# Further Comments On The Effects of Deformation on Isovector Electromagnetic and Weak Transition Strengths

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**Abstract**

We present a superior proof that the results for summed strength isovector dipole, spin dipole, and orbital dipole excitations are independent of deformations at the $\Delta$ N = 0 level. The effects of different oscillator frequencies in the x, y, and z directions are also considered.

## 1) INTRODUCTION

As has been previously noted [1], using the rotational model for $^{12}C$ and harmonic oscillator wave functions, the results for summed strength isovector dipole, spin-dipole, and orbital dipole excitations were independent of deformation in a $\Delta N = 0$ Nilsson model. We then presented a more general proof which did not require the use of the rotational model explicitly but rather did require that the valence nucleons were all in the $0p$ shell and that the mean square radius of $p_{1/2}$ and $p_{3/2}$ particles were the same (as they are with harmonic oscillator wave functions).

We here present a superior proof and make several points about dipole excitations. We consider excitations from the ground state of an $N = Z$ open shell nucleus (like $^{12}C$). We will assume the ground state has angular momentum $J = 0^+$. As in the original work we consider the operators ($rY^1_k t$, $r[Y^1 s]^\lambda_k t$, and $r[Y^1 \ell]^\lambda_k t$ .). Some of these operators arise in (p,n) reactions or neutrino absorption such as $\nu_e +^{12} C \rightarrow^{12} N + e^-$.

We show again in Table 1 the results of the summed strength in the asymptotic (oblate) limit and the spherical limit for the above operators in $^{12}C$. The results to individual final momenta $\lambda$ are different in these two limits, but the total summed strength is the same in these two limits.

For the ordinary dipole operator $rY'_k t_+$, the summed strength (SUM) , multiplied by $4\pi m\omega/\hbar$ is 27; for the spin dipole it is 20.25 and for the orbital dipole 48. We will soon explain why this is so.

## 2) THE NEW APPROACH

To see why the results for SUM are independent of the specific $0p$ configuration (or deformation) when spherical harmonic oscillators (H.O.) wavefunctions are used we note the following unique feature of dipole excitations: In the H.O. approximation there is only one excitation energy, $1\hbar\omega$. For the other modes this is not the case. For $E2$ transitions, the strength of which are highly dependent on deformation there are both $0\hbar\omega$ and $2\hbar\omega$ excitations; for $E3$ we have $1\hbar\omega$ and $3\hbar\omega$ excitations etc.

Since for $E1$ transitions there is only <u>one</u> excitation energy involved we can relate the summed strength to the energy weighted strength E.W.S.

$$SUM = E.W.S./\hbar\omega \tag{1}$$

The energy weighted strengths have been studied a great deal , and if we ignore, for the moment, the lack of commutivity of the potential energy with the various dipole operators, very simple results emerge.

Let us first show the electric dipole EWS referred to the center of mass, as given in Bohr and Mottelson[2]. They write the operator $M(E1, \mu) = e \sum_i (\frac{N-Z}{2A} - t_3(i))(rY^1_\mu)_i$ The 'classical' EWS for this operator is

$$EWS = \frac{9}{4\pi} \frac{\hbar^2}{2M} \frac{NZ}{A} \tag{2}$$

Which for $N = Z = \frac{A}{2}$ becomes

$$EWS = \frac{9}{32\pi} A \frac{\hbar^2}{M} \tag{3}$$

In our problem we have $t_+$ rather than $t_z$. Again going to the case of $N = Z = \frac{A}{2}$ , we have $M(E1, \mu) = e \sum_i (-t_+(i))(rY^1_\mu)_i$ The EWS is now expressed as

$$\frac{1}{2}[EWS(+) + EWS(-)] = \frac{9}{8\pi} \sum < 0|[zt_-, [-\frac{\hbar^2}{2M} \frac{d^2}{dz^2}, zt_+]]|0 > \tag{4}$$

where EWS(+) is the energy weighted strength for a process in which a neutron is changed into a proton and EWS(-) where a proton is changed into a neutron. Using the relations

$$[\frac{d^2}{dz^2}, zt_+] = 2\frac{d}{dz} t_+ \tag{5}$$

$$[z, \frac{d}{dz}] = -1 \tag{6}$$

$$[t_-, t_+] = -2t_z \tag{7}$$

We are reduced to

$$\frac{1}{2}[EWS(+) + EWS(-)] = \frac{9}{8\pi} \frac{\hbar^2}{M} \sum < 0|t_z + \frac{1}{2} + t_z 2z \frac{d}{dz}|0 > \tag{8}$$

We can easily compute $< 2z\frac{d}{dz} >$ by integration by parts (given real wave-functions).

$$< 2z\frac{d}{dz} >= \int \psi 2z \frac{d}{dz} \psi = I_T$$

$$\begin{aligned} I_T = \psi^2 2z - \int \psi(2\psi + 2z\frac{d\psi}{dz} \\ I_T = 0 - 2 - I_T \\ I_T = -1 \\ < 2z\frac{d}{dz} >= -1 \end{aligned} \tag{9}$$

This yields the simple result first derived by Lipparini and Stringari [3]

$$\frac{1}{2}[EWS(+) + EWS(-)] = \frac{9}{8\pi}\frac{\hbar^2}{M}\sum < 0|t_z + \frac{1}{2} - t_z|0 > \tag{10}$$

For N=Z we have

$$EWS(+) = EWS(-) = \frac{9}{16\pi}A\frac{\hbar^2}{M} \tag{11}$$

since in this case, $EWS(+) = EWS(-)$. For the SUM we obtain

$$SUM = \frac{EWS(rY_k^1 t_+)}{\hbar\omega} = \frac{9}{16\pi}A\frac{\hbar}{M\omega} \tag{12}$$

Finally we get

$$4\pi SUM\frac{M\omega}{\hbar} = 9\frac{A}{4} \tag{13}$$

This is the quantity given in Tables 1 and 2 of ref [1]. For $A = 12$ we get 27 for this quantity, confirming the results previously obtained. [1]

3) EFFECT OF DIFFERENT FREQUENCIES IN THE X,Y,AND Z DIRECTIONS

To take deformation effects further into account we introduce different frequencies in the x, y, and z directions. It can be shown that we get the correct result by making the following replacement in Eq(12).

$$\frac{1}{\hbar\omega} \rightarrow \frac{1}{3}(\frac{1}{\hbar\omega_x} + \frac{1}{\hbar\omega_y} + \frac{1}{\hbar\omega_z}) \tag{14}$$

To obtain this result we must not only consider excitations from 0p to higher shells but also excitations from 0s to 0p. Note that if the above expression (14) is expanded in terms of a deformation parameter $\delta$ there will be no linear terms.

To get an estimate of the size of this effect we use the self consistency conditions

$$\Sigma_x \omega_x = \Sigma_y \omega_y = \Sigma_z \omega_z \qquad (15)$$

where for $^{12}$C in the asymptotic limit

$$\begin{aligned} \Sigma_x = \Sigma_y &= 10 \\ \Sigma_z &= 6 \end{aligned} \qquad (16)$$

We define $\omega_0$ by $\omega_x \omega_y \omega_z = \omega_0^3$ and assume volume conservation, i.e. keep $\omega_0$ constant. We then find $\omega_x = 0.8434\omega_0$ and $\omega_z = 1.4057\omega_0$. We find $\frac{1}{3}(\frac{1}{\hbar\omega_x} + \frac{1}{\hbar\omega_y} + \frac{1}{\hbar\omega_z}) = \frac{1.0275}{\hbar\omega_0}$

There is a very small change in the overall strength. However $\frac{2}{3}$ of the strength is shifted down to $0.8434\hbar\omega_0$ and $\frac{1}{3}$ is shifted up to $1.4057\hbar\omega_0$. (Obviously the energy weighted strength does not change in this model.)

4) SPIN DIPOLE AND ORBITAL DIPOLE MODES

We next consider the spin-dipole mode and consider the EWSR in which only the kinetic energy is taken into account

$$EWS(spin\,multipole) = \sum_{\lambda M} \sum_i \frac{1}{2} < [[Y^L(i)s(i)]^{\lambda\dagger}_M, [\frac{p^2(i)}{2m}, [Y^L(i)s(i)]^{\lambda}_M] > \qquad (17)$$

$$= \sum_i \sum_{L,M,M_L,M_S,M_L',M_S'} (L1M_LM_S|\lambda M)(L1M_L'M_S'|\lambda M) \frac{1}{2} < [Y^{L\dagger}_{M_L}(i)s^\dagger_{M_S}(i), [\frac{p^2(i)}{2m}, Y^L_{M_L'}(i)s_{M_S'}(i) > \qquad (18)$$

Now since

$$\sum_{M,L} (L1M_LM_S|\lambda M)(L1M_L'M_S'|\lambda M) = \delta_{M_L,M_L'}\delta_{M_s,M_s'} \qquad (19)$$

We obtain

$$EWS(spin\, multipole) = \frac{1}{2}\sum_i \sum_{M_L} < [Y^{L*}_{M_L}(i), [\frac{p^2(i)}{2m}, Y^L_{M_L}(i)]] \sum s^\dagger_{M_s} s_{M_s} > \quad (20)$$

$$= \sum_i EWS(ordinary\, multipole) s(i) \cdot s(i) \quad (21)$$

Now $s(i) \cdot s(i)$ is equal to 3/4 for spin 1/2 particles. Hence $EWS$(spin multipole) = 3/4 $EWS$(ordinary multipole) For the spin dipole case in which there is only one excitation energy ($\hbar\omega$) the above relation also holds for summed strength. This was mentioned but not proven by Auerbach and Zamick[1].

As noted in their work and in Table 1 of the present work, the value of SUM for the spin-dipole case is 20.25 which is indeed 3/4 of the ordinary dipole 27.

For the orbital dipole case we replace $s(i) \cdot s(i)$ by $l(i) \cdot l(i)$. This differs from the spin-dipole case in the sense that $l(i) \cdot l(i)$ is state dependent with eigenvalue l(l+1) . The value of SUM in Table 1 (summed also over $\lambda$) is 48, all coming from the 0p shell. The value of SUM for the ordinary dipole coming from the 0p shell is 27-3=24. The factor of 2 is due to the fact that $l(l+1)$ equals two in the 0p shell.

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Table 1 Total isovector dipole strength in $^{12}C(\frac{4\pi m\omega}{\hbar} SUM(\lambda) 0^+ \rightarrow \lambda)$
in parenthesis are the strengths due to excitations from $0s$.

| Dipole | $rY^1_K t$ | |
|---|---|---|
| | Asymptotic | Spherical |
| $\lambda$ | | |
| 1 | 27 | 27 |

| Spin Dipole | $r[Y^1 s]^\lambda t$ | |
|---|---|---|
| $\lambda$ | | |
| 0 | 2.25 (0.25) | 3.25 (0.75) |
| 1 | 6.75 (0.75) | 8.25 (1.50) |
| 2 | 11.25 (1.25) | 8.75 (0) |
| Sum | 20.25 (2.25) | 20.25 (2.25) |

| Orbital Dipole | Dipole $r[Y^1 \ell]^\lambda t$ | |
|---|---|---|
| $\lambda$ | | |
| 0 | 0 (0) | 0 (0) |
| 1 | 14 (0) | 14 (0) |
| 2 | 34 (0) | 34 (0) |
| Sum | 48 (0) | 48 (0) |

References


1. L. Zamick and N. Auerbach, Nuclear Physics A 658, 285 (1999)
2. A. Bohr and B. Mottelson, Nuclear Structure, Vol. 1 and Vol. 2 (1969) and (1975), (Benjamin, New York, 1975).
3. W. Lipparini and S. Stringari, Phys. Rep. 175, 103 (1989)