Virtual Quantum Error Detection

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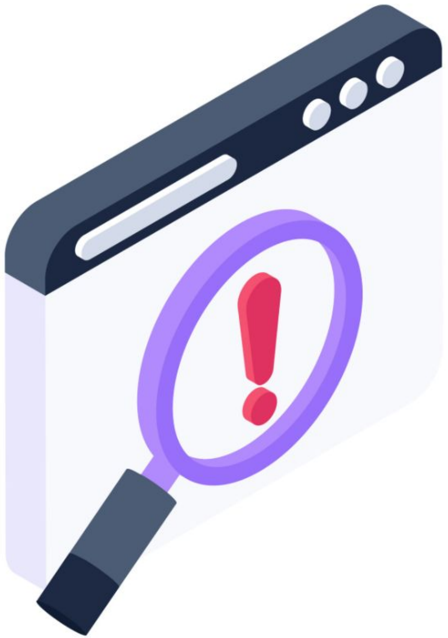


Introduction

A. Quantum Error Detection (QED) is used to enhance computation accuracy of logical qubits through ancilla qubits

B. QED requires expensive single-shot measurements (syndrome measurements)

C. Symmetry Expansion (SE) is a new error mitigation strategy that allows the calculation of the expval() of an observable in a noiseless state

D. Virtual Quantum Error Detection expands SE

Stabilizer Groups (S)

A Stabilizer groups are used to protect our circuit from noise

B S is generated by Generator sets (G), which are comprised of Pauli-Words

$IIZZZZ$
$IZZIIZZ$
$ZIZIZIZ$
$IIIXXXX$

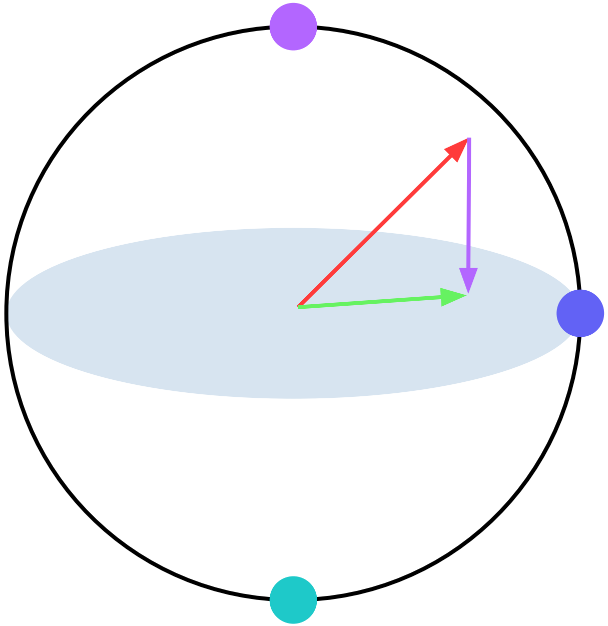
Pauli-Words

Example: S is similar to a "power set" of G

Generator Set (G)	Stabilizer Group (S)
$G_1 = I_0 \otimes Z_1 \otimes Z_2$ $G_2 = Z_0 \otimes Z_1 \otimes I_2$	$S_1 = I$ $S_2 = G_1$ $S_3 = G_2$ $S_4 = G_1 \otimes G_2$

Logical Codespace of a Stabilizer Group

$$\mathcal{C} = \{|\psi\rangle \,|\forall S_i \in \mathcal{S}, S_i |\psi\rangle = |\psi\rangle\}.$$



Syndrome Measurement

Detect physical errors by measuring generators G_i

Phase kick-back!

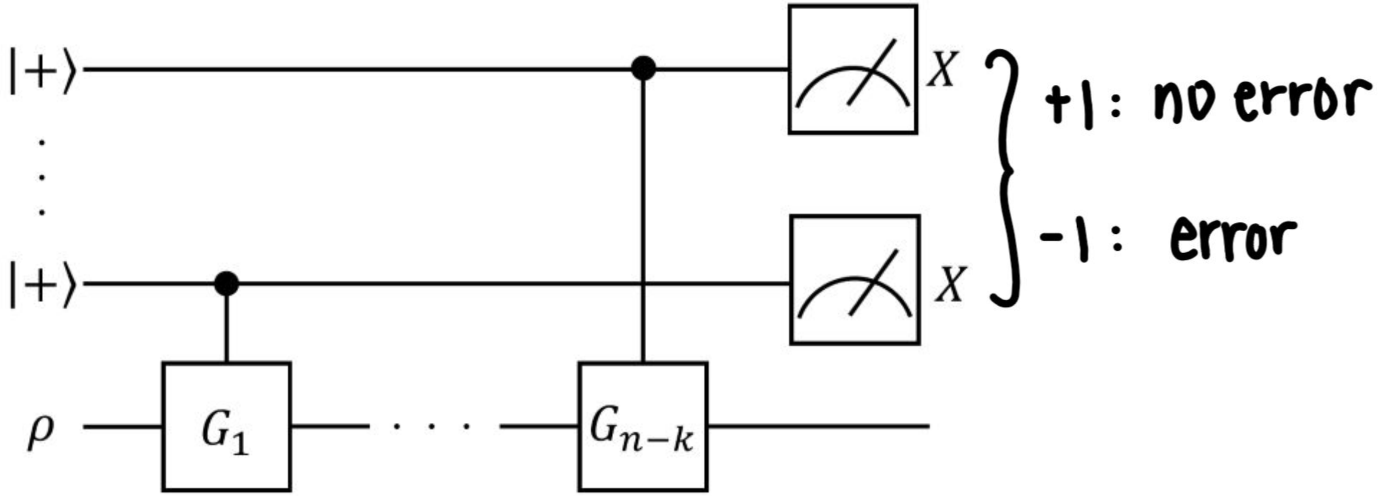


FIG. 1. Quantum circuit for quantum error detection (QED).

Symmetry Expansion

- Expectation value of an observable O for a noiseless state $\boldsymbol{\rho_{id}}$ from a noisy state $\boldsymbol{\rho}$

	Pure[illegible]e	Pure state ρ	Mixed state ρ
States	$\lvert\psi$[illegible]	$\rho = \lvert\psi\rangle\langle\psi\rvert$	$\rho = \sum_i p_i \lvert\psi_i\rangle\langle\psi_i\rvert$
Meas.*	$\lvert\langle\psi_i\lvert\psi\rangle\rvert^2$ $\langle\psi$[illegible]$\rangle$		

A ρ_{id} = ideal density matrix

B ρ = ρ_{id} with noise introduced

C ρ_{det} = ρ projected to C using P

$$\mathrm{tr}[\rho_{\mathrm{det}} O] = \frac{\sum_{S_i \in \mathcal{S}} \mathrm{tr}[\rho O S_i]}{\sum_{S_i \in \mathcal{S}} \mathrm{tr}[\rho S_i]}$$

Virtual Quantum Error Detection (VQED)

- Symmetry expansion can only be used right before measurement
- VQED allows us to calculate error-mitigated expectation values during circuit execution
- P = Projector to code space, ε = Noise , U = Logical Unitary Gate

	Pure state	Pure state ρ	Mixed state ρ
States	$\vert\psi\rangle$	$\rho = \vert\psi\rangle\langle\psi\vert$	$\rho = \sum_i p_i \vert\psi_i\rangle\langle\psi_i\vert$
Ops.	$\vert\psi'\rangle = U\vert\psi\rangle$	$\rho' = U\rho U^\dagger$	$\rho' = U\rho U^\dagger$

$$\rho_{\text{det}} = \frac{\rho'_{\text{det}}}{\text{tr}[\rho'_{\text{det}}]},$$

Implementation of VQED Circuit

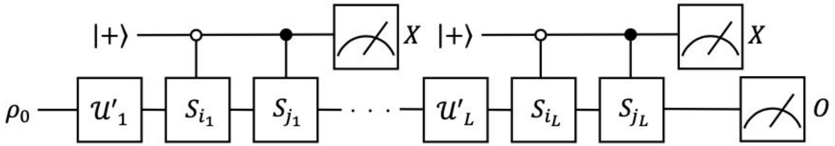


FIG. 2. Quantum circuit for virtual quantum error detection

The VQED Algorithm:

1. Choose area of interest. (# gates = L)
2. Randomly choose L pairs of stabilizers.
3. Run the circuit
4. Let a = expval(X_1 @ ... @ X_L), b = expval(O) * a
5. After N iterations return avg(b) / avg(a)

Pros of VQED

- No Syndrome Measurements
- Less qubit use
- Modular
- Higher fidelity

Cons of VQED

- Only outputs expectation values
- Slower - large expval calculations

Questions

Numerical Simulation