Solar Proton Burning Process Revisited within a Covariant Model Based on the Bethe-Salpeter Formalism

L.P. Kaptari[a,b], B. Kämpfer[a], E. Grosse[a]

[a]Forschungszentrum Rossendorf, PF 510119, 01314 Dresden, Germany

[b]Bogoliubov Laboratory of Theoretical Physics, JINR Dubna,

141980, Moscow reg., Russia

Abstract

A covariant model based on the Bethe-Salpeter formalism is proposed for investigating the solar proton burning process $pp \to De^+\nu_e$ and the near-threshold deuteron disintegration via electromagnetic and weak interactions. Results of numerical calculations of the energy dependence of relevant cross sections and the astrophysical low-energy cross section factor S_{pp} of the proton burning process are presented. Our results confirm previous canonical values, and the energy dependence of the S_{pp} factor is rather close to phenomenological extrapolations commonly adopted in computations of solar nuclear reaction rates.

I. Introduction: The proton-proton fusion reaction $pp \to De^+\nu_e$ plays an important role in understanding processes occurring in the Sun and in investigating the solar structure. This reaction is the first step in the chain of nuclear reactions producing the solar energy and neutrino fluxes. Unfortunately the corresponding cross section is far too low for a direct laboratory study. In this context reliable theoretical calculations and estimates of the reaction rates are of a great importance. Nowadays, the most precise estimates have been obtained within the framework of low-energy potential models [1–4]. The accuracy of the obtained results is believed to be of the order of few per cents or even better [2,5]. For instance, the low-energy cross section factor S_{pp} that determines the rates for the proton-proton fusion is estimated as $S_{pp} = 4.00(1 \pm 0.007) \times 10^{-25}$ MeV b [2]. While this number is considered as a canonical value, recent estimates [6], based on an effective relativistic field theory, claimed a considerably larger value being inconsistent with helioseismic data [7,8]. This has triggered a series of recalculations of S_{pp} within various approaches (cf. [9–11] and further references therein), which recover values of S_{pp} centered around the canonical one. However, there are some other indications [12] that S_{pp} may be 4.2×10^{-25} MeV b or even larger.

Calculations of the pp rates require the evaluation of two main ingredients: (i) the weak-interaction matrix element, and (ii) the overlap of the pp and deuteron wave functions. In most relativistic effective field theories a self-consistent treatment of the deuteron bound state is difficult, so one usually adopts the short-range effective theory [13] to avoid an explicit use of the deuteron wave function. Within such models one may achieve good fits of data in the zero energy limit, whereas a computation of matrix elements at finite values of the energy is not yet available.

In this paper we calculate the relevant matrix elements within a fully covariant model of the deuteron, based on the Bethe-Salpeter (BS) formalism. Within this model a variety of quantities, including the total and differential cross sections, energy dependences, angular distributions etc. are accessible. In the first part of this paper we introduce our formalism and consider as examples the processes of near-threshold deuteron disintegration by electromagnetic and weak interactions. The second part extends the formalism and is entirely dedicated to the solar pp burning processes. Within the relativistic impulse approximation a fully covariant expression for the S_{pp} factor is obtained.

II. Deuteron disintegration near thresholds: In this section we consider processes of the type $lD \to l'N_1N_2$, i.e. the deuteron disintegration by leptons in electromagnetic $(eD \to e'pn)$ and weak $(e^+D \to \nu pp)$ reactions near threshold. There the amplitudes fall down rapidly and the cross sections are governed by the phase space of final products.

The threshold energies are $E_{thr} = 2.7375$ MeV (eD reactions) and $E_{thr} = 0.93139$ MeV (weak interaction), where E_{thr} is the total energy of the incident lepton in the deuteron rest frame. We consider the electromagnetic disintegration process within the one-photon exchange approximation (cf. fig. 1a) and the weak positron-nucleon interaction (cf. fig. 1b) within the effective Fermi model. The deuteron vertex $D \to NN$ is treated in a covariant way within the BS formalism. The BS equation has been solved numerically [14], with realistic one-boson-exchange interactions, and suitably parameterized [15]. The solution describes fairly accurate the main low-energy and static characteristics of the deuteron, such as the deuteron binding energy, and its quadrupole and magnetic moments [16].

The invariant cross section of the process $D(l, l')NN$, depicted in fig. 1, has the form

$$d\sigma = \frac{(2\pi)^4 \delta^{(4)}\left(k_l + p_D - k'_l - p_{N_1} - p_{N_2}\right)}{2\sqrt{\lambda(s, M_d^2, \mu^2)}} |\mathcal{M}_{lD \to l'NN}|^2 \frac{d\mathbf{k'_1}}{(2\pi)^3 2\mathcal{E'}} \frac{d\mathbf{p'_1}}{(2\pi)^3 2E'_1} \frac{d\mathbf{p'_2}}{(2\pi)^3 2E'_2}, \quad (1)$$

where $\lambda(s, M_d^2, \mu^2) \equiv [s - (M_d + \mu)^2][s - (M_d - \mu)^2]$ is a kinematical factor with s as usual Mandelstam variable, M_d and μ are the deuteron and the electron (positron) masses, respectively. The 4-momenta of leptons are denoted as $k = (\mathcal{E}, \mathbf{k})$, while for hadron momenta the notation $p = (E, \mathbf{p})$ is used. The prime superscript specifies the momenta of the final particles. The subscript "2" denotes the spectator nucleon, i.e. the neutron in eD reactions and the non-interacting proton in e^+D processes (see fig. 1). The two nucleons in the final state are treated in the plane wave approximation, i.e. the effects of final state interaction are here disregarded and all calculations are performed within the relativistic impulse approximation.

The square of the invariant amplitude $\mathcal{M}_{lD \to l'NN}$ may be cast in the form $|\mathcal{M}_{lD \to l'NN}|^2 = S^2(Q^2) l_{\mu\mu} W^{\mu\nu}$, where $S(Q^2 = -(k_l - k'_l)^2)$ is the Feynman propagator of the intermediate exchanged boson (photon or W^+ boson), and the leptonic ($l_{\mu\nu}$) and hadronic ($W_{\mu\nu}$) tensors are defined by

$$l_{\mu\nu} = \frac{1}{2} \sum_{\text{spins}} \langle l'|\hat{j}_\mu|l\rangle \langle l|\hat{j}_\nu^+|l'\rangle = \frac{1}{2}\text{Tr}\left[(\hat{k}_1 + \mu)\Gamma_\mu(\hat{k'}_1 + \mu')\Gamma_\nu\right], \quad (2)$$

$$W_{\mu\nu} = \frac{1}{3} \sum_{\text{spins}} \langle N'_1 N'_2|\hat{J}_\mu|D\rangle \langle D|\hat{J}_\nu^+|N'_1 N'_2\rangle, \quad (3)$$

where $\hat{j}_\mu$ and $\hat{J}_\mu$ denote the leptonic and hadronic current operators, Γ_μ is the corresponding vertex function, related to the choice of interacting currents; $\hat{k}$ is used as a short hand notation for $\gamma^\mu k_\mu$. The electron-nucleon *electromagnetic* vertex in the on-mass-shell form, $\Gamma_\mu^{eN}(Q^2) = \gamma_\mu F_1(Q^2) + i\frac{\sigma_{\mu\alpha}Q^\alpha}{2m}\kappa F_2(Q^2)$, contains the electromagnetic form factors of the nucleon $F_{1,2}$, and is its anomalous magnetic moment κ. In view of the small transferred momenta and low initial energies, considered here, the second term may be safely

disregarded, and with high accuracy also the nucleon formfactor $F_1(Q^2)$ is replaced by unity. Then the electromagnetic part of the leptonic tensor is computed straightforwardly. For the *weak vertices* one may employ the Weinberg-Salam theory for weak interactions, however, in the considered energy range the effective Fermi model is appropriate. Thus

$$\hat{j}_\mu(x) = \bar{\psi}_e(x)\gamma_\mu\left(1 - \gamma_5\right)\psi_\nu(x), \tag{4}$$

$$\hat{J}_\nu(x)^{(\Delta S=0)} = \cos\theta_c\bar{\psi}_N(x)\frac{(\tau_1 - i\tau_2)}{\sqrt{2}}\gamma_\mu\left(1 - R\gamma_5\right)\psi_N(x), \tag{5}$$

and $S(Q^2) = \dfrac{G_F}{\sqrt{2}}$, and $R = 1.2681$ is the ratio of axial to vector coupling constants, $G_F = 1.16639 \times 10^{-5}\,\mathrm{GeV}^{-2}$ is the effective Fermi constant for the weak nucleon decay, $\theta_c = 0.24$ is the Cabibbo strangeness mixing angle ($\Delta S = 0$ means the strangeness conserving part of the current), $\psi_{l,N}(x)$ are the corresponding leptonic and nucleonic field operators, τ stands for the isospin Pauli matrices.

With virtue of the Mandelstam technique [17] for computing matrix elements within the BS formalism, the deuteron tensor $W_{\mu\nu}$ in eq. (3) receives the form

$$W_{\mu\nu} = \frac{1}{3}\left(p_2'^2 - m_2^2\right)\mathrm{Tr}\left[\bar{\Psi}^{BS}(p_1, p_2)\Gamma_\mu\left(\hat{p}_1' + m_1\right)\Gamma_\nu\Psi^{BS}(p_1, p_2)\left(\hat{p}_2' + m_2\right)\right], \tag{6}$$

where Ψ^{BS} is the deuteron amplitude as defined in refs. [16,18,19] and p_1 and $p_2 = p_2'$ are the momenta of the deuteron constituents with a mass m, i.e. the proton (m_p) or neutron (m_n) masses. By inspecting eq. (6) one observes that, due to the adopted mechanism in fig. 1, the spectator nucleon is on mass shell, and the first factor in eq. (6) is always zero. Nevertheless, the BS amplitudes are themselves singular when one nucleon is on mass shell with the singularity of each amplitude being of the order $(p_2^2 - m_2^2)^{-1}$. Consequently, the r.h.s. of eq. (6) is finite. In the general case the BS amplitude Ψ^{BS}, which is a 4×4 matrix, may be projected on a set of basis matrices and such an operation determines eight independent partial amplitudes [16,18]. However not all these amplitudes equally contribute to the deuteron matrix elements. One may use a basis set of spin-angular matrices [18] which allows to classify the partial amplitudes corresponding to their contributions in a full analogy with the non-relativistic case as ${}^3S_1^{++(--)}$, ${}^3D_1^{++(--)}$, ${}^3P_1^{+-(-+)}$, ${}^1P_1^{+-(-+)}$ waves in the spectroscopical notation (the superscripts denote the ρ spin of each constituent, cf. [16] for details). Then the ${}^3S_1^{++}$ and ${}^3D_1^{++}$ waves turn out to be a relativistic generalization of the non-relativistic S and D waves in the deuteron [16]. In the considered reactions the contribution of D waves vanishes near the threshold (as do the other amplitudes with at least one negative ρ spin), leaving us with contributions only from the ${}^3S_1^{++}$ waves. A direct evaluation of traces in eqs. (2, 6) results in analytical, covariant expressions for the cross section. For instance, the electromagnetic tensors read

$$l_{\mu\nu} W^{\mu\nu} = 16 M_d (2\pi)^3 \Phi^2(p_0, |\mathbf{p}_2|) \times$$
$$\left[(k_e p_1')(k' p_1) - \mu^2 (p_1 p_1') - (k_e' k_e) m_p^2 + (k_e' p_1')(k_e p_1) + 2\mu^2 m_p^2 \right], \tag{7}$$

where $\Phi(p_0, |\mathbf{p}_2|)$ is the covariant generalization of the deuteron wave function [16] within the BS formalism, which has the following form in the deuteron rest frame

$$\Phi(|\mathbf{p}_2|) = \frac{G_S(p_0, |\mathbf{p}_2|)}{(2\pi)^2 \sqrt{2M_d}(M_d - 2E_p)^2}, \qquad p_0 = M_d/2 - E_p, \quad E_p = \sqrt{m^2 + \mathbf{p}_2^2}, \tag{8}$$

where $G_S(p_0, |\mathbf{p}_2|)$ is the BS partial vertex corresponding to the $^3S_1^{++}$ amplitude [16] and m is an average value of the nucleon mass. Inserting eq. (7) into eq. (1) yields

$$d\sigma^{eD} = \frac{m_n}{E_2} \Phi^2(|\mathbf{p}_2|) \, d\mathbf{p}_2 \, \delta(p_1'^2 - m_p^2) \times \tag{9}$$

$$\left\{ \frac{4\alpha^2}{Q^4} \frac{4M_d}{m_p \sqrt{\lambda(s, M_d^2, \mu^2)}} \left((k_e p_1')(k' p_1) - \mu^2 (p_1 p_1') - (k_e' k_e) m_p^2 + (k_e' p_1')(k_e p_1) + 2\mu^2 m_p^2 \right) \frac{d\mathbf{k}_e'}{2\mathcal{E}'} \right\},$$

where the expression enclosed in the curly brackets together with the δ function is merely the cross section $d\sigma^{ep}$ of an electron scattering off a moving proton with the momentum p_1. Hence, for the electromagnetic eD disintegration processes the cross section may be written as

$$\sigma^{eD} = \int d\mathbf{p}_2 \frac{m_n}{E_2} \Phi^2(|\mathbf{p}_2|) \, d\sigma^{ep}(k, p_1). \tag{10}$$

By processing in an analogous way with the weak disintegration diagram displayed in fig. 1b, one obtains

$$\sigma_w^{e^+ D} = \int d\mathbf{p}_2 \frac{m_n}{E_2} \Phi^2(|\mathbf{p}_2|) \, d\sigma_w^{en}(k, p_1), \tag{11}$$

where, for convenience, the "elementary" cross section $d\sigma_w^{en}(k, p_1)$ for a hypothetical $e^+ n \to \nu p$ reaction has been introduced by

$$d\sigma_w^{en}(k, p_1) = \frac{G_F^2}{(2\pi)^2} \frac{4M_d \delta(p_1'^2 - m_p^2)}{m_p \sqrt{\lambda(s, M_d^2, \mu^2)}} \left\{ \left(R^2 + 1 \right) \left[(k_e p_1)(k_\nu' p_1') + (k_e p_1')(k_\nu' p_1) \right] \right.$$

$$\left. + \left(R^2 - 1 \right) m_p m_n (k_e k_\nu') - 2R \left[(k_e p_1')(k_\nu' p_1) - (k_e p_1)(k_\nu' p_1') \right] \right\} \frac{d\mathbf{k}_\nu'}{\mathcal{E}'}. \tag{12}$$

In computing eq. (11) the angular integration on $\mathbf{p}_2$ should be performed in only one-half of the hemisphere due to the two identical particles in the final state.

In fig. 2 results of the numerical calculation of the total cross section as a function of the initial energy of the electron (positron) in the laboratory frame are displayed. Above the electromagnetic disintegration threshold there is probably no way to experimentally separate the weak processes from the electromagnetic ones. Below the electromagnetic

and above the weak thresholds, the cross section of positron weak disintegration of the deuteron seems far too low to be measured under laboratory conditions. So one may conclude that at present an experimental investigation of the matrix element, relevant for the solar pp fusion, is still impossible also when using the cross channels. (This is in contrast to other reactions, cf. [20].) Only reliable theoretical calculations may be applied to estimate the nuclear reaction rates and to interpret the corresponding experimental neutrino data.

III. Solar proton burning process: Let us now consider the process of the two-proton fusion $p_1 p_2 \rightarrow \nu e^+ D$ at low relative energies. While this reaction looks rather similar to the processes in the previous section, here appears a peculiarity. Due to the relation $2m_p > M_d + \mu$ (with $m_p = 938.27231$ MeV, $M_d = 1875.61339$ MeV, deuteron binding energy $\varepsilon_D = 2.22455$ MeV, and $\mu = 0.510999$ MeV) there is no energy threshold for the pp fusion and, in principle, the fusion process may occur at arbitrarily small relative energies of the two protons. In this case, in the cross section (1) two factors will play the major role: (i) the two-proton flux ($\sim \sqrt{\lambda(s, m_p^2, m_p^2)}$) approaches rather rapidly zero and the computation of the cross section becomes problematic, (ii) a strong Coloumb repulsion between protons makes the amplitude $\mathcal{M}$ sharply decreasing and compensates singularities appearing from the vanishing flux. In order to avoid uncertainties connected to these circumstances one usually separates the contribution of Coulomb repulsion by factorizing the Coulomb part of the two-proton final state, which at low relative energies may be taken as $\psi_{pp} = \dfrac{\sqrt{2\pi\eta}}{e^{\pi\eta} - 1} \approx \sqrt{2\pi\eta} \, e^{-\pi\eta}$, where $\eta = \alpha/v$ is the Sommerfeld penetration parameter, v denotes the relative velocity of the protons, and $T_{pp} = m_p v^2/4$ is the corresponding relative kinetic energy; the exponent is known as the Gamov penetration factor [2]. Then one introduces the notion of the S_{pp} factor, which is determined only by the nuclear part of the interaction and is connected with the cross section by $\sigma \equiv \dfrac{S_{pp}(T_{pp})}{T_{pp}} e^{-2\pi\eta}$. Then the rate of pp fusion reactions can be written in the form [1,3] $\langle \sigma v \rangle_{pp} = 1.3005 \times 10^{-18} (2/T_6^2)^{1/3} f S_{pp}(T_{pp}) e^{-\tau}$ cm^{3}s^{-1}; here T_6 is the temperature in units of 10^6 K, $\tau \sim 15 - 40$ dominates the temperature dependence of the reaction rate, f is a screening factor calculated first by Salpeter [21], and S_{pp} is in MeV b. The S_{pp} factor is needed at the most probable interaction energy $T_{pp}^{(0)}$, which is in a range of 5 keV $\cdots$ 30 keV. However, in most analyzes in the literature, the value of S_{pp} is quoted at vanishing energy, obtained , e.g. within the low-energy effective theory [3,4,13]. Hence, for the sake of extrapolation to the finite value of $T_{pp}^{(0)}$, a good estimate of the associated derivatives is also required.

We now apply the same formalism as in the previous section, i.e. the relativistic impulse approximation, to estimate the low-energy behavior of S_{pp}. The effect of initial state

interaction is accounted for only in the Coulomb part of the interaction by adopting the corresponding part of the above two-proton wave function ψ_{pp}. A treatment of the nuclear part of the proton wave function as plane waves means that in the matrix element all transitions are allowed. Nevertheless, at low energies only the lowest partial waves contribute to the proton wave function and one may expect that the total cross section will be determined by the axial part of the current operator (5) and correspondingly by the super-allowed $0^+ \to 0^+$ Gamov-Teller transition.

In the limit $T_{pp} \to 0$, apart from the phase space volume, the value of the deuteron wave function $\psi(p_0, |\mathbf{p}|)$ at vanishing values of its arguments becomes important. As can be seen from eq. (8), at $|\mathbf{p}| \to 0$, $2E_p \to 2m$ the wave function $\psi(p_0, |\mathbf{p}|)$ displays a sharp maximum. Our solution of the BS equation [14] has been obtained by solving a system of coupled integral equations with Gaussian meshes with a minimum value of $|\mathbf{p}| = 0.310848493733$ MeV, so that an extrapolation to $|\mathbf{p}| = 0$ needs to be employed. This procedure causes some numerical uncertainty which we estimate to be about 10% in the final results.

By observing that in the center of mass of colliding protons $2\sqrt{\lambda(s, m_p^2, m_p^2)} = 4E_1 E_2\, v = sv$ and adopting the Gamow penetration factor, eq. (1) leads to

$$S_{pp}(T_{pp}) = \frac{G_F^2}{8s} \frac{\pi m_p}{(2\pi)^5} \int \frac{d\mathbf{P_d}}{E_d} \int \frac{d\mathbf{k}_\nu'}{\mathcal{E}_\nu'} \left|\mathcal{M}_{pp \to e^+\nu D}\right|^2 \delta(k_{e^+}^2 - \mu^2), \qquad (13)$$

where, due to the antisymmetrization of the initial two-proton state, the matrix element $\mathcal{M}_{pp \to e^+\nu D}$ is now determined by two Feynman diagrams, as depicted in fig. 3, with a relative minus sign. Correspondingly, the hadron tensor $W_{\mu\nu}$ consists of a direct and an interference term,

$$W_{\mu\nu}^{(dir)} = (p_2^2 - m_p^2)\mathrm{Tr}\left[\bar{\Psi}(p_1, p_n)\Gamma_\mu(\hat{p}_2 + m_p)\Gamma_\nu\Psi(p_1, p_n)(\hat{p}_2 + m_p)\right]$$
$$+ (p_1^2 - m_p^2)\mathrm{Tr}\left[\Psi(p_1, p_n)\Gamma_\nu(\hat{p}_1 - m_p)\Gamma_\mu\bar{\Psi}(p_2, p_n)(\hat{p}_1 - m_p)\right], \qquad (14)$$

$$W_{\mu\nu}^{(int)} = (p_2^2 - m_p^2)(p_1^2 - m_p^2)\mathrm{Tr}\left[\bar{\Psi}(p_2, p_n)\Gamma_\nu\Psi(p_1, p_n)\Gamma_\mu + \bar{\Psi}(p_1, p_n)\Gamma_\mu\Psi(p_2, p_n)\Gamma_\nu\right], \quad (15)$$

where Γ_μ is the vertex corresponding to the effective current operator (5). The direct part of the hadron tensor has exactly the same form as for the reactions $e^+ D \to \nu pp$ (cf. eq. (6)), resulting in an "elementary" cross section like that in eq. (12). The interference term is different and much more cumbersome, and for the sake of brevity we do not display it here.

In fig. 4 the energy dependence of S_{pp} computed by eq. (13) is displayed. The S_{pp} factor, starting from $T_{pp} + 0$ up to solar energies T_{pp}^0, displays a rather weak energy dependence. At very low kinetic energies the value of S_{pp} remains fairly constant, having

a value of 3.960×10^{-25} MeV b in agreement with previous results [2,5]. At larger energies, however, the S_{pp} factor increases rather rapidly. The two curves in fig. 4 correspond to two different methods of numerical calculations. In one case the deuteron wave function is replaced by its value at zero transferred momenta. To some extent this corresponds to calculations within low-energy effective potential models. The full line reflects results of calculations without such an approximation. Only at relatively high values of the kinetic energy, $T_{pp} > 20$ KeV, the effect of the deuteron wave function becomes sizeable.

Within low-energy effective theories the associated derivatives of the S_{pp} factor play an important role. The best estimates of the first derivative seem to be those quoted in ref. [2], $S'_{pp} = 4.48 \times 10^{-24}$ b, and the logarithmic one $S'_{pp}/S = 11.2$ MeV^{-1}. In figs. 5 and 6 we display our estimates for the energy dependence of these derivatives. The derivatives are computed at each value of T_{pp} by interpolating S_{pp} with a quadratic interpolation formula. From these pictures it is seen that within our approach both the derivative and the logarithmic derivative are below the estimates in [2] by $\sim 15 - 20\%$. However, at energies around $T_{pp}^{(0)}$ our results for the S_{pp} factor and the results of an interpolation with parameters from [2] are rather close to each other.

IV. Summary: We present a covariant model, based on the Bethe-Salpeter formalism, to investigate the energy dependence of the matrix elements relevant to the solar proton-proton burning process. Explicit covariant expressions for the cross section and the low-energy cross section factor S_{pp} are presented. Our numerical estimates of S_{pp} and its derivatives confirm previous results.

REFERENCES

[1] J.N. Bahcall, M.H. Pinsonneault, J. Wasserburg, Rev. Mod. Phys. **65** (1995) 885.

[2] E.G. Adelberger et al., Rev. Mod. Phys. **70** (1998) 1265.

[3] J. Carlson, D.O. Riska, R. Schiavilla, R.B. Wiringa, Phys. Rev. **C 44** (1991) 619.

[4] R. Schiavilla et al., Phys. Rev. **C 58** (1998) 1263.

[5] M. Kamionkowsky, J.N. Bahcall, Ap. J. **420** (1994) 884.

[6] A.N. Ivanov, N.I. Troitskaya, M. Faber, H. Oberhummer, Nucl. Phys. **A 617** (1997) 414, **A 625** (1997) 896.

[7] S. Degl'Innocenti, G. Fiorentini, B. Ricci, **astro-ph**/9707133.

[8] J.N. Bahcall, M. Kamionkowski, Nucl. Phys. **A 625** (1997) 893.

[9] A.N. Ivanov, H. Oberhummer, N.I. Troitskaya, M. Faber, **nucl-th**/9910021.

[10] X. Kong, F. Ravndal, **nucl-th**/9904066; Nucl. Phys. **A 656** (1999) 421.

[11] T.-S. Park, K. Kubodera, D.-P. Min, M. Rho, **astro-ph**/9804144.

[12] H. Schlattl, A. Bonanno, L. Paterno, **astro-ph**/9902354.

[13] D.B. Kaplan, M.J. Savage, M.B. Wise, Nucl. Phys. **B 534** (1998) 329; Phys. Rev. **C 59** (1999) 617.

[14] A.Yu. Umnikov, L.P. Kaptari, F.C Khanna, Phys. Rev. **C 56** (1997) 1700; Phys. Lett. **B 334** (1994) 163.

[15] A.Yu. Umnikov, Z. Phys. **A 357** (1997) 333.

[16] L.P. Kaptari, A.Yu. Umnikov, S.G. Bondarenko, K.Yu. Kazakov, F.C. Khanna, B. Kämpfer, Phys. Rev. **C 54** (1996) 986.

[17] S. Mandelstam, Proc. Roy. Soc. (London) **A 233** (1955) 123.

[18] J.J. Kubis, Phys. Rev. **D 6** (1972) 547;
M.J. Zuilhof, J.A. Tjon, Phys. Rev. **C 22** (1980) 2369.

[19] B.D. Keister, J.A. Tjon, Phys. Rev. **C 26** (1982) 578.

[20] N. Iwasaet et al., Phys. Rev. Lett. **83** (1999) 2910.

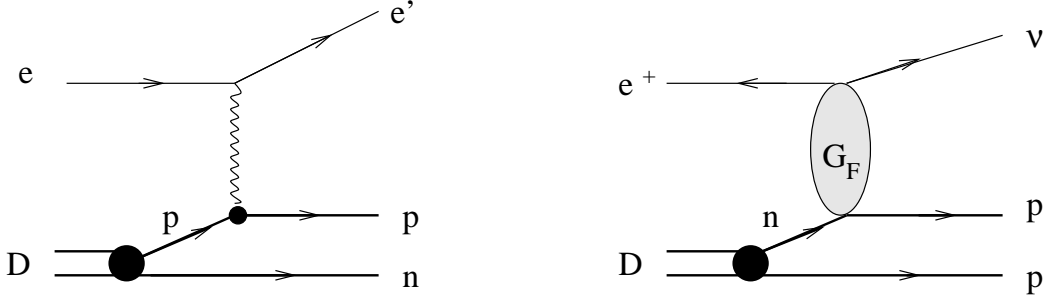[21] E.E. Salpeter, Phys. Rev. **88** (1952) 547.

FIG. 1. Feynman diagrams for electromagnetic (left panel) and weak (right panel) deuteron disintegration. The weak coupling constant and the propagator of the W^+ boson has been replaced by the effective Fermi constant G_F for the 4-point contact interaction.
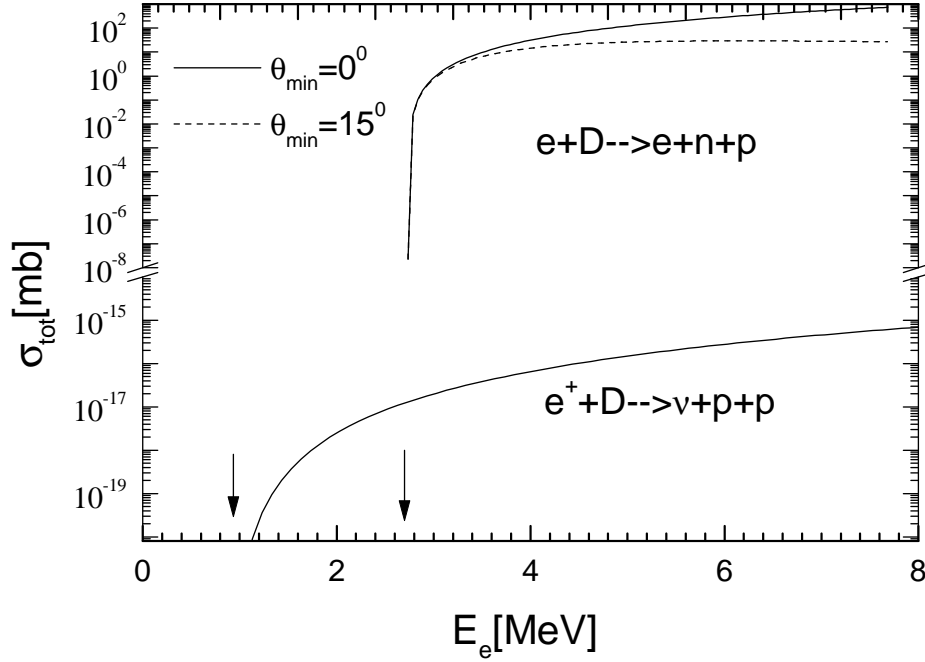

FIG. 2. The threshold behavior of the total cross section in electromagnetic and weak disintegrations of the deuteron. The arrows depict the corresponding thresholds. The dashed line displays the integrated $eD \rightarrow enp$ cross section when the outgoing electron polar angle Θ is larger than 15°.
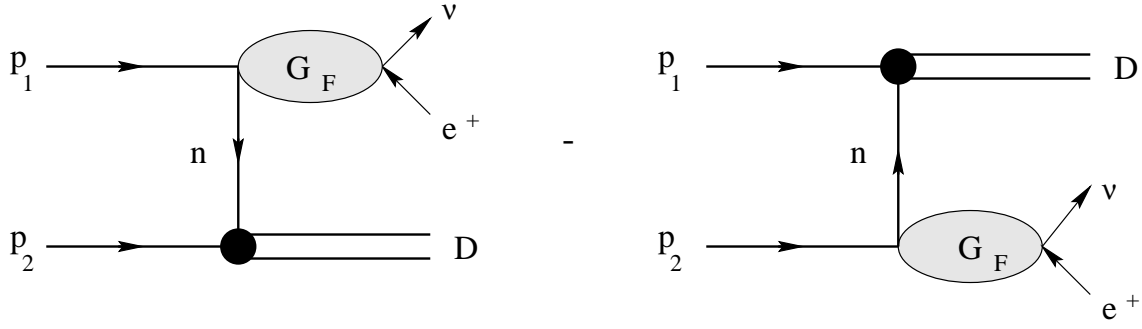
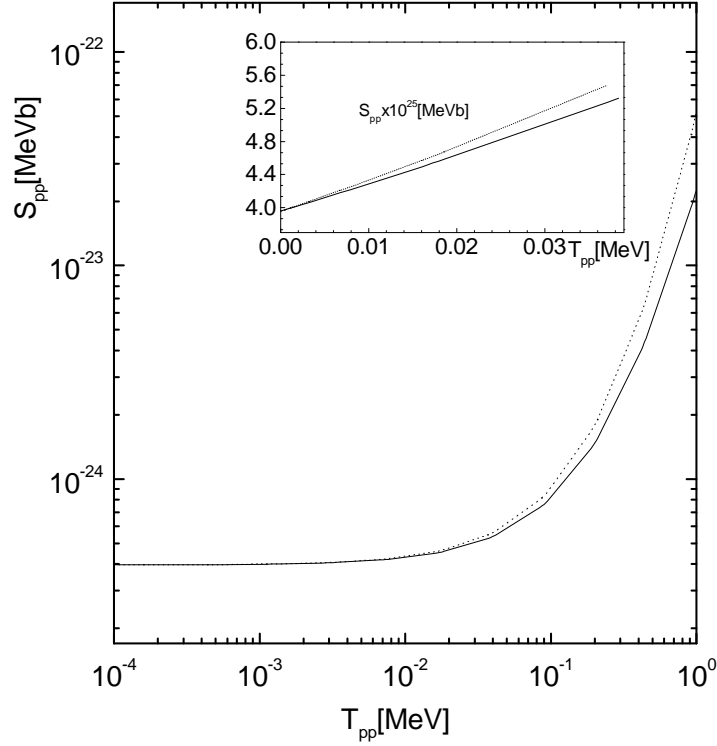FIG. 3. Feynman diagrams contributing to the processes $pp \rightarrow e^+\nu D$.

FIG. 4. The S_{pp} factor for the reaction $pp \rightarrow e^+\nu D$ as a function of the relative kinetic energy of incident protons. The full line is the result of a calculation by eq. (13), the dotted line is the result of a computation with the deuteron wave function taken as constant, $\Psi(p_0, |\mathbf{p}|) = \Psi(0, 0)$. The inset shows S_{pp} vs. T_{pp} in linear scales.
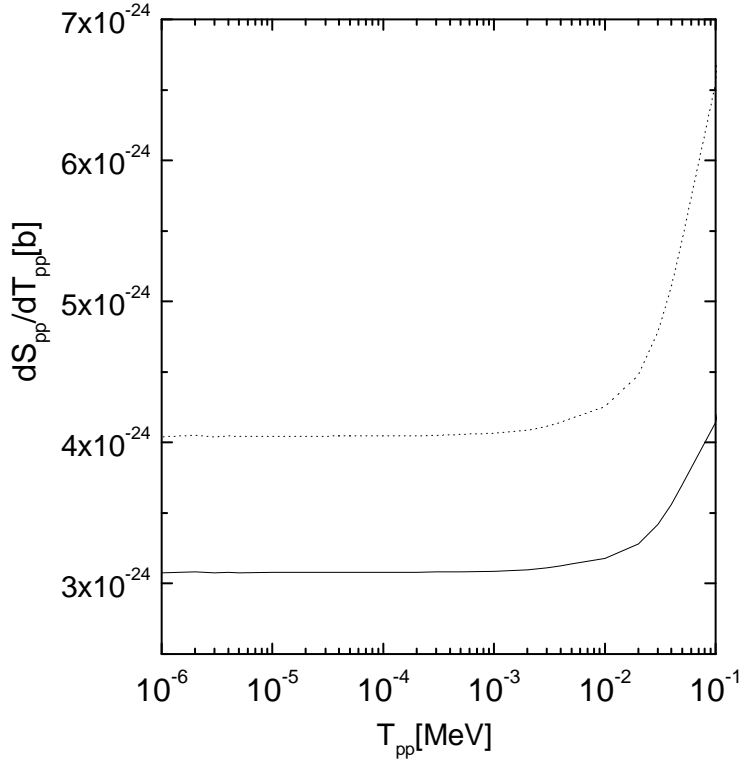
FIG. 5. The first derivative of the low-energy nuclear reaction cross section factor S_{pp}. Notation as in fig. 4
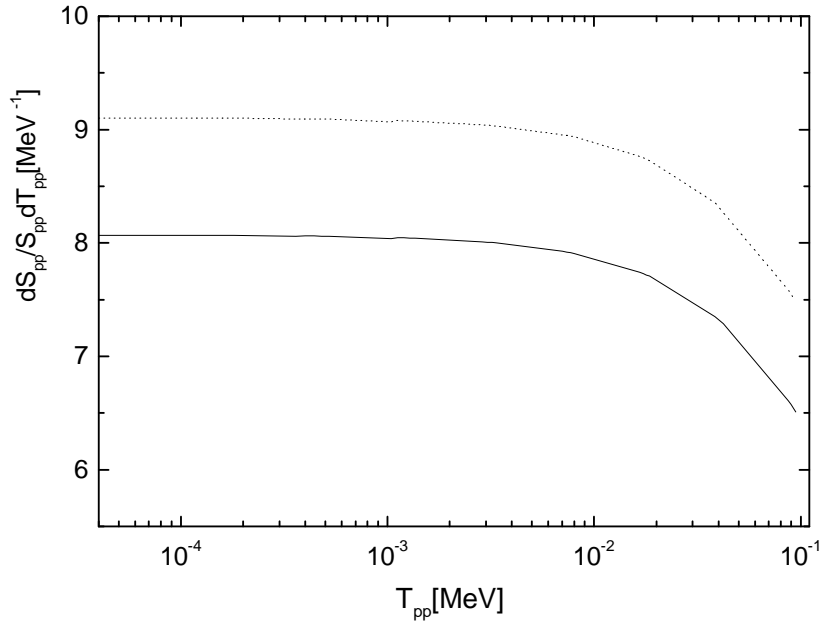

FIG. 6. The logarithmic derivative of the low-energy nuclear reaction cross section factor S_{pp}. Notation as in fig. 4