(\omega_j t) + \frac{i|g_j|^2}{4\omega_j^2} \{x^2 - x'^2\} \sin(2\omega_j t) \quad (7.23)$$

Consequently, the temporal behavior of the decoherence is determined by

$$F(x', x, t) = \exp \left\{ -(x - x')^2 \sum_{j=1}^N \frac{|g_j|^2}{2\omega_j^2} \sin^2(\omega_j t) + (x^2 - x'^2) \sum_{j=1}^N \frac{i|g_j|^2}{4\omega_j^2} \sin(2\omega_j t) \right\} \quad (7.24)$$

In the case of continuous spectrum, the sum $R(t) = \sum_{j=1}^N \frac{g_j^2}{2\omega_j^2} \sin^2(\omega_j t)$ can be re-expressed in terms of a spectrum distribution $\rho(\omega_k)$ as $R(t) = \int_0^\infty \frac{\rho(\omega_k) g_k^2}{2\omega_k^2} \sin^2 \omega_k d\omega_k$. From some concrete spectrum distributions, interesting circumstances may arise. For instance, when $\rho(\omega_k) = \frac{4}{\pi} \gamma / g_k^2$ the integral converges to a negative number proportional to time t , precisely, $S(t) = \gamma t$ [31].

Therefore, our analysis recovers the result

$$\rho(x, x', t) = \varphi(x) \varphi^*(x') e^{-\gamma t (x - x')^2} \exp[i\pi(x^2 - x'^2)s(t)] \quad (7.25)$$

for the reduced density matrix of the macroscopic object, which was obtained by Joos and Zeh [54] through the multi particle external scattering mechanism and by Zurek separately through Markov master equation. Here, $s(t) = \sum_{j=1}^N \frac{\sin(2\omega_j t)}{\pi \omega_j^2}$ is a time-dependent multi-period function. This shows that the norm of the decoherence factor is exponentially

decaying and as $t \rightarrow \infty$, the off-diagonal elements of the density matrix vanish simultaneously! Due to the presence of the oscillating factor $s(t)$ of multi-period, $\rho(x, x', t)$ seems very complicated. But on the other hand, the simple decaying norm of $\rho(x, x', t)$ can well serve to describe decoherence of the macroscopic object. Consider now a similar example by Joos and Zeh. We take a coherent superposition of two Gaussian wave packets of width d

$$\varphi(x) = \frac{1}{\sqrt[4]{8\pi d^2}} \left\{ \exp\left(-\frac{(x-a)^2}{4d^2}\right) + \exp\left(-\frac{(x+a)^2}{4d^2}\right) \right\} \quad (7.26)$$

The norm of the corresponding reduced density matrix

$$|\rho(x, x', t)| = \sum_{k,l=0}^1 P_{kl}(x, x', t) \quad (7.27)$$

contains 4 peaks

$$\begin{aligned} P_{11}(x, x', t) &= \frac{1}{\sqrt{8\pi d^2}} e^{-\gamma t(x-x')^2} \exp\left[-\frac{(x-a)^2}{4d^2} - \frac{(x'-a)^2}{4d^2}\right] \\ P_{10}(x, x', t) &= \frac{1}{\sqrt{8\pi d^2}} e^{-\gamma t(x-x')^2} \exp\left[-\frac{(x-a)^2}{4d^2} - \frac{(x'+a)^2}{4d^2}\right] \\ P_{01}(x, x', t) &= \frac{1}{\sqrt{8\pi d^2}} e^{-\gamma t(x-x')^2} \exp\left[-\frac{(x+a)^2}{4d^2} - \frac{(x'-a)^2}{4d^2}\right] \\ P_{00}(x, x', t) &= \frac{1}{\sqrt{8\pi d^2}} e^{-\gamma t(x-x')^2} \exp\left[-\frac{(x+a)^2}{4d^2} - \frac{(x'+a)^2}{4d^2}\right] \end{aligned} \quad (7.28)$$

centering respectively around the points (a, a) , $(a, -a)$, $(-a, a)$ and $(-a, -a)$ on $x-x'$ -plane. The heights are respectively $1/\sqrt{8\pi d^2}$, $e^{-4\gamma ta^2}/\sqrt{8\pi d^2}$, $e^{-4\gamma ta^2}/\sqrt{8\pi d^2}$ and $1/\sqrt{8\pi d^2}$. Obviously, two peaks with centers at $(a, -a)$ and $(-a, a)$ decays with time while the other two keep their heights constant. Fig.6. shows this time-dependent configuration at $t=0$, and a finite t . As $t \rightarrow \infty$, two off-diagonal terms P_{10} and P_{01} decay to zero so that the interference of the two Gaussian wave packets are destroyed. In this sense, we say that the pure state $\rho(x, x', t=0) = \int dx \varphi(x) \varphi^*(x') |x\rangle \langle x'|$ becomes a mixture

$$\rho(t) = \int dx \varphi(x) \varphi^*(x) |x\rangle \langle x| \quad (7.29)$$

in x -representation.

Interference of two plane waves of wave vector k_1, k_2 provides us another simplest example. Without decoherence induced by its internal motions or the external scattering, their

coherent superposition $\varphi(x) = \sqrt{\frac{1}{4\pi}}[e^{ik_1x} + e^{ik_2x}]$ yields a spatial interference described by the reduced density matrix

$$\begin{aligned} \rho_0(x, x', t) = & \frac{1}{4\pi} \{ e^{ik_1(x-x')} + e^{ik_2(x-x')} + \\ & \exp[i(\frac{k_1^2t - k_2^2t}{2m} + k_2x - k_1x')] + \exp[i(\frac{k_2^2t - k_1^2t}{2m} + k_1x - k_2x')] \} \end{aligned} \quad (7.30)$$

Under the influence of internal motions , it becomes

$$\rho(x, x', t) \approx \rho_0(x, x', t) e^{-\gamma t(x-x')^2}$$

for large mass. We see that the difference created by decoherence is only reflected in the off-diagonal elements, and the pure decoherence (without dissipation) does not destroy the interference pattern described by the diagonal term

$$\rho(x, x, t) = \rho_0(x, x, t) = \frac{1}{2\pi} \{ 1 + \cos[\frac{k_1^2t - k_2^2t}{2m} + (k_2 - k_1)x] \} \quad (7.32)$$

This simple illustration tells us that the present quantum decoherence mechanism may not have to do with the interference pattern of the first order coherence, but it does destroy the higher order quantum coherence: $\rho(x, x', t) \rightarrow 0$ as $t \rightarrow \infty$. In fact, Savage, Walls and Yu have shown that, due to the induced loss of energy, quantum dissipation is responsible for the disappearance of the interference pattern of the first order coherence . The influences of internal motions or external scattering on the decoherence of a macroscopic object may be very complicated. Intuitively, these dynamic effects should depend on the details of interaction between the collective variables and the internal and external degrees of freedom. Practically, we can classify these influences into two species, namely, quantum dissipation and quantum decoherence, and then study them separately by different models.

VIII. CONCLUDING REMARKS

We remark that the quantum decoherence of a small system, resulting from a transition of the entangling “large system” from quantum to classical, is certainly an irreversible process. This is because the density matrices $\rho_s(0)$ and $\rho_s(t)$ have different ranks for $t \neq 0$. Thus they can not be transformed into each other through an unitary time-evolution matrix. If we

only consider a closed system, as a postulate with certain classical elements in it, its WPC or quantum decoherence can not be derived from Schrödinger equation based on the basic laws of quantum mechanics. Since quantum mechanics was founded, physicists have wished to add this WPC postulate to the axiom system of quantum mechanics. von Neumann and Wigner made the first attempt and considered the measurement detector plus the measured system as a total system called a “universe” satisfying Schrödinger equation. They hoped that, projected on the system, the evolution of the “universe” leads to wave packet collapse naturally. However, because it did not take the macroscopic or classical character of detector into account [1,2], this approach brings philosophical difficulty: If the observation of the detector force the measured system to decohere, the detector must decohere in advance. So the second detector is needed to monitor the first one, and the third one is needed to monitor the second one and so on. By this argument in logic, a chain of detectors should be introduced in sequence (we usually call it von Neumann’s chain), and at the end of this von Neumann’s chain, there should exist a pair of eyes as a special detector, which is required to be classical.

In 1972, to avoid the introduction of this chain of detectors, Hepp and Coleman raised a dynamical description for the WPC via a simple exactly-solvable model. They emphasized that, if the macroscopic limit of the first detector is considered appropriately, detectors other than the first one are not necessary. Using the macroscopic character of the first detector is crucial to the solution of this problem. Later Namiki, Nakazato and Pascazio et al generalized this work to put forward various new models for quantum measurement [26,27]. In 1993, after carefully analyzing these models and taking the classical limit of detector into account, one of the authors (CPS) found that the essence implied by these models is a factorization structure [17,18]. By exact- solvable models and approximately-solvable models as well, it is shown that when the effective evolution of the detector can be factorized, in the macroscopic limit that the number of particles composing the detector approaches infinity, quantum decoherence or WPC will appear naturally.

Previously we also associated quantum decoherence problem with the requirement that the result of a measurement should be macroscopically observable so that an ideal entanglement happens dynamically [21]. But all discussions about the interaction induced quantum decoherence strongly rely on the particular forms of interaction, namely, the interaction [3] $H_I(q, x)$ of non-demolition type, which depends on the variable q of the measured system,

but commutes with its free Hamiltonian. Therefore, a fatal defect inherent in the previous works, including our own works, concerning the study of environment induced decoherence based on quantum measurement theory is that the question why should nature choose such a particular form of interaction remains unanswered. In some sense, the present work in this paper has well tackled this problem. Indeed, through the B-O adiabatic separation of the quantum and quasi-classical variables, we have demonstrated that in the adiabatic limit, the effective interaction reduced from a quite general coupling just takes such a particular form.

In a wide sense, the adiabatic entanglement can be well understood in the picture of coupled channels [50], which is an extensive generalization of B-O approximation. Consider a total system whose wave function depends on two set of variables, q and x . Let Q be an operator only acting on the function of q and has a complete set of eigen-vectors $\{|n\rangle\}$ with the corresponding eigenvalues v_n . Since $\{|n\rangle\}$ forms a complete basis of the Hilbert space of all functions of q , the total eigenfunction $\Psi_E(x, q)$ of the full Hamiltonian $H = H_E(x) + H_s(q) + H_I(x, q)$ with eigen-value E can be regarded as a function of q for a given x and then can be expressed as $\Psi_E(x, q) = \sum \phi_n(x)|n\rangle$. The *channel wave function* $\phi_n(x)$ is defined by the *coupled channel equations*

$$H_{nn}(x)\phi_n(x) + \sum_{m \neq n} H_{nm}(x)\phi_m(x) = E\phi_n(x) \quad (8.1)$$

The matrix elements $H_{mn}(x) = \langle m|H|n\rangle_q$ are defined in terms of the q -function space “integral”. Under a certain condition, if the off-diagonal elements *can be neglected physically*, an effective non-demolition Hamiltonian $H_{eff} = H_{E-eff}(x) + H_{s-eff} + H_{in}(x)$:

$$\begin{aligned} H_{E-eff} &= \text{diag.}[H_{11}^E(x), H_{22}^E(x), \dots, H_{dd}^E(x)] \\ H_{s-eff} &= \text{diag.}[\lambda_1, \lambda_2, \dots, \lambda_d], \\ H_{in}(x) &= \text{diag.}[H_{11}^s(x), H_{22}^s(x), \dots, H_d^s(x)] \end{aligned} \quad (8.2)$$

can be partially diagonalized in the ‘channel space’. Here $H_{mm}^A(x) = \langle m|H_A|m\rangle_q$ for $A = E, S$ and $\lambda_m = \langle m|H_s(q)|m\rangle_q$ are constants. Obviously, the non-demolition condition $[H_{s-eff}, H_{in}(x)] = 0$ holds as H_{s-eff} is a constant matrix. In the B-O approximation the channel operator Q is taken to be $Q[x] = H_s(q) + H_I(x, q)$, which is parametrized by x . The adiabatic condition maintains that, only the diagonal elements play a dominant role

and the off-diagonal elements *can be neglected for very small channel-channel coupling*[42]. Therefore, it can be concluded that there may exist a more universal mechanism beyond B-O approximation to realize the quantum decoherence dynamically originated from the basic interaction, which is related to the theory of coupled channels.

Finally we point out that the presence of non-demolition interaction [3] is only a necessary condition for quantum decoherence to appear. Sufficient conditions should include the requirement that the large system be classical so that its final states could be orthogonal to one another. In this paper, we have regarded the spin-system with a very large spin and the harmonic oscillator initially in a coherent state as classical objects. Then within the semi-classical framework, even in the case of a general potential motion, we are able to relate the macroscopic distinguishability of the quantum states of the large system to its classical limit behaviors. However, there are still vague points in the definition of the quantum-classical division for the large system. This problem is deeply rooted in the following more fundamental and more challenging issue: why or in what sense does a general large system behave classically. If we imagine that, beside the considered quantum system, there is another system coupling with the large system to decohere it, then the present problem will be trapped into an evil logic chain. One notices the difficulty here is very similar to that faced by von Neumann and Wigner about sixty years ago [1,2]. Though new experiments have been revitalizing the study of decoherence problem and progress is being made, it seems that there is still a long way to go to finally understand quantum irreversible process completely. To reach this goal, one should first find a satisfactory definition for the so called quantum-classical boundary. At present it is very unclear to us how to do this without recourse to particular physical systems.

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Appendix. Wei-Norman Algebraic Solution For $H = \frac{1}{2M}(\mathbf{p} - A)^2 + fx$

Let $U(t)$ be the evolution operator of a quantum system with the effective Hamiltonian

$$H = \frac{1}{2m}(\mathbf{p} - A)^2 + fx \quad (a1)$$

where A is a constant induced gauge potential. Neglecting the constant term, we can rewrite the effective Hamiltonian

$$H = \frac{1}{2m}\mathbf{p}^2 - \frac{A}{M}\mathbf{p} + fx \quad (a2)$$

as an element of the Lie algebra \mathfrak{L} generated by $\{p^2, p, x, 1\}$.

According to Wei-Norman's algebraic theorem [46], a solution of the Schroedinger equation for the evolution operator $U(t)$ must be an element belonging to the Lie group related to the Lie algebra \mathfrak{L} . Since the commutation relations are closed among these four elements, the solution $U(t)$ is assumed to have a factorized form

$$U(t) \equiv e^{\alpha(t)\mathbf{p}^2} e^{\beta(t)\mathbf{p}} e^{\gamma(t)x} e^{\mu(t)} \quad (a3)$$

Its Schroedinger equation defines an solvable system of equations about the time-dependent parameters $\alpha(t), \beta(t), \gamma(t)$ and $\mu(t)$:

$$\begin{aligned} \frac{d}{dt}\alpha(t) &= -\frac{i}{2M} \\ \frac{d}{dt}\beta(t) - 2i\alpha(t)\frac{d}{dt}\gamma(t) &= \frac{iA}{M} \\ \frac{d}{dt}\gamma(t) &= -if \\ \frac{d}{dt}\mu(t) - i\beta(t)\frac{d}{dt}\gamma(t) &= 0 \end{aligned} \quad (a4)$$

The solution is

$$\begin{aligned} \alpha(t) &= -\frac{it}{2M} = i\tilde{\alpha}(t) \\ \beta(t) &= -\frac{ift^2}{2M} + \frac{iAt}{M} = i\tilde{\beta}(t) \\ \gamma(t) &= -itf \\ \mu(t) &= -\frac{if^2t^3}{6M} + \frac{iAft^2}{2M} = i\tilde{\mu}(t) \end{aligned} \quad (a6)$$

The action of the evolution operator

$$U_k(t) \equiv e^{i\tilde{\alpha}(t)\mathbf{p}^2} e^{i\tilde{\beta}(t)\mathbf{p}} e^{-iftx} e^{i\tilde{\mu}(t)} \quad (a3)$$

transforms the initial state in momentum representation

$$\varphi(p, 0) = \langle p | \varphi(0) \rangle = \left(\frac{2a^2}{\pi}\right)^{\frac{1}{4}} e^{-a^2 p^2} \quad (a3)$$

into

$$\varphi(p, t) = \langle p | U(t) | \varphi(0) \rangle = \left(\frac{2a^2}{\pi}\right)^{\frac{1}{4}} e^{i\tilde{\mu}(t)} e^{i\tilde{\alpha}(t)p^2} e^{i\tilde{\beta}(t)p} e^{-a^2(p+ft)^2} \quad (a3)$$

Then in momentum representation we can easily calculate the overlap of two entangled states:

$$|\langle \varphi'(t) | \varphi(t) \rangle| = \exp[-a^2 t^2 (f^2 + f'^2)] \times \\ \left| \exp\left\{ \frac{-[\tilde{\beta}(t) - \tilde{\beta}'(t) + i2a^2 t(f + f')]^2}{8a^2} \right\} \right|$$

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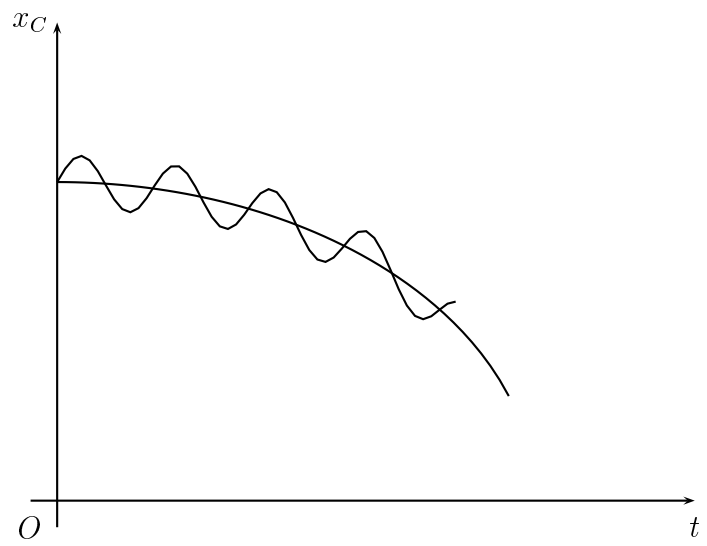


Fig.1:Classical orbit with quantum fluctuation.

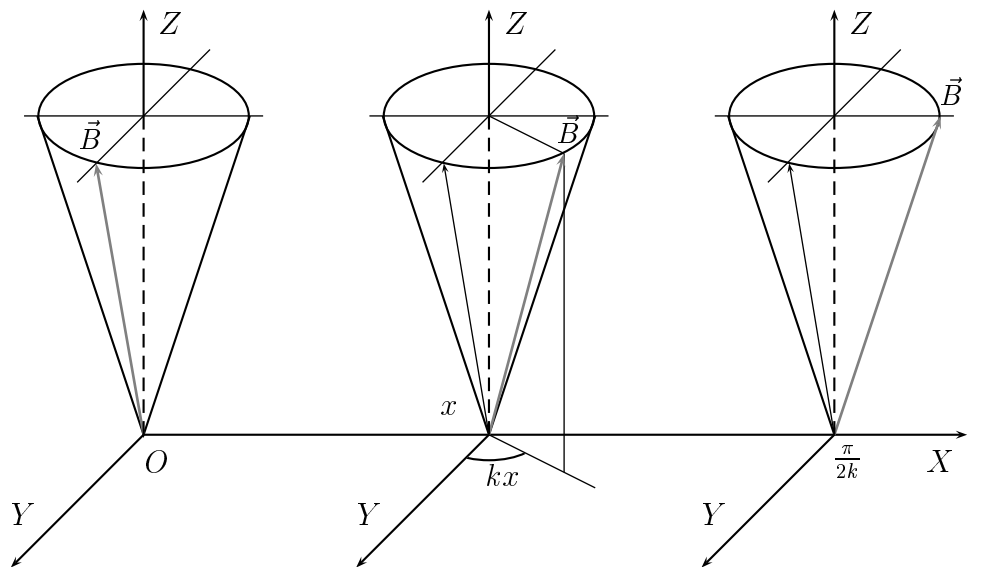


Fig.2: The configuration of a rotating magnetic field for Stern-Gerlach experiment.

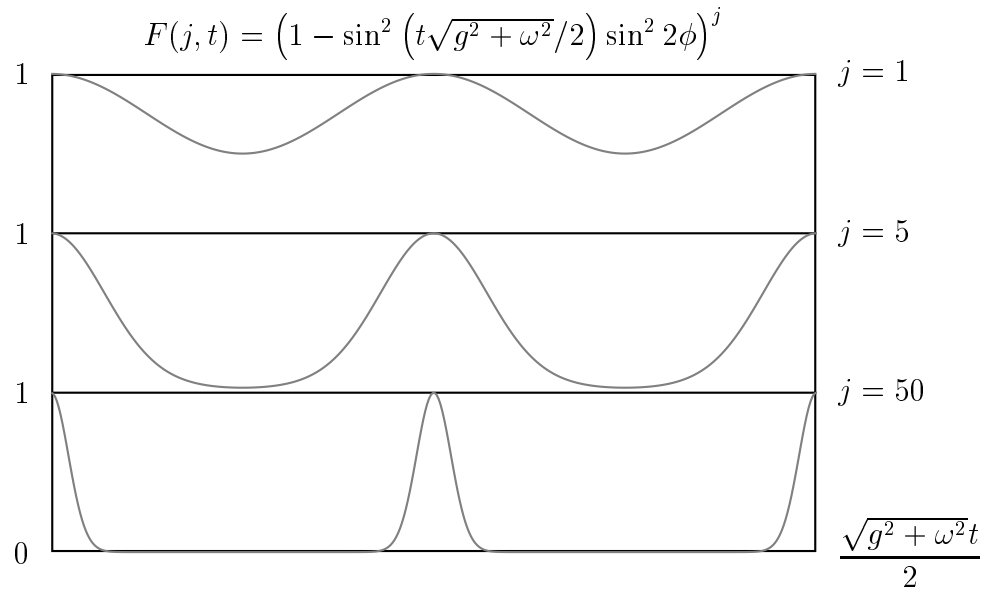


Fig.3:Disappear of non-diagonal elements of density matrix. Here $\sin^2 2\phi = 1/$

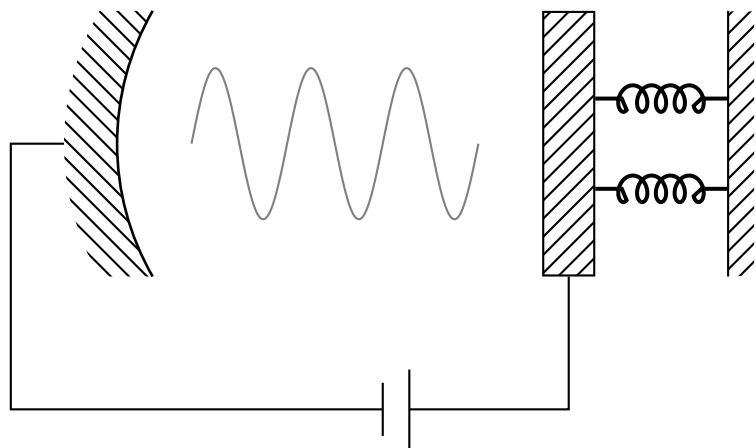


Fig.4: Cavity with a oscillating mirror.

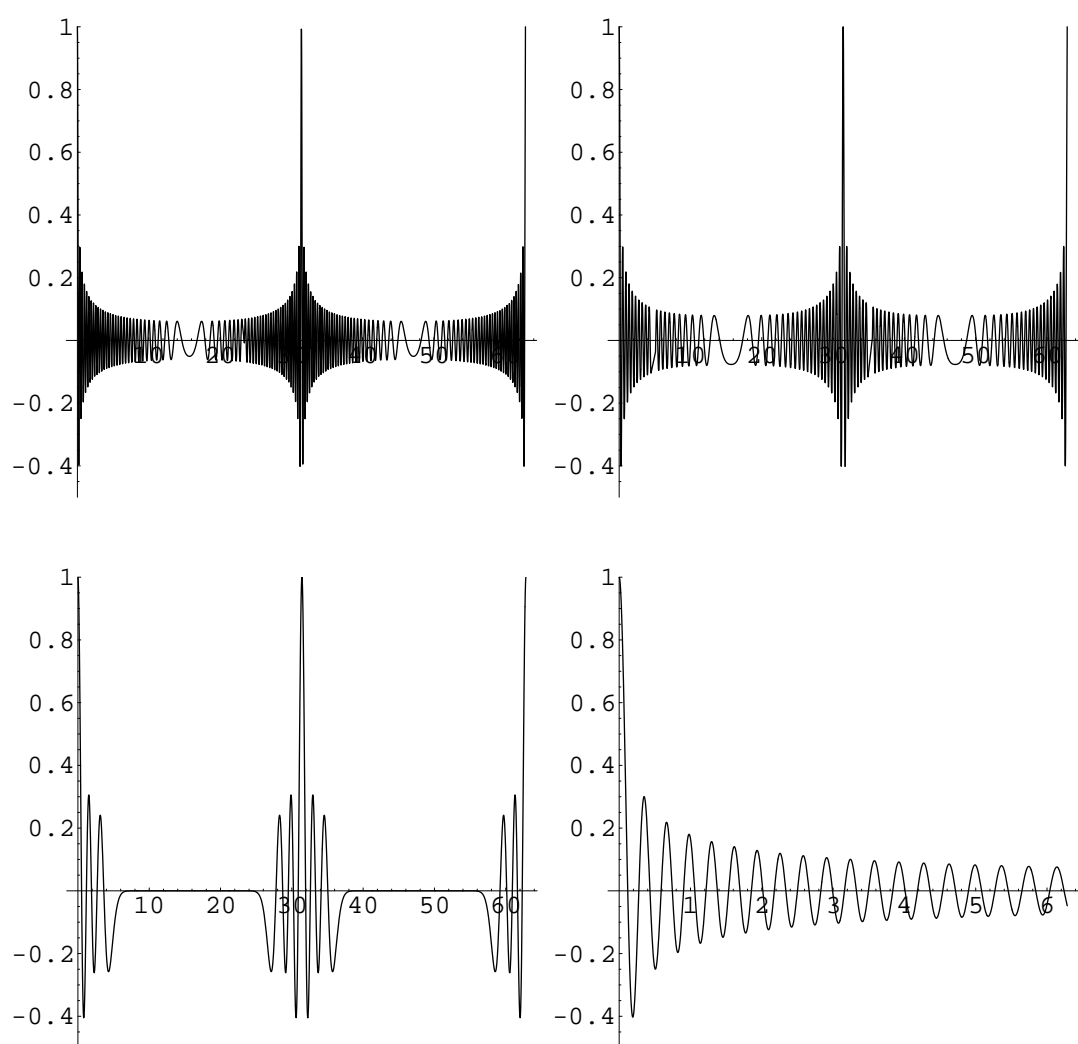


Fig.5:Time-dependency of the nondiagonal elements.

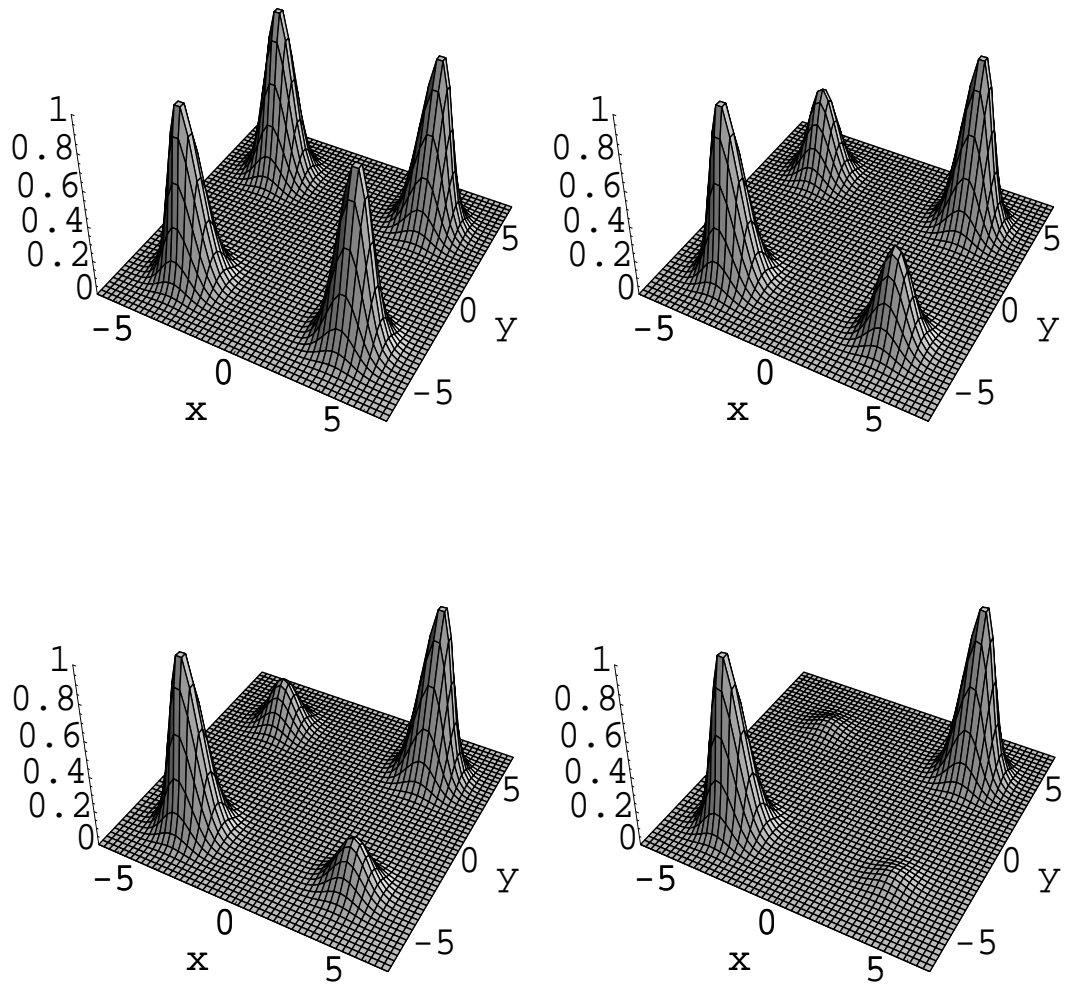


Fig6.Disappearance of the nondiagonal elements of the density matrix.