

# Freeze-out in hydrodynamical models in relativistic heavy ion collisions

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## 1. INTRODUCTION

In continuum and fluid dynamical models, particles, which leave the system and reach the detectors, can be taken into account via freeze-out (FO) or final break-up schemes, where the frozen out particles are formed on a 3-dimensional hypersurface in space-time. Such FO descriptions are important ingredients of evaluations of two-particle correlation data, transverse-, longitudinal-, radial- and cylindrical- flow analyses, transverse momentum and transverse mass spectra and many other observables. The FO on a hypersurface is a discontinuity, where the pre FO equilibrated and interacting matter abruptly changes to non-interacting particles, showing an ideal gas type of behavior.

The frequently used Cooper-Frye formula, the rapidity distribution, transverse momentum spectra, etc., – all include expression  $f_{FO}(x, p, T, n, u^\nu) p^\mu d\hat{\sigma}_\mu$ , where  $f_{FO}(x, p, T, n, u^\nu)$  is the post FO distribution which is unknown from the fluid dynamical model,  $d\hat{\sigma}_\mu$  is normal vector to the FO hypersurface. Those formulas work well for timelike  $d\hat{\sigma}^\mu$  ( $p^\mu d\hat{\sigma}_\mu > 0$ ). If  $d\hat{\sigma}_\mu$  is spacelike we count particles going backwards through FO front as well as outwards. The post FO distribution can not be a thermal one! In fact  $f_{FO}$  should contain only particles which cross the FO front outwards,  $p^\mu d\hat{\sigma}_\mu > 0$ , since the rescattering and back scattering are not allowed any more in the post FO side.

$$f_{FO}(x, p, T, n, u^\nu, d\hat{\sigma}_\mu) = 0, \quad p^\mu d\hat{\sigma}_\mu < 0. \quad (1)$$

Initially if we know the pre FO baryon current and energy-momentum tensor,  $N_0^\mu$  and  $T_0^{\mu\nu}$ , we can calculate locally, across a surface element of normal vector  $d\hat{\sigma}^\mu$  the post FO quantities,  $N^\mu$  and  $T^{\mu\nu}$ , from the relations:  $[N^\mu d\hat{\sigma}_\mu] = 0$  and  $[T^{\mu\nu} d\hat{\sigma}_\mu] = 0$ , where  $[A] \equiv A - A_0$ . One should also check that entropy is nondecreasing in FO. In numerical calculations the local FO surface can be determined most accurately via self-consistent iteration. Initial ideas to improve the Cooper-Fry FO description this way were suggested in refs. [1–3].

## 2. FREEZE-OUT DISTRIBUTION FROM KINETIC THEORY

Let us assume an infinitely long tube with its left half ( $x < 0$ ) filled with nuclear matter and in the right vacuum is maintained. We can remove the dividing wall at  $t = 0$ , and then the matter will expand into the vacuum. Continuously removing particles at the right end of the tube and supplying particles on the left end we can establish a stationary flow in the tube, where the particles will gradually freeze out in an exponential rarefaction wave propagating to the left in the matter. We can move with this front, so that we describe it from the reference frame of the front (RFF).

We assume that there are two components of our momentum distribution:  $f_{free}(x, \vec{p})$  and  $f_{int}(x, \vec{p})$ . However, we assume that at  $x = 0$ ,  $f_{free}$  vanishes exactly and  $f_{int}$  is an ideal Jüttner distribution, then  $f_{int}$  gradually disappears and  $f_{free}$  gradually builds up, according to the differential equations:

$$\begin{aligned} \partial_x f_{int}(x, \vec{p}) dx &= -\Theta(p^\mu d\hat{\sigma}_\mu) \frac{\cos \theta_{\vec{p}}}{\lambda} f_{int}(x, \vec{p}) dx, \\ \partial_x f_{free}(x, \vec{p}) dx &= +\Theta(p^\mu d\hat{\sigma}_\mu) \frac{\cos \theta_{\vec{p}}}{\lambda} f_{int}(x, \vec{p}) dx, \end{aligned} \quad (2)$$

where  $\cos \theta_{\vec{p}} = \frac{p^x}{p}$  in RFF. It expresses the fact that particles with momenta orthogonal to the FO surface leave the system with bigger probability than particles emitted at an angle.

Such a dramatically oversimplified model can reproduce cut Jüttner distribution as a post FO one, but it does not allow the complete FO – the interacting component of momentum distribution survives even if  $x \rightarrow \infty$ .

To improve our model we take into account rescattering within the interacting component, which will lead to re-thermalization and re-equilibration of this component. We

use the relaxation time approximation to simplify the description of the dynamics. Thus, the two components of the momentum distribution develop according to the differential equations:

$$\begin{aligned} \partial_x f_{int}(x, \vec{p}) dx = & -\Theta(p^\mu d\hat{\sigma}_\mu) \frac{\cos \theta_{\vec{p}}}{\lambda} f_{int}(x, \vec{p}) dx + \\ & + [f_{eq}(x, \vec{p}) - f_{int}(x, \vec{p})] \frac{1}{\lambda'} dx, \end{aligned} \quad (3)$$

$$\partial_x f_{free}(x, \vec{p}) dx = +\Theta(p^\mu d\hat{\sigma}_\mu) \frac{\cos \theta_{\vec{p}}}{\lambda} f_{int}(x, \vec{p}) dx. \quad (4)$$

The interacting component of the momentum distribution, described by eq. (3), shows the tendency to approach an equilibrated distribution with a relaxation length  $\lambda'$ . Of course, due to the energy, momentum and particle drain, this distribution  $f_{eq}(x, \vec{p})$  is not the same as the initial Jüttner distribution, but its parameters,  $n_{eq}(x)$ ,  $T_{eq}(x)$  and  $u_{eq}^\mu(x)$ , change as required by the conservation laws.

Let us assume that  $\lambda' \ll \lambda$ , i.e. re-thermalization is much faster than particles are freezing out, or much faster than parameters,  $n_{eq}(x)$ ,  $T_{eq}(x)$  and  $u_{eq}^\mu(x)$  change. Then  $f_{int}(x, \vec{p}) \approx f_{eq}(x, \vec{p})$ , for  $\lambda' \ll \lambda$ . For  $f_{eq}(x, \vec{p})$  we assume the spherical Jüttner form at any  $x$  including both positive and negative momentum parts with parameters  $n(x)$ ,  $T(x)$  and  $u_{RFG}^\mu(x)$ . (Here  $u_{RFG}^\mu(x)$  is the actual flow velocity of the interacting, Jüttner component, i.e. the velocity of the Rest Frame of the Gas (RFG) [1]). In this case the changes of conserved quantities due to particle drain or transfer can be evaluated for an infinitesimal  $dx$  and new parameters of  $f_{eq}(x + dx, \vec{p})$  can be found [4].

We would like to point out that, although for the spherical Jüttner distribution the Landau and Eckart flow velocities are the same, the change of this flow velocity calculated from the loss of baryon current and from the loss of energy current appear to be different  $du_{i,E,RFG}^\mu(x) \neq du_{i,L,RFG}^\mu(x)$ . This is a clear consequence of the asymmetry caused by the FO process as it was discussed in ref. [4,5]. This problem does not occur for the freeze-out of baryonfree plasma.

We performed the calculations, according to this model, for the baryonfree and massless gas [6,7]. We would like to note that now  $f_{int}(x, \vec{p})$  does not tend to the cut Jüttner distribution in the limit  $x \rightarrow \infty$ . Furthermore, we obtain that  $T \rightarrow 0$ , when  $x \rightarrow \infty$ . So,  $f_{int}(x, \vec{p}) = \frac{1}{(2\pi\hbar)^3} \exp[(\mu - p^\nu u_\nu)/T] \rightarrow 0$ , when  $x \rightarrow \infty$ . Thus, all particles freeze out in the improved model, but such a physical FO requires infinite distance (or time). This second problem may also be removed by using volume emission model discussed in [6]. The application of our onedimensional model to transverse expansion gives result showing in Fig. 1, which is in qualitative agreement with experiment.

Recent calculations have confirmed that the FO hypersurface idealization can be justified even in microscopic reaction models (like UrQMD or QGSM) for nucleon data in collisions of massive heavy ions.

The improvements presented here are essential and lead to non-negligible qualitative and quantitative changes in calculations including FO. Several further details and consequences of this improved approach have to be worked out still (e.g. [9]) to obtain more accurate data from the numerous continuum and fluid dynamical models used for the description of heavy ion reactions.

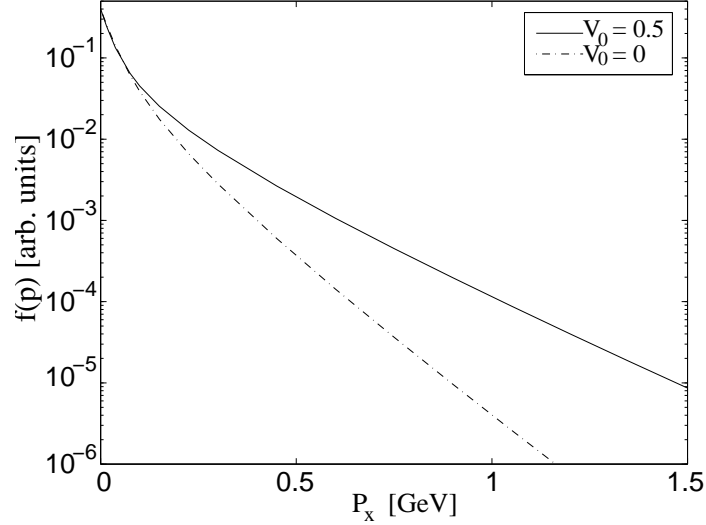


Figure 1. The local transverse momentum (here  $p_x$ ) distribution for baryonfree, massless gas at  $p_y = 0$ ,  $x = 100\lambda$  and  $T_0 = 130$  MeV. The transverse momentum spectrum is obviously curved due to the freeze-out process, particularly for large initial flow velocities. The apparent slope parameter increases with increasing transverse momentum. This behavior agrees with observed pion transverse mass spectra at SPS [8]. From [6].

## REFERENCES

1. K.A. Bugaev, Nucl. Phys. A606 (1996) 559.
2. L.P. Csernai, Zs.I. Lázár and D. Molnár, Heavy Ion Phys. 5 (1997) 467.
3. J.J. Neymann, B. Lavrenchuk and G. Fai, Heavy Ion Phys. 5 (1997) 27.
4. Cs. Anderlik, Zs. Lázár, V.K. Magas, L.P. Csernai, H. Stöcker and W. Greiner, (nucl-th/9808024) Phys. Rev. C 59 (99) 388.
5. Cs. Anderlik, L.P. Csernai, F. Grassi, W. Greiner, Y. Hama, T. Kodama, Zs. Lázár, V.K. Magas and H. Stöcker, (nucl-th/9806004) Phys. Rev. C 59 (1999) in press.
6. Cs. Anderlik, L.P. Csernai, F. Grassi, W. Greiner, Y. Hama, T. Kodama, Zs. Lázár, V.K. Magas and H. Stöcker, (nucl-th/9903045) Heavy Ion Physics 9 (1999) 193.
7. V.K. Magas, Cs. Anderlik, L.P. Csernai, F. Grassi, W. Greiner, Y. Hama, T. Kodama, Zs. Lázár and H. Stöcker, (nucl-th/9905054) accepted for publication in Phys. Lett. B.
8. Nu Xu, et al., (NA44), Nucl. Phys. A610 (1997) 175c. (pion spectra in Figs. 4 and 6.) P.G. Jones, et al., (NA49), Nucl. Phys. A610 (1997) 188. ( $h^-$  spectra in Fig. 2.)
9. K. A. Bugaev, M. I. Gorenstein, nucl-th/9903072.