

# Hands-On Heterogeneous Agent Macroeconomics

## Using the [Econ-ARK/HARK](#) Toolkit

Christopher D. Carroll

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Because Representative Agent (‘RA’) models were not useful for understanding much of what happened in the Great Recession, policymakers including Larry Summers (2011), Fed Chair Janet Yellen (2016), former IMF Chief Economist Olivier Blanchard (2016), ECB Governing Board Member Benoit Coeure (2013), and Bank of England Chief Economist Andy Haldane (2016) have suggested that incorporation of heterogeneity (for example, across borrowers and lenders) must be an essential part of the agenda in developing new and better models. In confirmation of that intuition, a number of recent papers, most notably Kaplan, Moll, and Violante (2018) and Krueger, Mitman, and Perri (2016), have developed models that include a realistic description of microeconomic heterogeneity, and have shown that such models can generate more sensible macroeconomic implications than RA models for important questions like the operation of fiscal and monetary policy.

This course will provide a hands-on introduction to the construction of models with ‘serious’ heterogeneity (that is, heterogeneity that matches the microeconomic facts that theory suggests *should* matter for macroeconomic outcomes like consumption dynamics); why such heterogeneous agent (‘HA’) models have implications different from those of RA models; and how existing HA models can be adapted to new questions. (‘Hands-On’ means that students with their own laptops will run the and experiment with the code that solves these models in class.)

The course will have two main elements: Lectures explaining the conceptual foundations of the models work; and hands-on demonstrations of live working versions of such models using the open-source [Econ-ARK/HARK](#) toolkit.

Students should bring a laptop on which they have permissions to install and run new software. Prior to class, on that laptop, students should have installed the [anaconda3](#) stack, which is a distribution of python 3 that includes a robust set of extra tools that are useful for doing computational work. A good guide to installing anaconda is [here](#).

(In “readings” below, starred readings are strongly suggested)

# 1 Preliminaries

Certain things must be done to prepare for running everything on your own computer

## 1.1 Option 1: Do Everything On Your Own Computer's OS

1. Install **Anaconda**: <https://docs.anaconda.com/anaconda/install>
2. Get Git

Get the command-line tool:

<https://atlassian.com/git/tutorials/install-git>

Get a GitHub Account

Download the GitHub Desktop App

– And connect it to your online GitHub account

3. **Install HARK**

- Follow the instructions for installing HARK for Anaconda

4. Clone the **DemARK** and **REMARK** repos

```
git clone https://github.com/econ-ark/DemARK
```

```
git clone https://github.com/econ-ark/REMARK
```

5. Using python from the command line:

```
pip install nose
```

```
python -c 'import HARK ; print(HARK.__file__)'
```

```
cd [root directory for HARK]
```

```
python -m nose
```

## 1.2 Option 2: Do Everything On a Virtual Machine

1. Find (or buy) free space of at least 100 GB
  - A FAST flash/pen drive is fine (not a slow one)
2. Follow the instructions [for installing your VM](#)

# 2 Motivation

Models with serious microfoundations yield fundamentally different conclusions than RA models about core questions in macroeconomics.

1. How monetary policy works

- HA channels account for most of the mechanism of monetary transmission
2. Whether fiscal policy works
    - ‘serious’ HA models are consistent with evidence of MPC’s of 0.5
  3. What made the Great Recession Great
    - RA models: Mostly a supply shock
    - HA models: Mostly a demand shock

Slides:

[Intro to Monetary Policy with Heterogeneity](#), Crawley (2019)

Readings:

- \* Ahn, Kaplan, Moll, Winberry, and Wolf (2017), Introduction, Conclusion
  - Compact and well written discussion of the state and progress of HA macro.
- \* Carroll and Crawley (2017), [Sections 1, 2, and 4](#)
  - This discussion of that paper puts the relationship of HA to RA models in context.

## 3 The Relation Between Micro and Macro Models

### 3.1 The Relation Between Micro and Macro Models

It has become clear in the last decade (and especially in the last few years) that the micro and macro parts of the HA macro problem are surprisingly independent. HA macro models can be solved by first using powerful tools to find the steady state of the micro problem, and then using other tools to calculate how the micro solution changes over the business cycle. Relatively small – or even linear – *changes* to even a highly nonlinear microeconomic problem can result in quantitatively plausible macroeconomic dynamics. p Readings:

Reiter (2009)

Boppart, Krusell, and Mitman (2018)

Auclert (2019)

- \* [Heterogeneity and Macro Modeling In Policymaking Institutions](#),

## 4 Micro Models

### 4.1 Micro Consumption Theory Refresher

The course will assume that students are familiar with standard quantitative tools for solving RA models, like DYNARE. The bulk of the “hands-on” part of the course will therefore involve learning and using tools for solving micro problems with ‘serious’ microfoundations.

#### 4.1.1 *The Infinite Horizon Perfect Foresight Model*

Kinds of Impatience: Absolute, Return, and Growth

Notes:

- Consumption Under Perfect Foresight and CRRA Utility
- The Certainty Equivalent Consumption Function

#### 4.1.2 *Consumption With Labor Income Uncertainty*

- Notes: A Tractable Model of Buffer Stock Saving
- Notebook: Interactive Demo

Readings

Carroll (2001)

#### 4.1.3 *Rate-Of-Return Uncertainty without Labor Income*

Under CRRA utility, without labor income risk:

1. The consumption function is linear
2. An increase in risk reduces consumption and the MPC

Notes:

Notes: Consumption out of Risky Assets

- (Unavoidably) Riskier Returns  $\Rightarrow$  lower  $C$  (if  $\rho > 1$ )

Portfolio Choice with CRRA Utility

- Greater financial risk  $\Rightarrow$  Smaller Portfolio Share ( $\varsigma \downarrow$ )

Consumption With Portfolio Choice:  $\sigma_{\mathbf{r}} \uparrow \Rightarrow \uparrow$  in  $C$  (!)

- You reduce your exposure to risk  $\varsigma \downarrow$
- Total eventual portfolio risk is *less* than before  $\sigma_{\mathbf{r}} \uparrow$

- Precautionary saving  $\downarrow$
- Net result:  $\sigma_{\mathbf{r}} \uparrow \Rightarrow C \uparrow$

Notebook: [ConsPortfolioModel](#)

- “Stockholding Puzzle” Bertaut and Haliassos (1997)

Origins: Merton (1969), Samuelson (1969)

#### 4.1.4 Habits

Notes:

- [Consumption Models with Habit Formation](#):
  - $\Delta \log c_{t+1} = (1 - \alpha)b_r + \alpha \Delta \log c_t$

## 5 Computational Tools

### 5.1 Vision for the Econ-ARK Project

- [Intro-To-Econ-ARK](#)

## 6 Hands-On Introduction

Here we will explain how to begin using the [Econ-ARK](#) toolkit for heterogeneous agent macro modeling. Students will be taught how to use the toolkit to solve increasingly sophisticated models, starting with partial equilibrium perfect foresight models and ending with some exercises using a full general equilibrium micro-macro model with idiosyncratic and aggregate risks.

### 6.1 A Gentle Introduction

This section builds our first simple models using the toolkit

#### 6.1.1 Perfect Foresight

Notebook: [A Gentle Introduction to HARK - Perfect Foresight](#)

#### 6.1.2 Adding ‘Serious’ Income Uncertainty

Notebook: [A Gentle Introduction to Buffer Stock Saving](#)

## 6.2 Liquidity Constraints, Precautionary Saving, and Impatience

1. The Growth Impatience Condition
2. Liquidity Constraints and Precautionary Saving
3. Impatience and Target Wealth

Notebook: [BufferStockTheory Problems](#)

## 6.3 ‘Serious’ Wealth Inequality

Notebook: [Micro-and-Macro-Implications-of-Very-Impatient-HHs](#)

References: Carroll, Slacalek, Tokuoka, and White (2017)

## 6.4 Matching the Distribution – of the MPC

Theory : [Figure 5](#), Carroll, Slacalek, Tokuoka, and White (2017)

Evidence : [Figure 10b](#), Crawley and Kuchler (2018)

## 6.5 Hands-On with Real HA Models

For an economy in steady state (that is, with constant factor prices like interest rates and wages), models with ‘serious’ income heterogeneity have been solvable in partial equilibrium since about 1990 (Zeldes (1989), Deaton (1991)). Calculating an equilibrium distribution of wealth that results from those policy functions and matching it to the total amount of observed wealth (and a corresponding interest rate) was first done by Hubbard, Skinner, and Zeldes (1994) using a supercomputer. Aiyagari (1994) proposed a radically simple model that did not attempt to match the distributions of wealth and income, but could be solved without a supercomputer.

In a rational expectations steady state, there are no expected *changes* in interest rates, wages, or the distribution. Aggregate fluctuations make calculation of an RE equilibrium massively more difficult, because:

1. Meaningful aggregate fluctuations will change the distribution of wealth and income
2. The amount of aggregate saving depends on how aggregate wealth and income are distributed
3. The amount of saving determines future factor prices
4. In principle, RE therefore requires that everyone know the entire distribution of wealth, income, and any other state variables in the population

The problem therefore suffers from a severe case of the “curse of dimensionality.” (That is, it’s *really* hard!). The first paper to tackle the problem was Krusell and Smith (1998). Work by Bayer and Luetticke (2018) builds on all of the prior work to construct a reasonable HANK model that can be solved in a few minutes on a laptop. The key contribution of Krusell and Smith (1998) was to discover that, in practice, highly accurate predictions of future aggregate states could be made using only the mean of the current aggregate capital stock

Notebook: [KrusellSmith.ipynb](#)

## 6.6 The Micro Steady State and Macro Fluctuations

A problem with solving methods using the original Krusell Smith method is that the computational challenge was so great that only the simplest such models could be solved, and the ability to construct standard tools like impulse response functions to aggregate shocks was very limited.

Reiter (2009) showed how to solve such problems several orders of magnitude faster; the essence of his idea was to solve the micro problem for the steady-state distribution, and then capture business cycle fluctuations by figuring out how to perturb the decision rules and the distribution appropriately.

Building on his work, the last few years have seen great further strides in speed and power of such tools.

References:

Reiter (2009)

Boppart, Krusell, and Mitman (2018)

Ahn, Kaplan, Moll, Winberry, and Wolf (2017)

Bayer and Luetticke (2018)

## 6.7 The Bayer-Luetticke Method

- Notebooks:
  - [OneAsset HANK Model](#)
  - [TwoAsset HANK Model](#)
  - [DCT-Copula-Illustration](#)

## 6.8 Other Literature

References:

[Monetary Policy Transmission with Many Agents](#), Crawley and Lee (2019)

Macprudential Policies in a Heterogeneous Agent Model of Housing Default,  
Khan (2019)

Redistribution, risk premia, and the macroeconomy, Kekre and Lenel (2019)

The Missing Intercept: A Sufficient Statistics Approach to General Equilibrium  
Effects, Wolf (2019)



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