ρ - nucleus bound states in Walecka model

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Possible formation of ρ nucleus bound state is studied in the framework of Walecka model. The bound states are found in different nuclei ranging from ^3He to ^{208}Pb. These bound states may have a direct bearing on the recent experiments on the photoproduction of ρ meson in the nuclear medium.

24.85.+p, 12.38.Qk, 14.40.Cs, 25.20.Lj

The properties of hadrons in the nuclear medium is a field of current interest. These properties at high density and/or temperature are important for the study of neutron stars and supernovae. The possible explanations of the experimental results of heavy ion collisions will also depend on the in-medium behaviour of the hadrons. The recent observation of enhanced dilepton production in the low invariant mass domain in heavy ion collider experiments [1] has triggered speculation [2] that the effective ρ- meson mass decreases in the nuclear medium. The studies using Chiral perturbation theory shows that even at finite densities there may be a partial restoration of chiral symmetry leading to the decrease of vector meson masses from their free values [3].

A number of experiments have been done to study the possible shift of the masses of vector mesons ω, η [4] and ρ [5]. The most notable experiments regarding ρ meson mass modification are $K^+ - {}^{12}C$ elastic cross section measurements at 800 MeV that have also revealed an enhancement which can be attributed to a shift in ρ^0 mass [6]; the need for shifted meson masses to explain the spin transfer in the $\vec{p}$-A scattering experiment at IUCF [7]; and photoproduction experiments at the INS facility in Tokyo via the reaction $^3He(\gamma, \pi^+, \pi^-)$ at photon energies below the free production threshold of 1083 MeV for the ρ^0 meson [8,9]. The anlysis of the experimental data yielded $m^*_{\rho^0}$ of 642$\pm$40 MeV/c^2 and 669$\pm$32 MeV/c^2 in the photon energies 800-880 MeV and 880-960 MeV respectively [10]. The reanalysis of the single and double Δ experiment [9] gave the ρ^0 mass to be 490$\pm$40 MeV/c^2 in the photon energy range 380-700 MeV [11]. The similar trend has been found by Bianchi *et al.* [12] who have developed a model based on hadronic fluctuations of the real photon to describe the total photonucleon and photonuclear cross sections. A decrease in ρ meson mass in different nuclei is found to be necessary to explain the experimental data.

The substantial change in the ρ meson mass led some authors to argue that such large decrease in mass can not be explained by the mean field picture of the nuclear matter [13]. These authors suggest that this decrease in mass should be taken as a signature of the partial restoration of chiral symmetry [14] in ground state nuclei. On the other hand, Bhattacharyya *et al.* [15] showed that the above conclusion is premature since a proper inclusion of the relevant interactions in a mean field description does give a large decrease in the ρ meson mass as suggested by the experimental results.

In a recent work Popendreou *et al.* [16] have shown that the reduction in ρ^0 meson mass can also be explained through a ρ^0- nucleus bound state. They found that the depth of the potential required to produce the bound state is consistent with that expected from Dirac phenomenology using Brown-Rho scaling. In the present report we have studied the formation of ρ^0-nucleus bound state using Walecka model.

The Walecka model is one of the most popular mean field model for nuclear matter [17]. Though there have been lot of modifications [18] this model still serves as the most effective mean field theory of nuclear matter. Starting from the Walecka model Lagrangian [15] one can write down the coupled differential equation for the fields in the mean field approximation. These equations are solved self consistently for different nuclei to get the density distributions. The ρ^0 meson mass is then evaluated as the pole of the propagator to get the mass variation with density as well as radius [15,19]. The evaluation of ρ^0 mass involves two coupling constants. One is the vector coupling of ρ^0 meson with nucleon (g_ρ) and the other is the tensor coupling of ρ^0 with nucleons (f_ρ or $c_\rho = f_\rho/g_\rho$). In general, one can not fix the value of the tensor coupling constant within the premise of Walecka model itself. So in practice, one can take g_ρ as well as c_ρ both from some other source like Bonn potential [20] or QCD sum rules [21] or keep the g_ρ as obtained within Walecka model and use c_ρ from other calculations [15]. As shown in [15], the reduction in the ρ^0 meson mass is different for different parameter sets. The maximum reduction in mass is obtained from Walecka model parametr set where as the QCD sum rule parameter set yields the minimum reduction. In the following we have used the parameter set $g_\rho = 8.912$ as obtained in Walecka model by fitting the asymmetric energy for nuclear matter and $f_\rho = 2.866$ [22] to describe the ρ-nucleus bound states. Similar values for f_ρ are obtained from Bonn potential [20] as well as QCD sum rule calculations [21]. The average ρ^0 masses in ^3He for this parameter set are 657 MeV and 600 MeV without and with tensor interaction respectively.

The ρ^0 meson consists of same flavour quark - antiquarks and as a result it is not expected to feel the Lorentz

Vector potential generated by the nuclear environment. The total potential felt by ρ^0 is then given by $m^*_{\rho^0}(r) - m_{\rho^0}$ where $m^*_{\rho^0}$ now depends on the position from the center of the nucleus. So in a nucleus the static ρ^0 meson field ϕ_ρ is given by,

$$[\nabla^2 + E_\rho^2 - m^{*2}_{\rho^0}]\phi_\rho = 0. \tag{0.1}$$

To incorporate the width of ρ^0 meson in our estimate for the bound states we assume the phenomenological form as suggested by Saito *et al.* [19].

$$\tilde{m^*_{\rho^0}} = m^{*2}_{\rho^0}(r) - \frac{i}{2}\{[m_{\rho^0} - m^*_{\rho^0}(r)]\gamma_\rho + \Gamma_\rho\} \equiv m^*_{\rho^0}(r) - \frac{i}{2}\Gamma^*_\rho(r) \tag{0.2}$$

where Γ_ρ =150 MeV is the width of ρ meson in free space. γ_ρ is treated as a phenomenological parameter chosen to describe the in-medium meson width Γ^*_ρ. So we actually solve the equation

$$[\nabla^2 + E_\rho^2 - \tilde{m^{*2}}_{\rho^0}(r)]\phi_\rho(r) = 0 \tag{0.3}$$

The above equation has been solved in the coordinate space using relaxation method [23]. This is done in the following way. First we separate the eqn.(0.3) in radial and angular parts. Then the wave function ϕ_ρ and energy E_ρ are written as $\phi_\rho = \phi^1_\rho + i\phi^2_\rho$ and $E_\rho = E^1_\rho + iE^2_\rho$, where the superscripts 1 and 2 denote the real and imaginary parts of the relevant quantities. Substituting these in the radial part of the wave equation one gets two second order coupled differential equations for real and imaginary part of the wave function. These are then solved for the real and imaginary part of the energy E_ρ. The single particle energy E can be defined in terms of complex eigenenergies E_ρ as

$$E_\rho = E + m_\rho - i\frac{\Gamma}{2} \tag{0.4}$$

The single particle energies $E = Re(E_\rho - m_\rho)$ and the full width Γ for different nuclei are given in Table 1 for two different parameter sets PI and PII. For PI and PII the vector coupling g_ρ is 8.912 whereas the tensor coupling $f_\rho =$ 2.866 [22] for PI and $f_\rho = 0$ for PII. The table shows that a non-zero γ_ρ increases the width where as the real part remains almost same. This is evident from eqn. (0.2) which shows that a non-zero γ_ρ increases the imaginary part of the potential. Similar effect has been found in case of ω-nucleus bound states [19] as well. The energies in table 1 show that we get very strongly bound ρ-nucleus states in our present formalism. Without the tensor coupling, the $E = -132.95$ MeV in the ^3He in $l = 0$ state. The similar value for the ρ bound state has been found in ref. [16]. On the other hand, we find that $E = -199.14$ MeV in ^3He in the presence of tensor coupling. This high binding is clearly the result of a large drop in the ρ^0 mass in the presence of tensor coupling. In other words, to get less binding one has to reduce the effect of tensor coupling or introduce new effects like form factor which will restrict the dramatic change in the ρ^0 mass.

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TABLE I. Single particle energy E and full width Γ for different nuclei for different values of γ_ρ for two different parameter sets ; PI $\equiv g_\rho = 8.912$ and $f_\rho = 2.866$; PII $\equiv g_\rho = 8.912$ and $f_\rho = 0$

Nuclei	γ_ρ	l	E(PI) (MeV)	Γ(PI) (MeV)	E(PII) (MeV)	Γ(PII) (MeV)
^3He	0	0	-199.14	133.87	-132.95	137.50
		1	-126.66	128.48	-68.95	134.02
	0.2	0	-200.15	177.70	-133.55	168.91
		1	-127.79	163.11	-69.62	156.91
	0.4	0	-201.32	221.84	-134.09	200.48
		1	-128.93	198.04	-70.05	179.99
^{12}C	0	0	-232.60	140.5	-162.15	142.82
		1	-191.12	133.90	-125.96	137.68
	0.2	0	-233.18	188.11	-162.45	176.79
		1	-192.00	176.81	-126.43	168.11
	0.4	0	-233.86	235.89	-162.74	210.83
		1	-193.01	219.96	-126.88	198.64
^{40}Ca	0	0	-254.05	144.92	-181.01	146.14
		1	-232.72	140.54	-162.30	142.74
	0.2	0	-254.36	195.00	-181.16	181.89
		1	-233.26	188.24	-162.58	176.84
	0.4	0	-254.73	245.17	-181.33	217.68
		1	-233.92	236.11	-162.87	211.01
^{90}Zr	0	0	-262.44	146.79	-188.35	147.56
		1	-249.47	143.77	-176.99	145.23
	0.2	0	-262.63	197.83	-188.44	183.97
		1	-249.84	193.41	-177.17	180.72
	0.4	0	-262.87	248.93	-188.55	220.40
		1	-250.29	243.17	-177.37	216.25
^{208}Pb	0	0	-266.14	147.95	-191.59	148.44
		1	-258.05	146.07	-184.50	146.99
	0.2	0	-266.26	199.31	-191.65	185.01
		1	-258.28	196.53	-184.62	182.97
	0.4	0	-266.41	250.71	-191.72	221.60
		1	-258.57	247.07	-184.76	218.98