~~Mee~~

The original pb

$$(\mathcal{P}) \begin{cases} \min_x \frac{1}{2} \|r - \phi x\|_2^2 \\ \text{s.t.} \\ \|x\|_1 \leq \eta \end{cases}$$

Regularized seq of pbs.

$$(\mathcal{P}_k) : \begin{cases} \min \frac{1}{2} \|r - \phi x\|_2^2 \\ \text{s.t.} \\ \|x\|_{1,k} \leq \eta \end{cases}$$

$$\|x\|_{1,k} = \sum_{i=1}^{n} |x_i|_k$$

Where $x = (x_1, \dots ; x_N)^T$

$$|x_i|_k = \frac{1}{k} \left[\ln(1 + e^{-kx_i}) + \ln(1 + e^{kx_i}) \right]$$

$(\mathcal{P}_k)$ convex and differentiable

Denote (x_k^*, λ_k^*) optimal sol to $(\mathcal{P}_k)$

Note λ_k^* derives from the Lagrangia and the KKT conditions

(1)

Objective: Prove that

$(x_k^*, \lambda_k^*) \to \underbrace{(x^*, \lambda^*)}_{\substack{\text{optimal sol} \\ \text{to } (\mathcal{P})}}$

Thm The solution (x_k^*, λ_k^*) to $(\mathcal{P}_k)$ converges as $k \to \infty$ to (x^*, λ^*) of $(\mathcal{P})$

Some techniqual requirements

Prop LDNN is reliant upon equality constrained pb.

Prop $(\mathcal{P}_k)$ reduce to

$$\begin{cases} \min \frac{1}{2} \|r - \phi x\|_2^2 \\ \text{s.t.} \\ \|x\|_{k,n} = \eta \end{cases}$$

provided that

$$\eta < \frac{\|\phi^T r\|_2}{\|\phi^T \phi\|_2} - \frac{2n \ln(2)}{k}$$

+

Rb is the reducing condition is never fullfilled in our case

$\eta = 30$ and $\dfrac{\|\phi^T r\|_2}{\|\phi^T \phi\|_2} \simeq 3,\ldots$

($N_z = 15$, magnitude $= \pm 2$)

We are interested in (P_k)

$\|x^*\|_{1,k} \simeq 31,94\ldots$

at $k = 354$ done

prenons : $\eta = 30$ et see the behaviour.

Use LPNN even if the equality-constraint is not valid. check

$\eta = 31,\ldots$ in res (P_k) and in 13) direct methods

$\eta = 30$ in (P_k) and direct methods

Dynamics for (P_k)

$$\frac{dx}{dt} = \phi^T(r - \phi x) - \lambda \nabla_x \|x\|_{1,k}$$

$$\frac{d\lambda}{dt} = \|x\|_{k,1} - \eta$$

$k = 354$; $\eta = 2$ values.

$r \to$ ok, $\phi \to$ ok

$\|x\|_{k,1}$ ok.

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in testim, I evaluate

$\|x^*\|_{1,k}$ $k = 1 \to 354$

at $k = 355 \to$ it becomes inf.

Then the appropriate k is $k = 354$ and $\eta = 31,94\ldots$

For this $\eta \to$ we have good results in direct methods.

$\nabla_x \|x\|_{1,k} \in \mathbb{R}^n$

Coms $\|x\|_{1,k} = \sum_{i=1}^{n} |x_i|_k$

$$\frac{\partial}{\partial x_i}\|x\|_{1,k} = \frac{\partial}{\partial x_i}|x_i|_k$$

$$= \frac{e^{kx_i}}{1+e^{kx_i}} - \frac{e^{-kx_i}}{1+e^{-kx_i}}$$

$$\frac{dx}{dt} = \left(\frac{dx_1}{dt} \cdots \cdots \frac{dx_n}{dt} \right)^T$$

$$\frac{d\lambda}{dt} \in \mathbb{R}$$