

An Approach to Measurement by Quantum-Stochastic-Parameter Averaged Bohmian Mechanics

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Abstract

A coarse-grained quantum operator technique is used along with the formalism of Bohmian mechanics endowed with stochastic character at the quantum level in order to address some central issues in the quantum theory of measurement. A surprisingly simple picture of decoherence and EPR correlations emerges from its use.

“It is the customary fate of new truths to begin as heresies and to end as superstitions”

Thomas Henry Huxley: *The Coming of Age of the Origin of Species*.

1 Introduction

The present work is devoted to the consistent removal from the quantum formalism of the postulate where it gets closest sheer contradiction; the so-called projection postulate. In its version for non-degenerate pure states, it prescribes that immediately after a measurement with outcome q , the quantum state must change as follows:

$$|\psi(t_2)\rangle = \langle\psi(t_1)|P_q|\psi(t_1)\rangle^{-1/2} P_q |\psi(t_1)\rangle \quad (1)$$

where $P_q = |q\rangle\langle q|$. Now, it is impossible to obtain (1) from the general quantum law of evolution

$$|\psi(t_2)\rangle = U(t_2, t_1) |\psi(t_1)\rangle \quad (2)$$

for whatever linear and unitary $U(t_2, t_1)$, as (1) is nonlinear in its vector argument $|\psi(t_1)\rangle$. One would try to escape contradiction by considering the denominator in (1) as a mere *convention* introduced in order to keep handy the statistical interpretation of quantum mechanics for individual systems even through the occurrence of such accidents as “measurements”. The problem is the natural relaxation of (1)

$$|\psi(t_2)\rangle = P_q |\psi(t_1)\rangle \quad (3)$$

is *also* an impossibility in combination with (2) because of its non-unitary character. Put bluntly: Copenhagen quantum mechanics (CQM) makes room for a precarious consistency thanks to an artificial decree for two contradictory assumptions to hold at different times and in different (but otherwise unspecified) contexts, thus getting the nearest to logical “collapse” a theory has ever been. But we are speaking of mathematical statements, so the point is mathematical consistency, not philosophical foginess.

It is perhaps ironical that all attempts to complete quantum mechanics have been haunted for decades by the issuing of different proofs, all of them consisting on some *reductio ad absurdum* argument, when this dazzling absurdity already is among Copenhagen’s quantum postulates. Such counter-arguments have always proved extremely fragile, as showed by J. S. Bell concerning Von Neumann’s infamous impossibility theorem. The reason, no doubt, is all of them are chained to one and the same burden of an excessively close identification between elements of reality or *pre-existing properties of a system* and *eigenvalues of certain operator*. The latter are only present in the linear level of the theory (already thinking in Bohmian terms,) which is precisely the heaviest burden one is relieved from when trying to extend quantum mechanics with additional variables, and the former are only present in our unredeemably deterministic minds. It is *only if* we (willingly) restrain ourselves from going beyond the quantum-operator level that we find our hands tied when trying to make a consistent and exhaustive (including every conceivable experiment) model of the world. It seems pertinent to point out that outcomes of experiments *are* always particle collisions

in screens or counters, and not operator eigenvalues. The fact that there is one possible detection for each operator eigenvalue we can think of, must be looked upon as a theorem of the additional variable¹ theory. And in this additional-variable theory there is no need for elements of reality to pre-exist before the experiment is performed because the theory may well be *contextual*. This forces, not only to feed $|\psi\rangle$ into the Kochen-Specker theorem [4], [28], [35] as part of the overall set of dynamical variables giving rise to the particular outcomes, but also the potential energy, which is related to the environment's dynamical state in a highly non-trivial way. In particular, discontinuity in the potential energy both in time and space and stochasticity may be involved in an essential way, and we know continuity and determinism are both essential ingredients of the Kochen-Specker argument (we will get back to this point in the concluding remarks).

In this work, I am facing the fact that neither (1) nor (3) are compatible with (2). In particular, I find that, when Bohmian mechanics is added to the quantum formalism, it *is* possible to contemplate measurements in a consistent way substituting formula (1) by

$$|\psi(t_2)\rangle = \sum_{\forall q \in \sigma(Q)} e^{-i\alpha_q} P_q |\psi(t_1)\rangle \quad (4)$$

where α_q are parameters both (i): stochastic in character, and (ii): formally included within the quantum formalism so that (4) comes from a certain resolute approximation in the Schrödinger equation. Of course, it will be necessary to complement the resulting image with additional variables in order to supply for the pointers which (4) obviously lacks or, in other words, to supply for the definiteness of the observed world. The essentials of this approach were already developed by Bohm[8],[40], and there are other precedents in the literature as concerns the assumption of the existence of a source of stochasticity at the quantum-dynamical level[30],[13]. What, I gather, is new in this approach[38] is the realisation that formula (4) for different elections of the operator Q , fits at least three paradigmatic situations in experiments performed with single particles: (1st): collimation, (2nd): deflection and (3rd): localisation.

To summarise, I will assume the following:

¹I intently avoid the malignant use of the word “hidden” for variables as obvious as the coordinates of a pixel on a screen.

1st: I do not set out to develop an encompassing description of the workings of the macroscopic world. I will adopt a much less ambitious program instead, concerning such things as “experiments”. For all their narrowness, such situations do have a valuable advantage: they are very much under human control. Thus, it is not a description of conscience or particles in a thermal bath etc., I am concerned with. It is Stern-Gerlach, EPR, double-slit, etc., *experiments*. There is, of course, an undeniable interest in crossing over from these to more general realms. My hope, however, is that once an elementary but consistent account of the relevant topics for simple, concrete, *repeatable* instances is presented, it will not be very difficult to believe that similar things must happen in a more general context (after all, features such as coherence are an exception, not a general rule).

The most important characteristic of these simplest of situations we call Stern-Gerlach, EPR, etc. is *not* that they are made up of few dynamical variables (which is actually not the case because macroscopic field sources stem from the presence of swarms of particles,) but the fact that they do not display a history-contingent[18] or branch-dependent character or, in other words, that they are *repeatable*. Furthermore, “repeatable” does not mean that we can guarantee for each and every dynamical variable to repeat its history every time we set up another run of the experiment. What it actually means is we can separate in a consistent and useful manner the dynamical variables we are not interested in and average over when the dust has settled. In doing this, we must end up with a mathematical construct that succeeds to statistically describe the behaviour of the whole collectivity. Another aspect of experiments, as opposed to more general situations, is they are limited by a series of specific *consistency conditions* that must be put in and duly justified. This point will be further developed later.

2nd: Although a description based on the density matrix is the minimal guaranteed on theoretical grounds[19], I assume that the density matrix is to be obtained from a more fundamental description based on state vectors (again, a view not universally agreed upon[2]). The density-matrix description arises, then, due to a combination of two things:

(i) Entanglement between the sub-system we are interested in and the rest of the world (including, if necessary, distant systems²)

²This is a reason (among others) to be extremely careful when ascribing the occurrence of decoherence to an “environment”. The part of the whole system responsible for the

(ii) Ignorance (or unconcern) on our part about the fine details of the latter's history.

This mechanism is sometimes described (see e.g., [41] for a brief exposition) in the following terms: if a typical pure entangled state of the whole system (\mathcal{S})+environment (\mathcal{E}) is $|\Psi_{\mathcal{SE}}\rangle = \sum_q c_q |q, \mathcal{E}_q\rangle$, then writing down the operator $W_{\mathcal{SE}} \equiv |\Psi_{\mathcal{SE}}\rangle \langle \Psi_{\mathcal{SE}}|$ produces

$$W_{\mathcal{SE}} = \sum_{q,q'} c_q c_{q'}^* |q, \mathcal{E}_q\rangle \langle q', \mathcal{E}_{q'}| \quad (5)$$

which, after summing over the “uninteresting” environmental degrees of freedom \mathcal{E}_q and assuming $\sum_q |\mathcal{E}_q\rangle \langle \mathcal{E}_q| = I_{\mathcal{E}}$, yields

$$W_{\mathcal{S}} = \sum_q |c_q|^2 |q\rangle \langle q| \quad (6)$$

while the most general $|\Psi_{\mathcal{SE}}\rangle$ displaying entanglement between our system (q -variables) and the rest of the world (\mathcal{E} -variables) would lead to

$$W_{\mathcal{SE}} = \sum_{q,q'} c_{q\mathcal{E}} c_{q'\mathcal{E}'}^* |q, \mathcal{E}\rangle \langle q', \mathcal{E}'| \quad (7)$$

But, as is easy to check by using the closure relation for the \mathcal{E} 's:

$$W_{\mathcal{S}} = \sum_{\mathcal{E}} \langle \mathcal{E} | W_{\mathcal{SE}} | \mathcal{E} \rangle = \sum_{q,q'} W_{qq'} |q\rangle \langle q'| \quad (8)$$

where $W_{qq'} = \sum_{\mathcal{E}} c_{q\mathcal{E}} c_{q'\mathcal{E}'}^*$. Conclusion: a more general pattern of entanglement between the system and the rest of the world than the “matching” one in (5) does *not* result in a decoherent expression for $W_{\mathcal{S}}$. Thus, while the punch-line of the previous argument is essentially correct, the ambiguous word “environment” can render it somewhat misleading. What is really essential to the argument is: a typical decoherent situation is one in which some extraneous system has so special a relation to our system of interest as to repeatedly interact with it through a selected collection of states $|\mathcal{E}_q\rangle$ that exactly replicates the system's (normal environment, “apparatus”, or whatever other technical name).

leading role in the appearance of decoherence in the “interesting” system can, in principle, be very far away.

However, if the viewpoint of attaching a non fundamental meaning to W is to be embraced, it is of the greatest importance to take to heart the *merely instrumental nature of the density matrix*. This operator is only an artifact I use in order to calculate probabilities relative to whatever experiment I decide to carry out, and is not an attribute of individual systems. It must be let for those who look at W as a fundamental description to lament, e. g., the impossibility to write it unambiguously in terms of state vectors.

3rd: Formula (1) is dismissed. Quantum evolution (whatever it ultimately represents) is always linear and unitary; monitored by the Schrödinger equation. I claim though, the soundness of making approximations *on it*, which must be duly justified. Furthermore, I will supplement dynamical quantum mechanics with the position of the particle and its trajectory in order to establish a more detailed dynamical theory.

The addition of the particle's position and trajectory will be realised here according to Bohmian mechanics, which is maximally abridged: It branches into two levels:

(BM1): Waves that evolve according to (2) with $U(t_2, t_1)$ a unitary linear operator being a function of a so-called CSCO (Complete Set of Commuting Operators).

(BM2): Wave-bound particles; the corresponding trajectories given by³

$$m\mathbf{v}(\mathbf{x}, t) = \hbar \nabla \text{Im}(\ln \psi(\mathbf{x}, t)) \quad (9)$$

where $\psi(\mathbf{x}, t) = \langle \mathbf{x} | \psi(t) \rangle$ is the unique solution of (2) furnished with the initial conditions implementing the specific experimental preparation and I assume $\arg \psi(\mathbf{x}, t) \in [0, 2\pi)$ to fix ideas, although none of the results depend on the particular Riemann sheet we place the logarithm in (due to the action of the gradient).

But if Bohmian mechanics is to crown its aspirations to explain⁴ CQM's

³Straightforward manipulations lead to this form from the nowadays favoured one $m\mathbf{v}(\mathbf{x}, t) = \hbar \text{Im}(\nabla \psi(\mathbf{x}, t) / \psi(\mathbf{x}, t))$ or $m\mathbf{v}(\mathbf{x}, t) = \hbar \text{Im}(\psi^*(\mathbf{x}, t) \nabla \psi(\mathbf{x}, t) / \psi^*(\mathbf{x}, t) \psi(\mathbf{x}, t))$ (see, e.g., [15]). The latter is more convenient as concerns arguments on probability flux and related matters. But in the present discussion, I will need the proposed form (9) because it involves the logarithm. The continuous occurrence of its complex-plane inverse; the exponential operator, in the present work makes this form compelling.

⁴Mind my use of the word “explain,” and not “supersede”. Bohmian mechanics cannot be dismissed on the grounds that it is incapable of getting “the same results with as little effort” as CQM. As long as we succeed to prove CQM to be but a user-friendly

famous FAPP validity, some demonstration is needed that the identification of squared amplitudes with probabilities is a sound procedure. This would imply, either a mathematical demonstration, (probably based on an analysis of Bohmian evolution *à la Liouville*) that Bohm's evolution leads to such a quantum equilibrium situation from arbitrary initial conditions, or else a more challenging step-by-step argument involving a discussion of every conceivable experimental *preparation* (they are not as many as it seems at first sight,) e.g., collimation, thermal bath at finite temperature, etc. by using simplifying auxiliary hypotheses specific to each example. This is an ongoing program of research in Bohmian mechanics that I will not deal with in the present work. I am just going to draw conclusions from the assumption of compliance with the quantum equilibrium condition.

But I will have to appeal to the reader's patience and engage (once more!) in some further excruciating use of trite quantum formalism. Though not earth-shattering by any means, there are some new insights in store for patient readers.

2 Brief Mathematical Notices.

My reasoning will very much rely on overlooked consequences of well-known mathematical facts, so it will be convenient to summarise these facts. Remember: provided a space of accessible states is specified, we can expand any self-adjoint operator F , such that $[F, Q] = 0$ as

$$F = \sum_{q \in \sigma(Q), \lambda} f(q, \lambda) P_{q, \lambda} \quad (10)$$

with $P_{q, \lambda} = |q, \lambda\rangle \langle q, \lambda|$ and some index λ accounting for degeneracy.

Several remarks to be made: (1) An offensively redundant wording of (10) : the fact that F is Q -commuting plainly means that F is a linear combination of q -projectors (the point is essential). (2) The summatory symbol could mean a Lebesgue integral for some sub-regions of the spectrum

version of the more fundamental Bohmian mechanics, the effort spent in using Bohmian mechanics to solve “typically quantum” problems can be compared to the effort spent in using general relativity to calculate the orbits of a free-falling marble. Though, it is certainly uncomfortable that no context has appeared as yet in which Bohmian mechanics goes *further* than CQM in the phenomenological battleground.

$\sigma(Q)$ and I will shift from one notation to the other somewhat relaxedly. (3) Not every operator acting on the space of states complies with (10), because an arbitrary operator could depend on further operators, not all of them commuting with Q . The extended set which does constitute a real operator basis is called an ISO (Irreducible Set of Observables).

Result (10) entitles us to the notation $f(Q)$. A so-called operator calculus based on this relation is possible. Dropping λ from the equation for the sake of simplicity, and keeping it always in mind as an eventual addendum:

$$f(Q) = \sum_{q \in \sigma(Q)} f(q) P_q \quad (11)$$

of which Q itself is a particular instance $Q = \sum_{q \in \sigma(Q)} q P_q$.

Further technical questions need not be detailed here. What I do need is to prompt a series of immediate corollaries of (11) for particular instances it contemplates. First, if $f(Q) = \exp iQ$

$$f(Q) = \exp iQ = \exp \left(\sum_{q \in \sigma(Q)} iq P_q \right) = \sum_{q \in \sigma(Q)} e^{iq} P_q \quad (12)$$

A direct application of (12) is provided by the action of $U(t_2, t_1)$ on an arbitrary state when the latter is expanded in eigenstates of a Hamiltonian with a bounding potential. Then we have: $Q = -(t_2 - t_1)H_{\text{bounding}}/\hbar$ and q is simply $-(t_2 - t_1)/\hbar$ times the energy of the corresponding bound state.

But one could foster hopes for the profitability of (12) to reach further than that, for it is the exponential operator of $-i(t_2 - t_1)H/\hbar$ which, in the most general context, carries quantum states from time t_1 to t_2 . Furthermore, (12) involves two among the most relevant elements in quantum theory of measurement; one is projectors, and the other is phase factors (which must have *something* to do with how decoherence comes about). Of course, in a general case I cannot take advantage of (12), because the dynamics is far too messy in terms of the operator Q I may be interested in (i.e., H does not commute with it). The whole idea of the present work is to show that actually, there are instances in which we *can* do such a simplification. In fact, as we will see, I can define an “ideal” measurement as a situation in which, at some crucial time, the dynamics undergoes a transitory stage characterised by (12)⁵.

⁵To be sure, an ideal measurement *is* implicitly defined in this way in the literature.

I introduce also the following device: I do not demand that the collection $\{P_q\}$ be *exhaustive*⁶, that is; even if $\sigma(Q)$ is continuous, I can perform a coarse-graining by integrating the point-like projectors over certain extended regions D_k ⁷. Then

$$P_k(Q) \equiv \int_{D_k} dq P_q \quad (13)$$

for a numerable partition $\{D_k\}_{k=1,\dots,n}$ of $\sigma(Q)$. Nothing prevents me from adopting a partition with an infinite number of projectors, but I keep it finite for reasons that will become apparent later. This finiteness will presumably force upon me to take some of the D_k infinite in extension (again, a feature that will have a suitable explanation). Then, if α_k are real numbers, and if I previously introduce

$$g(Q) = \sum_{k=1}^n i\alpha_k P_k(Q) \quad (14)$$

the application of (11) to $f = \exp \circ g$ renders formula (12) still validated as a new “coarse-grained” version

See e.g., an exhaustively documented reference such as [17] or the classics [5] or [40] in this concern. There you can find an ideal measurement characterised as one in which neither (1st): the weights $w_q = \sum_{\lambda} \langle q, \lambda | w | q, \lambda \rangle$ of the different eigenstates nor (2nd): the coherences $w_{\lambda\lambda'} = \sum_q \langle q, \lambda | w | q, \lambda' \rangle$ between Q -commuting variables are altered after the measurement process. This statement is equivalent to statement (22) that we will see next (theorem). And (12) is a further corollary resulting when the evolution formula is considered. The 2nd requirement amounts to requiring that the evolution operator factorises into a non-trivial part acting on the subspace spanned by Q -eigenstates times the identity operator acting on the subspace spanned by Λ -eigenstates, where λ is a generic eigenvalue corresponding to a certain operator Λ .

⁶Some authors call exhaustive any collection of projectors that sum up to the identity operator $\sum_k P_k = I$. We do not follow this use because it is more correct to call such sets complete (as is otherwise traditional,) and we do need to distinguish them from the really exhaustive ones. For us, $\{P_q\}$ is exhaustive if *any* operator $f(Q)$ admits an expansion: $f(Q) = \sum_{q \in \sigma(Q)} f(q) P_q$. This is in keeping with the difference in nuance between both words in ordinary English.

⁷Our coarse graining is meant to be relevant at a specific time. In particular, we will not use it to build a sequential product of operators or *history* for the observable in question. In this respect (and some more,) this approach differs from the loosely equivalent ones taken up by Griffiths, Gell-Mann-Hartle[18], Omnés[33], Zurek[41], etc.

$$\exp \left(i \sum_{k=1}^n \alpha_k P_k(Q) \right) = \sum_{k=1}^n e^{i\alpha_k} P_k(Q) \quad (15)$$

Mind the purposeful nature of the discrete index k . Its occurrence means that $f(Q)$ in this operator calculus is now a function of Q *through* the coarse-graining just introduced, and not an arbitrary function of Q . Any doubts that (15) is valid after such a procedure, are dissipated by directly expanding the power series plus use of orthogonality relations $P_k(Q)P_j(Q) = \delta_{kj}P_j(Q)$.

Another corollary of (12) that will prove useful: let us expand Q as $Q = R \otimes S$ for certain complete operator set R and S . The reader does not need a boring proof to believe

$$\exp \left(i \sum_{r,s} \alpha_{rs} P_r \otimes P_s \right) = \sum_{r,s} e^{i\alpha_{rs}} P_r \otimes P_s \quad (16)$$

but if suspicious again is invited to check it the hard way by direct expansion and using along the way identities as $(P_1 \otimes P_2)^\perp = P_1^\perp \otimes P_2^\perp + P_1 \otimes P_2^\perp + P_1^\perp \otimes P_2$. Be careful with formula (16) : I have found fairly seasoned postgraduates in theoretical physics taking offence at my emphasising it (because of its triviality⁸) that actually do fail to apply it correctly right away. For example, do not rush into:

$$\exp(i\alpha P_1 \otimes P_2) = e^{i\alpha} P_1 \otimes P_2 \quad (\text{false}) \quad (17)$$

for $P_1 + P_2 = I$, but mind instead that

$$\exp(i\alpha P_1 \otimes P_2) = e^{i\alpha} P_1 \otimes P_2 + P_1^\perp \otimes P_2^\perp + P_1 \otimes P_2^\perp + P_1^\perp \otimes P_2 \quad (\text{true}) \quad (18)$$

because any $\alpha_{rs} = 0$ in the l.h.s. of (16) transforms into $e^{i\alpha_{rs}} = 1$ in the r.h.s. of (16) (sorry for the lag to the cleverer people). Another operator identity I will also use is

$$\exp iQ \otimes R = \exp \left(\sum_{q \in \sigma(Q)} iq P_q \otimes R \right) = \sum_{q \in \sigma(Q)} P_q \otimes \exp(iqR) \quad (19)$$

⁸And trivial *is is not* in several ways: the reader may well be used to its common occurrence in matrix algebra. What is almost universally ignored is the validity of this kind of coarse-grained-version identities that makes them illuminating when applied within the position factor of the overall Hilbert space.

Finally, a further relation I will need is the Campbell-Hausdorff identity for two non-commuting operators A and B

$$\exp A \exp B = \exp \eta(A, B) \quad (20)$$

whose initial terms are

$$\eta(A, B) = A + B + \frac{1}{2} [A, B] + \frac{1}{12} [[A, B], B] + \frac{1}{12} [[B, A], A] + \dots \quad (21)$$

Later, I will make use of (21) in relation to the kinetic and potential-energy operators. The commutator of these operators produces a non constant, non- A nor B -commuting operator in general, rendering (20) almost unmanageable. But an approximation will be introduced and proved feasible for certain states upon which (20) will be made to act. The essence of such approximation is that the action of (21) upon such suitable states (that is, for appropriate values of certain parameters), yields successive commutators that can be made arbitrarily small, so I will not need to care about fine details of this formula (numeric factors and signs) as long as we keep in mind that every term is obtained by further commutation of a previous-order term with either A or B and multiplication by a numeric factor.

3 Coarse-Grained Evolution Formulae and Bohm's Approach to Measurement.

I am now going to take a glimpse at history with the help of the previous tools. In doing so, one realises that certain basic results of a long-established analysis in the quantum theory of measurement[8],[40] are but a trivial application of operator formula (12) (or (15) when a coarse-graining is required). I also notice that there is nothing to prevent me from applying Bohm's analysis to the particular instance of a position measurement. In fact, I can contemplate a localisation as a particular example among a series of other paradigmatic examples all of them cast into the same mathematical mold. But paying heed to the physics lore[5], a localisation is not just any old kind of measurement. It is *the* measurement. That is why, in the present work, this classical analysis by Bohm is taken as all but conclusive.

But let us shortly review Bohm's approach. It goes like this: if we are to "measure" the quantum variable Q , the dynamical approximation that is in force at a certain critical time during that process (to be further specified) is one that makes the Hamiltonian operator diagonal in Q^9 , that is, I consider some

$$H_{Q\text{-measuring}} = H(Q; Q\text{-commuting operators}) \quad (22)$$

It is unfortunate that no mathematical notation can justly enfazise the interesting features of the previous formula. The suffix in the l.h.s. of it does *not* imply Q -dependence. It implies whatever technical conditions the experimentalist has to comply with to make sure the experiment produces the results it has to. The r.h.s. does speak of Q -dependence, so the relation is very far from obvious. In order to properly understand how relation (22) comes about, it is far better to drop any kind of aprioristic reasoning (or blind faith in a messianic insight by its original proposer) and go instead one by one to the particular examples we know best. Thus, I use an inductive reasoning to see that (22) is in fact a good account of things going on at least in several standard situations, as long as they are seen as a limiting, well-behaved case¹⁰.

Consider, e.g., the Stern-Gerlach experiment. The first stage, in which a particle is selected that approaches the Stern-Gerlach window in the desired direction, constitutes a linear momentum measurement (preparation) in itself, with a state evolving in accordance with the free-evolution formula:

$$|\psi(t_2)\rangle = e^{-i\eta(t_2-t_1)\mathbf{P}^2/2m\hbar} |\psi(t_1)\rangle \quad (23)$$

⁹It is frequent to find an even more restrictive assumption in the literature, such as $H_Q = Q \otimes P_y$ (the apparatus being included in the analysis,) with P_y being a canonical momentum associated to the apparatus. This is an extension to a tensor-product space compatible with (22) that I will consider later in a slightly modified form. For the time being, I am focusing on how things look from the one-factor Hilbert space of the system that is being analysed.

¹⁰A further remark, though, is necessary: I must in this concern ignore several technical qualifications about different kinds of measurement that would obscure the point I am trying to bring to light, although they could prove relevant to some other effects. I mean those referred to as QND (quantum non demolition), 1st and 2nd kind measurements, etc. Thus, a general process of measurement could involve several stages, each sharply defined as concerns (22), and, consequently, each equally suitable for its application. Yet, some of these stages could stand for an example of QND etc., while others would not.

This is a nice example of spectral formula (12) as clearly seen by expanding:

$$|\psi(t_2)\rangle = \int d^3p |\mathbf{p}\rangle \langle \mathbf{p}| e^{-i\eta(t_2-t_1)\mathbf{P}^2/2m\hbar} |\psi(t_1)\rangle = \quad (24)$$

$$\int d^3p e^{-i\eta(t_2-t_1)\mathbf{P}^2/2m\hbar} |\mathbf{p}\rangle \langle \mathbf{p}| \psi(t_1)\rangle = \sum_{\mathbf{p}} e^{-i\eta(t_2-t_1)\mathbf{P}^2/2m\hbar} P_{\mathbf{p}} |\psi(t_1)\rangle \quad (25)$$

and consequently, of the expression already advanced (4). Being H (hence $U(t_2, t_1)$) diagonal in \mathbf{P} , I end up recovering the well-known free-evolution formula. Of course, you need not go this length for me to teach you free-quantum evolution. The point to highlight is (23) as being a particular instance for the application of (12).

Suppose now that at $t = t_3$, free evolution has accomplished its job of getting an incoming state fairly peaked in the momentum space and in the desired direction. This is the time when the quantum wave reaches the Stern-Gerlach window, at which moment an interaction impulsive in nature is triggered within the window (while $[t_1, t_2]$ is extremely large, the impulsive interval $[t_2, t_3]$ must be very short¹¹). This interaction is given account of by the magnetic-dipole term $H \propto \sigma_n B_n (\mathbf{x} \cdot \mathbf{n})$, so we have rule (22) again for the particular instance $Q = \sigma_n$. But take notice: the experimenter does not set out to comply with condition (22) as a procedural experimental prescription; the dipole Hamiltonian representing the experimenter's manipulations is *inevitably* diagonal in σ_n .

Subsequently, a time lapse from t_3 to a certain t_4 is necessary for the beam to reach a faraway screen. This last stage of the experiment requires (sticking to what we think is the surest commandment of quantum mechanical formalism, i.e., the Schrödinger equation!) an interaction that ideally, would look very much like $H \simeq V(\mathbf{x})$ (collision). But then rule (22) is in force once again for the election $Q = \mathbf{X}$.

Some further elaboration of the previous example showing how (22) cannot be too far off the mark in matters of measurement is given next, but

¹¹“Large” or “short” mean, of course, as compared to the relaxation time provided by the dispersion of a wave packet. In order to be definite, I can take $\Delta t = \Delta_{\psi} K(\mathbf{P}) / \left| \partial_t \langle K(\mathbf{P}) \rangle_{\psi} \right|$ in the spirit of the Mandelstam-Tamm interpretation of the uncertainty relation[29].

let us recall first as the last preliminary example that an energy *preparation* for an stationary state involves a dynamics monitored by a Hamiltonian that is diagonal (as couldn't be otherwise) in its own representation: $H = \sum_E E P_E \implies \exp[-i(t_2 - t_1)H/\hbar] = e^{-i(t_2 - t_1)E/\hbar} P_E$.

Thus, completely different experimental procedures coincide in mathematical form under a conceptual unification that involves both preparations and other, of more uncontrollable effects, measurements.

3.1 The Stern-Gerlach experiment.

Here I fill up some details of this already outlined most classical piece of theoretical analysis about measurement with the fresh feature of showing that it is simply a particular instance of identity (12). Let us take the spin-1/2 Hilbert space for neutral¹² paramagnetic particles, and $P_+ = |+\rangle\langle+|$, $P_- = |-\rangle\langle-|$. The z -component of spin is represented by the operator $\sigma_z = P_+ - P_-$. Given that the Hamiltonian interaction at the passing of the beam through the Stern-Gerlach window can be approximated by $H = -\mu_z \sigma_z B_z(z)$, after application of (12) to the particular case of $U(t_2 + \tau, 0)$ with t_2 being the time when the wave packet enters the window, and assuming an initial state approaching the window along the x -axis: $\langle \mathbf{x} | \psi(t_2) \rangle = e^{ip_0 x/\hbar} (\psi_+(\mathbf{x}) |+\rangle + \psi_-(\mathbf{x}) |-\rangle)$, I get

$$\langle \mathbf{x} | \Psi(t_2 + \tau) \rangle = e^{ip_0 x/\hbar} \left(e^{i\alpha} e^{iz\Delta p_z/\hbar} \psi_+(\mathbf{x}) |+\rangle + e^{-i\alpha} e^{-iz\Delta p_z/\hbar} \psi_-(\mathbf{x}) |-\rangle \right) \quad (26)$$

The deflection is given by both factors $e^{\pm iz\Delta p_z/\hbar}$ coming from Taylor-expanding the magnetic field inside the Stern-Gerlach window, with $\Delta p_z = \mu_z \partial B_z(z)/\partial z$ evaluated at $x = y = z = 0$ (the centre of the window). The remaining parameter $\alpha = \mu_z B_z(0)$ and other analogous appearing later will be interpreted (as Bohm[8] already did without still considering additional variables,) as giving rise to decoherence when Bohmian mechanics is added to the scheme. Bohmian mechanics postulates as a plausible guess (to be further consolidated theoretically) that the quantum equilibrium condition is universally in force. This implies that the wave function's modulus squared be used as a probability density for the *localisation* of the individual particles. But if such wave function is affected by further (stochastic) parameters,

¹²We put off inconvenient dragging Lorentz forces.

the next reasonable step to follow is to introduce some average over these parameters in addition to the quantum scalar product. At first, I will take this average quite relaxedly, simply justifying it upon the fact that such parameters generally appear as phase factors varying very rapidly in relation to all the remaining quantum parameters involved. Later, I will involve myself more deeply with this question and formally introduce such an average after assuming entanglement between the system under study and the apparatus. The mentioned additional average will contemplate, among other things, a Hilbert scalar product for the pointer variables of the apparatus. As a further historical remark, let us say that in none of Bohm's works is there an explicit reference to, nor any use of identity (12).

3.2 The special role of position measurements.

The next one is an example that really provides telltale clues in matters of interpretation and is a simple corollary of the coarse-grained version (15) of our identity. The reason why Bohm did not (to the best of my knowledge) pay attention to the possibility of this argument is to realise that, back then, it was not very fashionable to look at quantum mechanics in the light of V. Neumann's spectral analysis of yes-no observables, as it has been, e.g., in the sequel of works [21], [18], [33], etc.

To develop the argument, I use the impulsive approximation¹³ $H \simeq V(\mathbf{X})$. Furthermore, I pick the particular realisation of (15) with $Q = \mathbf{X}$, and adopt a convenient coarse graining $P_k(\mathbf{X}) \equiv \int_{D_k} d^3x |\mathbf{x}\rangle \langle \mathbf{x}|$ whose justification ultimately must be put to rest on the existence (whether on purpose or not) of a certain finite set of non-overlapping regions D_k where the particles can be captured ("detectors", you may think). This produces in position representation

$$\langle \mathbf{x} | P_k(\mathbf{X}) | \mathbf{x}' \rangle = \chi_{D_k}(\mathbf{x}) \delta(\mathbf{x} - \mathbf{x}') \quad (27)$$

But then, with the substitution $\alpha_k = -\eta_k(t_2 - t_1)/\hbar$:

¹³Strictly speaking, the impulsive approximation not only implies the aforementioned \mathbf{X} -dependence, but *at the same time*, the time dependence $\chi_{[t_1, t_2]}(t)$ implementing its brief validity in time. It is, of course, both possible and sterile here to be somewhat more formal and use Dyson's time-dependent evolution formula $\exp[-i/\hbar \mathcal{P} \int H(t) dt]$ with $H(t) = \chi_{[t_1, t_2]}(t) V(\mathbf{X})$.

$$\left\langle \mathbf{x} \left| \exp \left(i \sum_{k=1}^n \alpha_k P_k(\mathbf{X}) \right) \right| \mathbf{x}' \right\rangle = \delta(\mathbf{x} - \mathbf{x}') \sum_{k=1}^n e^{-i\eta_k(t_2-t_1)/\hbar} \chi_{D_k}(\mathbf{x}) \quad (28)$$

so that:

$$\psi(\mathbf{x}, t_2) = \sum_{k=1}^{\infty} e^{-i\eta_k(t_2-t_1)/\hbar} \chi_{D_k}(\mathbf{x}) \psi(\mathbf{x}, t_1) \quad (29)$$

To be more concrete, imagine now the particular realisation of (29) that models the interaction at a certain moment t_1 is precisely a sharp potential “basin” $V(\mathbf{x}) = \eta \chi_D(\mathbf{x})$, $\eta < 0$ or a “plateau” $\eta > 0$ where D is some particular D_k (a rough modelling of a definite “detection”). I would have

$$\psi(\mathbf{x}, t_2) = e^{-i\eta(t_2-t_1)/\hbar} \chi_D(\mathbf{x}) \psi(\mathbf{x}, t_1) + \chi_D^\perp(\mathbf{x}) \psi(\mathbf{x}, t_1) \quad (30)$$

Now eq. (30) can be qualified anything we want except neutral in matters of interpretation: it is sure to give a hard time to anyone asserting CQM is complete and willing to consider it seriously, and make the delight of a “hidden-variable” advocate. It simply is telling us that the wave function, when suddenly “hit” by an impulsive localised square potential does not change its associated local probability density $|\psi(\mathbf{x}, t)|^2$ at all!. It just shifts the relative phase between the inside component $\psi_{\text{in}}(\mathbf{x}) \equiv \chi_D(\mathbf{x}) \psi(\mathbf{x})$ and the outside one $\psi_{\text{ex}}(\mathbf{x}) \equiv \chi_D^\perp(\mathbf{x}) \psi(\mathbf{x})$ (self-explanatory definitions). It is, besides, unable by itself to provide an image of what a detection must be. But if someone is to dismiss the equation as nonsense, they should also explain why so unmistakable an approximation as this (no doubt that $H(t) = \chi_{[t_1, t_2]}(t) V(\mathbf{X})$ does represent however crudely a localising attempt) fails so catastrophically to embody the common-sense properties of localisation. I am going to take delight in it and show how, when combined with the general scheme of Bohmian mechanics, it explains the appearance and persistence of decoherence for times t corresponding to the arrival of the wave fronts at a faraway screen or detector. Moreover, I am going to show that, in combination with Bohmian mechanics, eq. (30) provides an image of particles bouncing off D by either absorbing ($\eta < 0$) or repelling ($\eta > 0$) the particle with an uncertain direction depending both on D and the state’s spacial profile. Finally, I am also going to get an unmistakable picture of quantum EPR “nonlocality” in the last section of this work.

With a more general potential, the plain impulsive approximation giving $\psi(\mathbf{x}, t_2) = e^{-iV(\mathbf{x})(t_2-t_1)/\hbar} \psi(\mathbf{x}, t_1)$, imprints a locally varying phase shift in the outgoing wave function or, in other words, changes its momentum distribution in a way easily analysed by applying the momentum operator to the outcoming wave. If $\tau = t_2 - t_1$, the momentum distribution at $t = t_2$ is given by

$$[\mathbf{P}\psi](\mathbf{x}, t_2) = -i\hbar \nabla \left[e^{-iV(\mathbf{x})\tau/\hbar} \psi(\mathbf{x}, t_1) \right] = \quad (31)$$

$$[-\tau \nabla V(\mathbf{x}) \psi(\mathbf{x}, t_1) - i\hbar \nabla \psi(\mathbf{x}, t_1)] e^{-iV(\mathbf{x})\tau/\hbar} \quad (32)$$

No mistake about the interpretation of this: the new momentum distribution is such that

$$\langle \psi(t_2) | \mathbf{P} | \psi(t_2) \rangle = \int dx \psi^*(\mathbf{x}, t_1) [-\tau \nabla V(\mathbf{x}) \psi(\mathbf{x}, t_1) - i\hbar \nabla \psi(\mathbf{x}, t_1)] = \quad (33)$$

$$= \langle \psi(t_1) | \mathbf{P} | \psi(t_1) \rangle - \tau \int dx \nabla V(\mathbf{x}) |\psi(\mathbf{x}, t_1)|^2 \quad (34)$$

Now, this comes from commonplace Hamiltonian dynamics and is not mysterious at all. The obvious interpretation of (34) in terms of probability distributions is: the wave function gets an additional “kick” in a direction that is generally uncertain as long as the exact profile of $V(\mathbf{x})$ is uncertain, depending on both the potential and the initial state’s spatial profile.

On the other hand, in terms of Bohmian mechanics we get, by means of (9)

$$m\mathbf{v}(\mathbf{x}, t_2) = \frac{\hbar}{2i} \frac{\psi^*(\mathbf{x}, t_1) \nabla \psi(\mathbf{x}, t_1) - \psi(\mathbf{x}, t_1) \nabla \psi^*(\mathbf{x}, t_1)}{\psi^*(\mathbf{x}, t_1) \psi(\mathbf{x}, t_1)} - \tau \nabla V(\mathbf{x}) = \quad (35)$$

$$m\mathbf{v}(\mathbf{x}, t_1) - \tau \nabla V(\mathbf{x}) \quad (36)$$

so the new linear momentum is the previous one (given by the Bohmian velocity field) plus a contribution $-\tau \nabla V(\mathbf{x})$. But this is not the way in which the approximation proves itself more useful (and plausible) to our purposes. The step previously considered of $V(\mathbf{x})$ assuming a square spatial profile

with the stochastic character borne by multiplying constants will prove itself better in a series of concerns as will be shown in what follows.

To fix ideas (and also because of its paradigmatic character,) I consider again a Stern-Gerlach experiment for a neutral particle with arbitrary spin s assuming $2s + 1$ possible values. First of all, I will reshape the general set of projector-observables introduced before to render them tailor-made to suit my experiment (here is the explanation of our previous coarse-graining). So they are now a finite series of yes-no observables $P_m(\mathbf{X})$, $m = -s, \dots, +s$, in the way of (27), corresponding to $2s + 1$ non-overlapping “detectors”, plus the complementary of the union $\cup_{k=-s}^{+s} D_m$, that is, $\left(\cup_{k=-s}^{+s} D_m\right)^c$ which will be denoted with the suffix “ex” for short. Each of these “detectors” corresponds to a region where the collision of a particle with precisely that spin z -projection is expected to hit. Then we have a partition of the identity in the position factor subspace of the whole Hilbert space given by

$$\sum_{m=-s}^{+s} P_m + P_{\text{ex}} = I \quad (37)$$

These tool-box operators constitute a drastic reduction as compared to the more general formulation in quantum mechanics as concerns the implementation of results by means of eigenvalues. It is a way of giving mathematical shape to the feature that the only eigenvalues truly relevant to this experiment when it comes to speaking of position measurements are certain, fixedly defined *position q-bits*. So, it is two essential elements we are integrating here; (1): impulsive interactions (dynamics) and (2): countable character in the possible outcomes.

As an example, the particular operator corresponding to the observable-proposition “ \mathbf{x} IS EXTERNAL TO D_{-s} ” would be represented by the operator $P_{-s}^\perp = P_{-s+1} + P_{-s+2} + \dots + P_s + P_{\text{ex}}$, and *its action on an eigenstate $|\psi\rangle$ of the projectors making up (37) would be*

$$P_{-s}^\perp |\psi\rangle \equiv (P_{-s+1} + P_{-s+2} + \dots + P_s + P_{\text{ex}}) |\psi\rangle = \begin{cases} 0 |\psi\rangle & \text{if } |\psi\rangle \text{ is within } D_{-s} \\ 1 |\psi\rangle & \text{if } |\psi\rangle \text{ is outside } D_{-s} \end{cases} \quad (38)$$

I can even consider, in principle, observables like: “ \mathbf{x} IS INTERNAL TO EITHER D_1 OR D_2 ”, i.e.:

$$P_{(-s \text{ or } -s+1)} |\psi\rangle \equiv (P_{-s} + P_{-s+1}) |\psi\rangle = \quad (39)$$

$$\begin{cases} 1 |\psi\rangle & \text{if } |\psi\rangle \text{ is either within } D_{-s} \text{ or } D_{-s+1} \\ 0 |\psi\rangle & \text{if } |\psi\rangle \text{ is outside both } D_{-s} \text{ and } D_{-s+1} \end{cases} \quad (40)$$

Still other observables of classically impossible realisation, like “**x** IS INTERNAL TO BOTH D_m AND $D_{m'}$ ” with $m \neq m'$, which is identically assigned the zero operator.

But example (38) leads us to the following: for this scheme to make any sense within the context of my particular experiment, I need to supplement this set of relevant projectors with a consistency condition on the states I am using as possible inputs and outputs. In physical terms, this means I have to aim my wave packets to the regions of detection and intently preclude any situation in which any of these packets would be, at the moment of detection “caught in between” D_m and $(D_m)^c$ for arbitrary m . This means that the incoming wave is made up as a coherent superposition of states that are either *completely within* or else *completely without* the detection regions D_m at the time designed for each wave packet to hit “its” detector. In other words, I will consider only states satisfying the *coincidence condition*:

$$P_m(\mathbf{X}) |\psi_m(t_{\text{loc}})\rangle = \delta_{mm'} |\psi_{m'}(t_{\text{loc}})\rangle \quad (41)$$

$$P_{\text{ex}}(\mathbf{X}) |\psi_m(t_{\text{loc}})\rangle = 0 \text{ for all } m \quad (42)$$

with

$$|\psi(t_{\text{loc}})\rangle = \sum_{m=1}^n c_m |\psi_m(t_{\text{loc}})\rangle \quad (43)$$

where t_{loc} stands for the time when the localisation is triggered. Mind the time dependence implied in (41)-(43), because I am focusing on experiments in which the initial state is prepared as a superposition of wave packets (Gaussian, to be more concrete). Thus

$$\langle \mathbf{x} | \psi(t) \rangle = \sum_{k=-s}^{+s} c_m \langle \mathbf{x} | \psi_m(t) \rangle = \quad (44)$$

$$\sum_{m=-s}^{+s} c_m (\sigma_m(t))^{-1/2} (2\pi)^{-1/4} e^{i\alpha_m} e^{-i\mathbf{p}_m \cdot \mathbf{x}/\hbar} e^{-(\mathbf{x}-\mathbf{x}_m(t))^2/4\sigma_m^2(t)} \quad (45)$$

where the parameters $\mathbf{p}_m = \mathbf{p}_0 + \Delta\mathbf{p}_m$, $\alpha_m = \mu_m B_z(0)$ and $\Delta p_m = \mu_m \partial B_z(z)/\partial z$ come from the previous stage of deflection as in the 2-state Stern-Gerlach experiment reviewed before. Of course, (41)-(43) do not determine $|\psi_m(t_{\text{loc}})\rangle$, so these are not eigenstates to be uniquely determined (corresponding to the fact that the set of localisation operators $P_m(\mathbf{X})$ is not exhaustive).

A still more restrictive condition (*strong coincidence condition*)¹⁴ to be used in what follows, is (41)-(43) plus

$$\left. \frac{\partial^{(r)}}{(\partial x_k)^r} \psi_m(\mathbf{x}, t_{\text{loc}}) \right|_{\partial D_m} = 0 \text{ for all } r \quad (46)$$

Although at first sight very strong a condition in a general setting, it is always possible to make it valid by making the regions D_m large enough. We can play quite freely with such parameters because of the non existence of a fundamental length parameter imposed upon us, the point of interest being to discuss a plausible scenario for macroscopic localisation.

Let us digress a little about the previous ideas: a quite disturbing problem the impulsive approximation suffers from is that it does not seem to embody *by itself* a macroscopic situation generally enough. The question whether a more genuinely macroscopic condition can be used is given a drastically simplifying answer by the model of *discontinuity in the potential energy*. This model, though extremely idealised in the mathematical side, when combined with the impulsive approximation seems a more credible candidate for such macroscopicity condition than the bare impulsive approximation. The following is suggested as a possible reason for this: It is a well-known argument in statistical mechanics that for non-analytic functions to occur as probability distributions (which is what characterises the coexistence of distinct

¹⁴Great caution is needed, of course, to handle this condition in order not to be led to inconsistencies, e.g.; the *strict* vanishing of the derivatives of the wave function to every order *plus* the requisite on the wave function to be analytic in \mathbf{x} , would lead for $\psi_m(\mathbf{x}, t)$ to be identically zero. Reference [25], is given as a cautionary note to be taken in this concern. I am using the conclusion of that analysis throughout; namely: the safest way to handle states whose derivatives are being assumed to vanish to every order in a certain region is by means of the propagator:

thermodynamic phases,) the intervention of an infinite number of microscopic mechanical states is strictly required. This is usually referred to as the problem of the thermodynamic limit. If, in an analogous way, the occurrence in quantum mechanics of sharp-edged potentials like $V(\mathbf{x}) = \sum_k \eta_k \chi_k(\mathbf{x})$ should come about because of the participation of very many particles each with infinitely many states interacting with my particle of interest, the previous model would constitute an intuitive *ex post facto* implementation of macroscopicity. In short, macroscopicity as concerns localisation would be embodied by the occurrence of discontinuous domains in the potential energy. But this discontinuity-macroscopicity correspondence should not be taken too far, because discontinuity in the potential energy is also a feature of typically non-macroscopic problems as, e.g., reflection on a potential wall. The coincidence condition plays a crucial role in that it implements the fact that it is presumably the passing of the particle by the region of detection what triggers the activation of such discontinuity domains. In contrast, reflection problems (a situation which is widely used in quantum experiments with no decoherence implied in it) require the assumption that such discontinuity domains are pre-existing in the particle's programmed path.

To show how the idea just exposed fits in without strictly having to make $K(\mathbf{p})$ go to zero as compared to $V(\mathbf{x})$ (which is what the usual form of the impulsive approximation would require,) I make use of (20) and (21), with $A = i\tau K(\mathbf{P})$, $B = -i\tau (K(\mathbf{P}) + V(\mathbf{X}))$, choosing $\hbar = 1$ and $t_2 - t_1 \equiv \tau$:

$$\exp i\tau K \exp -i\tau (K + V) = \exp \eta (i\tau K, -i\tau (K + V)) = \quad (47)$$

$$\exp \eta (i\tau K, -i\tau (K + V)) = \exp \left(-i\tau V + \frac{\tau^2}{2} [K, V] + \quad (48)$$

$$\frac{(i\tau)^3}{6} [[K, V], K] - \frac{(i\tau)^3}{12} [[K, V], V] + \dots \right) \quad (49)$$

But notice that, when $\langle \mathbf{x} | K(\mathbf{P}) | \mathbf{x}' \rangle = -(1/2m)(-i\nabla)^2 \delta(\mathbf{x} - \mathbf{x}')$, then

$$[K(-i\nabla), V(\mathbf{x})] \psi \propto (-i\nabla)^2 (V(\mathbf{x})\psi) - V(\mathbf{x})(-i\nabla)^2 \psi = \quad (50)$$

$$- \nabla^2 V(\mathbf{x})\psi - 2\nabla V(\mathbf{x}) \cdot \nabla \psi \quad (51)$$

The following terms are

$$[[K(-i\nabla), V(\mathbf{x})], K(-i\nabla)] \psi \propto \quad (52)$$

$$= 2\nabla V(\mathbf{x}) \cdot \nabla (\nabla^2 \psi) - \nabla^2 (\nabla^2 V(\mathbf{x})) \psi - 4\nabla (\nabla^2 V(\mathbf{x})) \cdot \nabla \psi - \quad (53)$$

$$4(\partial_i \partial_j V(\mathbf{x})) (\partial_i \partial_j \psi) \quad (54)$$

and another one for $[[K(-i\nabla), V(\mathbf{x})], V(\mathbf{x})] \psi$, etc. The moral of the former expansion is: the successive commutations are simply a sum of terms proportional to derivatives of $V(\mathbf{x})$ of order ≥ 1 , because the effect of commutation is to remove the zero-order derivative of $V(\mathbf{x})$ from $L[V(x)]$ with L being any polynomial differential operator with constant coefficients. Now, if the class of states we are considering is a superposition of say, wave packets $e^{-(\mathbf{x}-\mathbf{x}_m(t_{loc}))^2/4\sigma_m^2(t_{loc})}$ ¹⁵ satisfying the strong coincidence condition at $t = t_{loc}$ (negligibly small derivatives at the boundary,) with $V(\mathbf{x}) = \sum_k \eta_k \chi_{D_k}(\mathbf{x})$, it is immediate that such derivatives (proportional to delta functions in the coordinates normal to the potential wall) are approximately zero at this boundary.

For this very special instance, then, I can proceed as follows

$$\exp(-i\tau \left(K(-i\nabla) + \sum_m \eta_m \chi_{D_m}(\mathbf{x}) \right)) \sum_{m'} c_{m'} \psi_{m'}(\mathbf{x}, t) \simeq \exp(-i\tau K(-i\nabla)) \times \quad (55)$$

$$\exp \left(-i\tau \sum_m \eta_m \chi_{D_m}(\mathbf{x}) \right) \sum_{m'} c_{m'} \psi_{m'}(\mathbf{x}, t) = \exp(-i\tau K(-i\nabla)) \times \quad (56)$$

$$\sum_{m, m'} e^{-i\tau \eta_m} c_m \chi_{m'}(\mathbf{x}) \psi_m(\mathbf{x}, t) = \exp(-i\tau K(-i\nabla)) \sum_{m, m'} e^{-i\tau \eta_m} c_m \delta_{mm'} \psi_{m'}(\mathbf{x}, t) = \quad (57)$$

¹⁵I need to use functions that are suitable for the application of delta distributions (that is; polynomially bounded).

$$\exp(-i\tau K(-i\nabla)) \sum_m e^{-i\tau\eta_m} c_m \psi_m(\mathbf{x}, t) = \sum_k e^{-i\tau\eta_m} c_m \psi_m^{(\text{free})}(\mathbf{x}, t + \tau) \quad (58)$$

where $\psi_m^{(\text{free})}(\mathbf{x}, t + \tau)$ are wave packets free-propagated from the original ones $\psi_m(\mathbf{x}, t)$. (58) plays the role of an extended impulsive approximation. It is valid only when the wave packets I am using are such that, whenever impulsive square potentials are activated, the m^{th} wave front is well within the m^{th} potential box. If I do not enforce this condition and allow, e.g., for the waves packets to face potential walls pre-existing in their path, then they would respond to a typical reflection model, whose effects are completely different as we know. On the other hand, any intermediate situation would be far more involved by using the Campbell-Hausdorff identity.

3.3 Supplementary conditions for the measurement of position. The two-slit experiment

I use now this classical experimental test in order to illustrate the continuous need in the quantum theory of measurement for the introduction of auxiliary hypotheses in the form of consistency conditions whenever a particular experiment is proposed. I must expect conditions of this kind to be associated in an inextricable way to the nature of each experiment we design. The reason is, of course, that I am not dealing with a fundamental theory and need to take into account whatever characteristics are imposed by me rather than universally present. Another reason for choosing this example is that it illustrates very well an essential feature of typical quantum experiments, namely; the persistence of decoherence¹⁶ when a further localisation is performed after a previous localisation has already been carried out in the past.

In this case, the wave is a superposition of two partial waves that, after coming through the double slit setting, at the moment of being localised in detectors separated in such a way as to be able to discern between both alternatives of passage can be written as:

¹⁶Actually, the argument concerning the persistence of decoherence after a localisation has taken place can only be accounted for once the quantum equilibrium condition is assumed. The present one is a preliminary point to even start talking about persistence of decoherence for, if the different partial waves do not overlap in the course of subsequent evolution, it doesn't even make sense to talk about decoherence, as explained here.

$$\psi(\mathbf{x}, t_{\text{loc}}) = \psi_1(\mathbf{x}, t_{\text{loc}}) + \psi_2(\mathbf{x}, t_{\text{loc}}) \quad (59)$$

But something absolutely essential for this experiment to function properly is that both $\psi_k(\mathbf{x}, t)$ be nonoverlapping at the time of localisation:

$$\psi_1(\mathbf{x}, t_{\text{loc}})\psi_2(\mathbf{x}, t_{\text{loc}}) \simeq 0 \quad (60)$$

which, somewhat more rigorously means

$$\int d^3x |\psi_1(\mathbf{x}, t_{\text{loc}})|^2 |\psi_2(\mathbf{x}, t_{\text{loc}})|^2 \ll 1 \quad (61)$$

Otherwise, the experiment would not be discerning between both alternatives of passage. The problem is that, as long as condition (60) is accurately satisfied, the decoherence condition

$$|\psi(\mathbf{x}, t_{\text{loc}})|^2 = |\psi_1(\mathbf{x}, t_{\text{loc}})|^2 + |\psi_2(\mathbf{x}, t_{\text{loc}})|^2 \quad (62)$$

is empty because, as long as both supports do not overlap, condition (62) and the coherent one

$$|\psi(\mathbf{x}, t_{\text{loc}})|^2 = |\psi_1(\mathbf{x}, t_{\text{loc}})|^2 + |\psi_2(\mathbf{x}, t_{\text{loc}})|^2 + 2\text{Re}(\psi_1(\mathbf{x}, t_{\text{loc}})\psi_2^*(\mathbf{x}, t_{\text{loc}})) \quad (63)$$

are one and the same. Thus, it is not enough to impose a coincidence condition to enforce decoherence effects. I need to perform a second localisation as well, and *at a distance sufficiently far removed from the first one*. This second localisation must be performed far enough from the first one for free evolution to extend the partial waves' supports and make both to overlap in a significant amount.

Thus, by applying the free evolution formula so that, if $\psi(\mathbf{x}, 0) = \psi_1(\mathbf{x}, 0) + \psi_2(\mathbf{x}, 0)$, $U(t, t_{\text{loc}} + \tau) = U^{(\text{free})}(t, \tau)$, $U(t_{\text{loc}}, 0) = U^{(\text{free})}(t_{\text{loc}}, 0)$, and $U(t_{\text{loc}} + \tau, t_{\text{loc}}) = \exp(-i\tau\eta_1/\hbar\chi_{D_1}(\mathbf{x}) - i\tau\eta_2/\hbar\chi_{D_2}(\mathbf{x}))$ I obtain

$$\psi(\mathbf{x}, t) = \langle \mathbf{x} | U(t, t_{\text{loc}} + \tau) U(t_{\text{loc}} + \tau, t_{\text{loc}}) U(t_{\text{loc}}, 0) | \psi(0) \rangle = \quad (64)$$

$$e^{-i\tau\eta_1/\hbar} \int_{D_1} d^3x' D^{(\text{free})}(\mathbf{x}, t; \mathbf{x}', \tau) \psi_1(\mathbf{x}', 0) + \quad (65)$$

$$e^{-i\tau\eta_2/\hbar} \int_{D_2} d^3x' D^{(\text{free})}(\mathbf{x}, t; \mathbf{x}', \tau) \psi_2(\mathbf{x}', 0) \quad (66)$$

That is,

$$\psi(\mathbf{x}, t) = e^{-i\tau\eta_1/\hbar}\psi_{1,\text{gap}}^{(\text{free})}(\mathbf{x}, t) + e^{-i\tau\eta_2/\hbar}\psi_{2,\text{gap}}^{(\text{free})}(\mathbf{x}, t) \quad (67)$$

where

$$\psi_{k,\text{gap}}^{(\text{free})}(\mathbf{x}, t) \equiv \int_{D_k} d^3x' D^{(\text{free})}(\mathbf{x}, t; \mathbf{x}', \tau) \psi_k(\mathbf{x}', 0) \quad (68)$$

with $k = 1, 2$. In this approximation, both $\psi_{k,\text{gap}}^{(\text{free})}(\mathbf{x}, t)$; $k = 1, 2$, display a time lag with respect to what their values would be if no localising interaction had been present in their path. It is the strong coincidence condition what restores the usual timing, because it does not suppress the free-evolution factor from the whole evolution operator:

$$\psi(\mathbf{x}, t) = e^{-i\tau\eta_1/\hbar}\psi_1^{(\text{free})}(\mathbf{x}, t) + e^{-i\tau\eta_2/\hbar}\psi_2^{(\text{free})}(\mathbf{x}, t) \quad (69)$$

Needless to say, the interesting feature of (69) is the fact that it stems from approximations made on the linear and unitary quantum evolution equation and is not a make-up.

4 The EPR experiment for the singlet state of two identical particles.

When reading about nonlocality in the literature we face a nagging difficulty not having so much to do with any real intricacy as with the sheer ambiguity with which such term is generally burdened. An expansion of the dictionary is not always what is needed, but here it is strictly necessary. It seems thus feasible to look at Bell's definition of nonlocal correlation $p(a, b) \neq p(a)p(b)$ for respective outcomes a and b of a certain experiment (which are causally separated by design) as one that is *minimal* in some sense, because it rests on the more fundamental mathematical definition of statistical dependence between two stochastic variables. In this view, nonlocality is implied nominally, because of what I declare a and b to be. I will know this property under the quite natural term of *Bell nonlocality* (BNL,) because, while it seems fair to call it nonlocality in *some* sense, it is presumably weaker than that which would be inferred from the physical picture of waves propagating superluminally.

There is, to be sure, no actual need for the wave packets to actually propagate faster than light to reach one another's support, for such influence to occur. This, at least, as long as two conditions are satisfied. Namely;

(1) The involvement of a many-particle phase space in the dynamical equations.

(2) Entanglement between the interacting subsystems.

But it is also true that I would like to have at hand some intuitive picture of how a mechanism so counter-intuitive in relativistic terms can take place without any harm done to the principle of relativity. And I would like to picture it in terms of the evolution equation of my theory. This is, of course, no other than the Schrödinger equation, and work has been made on it up to this point so that it can directly produce the desired picture.

Consider the Stern-Gerlach experiment for two neutral paramagnetic particles in the $\frac{1}{2} \otimes \frac{1}{2}$ spin space. I will always label the eigenstates of σ_n with an index so I can refer without ambiguity to a particular spin observable irrespective of the direction defining it. Thus, e.g., σ_z has as eigenstates $|+_z\rangle$ and $|-_z\rangle$, $\sigma_{\mathbf{n}}$ has $|+_{\mathbf{n}}\rangle$, $|-_{\mathbf{n}}\rangle$, etc. I also omit unnecessary particle indexes in the spin variables, the index being implicit in the tensor-product ordering. I will focus on the stage of the experiment when both wave packets reach the deflection window, and I will call that time t_{def} in the laboratory frame (which happens to coincide with the CM frame). If both particles are at the (initial) moment corresponding to the decay (the “starting gun” of the experiment) into a singlet state $\langle \mathbf{x}_1, \mathbf{x}_2 | \Psi(0) \rangle = 2^{-1/2} \psi(\mathbf{x}_1, \mathbf{x}_2; 0) (|+_z, -_z\rangle - |-_z, +_z\rangle)$, after a fairly long trip in opposite directions, they reach the configuration $2^{-1/2} \psi(\mathbf{x}_1, \mathbf{x}_2; t_{\text{def}}) (|+_z, -_z\rangle - |-_z, +_z\rangle)$. At that moment both have reached their respective windows and are simultaneously (in the LAB-frame) being acted upon by the respective dipole term. But let us suppose the first wave packet reaches its window slightly *before* the second one. We write the corresponding interaction as $H = \eta P_{+_z} \otimes I$, then, by means of (15) with $R = \sigma_z$ and $S = \sigma_x$

$$U(t_{\text{def}} + \tau, t_{\text{def}}) = e^{i\alpha(z_1)} P_{+_z} \otimes P_{+_x} + e^{i\alpha(z_1)} P_{+_z} \otimes P_{-_x} + P_{-_z} \otimes P_{+_x} + P_{-_z} \otimes P_{-_x} \quad (70)$$

with $\alpha(z) = -(\eta + z_1 \Delta p_{z_1}) \tau / \hbar$. The second factor of the r.h.s. has been expanded in terms of σ_x -eigenstates in order to see how the deflection of the first wave packet following σ_z affects the possible values for σ_x corresponding

to the second wave packet (we know the attempt to overcome Heisenberg's incompatibility at a distance is the sticking point since EPR times). The action of (70) on the state at $t = t_{\text{def}}$ is such that

$$\sqrt{2} \times |\Psi(t_{\text{def}} + \tau)\rangle = \sqrt{2} \times U(t_{\text{def}} + \tau, t_{\text{def}}) |\Psi(t_d)\rangle = \quad (71)$$

$$e^{i\alpha(z_1)} \langle +_x, -_z \rangle |+_z, +_x\rangle + e^{i\alpha(z_1)} \langle -_x, -_z \rangle |+_z, -_x\rangle - \quad (72)$$

$$\langle +_x, +_z \rangle |-_z, +_x\rangle - \langle -_x, +_z \rangle |-_z, -_x\rangle = \quad (73)$$

$$\frac{1}{\sqrt{2}} \left(e^{i\alpha(z_1)} |+_z, +_x\rangle - e^{i\alpha(z_1)} |+_z, -_x\rangle + |-_z, +_x\rangle + |-_z, -_x\rangle \right) \quad (74)$$

Now, the function I handle in QM to generate the probabilities of the different outcomes for the observable $\sigma_z \otimes \sigma_x$ of the composite system at time t is $|\langle s_z, s_x | \Psi(t) \rangle|^2$. But it is interesting to see what happens if I calculate from this formula the marginal probabilities for the different outcomes s_x of the second particle before (making $\alpha = 0$) and after (making $\alpha \neq 0$) the first particle's σ_z -deflection:

$$\sum_{s_z} |\langle s_z, \pm_x | \Psi(t_{\text{def}} + \tau) \rangle|^2 = |\langle +_z, \pm_x | \Psi(t_{\text{def}} + \tau) \rangle + \langle -_z, \pm_x | \Psi(t_{\text{def}} + \tau) \rangle|^2 = \quad (75)$$

$$= \frac{1}{2} (1 \mp \cos \alpha(z_1)) \quad (76)$$

With the substitution $\alpha = 0$ I recover what I would obtain if I calculated the marginal probabilities at time t_{def} , that is

$$\left| \sum_{s_z} \langle s_z, -_x | \Psi(t_{\text{def}}) \rangle \right|^2 = \sum_{s_z} |\langle s_z, -_x | \Psi(t_{\text{def}}) \rangle|^2 = \frac{1}{2} \quad (77)$$

The general case of $\alpha \neq 0$ I can only interpret in terms of probabilities provided I average over the stochastic parameter $\alpha(z_1)$ (in the way previously exposed when dealing with the Stern Gerlach experiment). Both (76) and (77) serve me to compare the probabilities before ($\alpha = 0$) and after ($\alpha \neq 0$)

the interaction has taken place (disregarding the effects of the free-evolution factor from the whole evolution operator). “Before”, I had¹⁷

$$\left| \sum_{s_z} \langle s_z, +_x | \Psi(t_{\text{def}}) \rangle \right|^2 = 0 \quad (78)$$

$$\left| \sum_{s_z} \langle s_z, -_x | \Psi(t_{\text{def}}) \rangle \right|^2 = 1 \quad (79)$$

and “after”, I have (provided I average to zero the trigonometric terms appearing in (76) and such average is represented by an overbar)

$$\overline{\sum_{s_z=+,-} |\langle s_z, +_x | \Psi(t_{\text{def}} + \tau) \rangle|^2} = \overline{\sum_{s_z} |\langle s_z, -_x | \Psi(t_{\text{def}} + \tau) \rangle|^2} = \frac{1}{2} \quad (80)$$

which means that the probabilities for the different outcomes of s_x for the second particle have changed as a consequence of having measured s_z for the first particle even though both are causally disconnected. Now, I new since Bell’s analysis[3] based just on quantum probabilities (but without assuming QM dynamics) that this nonlocality was necessary (exception made of possible loopholes). I have just presented a dynamical discussion of how such a bizarre physical phenomenon can come about with no need to strand into the allegedly inconsistent soil of nonlocal dynamics. The reason can be pinned down to what I mentioned before: a combination of a multi-particle phase space plus the involvement of entangled states. All through this process, causality is not even touched, as the only way to check this quantum change in the multiparticle-quantum-phase space is only after having cropped up the results (at which time, the causality time frame has obviously become outdated).

¹⁷I still have to 1st: sum the amplitudes and 2nd: square, because I still “do not know” what decoherence is about. It is the process of average, that I will introduce later more formally, what entails the habitual decoherence rule.

5 Quantum-Stochastic-Parameter averaged Bohmian Mechanics

I am now going to cover details that remained somewhat loose before. In doing so, I will show why one is able to be sloppy in not considering further entanglement between the system of interest and the rest of the world without any harm done to the essential ideas to be extracted. Furthermore, I will show the reason for the previous average over stochastic phases. The idea will be presented in the context of Bohmian mechanics.

Let us consider our system \mathcal{S} , along with a second system \mathcal{A} , with \mathcal{S} allowing a spectral expansion like the one giving rise to (15) and \mathcal{A} a somewhat different one in that we make room for an additional subspace (the one orthogonal to the rest of the pointer states). That is, I assume the existence of a spectral expansion for \mathcal{A} of the form

$$\sum_{q=1}^n P_q^{(\mathcal{A})} + P^{(\mathcal{A})\perp} = I^{(\mathcal{A})} \quad (81)$$

while indulging the fiction that the space of states of \mathcal{A} has dimension $n+1$, which allows to simplify the writing, while the corresponding generalisation would require the addition of an index¹⁸. The initial state of the whole system+apparatus adopts the form

$$|\Psi(0)\rangle = \sum_{q=1}^n c_q |\psi_q\rangle \otimes |\Phi_0\rangle \quad (82)$$

where the system under experimental test is assumed to have undergone a previous stage of preparation, and I am going to suppose the apparatus' initial state as having the most general form possible (known as *absolute standard*):

¹⁸Such additional index would not only freight the notation, but also have a somewhat obscure meaning, as it would not necessarily conform to any physically sensible eigenstate expansion. The reason is the CSCO I have chosen to expand \mathcal{A} 's space starts with the “unnatural” $Q^{(\mathcal{A})} \equiv \sum_q q |\mathcal{A}_q\rangle \langle \mathcal{A}_q|$, which is fair enough to describe \mathcal{A} in relation to \mathcal{S} , but not to exhaustively describe \mathcal{A} as a system of its own. Furthermore, the addition of a further CSCO to $Q^{(\mathcal{A})}$, would make me end up with a redundant description, thus risking inconsistency at every step. The set of observables adjoined to $Q^{(\mathcal{A})}$ in order to reach completion is, thus, “unnatural” *by construction*.

$$|\Phi_0\rangle = |\Phi^\perp\rangle + \sum_{q=1}^n a_q |\mathcal{A}_q\rangle \quad (83)$$

Thus, either the apparatus' transition to a pointer state as well as its remaining on the neutral state, unaffected, are possible. We have also (for all q)

$$\langle \Phi^\perp | \mathcal{A}_q \rangle = 0 \quad (84)$$

$$P_q^{(A)} |\Phi^\perp\rangle = 0 \quad (85)$$

The condition setemming from demanding normalisation for both the total state and the $|\mathcal{A}\rangle$'s will be ignored. So far, that is the general scenario. But let us be more concrete.

We recall now the standard scheme in the literature when it comes to consider the apparatus. The following interaction Hamiltonian is proposed for illustrative purposes

$$H = Q \otimes P_y \quad (86)$$

where $P_y = -i\partial/\partial y$ is the canonical operator corresponding to the one-dimensional pointer coordinate y . Then, if $\langle \mathbf{x}, y | \Psi(0) \rangle = \sum_{q=1}^n c_{q,\lambda} \psi_{q,\lambda}(\mathbf{x}) \Phi_0(y)$ (restoring a possible degeneracy in the formulae,) we get

$$\langle \mathbf{x}, y | \Psi(\tau) \rangle = \sum_{q=1}^n \sum_{\lambda} c_{q,\lambda} \psi_{q,\lambda}(\mathbf{x}) \Phi_0(y - q\tau) \quad (87)$$

So different outputs q induce different pointers in the global state. The pointers are simply wave functions recoiled from their original position (think of y as an angle or maybe as a location in a grid). I am going to modify slightly (86) so that it fits the previous discussion in what concerns the measured system's space of quantum states. That is, instead I will consider

$$H = \sum_k \eta_k P_k(Q) \otimes P_y \quad (88)$$

This amounts to changing the “measuring rule” $Q = \sum_q q P_q$ to a yes-no and stochastically affected “measuring rule” $f(Q) = \sum_k \eta_k P_k(Q)$. Now, in place of (87), I would have, by applying (19)

$$\Psi_{\{\eta,y\}}(\mathbf{x}, y; \tau) = \langle \mathbf{x}, y | \Psi_{\{\eta,y\}}(\tau) \rangle = \sum_{k=1}^n \sum_{\lambda} c_{k,\lambda} \psi_{k,\lambda}(\mathbf{x}) \Phi_0(y - \eta_k \tau) \quad (89)$$

where η runs over all η_k . Now, Bohmian mechanics prescribes that in the long run, the particle guided by the quantum wave will follow the statistical pattern of position variables \mathbf{x} given by the modulus of the wave function squared. But in order to be able to write a proper wave function, I must include \mathcal{A} , so the expression to be applied the quantum equilibrium condition to is $|\Psi_{\{\eta,y\}}(\mathbf{x}, \lambda, y; t)|^2$. With this, my probability density gets affected by the stochastics parameters η and y related to \mathcal{A} :

$$\rho_{\{\eta,y\}}(\mathbf{x}; t) \equiv |\Psi_{\{\eta,y\}}(\mathbf{x}, y; t)|^2 = \quad (90)$$

$$\sum_{k,k'=1}^n \sum_{\lambda,\lambda'} c_{k,\lambda} c_{k',\lambda'}^* \psi_{k,\lambda}(\mathbf{x}) \psi_{k',\lambda'}^*(\mathbf{x}) \int dy \Phi_0(y - \eta_k \tau) \Phi_0^*(y - \eta_{k'} \tau) \quad (91)$$

If only \mathcal{A} had its dynamical variables fixed, I would be finished, but the question is these variables have a stochastic character that entails some kind of average in order to obtain the probabilities for sub-system S . In this respect, it is very important to keep in mind that the sample space to be averaged over is the one made up of all the η 's and all the y 's (corresponding to stochastic quantum excitations that could be activated *in the same run* of the experiment). Any such average (which has to run over both η and y) will be denoted by an overbar and will be defined in the following way

$$\overline{u_{\{\eta,y\}}} \equiv \frac{\int d\eta_{k_1} \cdots \int d\eta_{k_n} \int dy \pi(\eta_{k_1}, \cdots, \eta_{k_n}, y) u_{\{\eta,y\}}}{\int d\eta_{k'_1} \cdots \int d\eta_{k'_n} \int dy \pi(\eta_1, \cdots, \eta_n, y)} \quad (92)$$

for any function u of the arguments η and y , and certain admissible density $\pi(\eta_1, \cdots, \eta_n, y)$ for the quantum stochastic parameters. Now, if $\Phi_0(y)$ behaves like a quasi-classical wave function, I must expect it to oscillate very rapidly for sizable variations of its *spacial* argument y , when the stochastic *energy* parameters η_k also change in a characteristically macroscopic range. The last means that the density π must display a very slow variation as compared to Φ_0 . This quasi-classical behaviour is implemented by

the vanishing of the correlation between the values of \mathcal{A} 's factor in the global wave function when the quantum stochastic parameters run over their typical values and certainly depends on the stochastic parameters' fluctuations: $\overline{\Phi_0(y - \eta_k \tau) \Phi_0^*(y - \eta_{k'} \tau)} = \delta_{kk'}$. With this, and provided π is normalised, one is led to:

$$\left| \overline{\Psi_{\{\eta, y\}}(\mathbf{x}, \lambda, y; t)} \right|^2 = \sum_{k=1}^n c_{k, \lambda} c_{k, \lambda'}^* \psi_{k, \lambda}(\mathbf{x}) \psi_{k, \lambda'}^*(\mathbf{x}) \overline{\Phi_0(y - \eta_k \tau) \Phi_0^*(y - \eta_{k'} \tau)} = \quad (93)$$

$$\sum_{k=1}^n \sum_{\lambda, \lambda'} c_{k, \lambda} \psi_{k, \lambda}(\mathbf{x}) c_{k, \lambda'}^* \psi_{k, \lambda'}^*(\mathbf{x}) \quad (94)$$

which is decoherent *with respect to the coarse-grained* $P_k(Q)$, but displays interferences with respect to the λ variables to which the experiment is blind (as obtained from the usual convention implied in the projection postulate). Attention must be paid to the words *with respect to the coarse-grained* $P_k(Q)$, because decoherent or not depends on what observable I am considering to measure next. Thus, if I make up my mind to use the outcoming state (94) as an *incoming* state in a further experiment designed for measuring say, R with $[Q, R] \neq 0$ then, being W diagonal in Q *thus* not in R , *thus* not in a new R -dependent coarse-graining $P_j(R)$, I find myself involved with coherences again.

6 Concluding remarks

Let us briefly review the whole idea of the present work: If I want a measurement to be “perfect”, it must neither blur nor bias the probability pattern of the collectivity $p(\mathbf{x})$ previous to the measurement. But if this is to be true and the quantum equilibrium condition is satisfied: $p(\mathbf{x}) = |\sum_{\lambda} \langle \mathbf{x}, \lambda | \psi \rangle|^2$ (while retaining linear and unitary evolution for the quantum waves,) any operator implementing this quantum change must be diagonal in \mathbf{x} . This, in turn, makes inescapable the form $\sum_{\mathbf{x}} e^{-i\eta \mathbf{x} \tau} P_{\mathbf{x}}$ of which the coarse-grained version previously introduced is simply a rough approximation suitable for use in the position representation but related to a more general observable Q .

Finally, Bohmian mechanics provides us with the pointers, while a reasonable average over the stochastic parameters involved produces the familiar coherence-loss result.

But, on the mathematical side, this is basically a work about the exponential operator. If the handling of the exponential evolution operator is to be trusted in any quantum mechanical context from molecular physics to QCD, I am placing the bet in that its form and properties must also have much to say in connection with this long-standing problem. The route followed in this respect is little short of compelling, the only thing to be contended being the roughness of the different approximations. More speculative is the assumption that Bohmian mechanics be the alternative to be followed when it comes to introducing additional variables.

But, irrespective of the final verdict of nature about Bohmian mechanics, there are several common assertions about the theory that are in sore need of being rebutted. One of them is the idea that Bohmian mechanics is no good because it is “nonlocal”. That ordinary CQM *is* already nonlocal because of (1) should be enough to bash away this popular opinion forever. But there is more and is well-known long ago: nonlocality is inferred from quantum probabilities (provided, in turn, by quantum waves). In consequence, we should expect that it did not depend on the particular model of additional variables we introduce. This peculiar quantum or “weak” nonlocality (BNL) must be explained, and not argued against. We should look for a picture of it in wave dynamics, and not in the additional-variable level. That is exactly the program I have followed in the present work.

As to the various theorems periodically launched in order to dispossess any additional-variable model from its consistency claims, it must be said that any such attempt must be aimed at whatever Bohmian mechanics does not succeed to explain, not at phenomena that Bohmian mechanics *has no difficulty at all* to explain. I mean, of course, spin measurements. Actually, there is no need for a model of additional variables to determine any pre-existing values of an arbitrary projection of spin, or of any other “inner” quantum variable for that matter, *previous* to the measurement (“*beables do not need to be EPR’s elements of reality!*”). Curiously enough, one of Bohr’s famous admonishments offers a helping hand to the additional-variable idea in this concern: it is the stochastic nature of the measurement process which, (in conjunction with the additional-variables’ evolution, that is) leads to one or other particular outcome for the selected quantum observable. Thus, *a*

unique initial value for all the dynamical variables (a particular position for the particle and as a result, a given velocity, plus a unique initial quantum state, plus a unique initial value for the potential energy) leads to *one or other* particular outcome *depending on* what particular deflecting dipole-term we decide to introduce next and on the precise value of the stochastic parameters involved. This is possible in the face of Köchen-Specker’s rigorous result, because Bohmian mechanics qualifies as an inherently *contextual* additional-variable model (a possibility that the Köchen-Specker theorem does not preclude). But the key to the catch-word “contextual” (which otherwise would be nothing but a fancy philosophy-laden name,) lies in the existence of discontinuous and stochastic changes in the environment. That any quantum-orthodoxy advocate would find unpalatable such discontinuity and stochasticity while accepting both features in the shameful form expressed in (1) would be ironical to the extreme. However, while it is true that I have not proved the mentioned discontinuity as an inevitable consequence of the quantum evolution equation in the present work, I have presented fairly plausible arguments for its occurrence: assemblies of many-particle systems with infinitely many quantum states as dynamical building blocks of the quantum context giving rise to discontinuous patterns of evolution is, to say the least, not impossible.

Another objection frequently presented to Bohmian mechanics is that its inception in physics does not justify the effort invested when CQM already gets the same results with much less effort. It is a question here of deciding between mathematical simplicity and logical consistency; but logical consistency should never be sacrificed on behalf of mathematical simplicity (this point has been addressed already in a footnote and is certainly not new in physics).

However, the most serious problem Bohmian mechanics has to face is that, probably, it is not the whole story. It is much too naive to be the whole story. This is because theories with particles are especially awkward when it comes to discuss symmetries (this is known since the demise of Lorentz’s theory of the electron (of which Dirac’s theory was a further sophistication in a similar spirit) followed by unfruitful attempts mainly by Poincaré to overcome the difficulties with a model of spatially extended electron with “inner” cohesion forces). Symmetries are so much better discussed in terms of fields, and we know symmetries are an essential ingredient of modern fundamental theories. In this respect, there is a possibility that Bohmian mechanics be

but a simpleminded model of a highly nontrivial *nonlinear* field theory in which symmetries are obvious. This would-be field theory splitting, in turn, into two separate modes of evolution: one of them made up of stable lumps and the other made up of Schrödinger waves. This possibility would presumably have to face the difficulty of sorting out problems with renormalizability in the relativistic version and seems quirky in that linear waves affect lumps, and not the other way about. But another more plausible possibility is that Bohmian mechanics be a “pointlike version” of a more fundamental theory of strings, branes or extended objects in general (if not for additional location variables more concrete than the location furnished by a dispersing wave, what sense does it make to speak of any kind of *spatial* substructure beyond the level of quantum waves?...or are strings and branes to be nothing but “stringy” or “brany” quantum numbers, “surrealistically” related to localisation within the wave?). Neither of both suggestions, of course, are logical necessities and either could be plagued by technical difficulties, but they stress the point that Bohmian mechanics, peculiar though it is, is probably analyzable in more cogent terms than its crude original form seems to impose.

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