

What Dimensions Do the Time and Space Have: Integer or Fractional?

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A theory of time and space with fractional dimensions (FD) of time and space (d_α , $\alpha = t, \mathbf{r}$) defined on multifractal sets is proposed. The FD is determined (using principle of minimum the functionals of FD) by the energy densities of Lagrangians of known physical fields. To describe behaviour of functions defined on multifractal sets the generalizations of the fractional Riemann-Liouville derivatives $D_t^{d(t)}$ are introduced with the order of differentiation (depending on time and coordinate) being equal the value of fractional dimension. For $d_t = \text{const}$ the generalized fractional derivatives (GFD) reduce to ordinary Riemann-Liouville integral functionals, and when d_t is close to integer, GFD can be represented by means of derivatives of integer order. For time and space with fractional dimensions a method to investigate the generalized equations of theoretical physics by means of GFD is proposed. The Euler equations defined on multifractal sets of time and space are obtained using the principle of the minimum of FD functionals. As an example, a generalized Newton equation is considered and it is shown that this equation coincide with the equation of classical limit of general theory of relativity for $d_t \rightarrow 1$. Several remarks concerning existence of repulsive gravitation are discussed. The possibility of geometrization all the known physical fields and forces in the frames of the fractal theory of time and space is demonstrated.

01.30.Tt, 05.45, 64.60.A; 00.89.98.02.90.+p.

I. INTRODUCTION

The problem concerning the nature of space and time is one of the most interesting problems of the modern physics. Are the space and time continuous? Why is time irreversible? What dimensions do space and time have? How is the nature of time in the equations of modern physics is reflected? Different approaches (quantum gravity, irreversible thermodynamics, synergetics and others) provide us with different answers to these questions. In this paper the hypothesis about a nature of time and space based on an ideas of the fractal geometry [1] is offered. The corresponding mathematical methods this hypothesis makes use of are based on using the idea about fractional dimensions (FD) as the main characteristics of time and space and in connection with this the generalization of the Riemann-Liouville fractional derivatives are introduced. The method and theory are developed to describe dynamics of functions defined on multifractal sets of time and space with FD.

Following [2], we will consider both time and space as an only material fields existing in the Universe and generating all other physical fields. Assume that each of them consists of a continuous, but not differentiable bounded set of small elements. Let us suppose a continuity, but not a differentiability, of sets of small time intervals (from which time consist) and small space intervals (from which space consist). First, let us consider set of small time intervals S_t (for the set of small space intervals the way of reasoning is similar). Let time be defined on multifractal set of such intervals (determined on the carrier of a measure \mathcal{R}_t^n). Each of intervals of this set (further we

use the approximation in which the description of each multifractal interval of these sets will be characterized by middle time moment t and refer to each of these intervals as "points") is characterized by global fractal dimension (FD) $d_t(\mathbf{r}(t), t)$, and for different intervals FD are different(because of the time dependence and spatial coordinates dependence of d_t). For multifractal sets S_t (or S_r) each set is characterized by global FD of this set and by local FD of this set(the characteristics of local FD of time and space sets in this paper we do not research). In this case the classical mathematical calculus or fractional (say, Riemann - Liouville) calculus [3] can not be applied to describe a small changes of a continuous function of physical values $f(t)$, defined on time subsets S_t , because the fractional exponent depends on the coordinates and time. Therefore, we have to introduce integral functionals (both left-sided and right-sided) which are suitable to describe the dynamics of functions defined on multifractal sets (see [1]). Actually, this functionals are simple and natural generalization the Riemann-Liouville fractional derivatives and integrals:

$$D_{a+,t}^d f(t) = \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(t')dt'}{\Gamma(n-d(t'))(t-t')^{d(t')-n+1}} \quad (1)$$

$$D_{b-,t}^d f(t) = (-1)^n \left(\frac{d}{dt}\right)^n \int_t^b \frac{f(t')dt'}{\Gamma(n-d(t'))(t'-t)^{d(t')-n+1}} \quad (2)$$

where $\Gamma(x)$ is Euler's gamma function, and a and b are some constants from $[0, \infty)$. In these definitions, as usually, $n = \{d\} + 1$, where $\{d\}$ is the integer part of d

if $d \geq 0$ (i.e. $n - 1 \leq d < n$) and $n = 0$ for $d < 0$. Functions under the integral sign we will consider to be generalized functions defined on the space of finite functions [4]. Similar expressions can be written down for GFD of functions $f(\mathbf{r}, t)$ with respect to spatial variables \mathbf{r} , with $f(\mathbf{r}, t)$ being defined on the elements of set $S_{\mathbf{r}}$ whose dimension is $d_{\mathbf{r}}$.

For an arbitrary $f(t)$ it is useful to expand the generalized function $1/(t - t')^{\varepsilon(t')}$ under the integral sign in (1)-(2) into a power series in $\varepsilon(t')$ when $d = n + \varepsilon$, $\varepsilon \rightarrow +0$ and write

$$D_{a+,t}^d f(t) = \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(t')}{\Gamma(n - d(t'))(t - t')} \times (1 + \varepsilon(t') \ln(t - t') + \dots) dt' \quad (3)$$

$$D_{b-,t}^d f(t) = (-1)^n \left(\frac{d}{dt}\right)^n \int_t^b \frac{f(t')}{\Gamma(n - d(t'))(t' - t)} \times (1 + \varepsilon(t') \ln(t' - t) + \dots) dt' \quad (4)$$

Taking into account that all functions here are real functions and $1/t = P(1/t) \pm \pi i \delta(t)$, singular integrals here can be defined through the rule

$$\int_0^t \frac{f(t')}{t - t'} dt' = a f(t) \quad (5)$$

where a is a real regularization factor. A good agreement of (3)-(4) with the exact values given by expressions (1)-(2) can be obtained at large time by fitting the value of a .

Instead of usual integrals and usual partial derivatives, in the frames of multifractional time hypothesis it is necessary to use GFD operators to describe small alteration of physical variables. These functionals reduce to ordinary integrals and derivatives if space and time dimensions are taken to be integer, and coincide with the Riemann-Liouville fractional operators if $d_i = \text{const}$. If fractional dimension can be represented as $d_i = n + \varepsilon_i(\mathbf{r}(t), t)$, $|\varepsilon| \ll 1$, it is also possible to reduce GFD to ordinary derivatives of integer order. Here we show this only for the case when $d = 1 - \varepsilon < 1$

$$D_{0+}^{1+\varepsilon} f(t) = \frac{\partial}{\partial t} \int_0^t \frac{\varepsilon(\tau) f(\tau) d\tau}{\Gamma(1 + \varepsilon(\tau))(t - \tau)^{1-\varepsilon(\tau)}} \quad (6)$$

$$\approx \frac{\partial}{\partial t} \int_0^t \frac{\varepsilon(\tau) f(\tau) d\tau}{(t - \tau)^{1-\varepsilon(\tau)}}$$

Though for $\varepsilon \neq 0$ the last integral is well defined and is real-valued, expanding it in power series in ε leads to singular integrals like (5)

$$A = \int_0^t \frac{\varepsilon(t') f(t')}{t - t'} dt'$$

To regularize this integral we will consider it to be defined on the space of finite main functions $\varphi(t')/2\pi i$ and take the real part of the common regularization procedure

$$A = a\varepsilon(t)f(t) \quad (7)$$

Thus we obtain

$$D_{0+}^{1+\varepsilon} f(t) = \frac{\partial}{\partial t} f(t) + \frac{\partial}{\partial t} [a\varepsilon(\mathbf{r}(t), t)f(t)] \quad (8)$$

where a is a regularization parameter. For the sake of independence of GFD from this constant it is useful in the following to choose β_i (on which ε depends linearly) proportional to a^{-1} . It can be shown that for large t the exact expressions for the terms in (1)-(2) proportional to ε are very close to the approximate expression given by (8) provided a special choice for the parameter α is made ($t = t_0 + (t - t_0)$, $t - t_0 \ll t_0$, $\alpha \sim \ln t \sim \ln t_0$)

II. EQUATIONS OF PHYSICAL THEORIES IN MULTIFRACTAL TIME AND SPACE

Equations describing dynamics of physical fields, particles and so on can be obtained from the principle of minimum of fractional dimensions functionals. To do this, introduce functionals of fractional dimensions of space and time $F_{\alpha}(\dots|d_{\alpha}(\mathbf{r}))$, $\alpha = t, \mathbf{r}$. These functionals are quite similar to the free energy functionals, but now it is fractional dimension (FD) that plays the role of an order parameter (see also [2]). Assume further that FD d_{α} is determined by the Lagrangian densities $L_{\alpha,i}$, ($i = 1, 2, \dots$, $\alpha = t, \mathbf{r}$) of all the fields $\psi_{\alpha,i}$, describing the particles and $\Phi_{\alpha,i}$ describing the interactions in the point (\mathbf{r})

$$d_{\alpha} = d_{\alpha}[L_{\alpha,i}(\mathbf{r}, t)] \quad (9)$$

Equations that govern d_{α} behavior can be found by minimizing this functional and lead to the Euler's equations written down in terms of GFD defined in (1)-(2)

$$D_{+,L_{\alpha,i}(x)}^{d_{\alpha}} d_{\alpha} - D_{-,x}^{d_{\alpha}} D_{+,L'_{\alpha,i}(x)}^{d_{\alpha}} d_{\alpha} = 0 \quad (10)$$

Substitution in this equation GFD for usual derivatives and specifying the choice for F dependence on d_{α} and relations between d_{α} and L_{α} (the latter can correspond to the well known quantum field theory Lagrangians) makes possible to write down the functional dependence $F[L]$ in the form (a, b, c are unknown functions of L or constants, L_0 is infinitely large density of the measure carrier \mathcal{R}^n energy)

$$F(\dots|d_{\alpha}) = \int dL_{\alpha} \left\{ \frac{1}{2} [a(L_{\alpha}) \frac{\partial d_{\alpha}}{\partial L_{\alpha}}]^2 + \frac{b(L_{\alpha})}{2} (L_{\alpha} - L_{\alpha,0}) d_{\alpha}^2 + c(L_{\alpha}) d_{\alpha} \right\} \quad (11)$$

or

$$F(\dots|d_{\alpha}) = \int d^4 L_{\alpha} \left\{ \frac{1}{2} [a(L_{\alpha}) \frac{\partial d_{\alpha}}{\partial L_{\alpha}}]^2 + \frac{b(L_{\alpha})}{2} (L_{\alpha} - L_{\alpha,0}) d_{\alpha}^2 + \frac{1}{4} c(L_{\alpha}) d_{\alpha}^4 \right\} \quad (12)$$

The equations that determine the value of fractional dimension follow from taking the variation of (11)-(12) and read

$$\frac{\partial}{\partial L} \left(a(L) \frac{\partial d_{t,\alpha}}{\partial L} \right) + b(L)(L - L_0)d_\alpha + c(L)d_\alpha^2 = 0 \quad (13)$$

or

$$\frac{\partial}{\partial L_\alpha} \left(a(L_\alpha) \frac{\partial d_{t,\alpha}}{\partial L_\alpha} \right) + b(L_\alpha)(L_\alpha - L_{0,\alpha})d_\alpha^2 + c(L_\alpha)d_\alpha^4 = 0 \quad (14)$$

For nonstationary processes one have to substitute the time derivative of d_α into the right-hand side of Eqs.(13)-(14). Neglecting the diffusion of d_α processes in the space with energy densities given by the Lagrangians L we can define $L_\alpha - L_{\alpha,0} = \tilde{L}_\alpha \ll L_{\alpha,0}$ with \tilde{L}_α having sense of over vacuum energy density and for the simplest case (13) gives ($\alpha = t$, $L_{t,i} \equiv L_i$)

$$d_t = \tilde{L}_t = 1 + \sum_i \beta_i L_i(t, \mathbf{r}, \Phi_i, \psi_i) \quad (15)$$

More complicated dependencies of d_α on $L_{\alpha,i}$ are considered in [2]. Note that relation (15) (and similar expression for $d_{\mathbf{r}}$ does not contain any limitations on the value of $\beta_i L_i(t, \mathbf{r}, \Phi_i, \psi_i)$ unless such limitations are imposed on the corresponding Lagrangians, and therefore d_t can reach any whatever high or small value.

The principle of fractal dimension minimum, consisting in the requirement for F_α variations to vanish under variation with respect to any field, in this theory produce the principle of energy minimum (for any type of fractional dimension dependency on the Lagrangian densities). It allows to receive Euler's-like equations with generalized fractional derivatives for functions $f(y(x), y'(x))$, that describe behaviour of physical value f depending on physical variables y and their generalized fractional derivatives $y' = D_{+,x}^{d_\alpha} f$

$$\delta F_{t,y_i} \sim \delta d_{t,y_i} = 0 \quad (16)$$

$$\delta_{y_i} d_\alpha(f) = \delta_{y_i} L_{\alpha,i}(f) = 0, \quad \alpha = \mathbf{r}, t \quad (17)$$

$$D_{+,y_i(x)}^{d_\alpha} f - D_{-,x}^{d_\alpha} D_{+,y'_i(x)}^{d_\alpha} f = 0 \quad (18)$$

The boundary conditions will have the form

$$D_{+,y'_i(x)}^{d_\alpha} f \Big|_{x_0}^{x_1} = 0 \quad (19)$$

In these equations the variables x stand for either t or \mathbf{r} (the latter takes into account fractality of spatial dimensions), $y_i = \{\Phi_i, \psi_i\}$, ($i = 1, 2, \dots$), $L_{\alpha,i}$ are the Lagrangian densities of the fields and particles. Here f can be of any mathematical nature (scalar, vector, tensor, spinor, etc.), and modification of these equations for functions f of more complicated structure does not encounter

any principal difficulties. As Lagrangians $L_{\alpha,i}$ one can choose any of the known in the theoretical physics Lagrangians of fields and their sums, taking into account interactions between different fields.

From Eq.(17) it is possible to obtain generalizations of all known equations of physics (Newton, Shroedinger, Dirac, Einstein equations and etc.), and the similar equations for fractional space dimensions ($\alpha = \mathbf{r}$). Such generalized equations extend the application of the corresponding theories for the cases when time and space are defined on multifractal sets, i.e. these equations would describe dynamics of physical values in the time and the space with fractional dimensions. The Minkowski-like space-time with fractional (fractal) dimensions for the case $d_t \sim 1$ can be defined on the flat continuous Minkowski space-time (that is, the measure carrier is the Minkowski space-time \mathcal{R}^4). These equations can be reduced to the well known equations of the physical theories for small energy densities, or, which is the same, for small forces ($d_t \rightarrow 1$) if we neglect the corrections arising due to fractality of space and time dimensions (a number of examples from classical and quantum mechanics and general theory of relativity were considered in [2]). For statistical systems of many classical particles the GFD help to describe an influence of fractal structures arising in systems on behavior of distribution functions.

III. GENERALIZED NEWTON EQUATIONS

Below we write down the modified Newton equations generated by the multifractal time field in the presence of gravitational forces only

$$D_{-,t}^{d_t(r,t)} D_{+,t}^{d_t(r,t)} \mathbf{r}(t) = D_{+,r}^{d_r} \Phi_g(\mathbf{r}(t)) \quad (20)$$

$$D_{-,r}^{d_r} D_{+,r}^{d_r} \Phi_g(\mathbf{r}(t)) + \frac{b_g^2}{2} \Phi_g(\mathbf{r}(t)) = \kappa \quad (21)$$

In (21) the constant b_g^{-1} is of order of the size of the Universe and is introduced to extend the class of functions on which generalized fractional derivatives concept is applicable. These equations do not hold in closed systems because of the fractality of spatial dimensions, and therefore we approximate fractional derivatives as $D_{0+}^{d_r} \approx \nabla$. The equations complementary to (20)-(21) will be given in the next paragraph. Now we can determine d_t for the distances much larger than gravitational radius r_0 (for the problem of a body's motion in the field of spherical-symmetric gravitating center) as (see (11) and [5] for more details)

$$d_t \approx 1 + \beta_g \Phi_g \quad (22)$$

Neglecting the fractality of spatial dimensions and the contribution from the term with b_g^{-1} , and taking $\beta_g = 2c^{-2}$, from the energy conservation law (approximate

since our theory and mathematical apparatus apply only to open systems) we obtain

$$\left[1 - \frac{2\gamma M}{c^2 r}\right] \left(\frac{\partial r(t)}{\partial t}\right)^2 + \left[1 - \frac{2\gamma M}{c^2 r}\right] r^2 \left(\frac{\partial \varphi(t)}{\partial t}\right)^2 - \frac{2mc^2}{r} = 2E \quad (23)$$

Here we used the approximate relation between generalized fractional derivative and usual integer-order derivative (10) with $a = 0.5$ and notations corresponding to the conventional description of motion of mass m near gravitating center M . The value $a = 0.5$ follows from the regularization method used and alters if we change the latter. Eq.(23) differs from the corresponding equation in general theory of relativity by presence of additional term in the first square brackets. This term describes velocity alteration during gyration and is negligible while perihelium gyration calculations. If we are to neglect it, Eq.(23) reduces to the corresponding classical limit of equations of general relativity equation. For large energy densities (e.g., gravitational field at $r < r_0$) Eqs.(7) contain no divergences [2], since integrodifferential operators of generalized fractional differentiation reduce to generalized fractional integrals (see (1)).

Note, that choosing for fractional dimension d_r in GFD $D_{0+}^{d_r}$ Lagrangian dependence in the form $L_{r,i} \approx L_{t,i}$ gives for (17) additional factor of 0.5 in square brackets in (23) and it can be compensating by fitting factor β_g .

IV. FIELDS ARISING DUE TO THE FRACTALITY OF SPATIAL DIMENSIONS ("TEMPORAL" FIELDS)

If we are to take into account the fractality of spatial dimensions ($d_x \neq 1$, $d_y \neq 1$, $d_z \neq 1$), Eqs.(17)-(19), we arrive to a new class of equations describing certain physical fields (we shall call them "temporal" fields) generated by the space with fractional dimensions. These equations are quite similar to the corresponding equations that appear due to fractality of time dimension and were given earlier. In Eqs.(10)-(12) we must take $x = \mathbf{r}$, $\alpha = \mathbf{r}$ and fractal dimensions $d\mathbf{r}(t(\mathbf{r}), \mathbf{r})$ will obey (14) with t being replaced by \mathbf{r} . For example, for time $t(\mathbf{r}(t), t)$ and potentials $\Phi_g(t(\mathbf{r}), \mathbf{r})$ and $\Phi_e(t(\mathbf{r}), \mathbf{r})$ (analogues of the gravitational and electric fields) the equations analogous to Newton's will read (here spatial coordinates play the role of time)

$$D_{-,r}^{d_r(r,t)} D_{+,r}^{d_r(r,t)} t(\mathbf{r}) = D_{+,t}^{d_t} (\Phi_g(t(\mathbf{r})) + e_r m_r^{-1} \Phi_e(t(\mathbf{r}))) \quad (24)$$

$$D_{-,t}^{d_t} D_{+,t}^{d_t} \Phi_g(t(\mathbf{r})) + \frac{b_{gt}^2}{2} \Phi_g(t(\mathbf{r})) = \kappa_r \quad (25)$$

$$D_{-,t}^{d_t} D_{+,t}^{d_t} \Phi_e(t(\mathbf{r})) + \frac{b_{et}^2}{2} \Phi_e(t(\mathbf{r})) = e_r \quad (26)$$

These equations should be solved together with the generalized Newton equations (20)-(21) for $\mathbf{r}(t(\mathbf{r}), \mathbf{r})$.

With the general algorithm proposed above, it is easy to obtain generalized equations for any physical theory in terms of GFD. From these considerations it also follows that for every physical field originating from the time with fractional dimensions there is the corresponding field arising due to the fractional dimension of space. These new fields were referred to as "temporal fields" and obey Eqs.(14)-(16) with $x = \mathbf{r}$, $\alpha = \mathbf{r}$. Then the question arises, do these equations have any physical sense or can these new fields be discovered in certain experiments? I want to pay attention on the next fact: if $L_{t,i} \approx L_{r,i}$ no new fields are generated. This is the case when fractal dimension of time and space $d_{t,br}$ can not be divided on $d_t + d_r$, the time and space fractal sets can not be divided too. The FD time and space are common and defined by value given by L_i (the latter can be chosen in the form of usual Lagrangians in the known theories).

V. CAN REPULSIVE GRAVITATIONAL FORCES EXIST?

In general theory of relativity no repulsive gravitational forces are possible without a change of the Riemann space curvature (metric tensor changes). But in the frames of multifractal time and space model, even when we can neglect the fractality of spatial coordinates, from (24) it follows (for spherically-symmetric mass and electric charge distributions)

$$m_r \frac{\partial^2 t(\mathbf{r})}{\partial \mathbf{r}^2} = \frac{\partial}{\partial t} \sum_i \Phi_i(t) \approx -\frac{m_r k_r}{c^2 t^2} \pm \frac{e^2}{ct^2} \quad (27)$$

with accuracy of the order of b^2 . Here m_r is the analogue of mass in the time space and corresponds to spatial inertia of object alteration with time changing (it is possible that m_r coincides with ordinary mass up to a dimensional factor). Eq.(27) describes the change of the time flow velocity from space point to space point depending on the "temporal" forces and indicates that in the presence of physical fields time does not flow uniformly in different regions of space, i.e the time flow is irregular and heterogeneous (see also Chapter 5 in [2]). Note, that introducing equations like (27) in the time space is connected with the following from our model consequences (see (17)-(19)) about equivalence of time and space and the possibility to describe properties of time (a real field generating all the other fields except "temporal") by the methods used to describe the characteristics of space. Below we will show that taking into consideration usual gravitational field in the presence of its "temporal" analogues gives way to the existence of gravitational repulsion proportional to the third power

of velocity. Indeed, the first term in the right-hand side of (27) is the analogue of gravity in the space of time ("temporal" field). Neglecting fractional corrections to the dimensions and taking into account both usual and "temporal" gravitation, Newton equations have the form

$$m \frac{d^2 \mathbf{r}}{dt^2} = \mathbf{F}_r + \mathbf{F}_t = -\frac{\gamma m M}{r^2} + \frac{m_r k_r}{ct^2} \left(\frac{d\mathbf{r}}{dt} \right)^3 \quad (28)$$

The criteria for the velocity, dividing the regions of attraction and repulsion reads

$$\left(\frac{d\mathbf{r}}{dt} \right)^3 = \left(\frac{\gamma m M}{r^2} \right)^{-1} \frac{m_r k_r}{ct^2} \quad (29)$$

Here $\mathbf{r}(t)$ must also satisfy Eq.(20). Introducing gravitational radii r_0 and t_0 (the latter is the "temporal" gravitational radius, similar to the conventional radius r_0), we can rewrite (23) as follows

$$\left| \frac{d\mathbf{r}}{dt} \right| = c \sqrt[3]{c \frac{t^2}{r^2} \frac{t_0}{r_0}} \quad (30)$$

In the last two expressions r is the distance from a body with mass m to the gravitating center, t is the time difference between the points where the body and the gravitating center are situated, $m_r = m/c$, $\kappa_r = \kappa_t/c$. If we admit that r_0 and t_0 are related to each other as $r_0 = t_0/c$, the necessary condition for the dominance of gravitation repulsion will be $c < rt^{-1}$. It is not clear whether this criteria is only a formal consequence of the theory or it has something to do with reality and gravitational repulsion does exist in nature. What is doubtless, that in the frames of multifractal theory of time and space it is possible to introduce (though, may be, only formally) dynamic gravitational forces of repulsion (as well as repulsive forces of any other nature, including nuclear).

VI. THE GEOMETRIZATION OF ALL PHYSICAL FIELDS AND FORCES

The multifractal model of time and space allows to consider the fractional dimensions of time d_t and space d_r (or undivided FD d_{tr} as the source of all physical fields (see (11)) (including, in particular, the case when flat (not fractal) Minkowski space-time \mathcal{R}^4 is chosen as the measure carrier). From this point of view, all physical fields are consequences of fractionality (fractality) of time and space dimensions. So all the physical fields and forces are exist in considered model of multifractal geometry of time and space as far as the multifractal fields of time and space are exists. Within this point of view, all physical fields are real as far as our model of real multifractal fields of time and space correctly predicts and describes the physical reality. But since in this model all the fields are determined by the value of fractal dimension of time and space, they appear as geometrical characteristics of

time and space (12-14). Therefore there exists a complete geometrization of all physical fields, based on the idea of time and space with (multi)fractional dimensions, the hypothesis about minimum of functional of fractal dimensions and GFD calculus used in this model. The origin of all physical fields is the result and consequence of the appearing of the fractional dimensions of time and space. One can say that a complete geometrization of all the fields that takes place in our model of fractal time and space is the consequence of the inducing (and describing by GFD) composed structure of multifractal time and space as the multifractal sets of multifractal subsets S_t and S_r with global and local FD. The fractionality of spatial dimensions d_r also leads to a new class of fields and forces (see (17)-(19) with $\alpha = r$). For the special case of integer-valued dimensions ($d_t = 1$, $d_r = 3$) the multifractal sets of time and space S_t and S_r coincide with the measure carrier \mathcal{R}^4 . From (14) it follows then that neither particles nor fields exist in such a world. Thus the four-dimensional Minkowski space becomes an ideal physical vacuum (for FD $d_\alpha > 1$ the exponent of R^n has value $n > 4$). On this vacuum, the multifractal sets of time and space (S_t and S_r) are defined with their fractional dimensions, and it generates our world with the physical forces and particles.

Now the following question can be asked: what is the reason for the dependence in the considered model of fractal theory of time and space of fractionality of dimensions on Lagrangian densities? One of the simplest hypothesis seems to assume that the appearing of fractional parts in the time and space dimensions with dependence on Lagrangian densities originates from certain deformations or strains in the spatial and time sets of the measure carrier caused by the influence of the real time field on the real space field and vice versa (generating of physical fields caused by deformations of complex manifolds defined in twistor space is well known [6]). Assuming then that multifractal sets S_t and S_r are complex manifolds (complex-valued dimensions of time and spatial points can be compacted), deformation, for example, of complex-valued set S_t under the influence of the spatial points set S_r would result in appearing of spatial energy densities in time dimension, that is generating of physical fields (see [6]). Fractional dimensions of space appearing (under the influence of set S_t deformations) yields new class of fields and forces (or can also not yield). It can be shown also that for small forces (e.g., for gravity - at distances much larger than gravitational radius) generalized fractional derivatives (1)-(2) can be approximated through covariant derivatives in the effective Riemann space [2] and covariant derivatives of the space of the standard model in elementary particles theory [7] (with the corrections taking into account fields generating and characterizing the openness of the world in whole [8,9]). All this allows to speak about natural insertion of the offered mathematical tools of GFD, at least for $\varepsilon \ll 1$, in the structure of all modern physical theories (note here, that the theory of gravitation as the theory of real fields

with a spin 2 is invented in [10]). Note also, that number of problems within the framework of the theory of multifractal time and space (classical mechanics, nonrelativistic and relativistic quantum mechanics) were considered in [2,9,11].

VII. CONCLUSIONS

In our model we postulate the existence of multifractal space and time and treat vacuum as \mathcal{R}^n space which is the measure carrier for the sets of multifractal time and space. Fractionality of time dimension leads then to appearing of space-time energy densities $L(\mathbf{r}(t), t)$, that is generating of the known fields and forces, and fractionality of space dimensions gives new time-space energy densities $L(t(\mathbf{r}), \mathbf{r})$ and a new class of "temporal" fields. Note, that the roles of d_t and $d_{\mathbf{r}}$ in distorting accordingly space and time dimensions is relative and can be interchanged. Apparently, one can consider the "united" dimension $d_{t,\mathbf{r}}$ - the dimension of undivided onto time and space multifractal continuum in which time and coordinates are related to each other by relations like those for Minkowski space, not using the approximate relation utilized in this paper $d_{t,\mathbf{r}} = d_t + d_{\mathbf{r}}$. Moreover in some cases it seems to be even impossible to separate space and time variables, and then d_t and $d_{\mathbf{r}}$ can be chosen to be equal to each other, i.e., there would be only one fractional dimension $d_t = d_{r_{x,y,z}} = 1 + \sum \beta_i L_i(\mathbf{r}(t), t; t(\mathbf{r}), \mathbf{r})$ describing the whole space-time. In this case one would have to calculate generalized fractional derivatives from the same Lagrangians, and new "temporal" fields will not be generated.

The considered model of multifractal time and space offers a new look (both in mathematical and philosophical senses) onto the properties of space and time and their description and onto the nature of all the fields they generate. This gives way to many interesting results and conclusions, and detailed discussion of several problems can be found in [2]. Here we restrict ourselves with only brief enumerating of the most important ones.

a) The model does not contradict to the existing physical theories. Moreover, it reduces to them when the potentials and fields are small enough, and gives new predictions (free of divergencies in most cases) for not small fields. Though, the question about applicability of the proposed relation between fractal dimension and Lagrangian densities still remains open.

b) We consider time and space to be material fields which are the basis of our material Universe. In such a Universe there exist absolute frames of reference, and all the conservation laws are only good approximations valid for fields and forces of low energy density since the Universe is an open system, defined on certain measure carrier (the latter probably being the 4-dimensional Minkowski space). Smallness of fractional corrections to the value of time dimension in many cases (e.g., on the

Earth's surface it is about $d_t - 1 \sim 10^{-12}$) makes possible to neglect it and use conventional models of the physics of closed systems.

c) The model allows to consider all fields and forces of the real world as a result of the geometrization of time and space (may be more convenient the term "fractalization" of time and space) in the terms of fractal geometry. It is fractional dimensions of time and space that generate all fields and forces that exist in the world. The model introduces a new class of physical fields ("temporal" fields), which originates from the fractionality of dimensions of space. These fields are analogous to the known physical fields and forces and can arise or not arise depending on certain conditions. Thus the presented model of time and space is the first theory that includes all forces in single theory in the frames of fractal geometry. Repeat once more: the model allows to consider all the fields and forces of the world as the result of geometrization including them in FD of time and space. It is non-integer dimensions of time and space that produce the all observable fields. The new class of fields naturally comes into consideration, originating solely from the fractionality of space dimensions and with the equations similar to those of the usual fields. The presented model of space and time is the first theory that allows to consider all physical fields and forces in terms of a unique approach.

d) Basing on the multifractal model of time and space, one can develop a theory of "almost inertial" systems [11,13,14] which reduces to the special theory of relativity when we neglect the fractional corrections to the time dimension. In such "almost inertial" frames of reference motion of particles with any velocity becomes possible.

e) On the grounds of the considered fractal theory of time and space very natural but very strong conclusion can be drawn: all the theory of modern physics is valid only for weak fields and forces, i.e. in the domain where fractional dimension is almost integer with fractional corrections being negligibly small.

f) The problem of choosing the proper forms of deformation that would define appearing of fractional dimensions also remains to be solved. So far there is no clear understanding now which type of fractal dimensions we must use, d_t and $d_{\mathbf{r}}$ or $d_{t,\mathbf{r}}$. Obviously, solving numerous different problems will depend on this choice as the result of different points of view on the nature of multifractal structures of time and space.

The author hopes that new ideas and mathematical tools presented in this paper will be a good first step on the way of investigations of fractal characteristics of time and space in our Universe.

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