# Strongly damped nuclear collisions: zero or first sound?

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## Abstract

The relaxation of the collective quadrupole motion in the initial stage of a central heavy ion collision at beam energies  $E_{lab} = 5 \div 20$  AMeV is studied within a microscopic kinetic transport model. The damping rate is shown to be a non-monotonic function of  $E_{lab}$  for a given pair of colliding nuclei. This fact is interpreted as a manifestation of the zero-to-first sound transition in a finite nuclear system.

#### I. INTRODUCTION

The phenomenon of strong dissipation in the collective motions of heated nuclear systems is a challenging problem for the nuclear transport theory. Following the analogy with other Fermi liquids, like the liquid <sup>3</sup>He [1,2], one expects also in nuclear systems two types of collective modes.

At small temperatures the mean field dominated modes should exist, from the possible violatation of the local thermal equilibrium due to the strong Pauli blocking which inhibits two-body collisions (the Landau zero sounds). This is actually the main nature of isoscalar

giant resonances, where indeed the collective energy is essentially given by the amount needed to deform the Fermi sphere in momentum space [3,4]. Isovector giant resonances are also mean field modes, this time of plasmon type since we can have both a distortion and a shift of the neutron (proton) Fermi spheres.

At high temperatures the usual hydrodynamical collective modes (first sounds) should propagate [5]. In an infinite Fermi liquid with strong repulsion (Landau parameter  $F_0 \gg 1$ ) the two sounds are clearly distinguishable, since the damping rate has a maximum as a function of the temperature [1,2]. However, in nuclear matter the transition between the two sounds is expected to be smeared-out since the Landau parameter  $F_0$  is small in absolute value [6].

The presence of this transition is still an open problem in nuclear dynamics. Only the isovector giant dipole resonances can be experimentally studied with sufficient accuracy in heated nuclei. The point is that for this kind of two-component mode the transition has some special features that make it difficult to observe a clear signature [7].

Aim of this work is to study the transition in the collective nuclear dynamics looking at the evolution of the damping mechanism of large amplitude quadrupole oscillations in fusion processes, in a microscopic kinetic model. Similar attempts have been recently performed in fission dynamics studies [8,9]. We will also follow the temperature dependence of the attenuation of the collective mode.

In the ref.s [8,9] a reduced friction coefficient

$$\beta = \frac{1}{E_{kin}^{coll}} \left( \frac{dE}{dt} \right)_{diss},\tag{1}$$

where  $E_{kin}^{coll}$  is the collective kinetic energy of the dinuclear system (DNS) and  $(dE/dt)_{diss}$  is the dissipation rate of the total collective energy  $E = E_{kin}^{coll} + E_{pot}$ , has been extracted from the measurements of the prescission neutron multiplicity in fast fission reactions. It was shown that in the temperature region  $T = 2 \div 3$  MeV the reduced friction coefficient  $\beta$  is very large ( $\beta \sim 10 \div 100 \cdot 10^{21} \text{ s}^{-1}$ ), result not explained with a one-body dissipation mechanism only.

In the present work we have performed some kinetic transport studies of the nuclear dynamics. Transport equations describe a self-consistent mean field dynamics coupled to two-body collisions and so we expect to see in a natural way the transition between the two sound propagations. We use the Boltzmann-Nordheim-Vlasov (BNV) [10] procedure to simulate the phase space dynamics, which has been quite successful in predicting mean properties of heavy ion collisions at medium energies. In particular we study central nucleus-nucleus collisions leading to fusion at beam energies  $E_{lab} < 21$  AMeV in order to extract the coefficient  $\beta$  as a function of the temperature from the damping of the collective quadrupole oscillations of the formed di-nuclear system (DNS). Density dependent effective interactions of Skyrme type (SKM [11,12]) and an averaged free nucleon-nucleon cross section,  $\sigma = 40mb$  ([12]) have been used.

The structure of the work is as follows. In Sect. 2 the procedure of the extraction of the reduced damping coefficient  $\beta$  from the time dependence of the quadrupole moment given by the BNV model is described. In Sect. 3 we present a novel method for determination of the temperature based on the energy conservation and on the BNV evolution of the potential energy. Sect. 4 contains our results on the  $\beta(T)$  and their interpretation in terms of two-body and one-body dissipation mechanisms. Summary and conclusions are given in Sect. 5.

# II. EXTRACTION OF THE REDUCED FRICTION COEFFICIENT FROM NUCLEAR KINETIC EQUATIONS

According to Eq. (1), the reduced friction coefficient  $\beta$  can be, in principle, calculated directly, once the phase-space distribution function  $f(\mathbf{r}, \mathbf{p}, t)$  is known from the BNV output, since:

$$E_{kin}^{coll}(t) = \int d\mathbf{r} \, \frac{m\langle v(\mathbf{r}, t)\rangle^2}{2} \, \rho(\mathbf{r}, t) \,\,, \tag{2}$$

$$\langle \mathbf{v}(\mathbf{r},t) \rangle = \frac{1}{\rho} \int d\mathbf{p} f(\mathbf{r},\mathbf{p},t) \frac{\mathbf{p}}{m} ,$$
 (3)

$$\rho(\mathbf{r},t) = \int d\mathbf{p} f(\mathbf{r},\mathbf{p},t) , \qquad (4)$$

$$E_{pot}(t) = E_{pot}^{int}(t) + \frac{3}{5} \int d\mathbf{r} \,\epsilon_F(\rho)\rho , \qquad (5)$$

$$E_{pot}^{int}(t) = \int d\mathbf{r} \,\epsilon_{m.f.}(\rho) + E_{coul}(t) , \qquad (6)$$

where  $\epsilon_F(\rho) = \hbar^2/(2m)(3\pi^2\rho/2)^{2/3}$  is the Fermi energy,  $\epsilon_{m.f.}(\rho)$  is the nuclear mean field interaction energy density and  $E_{coul}$  is the Coulomb energy. From Eq.(2) we see that the beam energy is not giving contribution to the collective kinetic energy.

However, in practice, the calculation of the collective kinetic energy  $E_{kin}$  is quite ambiguous in the test particle technique due to the strong dependence on the width of the gaussians representing the test particles [10].

We calculate then the coefficient  $\beta$  from the time evolution of the quadrupole moment of the DNS:

$$Q_{zz}(t) = \int d\mathbf{r} \, q_{zz}(\mathbf{r}) \rho(\mathbf{r}, t), \qquad q_{zz}(\mathbf{r}) = 2z^2 - x^2 - y^2 . \tag{7}$$

For a damped periodical motion (see Appendix)

$$Q_{zz}(t) \propto \exp(-i\omega t), \quad \omega = \omega_R + i\omega_I, \quad \omega_I < 0,$$
 (8)

the coefficient  $\beta$  is proportional to the imaginary part of the frequency  $\omega$ :

$$\beta = -4\omega_I \ . \tag{9}$$

The curves in Fig. 1 show the time evolution of  $Q_{zz}$  in the case of central collisions of  $^{64}\text{Ni} + ^{238}\text{U}$  at beam energies  $E_{lab} = 6.53 \div 20.53$  AMeV. The quadrupole moment quickly approaches a minimum at  $t = 100 \div 200$  fm/c. Afterwards the  $Q_{zz}$  starts to grow again, and after a time interval  $\Delta t = 50 \div 100$  fm/c it saturates. This saturation of the  $Q_{zz}$  is caused by an almost complete lost of the collective energy of the DNS. One can, therefore, qualitatively estimate the  $\beta$  coefficient just assuming that  $|\omega_I| \simeq \omega_R = 2\pi/t_{osc}$ , where  $t_{osc} \simeq 100$  fm/c is the period of oscillations. That gives  $\beta = 4|\omega_I| \simeq 70 \cdot 10^{21}$  s. This value is in agreement with the results obtained in Refs. [8,9]. However, we would like to stress that in Refs. [8,9] the

damping coefficient was extracted from the outgoing part of the trajectory of the DNS from compact mononucleus shape to scission during a time interval  $\sim 30000$  fm/c. Our analysis is concentrated on the part of the trajectory in vicinity of the mononucleus shape and the corresponding time scale  $\sim 300$  fm/c is much shorter.

In order to get the values of the coefficient  $\beta$  we fit a part of the curve  $Q_{zz}(t)$  to the function

$$Q_{zz}^{fit}(t) = \mathcal{A} + \mathcal{B}\sin(\omega_R t + \phi_0)\exp(-|\omega_I|t) . \tag{10}$$

The upper time limit of the fitting region is chosen at the second minimum of the quadrupole moment. The lower time limit is given by the earliest time when the same value of the  $Q_{zz}$  as in the second minimum is reached. This definition of the time limits, from one hand, corresponds approximately to the selection of a full oscillation in vicinity of the compact mononucleus shape. On the other hand, the stage of strong dissipation from the first minimum to the first maximum is completely in the fitting region. The best fit functions of Eq. (10) are shown by full dots in Fig.1. The fitting parameters  $\mathcal{A}$ ,  $\mathcal{B}$ ,  $\omega_R$ ,  $\phi_0$ ,  $\omega_I$  and the corresponding coefficient  $\beta=4|\omega_I|$  for central collisions of  $^{64}$ Ni +  $^{238}$ U at various beam energies are collected in the Table. We see that the damping increases with the increasing collision energy until  $E_{lab}=10.53$  AMeV. This fact can be understood already from Fig. 1, since with increasing beam energy a larger  $\Delta Q_{zz}=Q_{zz}^{saturation}-Q_{zz}^{minimum}$  is damped during a shorter time interval. But at higher energies  $E_{lab}>10.53$  AMeV the damping starts to decrease, as we can see from the presence of quadrupole vibrations at later times t>200 fm/c (Fig. 1). Eventually at  $E_{lab}>14.53$  AMeV a kind of saturation seems to be reached.

### III. DETERMINATION OF THE TEMPERATURE FROM BNV SIMULATIONS

A direct way to extract the temperature is to fit the local momentum distribution given by the BNV model to a  $T \neq 0$  Fermi distribution. However, in the case of low-energy nuclear collsions studied in present work, this direct method is not appropriate just because its accuracy of  $\sim 1$  MeV is not enough. Therefore we will follow a procedure based on the conservation of the energy and on the time evolution of the potential energy given by the BNV. The thermal excitation energy of the DNS is (c.f. [13]):

$$E_{therm}^* = E_{kin}^{c.m.} - E_{coul} - E_{rot} - \Delta E_{pot} , \qquad (11)$$

where  $E_{kin}^{c.m.} = E_{lab}A_1A_2/(A_1+A_2)$  is the center-of-mass kinetic energy,  $E_{coul} = e^2Z_1Z_2/(R_1+R_2+3.5)$  is the Coulomb energy,  $R_i = 1.2A_i^{1/3}$  (fm) i=1, 2 are the nuclear radii,  $E_{rot} = \hbar^2L^2/(2\Theta)$  is the rotational energy with L being the angular momentum (in  $\hbar$  units) and  $\Theta = [\frac{2}{5}(A_1R_1^2+A_2R_2^2)+A_1A_2/(A_1+A_2)(R_1+R_2)^2]m_{nuc}$  being the momentum of inertia  $(m_{nuc})$  is the nucleon mass). The last term  $\Delta E_{pot}$  in the r.h.s. of Eq.(11) is the difference between the values of the potential energy just before and after the overlapping of the density profiles. The origin of this term is mostly from the sharp decrease of the nuclear surface energy when the two nuclei touch each other. In Fig. 2 we show the time dependence of the potential energy per nucleon in the central collision of  $^{64}$ Ni +  $^{238}$ U at 10.53 AMeV. After some small increase until  $t \simeq 20$  fm/c due to the Coulomb contribution the total potential energy quickly drops by about 1 MeV/nucleon reaching the minimum at  $t \simeq 60$  fm/c. We define  $\Delta E_{pot}$  as

$$\Delta E_{pot} = E_{pot}^{min} - E_{pot}^{max} , \qquad (12)$$

where  $E_{pot}^{min}$  and  $E_{pot}^{max}$  are the first minimum and the first maximum of the potential energy.

In the Table we report the values of the potential energy "jumps"  $\Delta E_{pot}$ , the excitation energies and corresponding temperatures  $T = \sqrt{E_{therm}^*/a}$ ,  $a = A/8 \text{ MeV}^{-1}$  for the <sup>64</sup>Ni + <sup>238</sup>U central collisions at various beam energies<sup>1</sup>. Our temperatures are higher than those obtained in Ref. [8]. In particular, for the Ni+U collision at 6.53 AMeV we have T = 3.1 MeV and the authors of Ref. [8] report T = 2.4 MeV. This difference is mainly explained by the fact, that the temperatures in Ref. [8] are obtained from the outgoing stage of the

<sup>&</sup>lt;sup>1</sup> In the case of the central collisions  $E_{rot} = 0$  in Eq. (11)

reaction on much larger time scales as compared to our study. A further contribute to some temperature overshooting is coming from pre-equilibrium emissions, neglected in the energy balance Eq.(11). This effect, although expected to increase with beam energy, can be still considered quite small in this energy range.

#### IV. TEMPERATURE DEPENDENCE OF THE DISSIPATION

Fig. 3 shows the temperature dependence of the reduced friction coefficient  $\beta$ . The BNV results are shown by solid line with dots. The coefficient  $\beta$  first increases with temperature reaching the maximum at  $T \simeq 4.5$  MeV and then at higher temperatures it decreases. This signal is indeed quite robust and cannot be related to some overestimation of the temperatures at higher beam energies as discussed at the end of the previous section.

One can interpret the results of the full transport calculations within the approach of Ref. [5], based on the analytical solution of the linearized Landau-Vlasov equation

$$\left(\frac{\partial}{\partial t} + \frac{\mathbf{p}}{m^*} \frac{\partial}{\partial \mathbf{r}}\right) \delta f(\mathbf{r}, \mathbf{p}, t) - \frac{\partial \delta U}{\partial \mathbf{r}} \frac{\partial f_{\text{eq}}(\mathbf{p})}{\partial \mathbf{p}} = I_{coll}[\delta f] ,$$
(13)

where  $\delta f$  and  $f_0$  are the perturbation and the equilibrium value of the phase space distribution function,

$$\delta U(\mathbf{r}, \mathbf{p}, t) = \frac{1}{N(0)} \int \frac{g d\mathbf{p}'}{(2\pi\hbar)^3} (F_0 + F_1 \hat{p}\hat{p}') \delta f(\mathbf{r}, \mathbf{p}', t)$$
(14)

is the mean field perturbation  $(\hat{p} \equiv \mathbf{p}/p, \hat{p}' \equiv \mathbf{p}'/p')$ , g = 4 is the spin-isospin degeneracy of a nucleon,  $N(0) = gm^*p_F/(2\pi^2\hbar^3)$  is the level density at zero temperature,  $F_0$  and  $F_1$  are the Landau parameters and  $m^* = m/(1 + F_1/3)$  is the effective mass.

The collision integral in the r.h.s. of Eq. (13) is taken in the relaxation time approximation:

$$I_{coll}[\delta f] \simeq -\frac{1}{\tau} \delta f_{|l \ge 2} ,$$
 (15)

where

$$\delta f_{|l\geq 2}(\mathbf{p}) \equiv \sum_{l\geq 2} \sum_{m=-l}^{l} Y_{lm}(\hat{p}) \int d\Omega_{\hat{p}'} Y_{lm}^*(\hat{p}') \, \delta f(\mathbf{p}')_{|p'=p}$$

$$\tag{16}$$

is the part of the perturbation containing the quadrupole and higher multipolarity distortions of the Fermi surface. The effective relaxation time  $\tau$  includes two- and one-body dissipation contributions:

$$\tau^{-1} = \tau_{2body}^{-1} + \tau_{1body}^{-1} . \tag{17}$$

The relaxation time  $\tau_{2body}$  was calculated in Ref. [7] for various choices of a nucleon-nucleon scattering cross section:

$$\tau_{2body}^{-1} = T^2/\kappa \tag{18}$$

with  $\kappa \simeq 1900 \text{ MeV}^2\text{fm/c}$  for the isotropic energy independent isospin-averaged nucleonnucleon scattering cross section  $\sigma_{NN} = 40 \text{ mb}$ . For the one-body relaxation time we have used the wall-and-window formula (c.f. Ref. [5]):

$$\tau_{1body}^{-1} = \frac{\overline{v}}{2R\xi} , \qquad (19)$$

where  $R = 1.2A^{1/3}$   $(A = A_1 + A_2)$  is the radius of a mononucleus composed of the two colliding nuclei,

$$\overline{v} = \frac{3}{4} v_F \left[ 1 + \frac{\pi^2}{6} \left( \frac{T}{\epsilon_F} \right)^2 \right] \tag{20}$$

is an average velocity of nucleons, and  $\xi$  is a numerical factor which depends on the multipolarity and on the isospin of a collective mode. We have chosen a value  $\xi = 1.85$ , which corresponds to the isoscalar quadrupole mode in the scaling wall model (see Ref. [5] and refs. therein).

The solution of Eq. (13) inside a nucleus with uniform nonperturbed density can be found as a superposition of plane waves. In this case, as it was shown in Ref. [15], the intrinsic width of a giant multipole resonance  $\Gamma = 2\hbar/\tau_{rel}$ , where  $\tau_{rel}$  is the relaxation time of the distribution function  $\delta f$  ( $\delta f \propto \exp(-i\omega t)$ ,  $\omega = \omega_R + i\omega_I$ ,  $\omega_I = -1/\tau_{rel}$ ), can be expressed as

$$\Gamma \simeq 2q\hbar\omega_R \frac{\omega_R \tau}{1 + q(\omega_R \tau)^2} \tag{21}$$

with  $q = [\frac{5}{2}(1+F_0)(1+F_1/3)]^{-1}$ . Eq. (21) describes both well known [1] zero sound ( $\Gamma \propto \tau^{-1}$ ,  $\omega_R \tau \gg 1$ ) and first sound ( $\Gamma \propto \tau$ ,  $\omega_R \tau \ll 1$ ) regimes. In our calculations we have put the Landau parameters  $F_0 = 0.2$ ,  $F_1 = 0$ . The frequency has been chosen as  $\hbar \omega_R = 64.7 A^{-1/3}$  MeV ( $A = A_1 + A_2$ ) corresponding to the giant quadrupole resonance.

The solid line in Fig. 3 shows the friction coefficient

$$\beta_{ZFST} = 2\Gamma/\hbar \tag{22}$$

with  $\Gamma$  given by Eq. (21). We see that Eqs. (21), (22) agree qualitatively with the BNV model, but the analytical calculation gives a more smeared-out transition between the two sounds.

In the limit of the zero sound the coefficient  $\beta$  is

$$\beta_{ZS} = 4/\tau = \beta_{ZS}^{2body} + \beta_{ZS}^{1body} , \qquad (23)$$

where  $\beta_{ZS}^{2body} = 4/\tau_{2body}$ ,  $\beta_{ZS}^{1body} = 4/\tau_{1body}$ . This simple formula (dot-dashed line in Fig. 3) is quite close to the BNV results at  $T \leq 4.5$  MeV. However, either the two-body (dashed line) or one-body (dotted line) contributions taken separately are strongly underpredicting the BNV friction coefficient.

In order to better understand the relative importance of the two-body and one-body mechanisms we have also performed the BNV calculations at  $E_{lab} = 6.53$ , 8.53, 10.53, 12.53 and 14.53 AMeV switching off the collision term, corresponding to a pure Vlasov evolution. Fig. 4 shows the comparison of the  $Q_{zz}$  time evolution with and without collision term for the Ni+U collision at 14.53 AMeV. In the case without collisions the damping is much reduced: we observe several large-amplitude oscillations of the quadrupole moment. We have also checked that in the Vlasov evolution the oscillations at later times, t > 200 fm/c, are present for all the other studied beam energies. In Fig. 3, the coefficient  $\beta$  in the case of collisionless dynamics is shown by full squares. The damping slowly increases with

temperature and saturates at T=5 MeV. As expected, the system is always in the region of the zero sound. The absolute value of the reduced friction coefficient in the collisionless case is  $3 \div 5$  times less than for a full BNV calculation and close to the wall-and-window value  $4/\tau_{1body}$ .

#### V. SUMMARY AND CONCLUSIONS

The reduced friction coefficient was extracted for the initial stage ( $\sim 300 \text{ fm/c}$ ) of central  $^{64}\text{Ni} + ^{238}\text{U}$  collisions at beam energies  $E_{lab} = 6 \div 21 \text{ AMeV}$  in the framework of the BNV transport model. The quadrupole moment  $Q_{zz}$  has been chosen as a relevant collective variable. The quadrupole time evolution shows overdamped oscillations with a damping rate proportional to the friction coefficient. Two-body collisions play a major role in the damping of  $Q_{zz}$ .

As a function of the beam energy, the damping rate has a maximum at  $E_{lab} \simeq 10$  AMeV. The corresponding temperature of the DNS, neglecting particle emissions, is 4.6 MeV. We interpret, therefore, this temperature as a transition temperature from the zero-to-first sound propagation.

This result agrees with earlier calculations of V.M. Kolomietz et al. (Ref. [5]), where a value of the transition temperature  $4 \div 5$  MeV was deduced on the basis of the analytical solution of the linearized BUU equation in the case of isoscalar giant resonances of multipolarities l=0 and 2 in a hot nucleus.

Our calculations are overpredicting the transition temperature  $2 \div 2.5$  MeV that follows from the analysis of the prescission neutron multiplicities by J. Wilczyński et al. (Ref. [8]). We have to stress that the considered collective modes are different, in our analysis quadrupole oscillations in the entrance channel dynamics while in the ref. [8] the fission mode in the exit channel. In particular a relevant variance is on the time scales of the two modes, with a much larger proper time for the fission dynamics. Following the simple condition  $\omega_R \tau \simeq 1$  for the transition, we roughly get a  $T_{tr} \propto \sqrt{\omega_R}$  and then we can expect quite smaller transition temperatures for the fission modes. Moreover, as already stressed before, likely our temperature assignements are a little overestimated, particularly for the higher beam energies, due to the lack of pre-equilibrium emissions in the energy balance.

Looking at the BNV simulations, the transition from zero-to-first sound appears as a presence of quadrupole vibrations at relatively late times, t > 200 fm/c (see Fig. 1), for beam energies above 10 AMeV. These vibrations would accelerate the fission of the produced DNS that should be observed experimentally, as an increase of fast-fission cross sections.

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#### **APPENDIX**

In this Appendix we derive the relation (9) between the imaginary part  $\omega_I$  of the complex frequency  $\omega = \omega_R + i\omega_I$  and the reduced damping coefficient  $\beta$  of Eq. (1).

From the Eq. (7) and the continuity equation  $\dot{\rho} = -\nabla(\mathbf{v}\rho)$  we get:

$$\dot{Q}_{zz} = \int \rho \mathbf{v} \nabla q_{zz} d^3 r \ . \tag{24}$$

In the case of the small-amplitude damped periodical motion of the kind

$$\mathbf{v}(\mathbf{r},t) = \delta \mathbf{v}_0(\mathbf{r}) \exp(-i\omega t) \tag{25}$$

the Eq. (24) can be linearized with respect to small values of  $\delta v_0$ :

$$\dot{Q}_{zz} \simeq \left[ \rho_0 \int \delta \mathbf{v}_0(\mathbf{r}) \nabla q_{zz} d^3 r \right] \exp(-i\omega t) \equiv \dot{Q}_{zz}(t=0) \exp(-i\omega t) ,$$
 (26)

where  $\rho_0$  is the nonperturbed density ( $\rho = \rho_0 + \delta \rho$ ). The general solution of Eq. (26) is

$$Q_{zz}(t) = \text{const} + \frac{i\dot{Q}_{zz}(0)}{\omega} \exp(-i\omega t) . \qquad (27)$$

Therefore in a linear approximation, over  $\delta v_0$ , the quadrupole moment reveals oscillations with the same frequency  $\omega = \omega_R + i\omega_I$  as the velocity field  $\mathbf{v}(\mathbf{r}, t)$ .

The time derivative of the collective kinetic energy (2) is:

$$\dot{E}_{kin} \simeq m\rho_0 \int \text{Re}(\mathbf{v}) \text{Re}(\dot{\mathbf{v}}) d^3 r = m\rho_0 \left[ \int \delta v_0^2(\mathbf{r}) d^3 r \right] \cos(\omega_R t) (\omega_I \cos(\omega_R t) - \omega_R \sin(\omega_R t)) \exp(2\omega_I t) . \tag{28}$$

Averaging Eq. (28) over the period of oscillations we come to the relation:

$$\overline{\dot{E}_{kin}} \simeq \omega_I \rho_0 m \int \overline{(\text{Re}(\mathbf{v}))^2} d^3 r = 2\omega_I \overline{E_{kin}} , \qquad (29)$$

where we have dropped the therm  $\propto \cos(\omega_R t) \sin(\omega_R t) \exp(2\omega_I t)$  which changes the sign during the period of oscillation. According to the virial theorem for harmonic oscillators

$$\overline{E_{kin}} = \overline{E_{pot}} = \frac{1}{2}\overline{E} , \qquad (30)$$

where E is the total collective energy. Therefore

$$\overline{\dot{E}} \simeq 4\omega_I \overline{E_{kin}} \ .$$
 (31)

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Table Fit parameters from Eq. (10); potential energy "jumps"  $\Delta E_{pot}$ ; excitation energies and temperatures of the DNS Eqs. (11),(12); for central collisions <sup>64</sup>Ni + <sup>238</sup>U at various beam energies  $E_{lab}$ .

$E_{lab}$	$\mathcal{A}$	$\mathcal{B}$	$\omega_R$	$\phi_0$	$\omega_I$	$-\Delta E_{pot}$	$E^*$	Т
(AMeV)	$(Afm^2)$	$(Afm^2)$	c/fm		c/fm	AMeV	AMeV	MeV
6.53	33.81	24.34	0.024	-6.537	0.0133	0.88	1.2	3.1
8.53	25.60	39.94	0.030	-6.169	0.0175	1.28	1.9	3.9
10.53	21.46	38.45	0.036	-6.197	0.0180	1.68	2.7	4.6
12.53	18.34	31.13	0.038	-6.091	0.0163	2.10	3.4	5.2
14.53	16.42	27.03	0.039	-5.906	0.0147	2.28	3.9	5.6
16.53	14.33	23.50	0.040	-5.843	0.0137	2.46	4.4	6.0
18.53	12.89	26.73	0.039	-5.589	0.0141	2.68	5.0	6.3
20.53	11.14	28.05	0.038	-5.486	0.0147	2.74	5.4	6.6

#### FIGURE CAPTIONS

- Fig. 1 Solid lines time dependence of the quadrupole moment  $Q_{zz}$  for <sup>64</sup>Ni + <sup>238</sup>U central collisions at beam energies (from top to bottom)  $E_{lab} = 6.53$ , 8.53, 10.53, 12.53, 14.53, 16.53 and 20.53 AMeV. The value of the quadrupole moment at t=0 is always 85 fm<sup>2</sup>/nucleon. Full circles show the best fits obtained using Eq. (10) for each collision energy. See text after Eq. (10) for a definition of the fit regions.
- Fig. 2 Potential interaction energy per nucleon Eq. (6) as a function of time for the  $^{64}$ Ni(10.53 AMeV) +  $^{238}$ U central collision.
- Fig. 3 Temperature dependence of the reduced friction coefficient  $\beta$ . BNV calculations with (without) collision term are shown by full circles (squares) connected with thin solid line to guide eye. Errorbars are due to the ambiguity caused by a finite number of test particles. The analytical result of Ref. [5] for the Giant Quadrupole Resonance in a hot nucleus (see Eqs. (21), (22)) is shown by thick solid line. The two- and one-body contributions and their sum (see Eq. (23)) in the zero sound limit are shown by dashed, dotted and dash-dotted lines respectively.
- Fig. 4 Comparison of the time evolution of the quadrupole moment for <sup>64</sup>Ni(14.53 AMeV)
  + <sup>238</sup>U central collision calculated with (solid line) and without (dashed line) collision term.

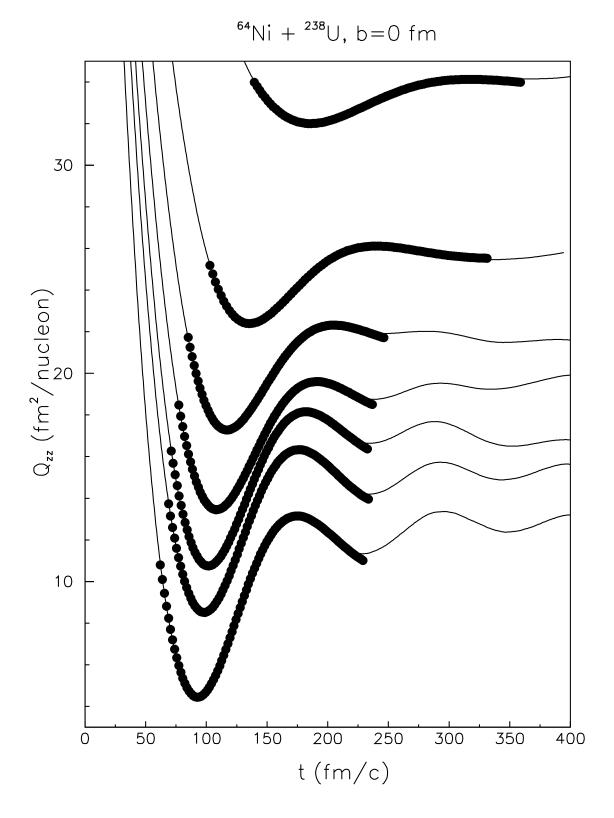


FIG. 1.

