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Identification of fractal properties and upscaling of parameters of a layered-heterogeneous reservoir

Identification of fractal properties and parameters upscaling of layered heterogeneous medium

IN Abdulin[1], VA Baikov[2,1]
[1]Ufa State Aviation Technical University, RF, Ufa
[2]RN-BashNIPIneft LLC, RF, Ufa

Email: Baikov@bnipi.rosneft.ru

keywords:hydrodynamic modeling, identification of parameters, layered inhomogeneous medium, upscaling, permeability, fractals, power laws

Coarsening of computational spatial grids is one of the main ways to reduce the cost of computing resources in geological and hydrodynamic modeling of hydrocarbon reservoirs. The procedure of overriding reservoir properties in an upsized cell of the computational grid is called upscaling (averaging). The quality of this procedure is determined by the degree of predictive capability decreasing of applied models. The traditional way for determining the average value as the arithmetic mean is not always applicable in practice, since it does not take into account the spatial heterogeneity of the averaged values distribution. In this paper, we consider the case of a reservoir with formation reservoir properties (permeability and porosity) values close to power functions of the spatial variable. Proximity of reservoir properties to power function indicates to a fractal inhomogeneity of the porous medium. The power-law upscaling procedure is proposed for this case. An initial-boundary-value problem for a one-dimensional fractal model of unsteady-state filtration is considered. investigations and investigations of the fractal investigations. The proposed methods tested on data from one of the fields in Western Siberia. A comparative analysis with the arithmetic mean method is performed on permeability data. The proposed techniques have a potential for use in reservoir engineering and monitoring. An initial-boundary-value problem for a one-dimensional fractal model of unsteady-state filtration is considered. investigations and investigations of the fractal investigations. The proposed methods tested on data from one of the fields in Western Siberia. A comparative analysis with the arithmetic mean method is performed on permeability data. The proposed techniques have a potential for use in reservoir engineering and monitoring. An initial-boundary-value problem for a one-dimensional fractal model of unsteady-state filtration is considered. investigations and investigations of the fractal investigations. The proposed methods tested on data from one of the fields in Western Siberia. A comparative analysis with the arithmetic mean method is performed on permeability data. The proposed techniques have a potential for use in reservoir engineering and monitoring.

I.N. Abdulin[1], V.A. Baykov[2.1], Ph.D.

[1]Ufa state aviation technical university

[2]OOO RN-BashNIPIneft

Contact address:Baikov@bnipi.rosneft.ru

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ABOUT one of the key stages in constructing a geological and technical reservoir model is hydrodynamic modeling, in which they use computational software packages – simulators. When performing multivariant hydrodynamic calculations, an important factor is the calculation time, which directly depends on the number of spatial cells in the model. In many cases, to significantly reduce the calculation time of the model, its upscaling (averaging) is carried out - the transition in the model from a fine computational grid to a larger one.

The correctness of upscaling is largely determined by the choice of the method for calculating the average value of the well data set in the considered cell of the computational grid. The simplest and most common are weighted average methods with the same weights. However, if the well data distributions have heterogeneous inclusions on the scale of the considered cell, then the use of such methods can lead to a significant decrease in the quality of geological and hydrodynamic modeling. The presence of such heterogeneities may indicate the presence of complex structures in the geological structure of the reservoir. One of the methods to improve the modeling of objects with complex structures is a method that takes into account their fractal properties.

The use of fractals in applied problems originates in the works of B. Mandelbrot [1]. It was noticed that not only purely abstract objects, but also geometric structures formed in the course of natural phenomena can have fractal properties. Fractals have found application in underground hydrodynamics – in describing the formation of viscous fingers in porous media [2], which are of a fractal nature, in describing percolation effects [3], and in many other problems [1].

One way to represent physical processes in fractal media is to use fractional derivatives [4]. Fractional-order equations can describe processes that have residual memory, which can arise due to the fractality of the medium. In [5], this and other methods of taking into account the fractal properties of a medium during the motion of a fluid in a porous medium are considered. A more extensive review of methods that take into account the fractality of the medium is presented in [6].

Another way to describe the filtration process in a fractal medium is to use the fluid transfer equation. In particular, for a weakly compressible fluid in the plane-parallel case, it has the form [7]

$$\frac{\partial}{\partial t}\left(m x^{D-1} p\right) - \frac{\partial}{\partial x}\left(K x^{D-1}\frac{\partial p}{\partial x}\right) = 0, \qquad (1)$$

Where t, x - time and space, respectively coordinate; m – porosity; - is a constant value, depending on the fractal properties of the medium (m-x^{D-1} – spatial distribution of pores); D – fractal dimension of the environment; p - pressure; - - viscosity; K, - are constant values describing the anomalous conductivity of the fractal medium.

The practical use of this model is difficult due to the problem of empirical identification of the parameters responsible for taking into account fractal properties. In [8], to obtain a model of this process, an approach was proposed with the replacement of constant values of porosity and permeability properties (PPP) with power-law dependences on the spatial coordinate. The determination of the parameters of these dependences is proposed to be carried out by averaging the fields of reservoir properties using Fourier analysis. The article proposes and considers a procedure for identifying fractal parameters by permeability as values at which the proximity functional of a solution with a power-law permeability function to a solution with a piecewise constant permeability function reaches a minimum (field data).

Power Laws for FES in Media with Fractal Properties

Let us consider an object of a fractal shape, along which a substance with a unit physical density is distributed. In this case, the mass of the object will obey the B. Mandelbrot relation $M \sim L^D$ (M - the mass of the object; L is the size of the averaging area) [18]. It follows from this relation that the average density of an object depends on the dimensions of the spatial region of averaging and is estimated by the expression $- \sim L^{D-E}$ (E - once-dimensionality of the averaging area, in the framework of this work E=1). It follows that $-(x) = -x^{D-E}$, - - const.

In geological and hydrodynamic modeling, the analogue of void distribution density is porosity. Permeability can be a power function of porosity [9]. On the other hand, porosity m and abso- fierce permeability k are purely geometric porous medium [10]. Accordingly, it can be assumed that there is a power-law dependence between the distributions of these parameters, which corresponds to media with fractal properties. In [8], based on the corollary of the B. Mandelbrot relation, the expressions

$$m = \bar{m} x^{D_m - E}, \tag{2}$$

$$k = k \cdot x^{D_k - E}, \tag{3}$$

Where D_m, m – fractal indicators for porosity; D_k, $\tilde{k}$ – fractal permeability indicators.

Quantities D_m And D_k associated with the dimension of the fractal environment as follows:

$$D_m = D, D_k = D - -. \tag{4}$$

Statement of the problem of identification of the fractal properties of the medium

It follows from the comparison theorem for the filtration equation with respect to the porosity function [11] that its best approximation corresponds to the best approximation of the pressure value p. According to this D_m And m we will choose based on the condition of the best approximation of this function on the entire segment of the described problem.

Let us consider two one-dimensional models of filtration of a single-phase flow in an inhomogeneous medium with equal power functions of porosity. In the first case, the permeability is an increasing piecewise constant function; in the second case, it is a power function of the spatial coordinate. In this section, we propose a technique for determining the parameters of the power law for permeability based on minimizing the measure of similarity between the pressure values of both models.

Task A. Boundary Value Problem for a Nonstationary Model filtration, where the permeability $k(x)$ is piecewise permanent function

$$\bar{m} x^{D_m-1} \frac{\partial p}{\partial t} - \frac{\partial}{\partial x}\left(\frac{k(x)}{\mu}\frac{\partial p}{\partial x}\right) = 0,$$
$$x \in (l, L) \setminus X_{gr}, \; l < x < L, \; 0 < t \le T$$
$$p(x,0) = p_0(x), \; p(l,t) = p_l(t), \; p(L,t) = p_L(t), \tag{5}$$

Where $X_{gr} = \{x_{i\,gr}, i = 1, \ldots, N\}$ – set of change points $k(x)$.

It is assumed that at the points of separation of media with different permeabilities, the conditions of "hard contact" are observed. Then at $x \in X_{gr}$ conditions are met pressure and flow jerkiness: $p(x)\big|_{x=x^i_{gr}-0} = p(x)\big|_{x=x^i_{gr}+0}$,

$$k(x)p_x(x)\big|_{x=x^i_{gr}-0} = k(x)p_x(x)\big|_{x=x^i_{gr}+0}, \; i = 1, \ldots, N.$$

Task B. Initial-boundary value problem for non-stationary mode of filtering in a fractal environment

$$\bar{m} x^{D_m-1} \frac{\partial \bar{p}}{\partial t} - \frac{\partial}{\partial x}\left(\frac{\bar{k}(x)}{\mu}\frac{\partial \bar{p}}{\partial x}\right) = 0, \; \bar{k}(x) = b x^a,$$
$$\bar{p}(x,0) = p_0(x), \; \bar{p}(l,t) = p_l(t), \; \bar{p}(L,t) = p_L(t). \tag{6}$$

Where a, b - coefficients.

Variational problem B . The task of determining the coefficient patients a And b in function $k(x)$ (problem B) from the condition mi- minimization of the measure of proximity of solutions to initial-boundary value problems A and B. As a measure of proximity, the integral from 0 to T sums of squared deviations from each other pressure ion, the rate of its change according to t and flow on the segment [l, L]. Functional proximity of solutions

$$F(a,b) = \int_0^T \int_l^L \left[\left(p(t,x) - \bar{p}(t,x,a,b)\right)^2 + \left(\frac{\partial p}{\partial x} - \frac{\partial \bar{p}}{\partial x}\right)^2 + \left(\frac{\partial p}{\partial t} - \frac{\partial \bar{p}}{\partial t}\right)^2 \right] dx\, dt \to \min. \tag{7}$$

Carrying out the identification procedure using field data

To improve the accuracy of calculations, we consider a particular case of the functions $p_0(x), p_l(t), p_L(t)$ for which the initial boundary value problem A has an analytical solution. This case is satisfied by the linear boundary conditions $p_l(t) = p_{l1} t + p_{l2}, p_L(t) = p_{L1} t + p_{L2}$ (choice $p_0(x)$ is carried out in accordance with the consistency with the conditions at the edges and "hard contact"), for them the solution of problem A has the form

$$p(t,x) = \begin{cases} A_1(x^{D_m-1} - 2B_1 t) - C_1, \; l > x > x_{1\,gr}, \\ A_2(x^{D_m-1} - 2B_2 t) - C_2, \; x_{1\,gr} > x > x_{2\,gr}, \\ \ldots, \\ A_{N-1}(x^{D_m-1} - 2B_{N-1} t) - C_{N-1}, \; L > x > x_{gr}^{N}, \end{cases} \quad (8)$$

where constants $A_i, B_i, C_i, (i=1, \ldots, N+1)$ according to equation number, edge conditions $x = l$ and not- flow discontinuities satisfy the following relations:

$$\begin{gathered} p_{l1} = 2AB_{eleven}D_m(D_m-1), p_{l2} = A_1 l^{D_m-1} - C_1; \\ A_i(x_{i\,gr})^{D_m-1} - C_i = A_{i-1}(x_{i\,gr})^{D_m-1} - C_{i-1}, \\ A_i B_i = A_{i-1} B_{i-1}, i=1,\ldots,N; \; B_i = D_i(D_i-1) k_i / m_i (m_m \; --), \quad i=1,\ldots,N-1. \end{gathered} \quad (9)$$

The data of gamma-ray logging of a well at one of the fields in Western Siberia were taken as initial data. Quantities $k(x)$, $m(x)$ reconstructed according to karo- , in accordance with the dependences obtained in [13], and normalized with respect to the maximum value.

For the considered data from the condition of the best approximation on the entire segment $\int_l^L (m(x) - m \, x^{D_m-1})^2 dx \to \min$

received: $\tilde{m} = 0.96$ $D_m = 0.04$.

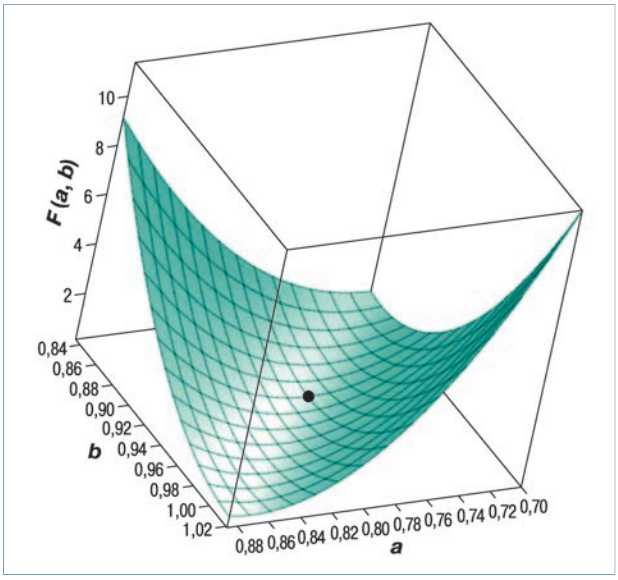
The rest of the model parameters brane following soothing way: $T = 100, l = 0, L = 1, - = 1, p_0(x) = p_0(x), p_l(t) = = p_l(t) = t, p_L(t) = p_L(t) = t + 10$.

The integrals in the considered functionals were calculated by the left rectangle method on a uniform grid. Problems B and C were solved numerically using the built-in procedures of the Maple mathematical package.

On fig. 1 shows the graph of the functional $F(a,b)$, which reaches a minimum of 0.14 (marked with a bold dot) at $a \approx 0.78$, $b \approx 0.914$.

The authors found that the functionality $F(a,b)$ reaches minimum at values of parameters close to the point nimum functionality $(c,d) - \int_l^L (k(x) - \bar{k}(x;c,d))^2 dx$.

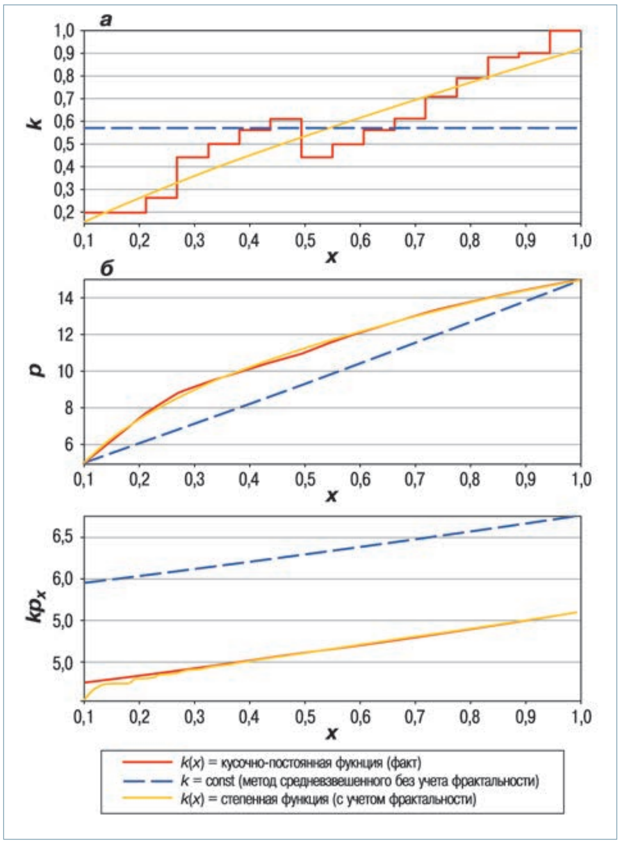
Deviations received values parameters $a_{min} = 0.78$, $b_{min} = 0.914$ from parameter values $c_{min} = 0.8$, $d_{min} = 0.93$ of this functional are comparable with the error of the numerical calculation of the integrals and the numerical solution of problems B and C, and the deviation of the value $F(a_{min}, b_{min})$ from $F(c_{min}, d_{min})$ few. Thus,



Rice. 1. Graph of the solution proximity functional $F(a, b)$

in the considered case, the variational problem B is equivalent to task tape $G(a, b) \to \min$.

On fig. 2 shows graphs of permeability k, pressure- nia p and flow on the segment kp_x, obtained by solving with zero values of the exponent in the functions porosity and permeability (excluding fractality) and problems B, A ($\tilde{m} = 0.96$ $D_m = 0.04$ $\tilde{k} = b_{min}$, $D_k = a_{min}$). From rice. 2, b, c it can be seen that the choice of the permeability value as a power function with parameters from condition (4) instead of the arithmetic mean (constant) value at quantities



Rice. 2. Comparison of approximation methods (for t=5 c) permeability functions k(A), pressure R(b) and flow kp_x(V)

Upscaling procedure for a medium with fractal properties

Upscaling is based on the procedure for calculating the average value in an enlarged spatial cell. The assumption of the isotropy of the medium with average values of reservoir properties often leads to unreliable results of hydrodynamic modeling. In particular, as shown in the previous section, this case includes the case of permeability with a nonzero exponent of the approximating power law. This section proposes a modification of the arithmetic mean method, the weighted mean arithmetic method, taking into account the fractal inhomogeneity of the medium.

Given the power-law inhomogeneity with the exponent $b$distribution of magnitude$f$, can be carried out modification of the arithmetic mean method. Obviously, in this method, in contrast to the standard method, the weights of the averaging elements will not be uniformly distributed. The resulting modification has the form

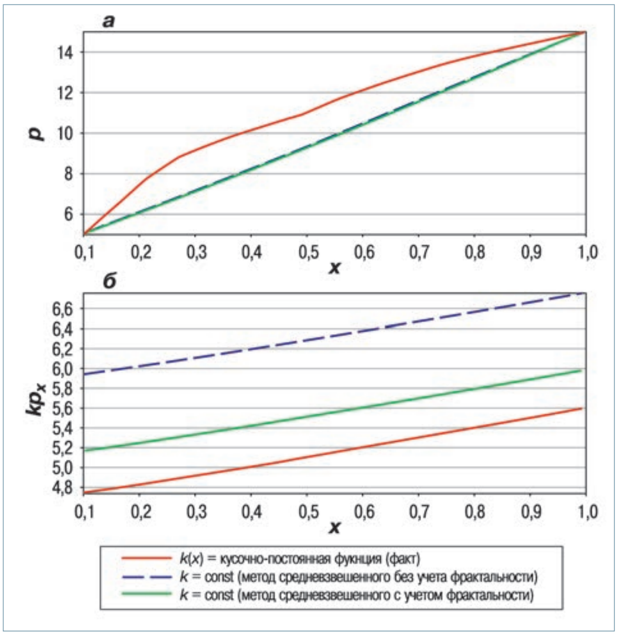
$$\bar{f} = \frac{\sum_{i=1}^{N-1} \left(x_{i+1}^{b-1} - x_i^{b-1}\right) f_i}{\left(x_N^{b-1} - x_1^{b-1}\right)}. \qquad (10)$$

To test the obtained modification of the averaging procedure, the permeability value was taken; as an example of calculation, the initial data from the previous section were considered.

And using the arithmetic mean method, we obtaink_1= 0.57, according to the method _weighted average taking into account fractal environmentk_2= 0.5.

For pressure values (Fig. 3,*A*), the modification did not lead to significant changes. However, reducing the error in the calculation of the flow value by means of y

20% (cm



Rice. 3. Comparison of weighted average methods with and without taking into account the fractality of the pressure function*R*at*t*=5 s (*A*) and flow*kpx*(*b*)

conclusions

1. A technique for empirical identification of the parameters of a filtration model in a fractal medium is proposed based on the minimum of the proximity functional to a solution with a piecewise constant permeability function (field data).

2. The problem of identifying parameters using the proposed technique is equivalent to a simpler problem of identifying parameters based on the best approximation of a piecewise constant function by a power function on the entire interval.

3. A modified procedure for averaging (upscaling) FES is proposed, which takes into account their power (fractal) distribution law.

4. When testing the proposed procedure on the field data of one of the wells in Western Siberia, it was found that the modification of upscaling led to an improvement in the correspondence of the flow value to the actual value by 20%. This indicates a great potential for the application of this technique in the design and monitoring of field development.

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