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Collaborations & Expertise:



Topic: Morphological Image Processing

Instructor: Abdul Qayyum, PhD

Class: MCSV

University of Burgundy, France

Outlines of Lecture:

In this lecture we will consider

- What is morphology?
- Simple morphological operations(Dilation, Erosion)
- Explanation of Dilation and Erosion with simple and detail examples
- Compound operations(Opening, closing)
- Boundary Extraction using morphological operation
- Noise Detection by Dilation and Erosion
- Region Filling
- Connected Components
- Skeletonization
- MATLAB based Implementation

What is Mathematical Morphology?

Morphological image processing (or *morphology*) describes a range of image processing techniques that **deal with the shape (or morphology) of features in an image**. Morphological operations are typically applied to remove imperfections introduced during segmentation, and so typically operate on bi-level images.

It is:

- Nonlinear
- Built on Minkowski set theory
- Part of the theory of finite lattices,
- For image analysis based on shape
- Extremely useful, yet not often used

What is Mathematical Morphology?

- Morphological operations are used as a step in image processing and analysis.
- It is used to modify the shape of objects in an image, by using local operations.
- It can be used to remove unwanted effects in segmentation post-processing
 - Remove small objects (that is assumed to be noise)
 - Smooth the edges of larger objects
 - Fill holes in objects
- Link objects together
 - It can be used as a part of object description and analysis
 - Locate the boundary of objects
 - Thin objects
 - Locating objects with a certain structure
 - Locating patterns in an image
- The operations are small, and often very fast.

Uses of Mathematical Morphology

- 1 image enhancement
- 2 image segmentation
- 3 image restoration
- 4 edge detection
- 5 texture analysis
- 6 particle analysis
- 7 feature generation
- 8 skeletonization
- 9 shape analysis
- 10 image compression

- 11 component analysis
- 12 curve filling
- 13 general thinning
- 14 feature detection
- 15 noise reduction
- 16 space-time filtering

Quick Example

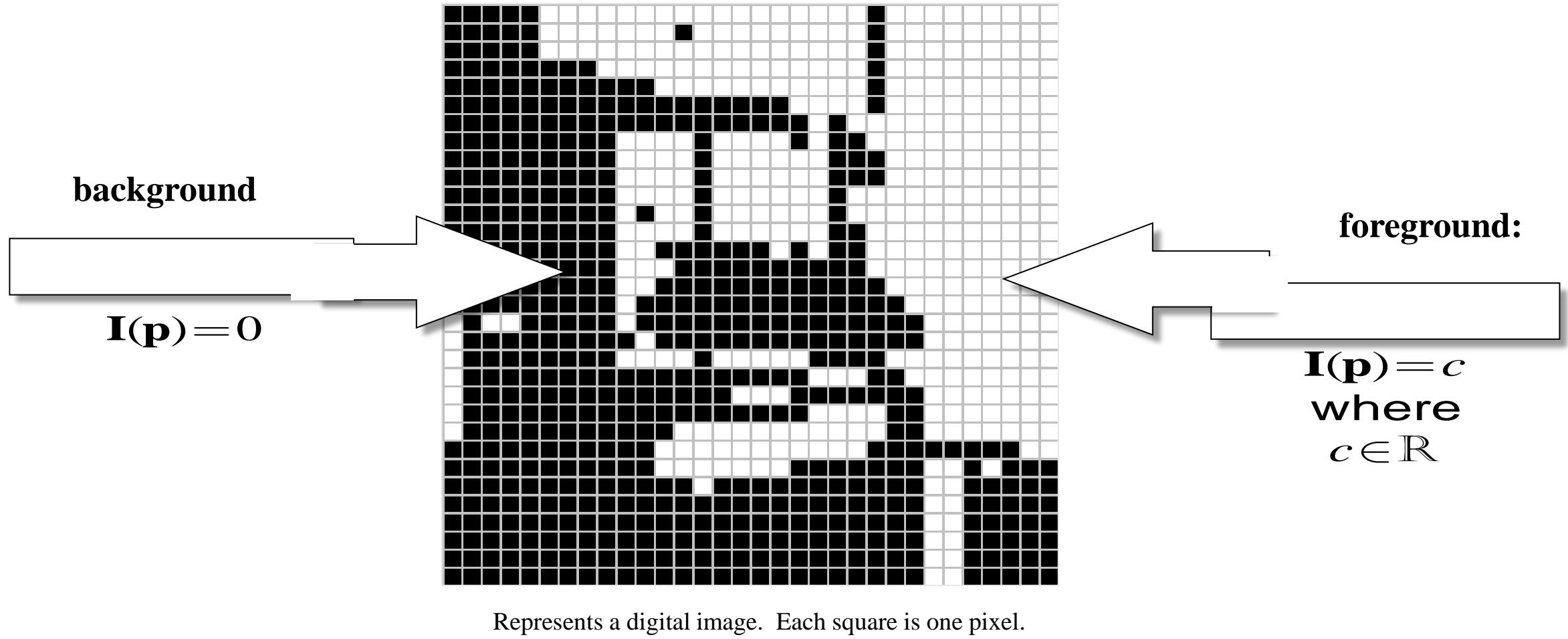


Image after segmentation



Image after segmentation and
morphological processing

A Binary Image:



Support of an Image

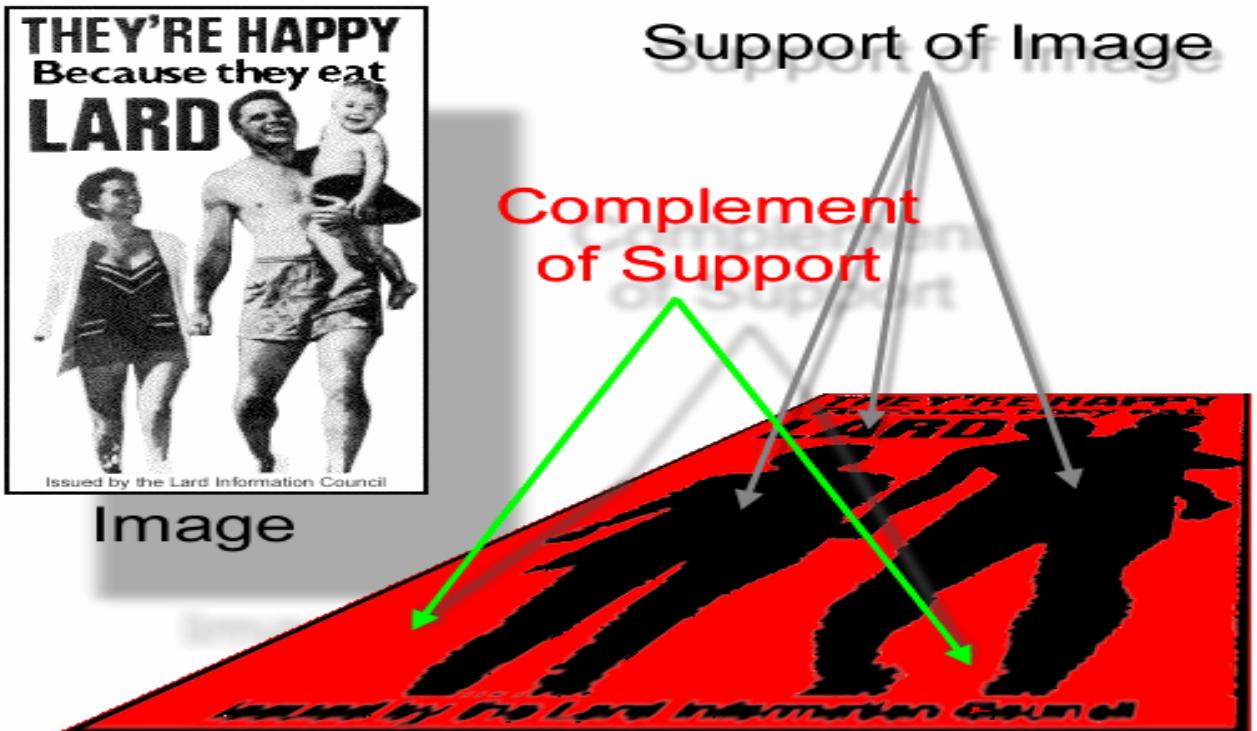
The support of a binary image is often defined as the set of locations of the foreground pixels. I.e.,

$$\text{supp}\{\mathbf{I}\} = \left\{ \mathbf{p} = (r, c) \in S_p \mid \mathbf{I}(\mathbf{p}) = v_{fg} \right\}.$$

In that case, the complement of the support is the set of background pixel locations within the image plane.

$$\{\text{supp}\{\mathbf{I}\}\}^c = \left\{ \mathbf{p} = (r, c) \in S_p \mid \mathbf{I}(\mathbf{p}) = v_{bg} \right\}.$$

But other times, the support is defined as the set of all pixel locations in the image both foreground and background.



Source: Lecture Notes on Mathematical Morphology: Binary Images by Richard Alan Peters

Structuring Elements

Structuring elements can be any size and make any shape.

However, for simplicity we will use rectangular structuring elements with their origin at the middle pixel

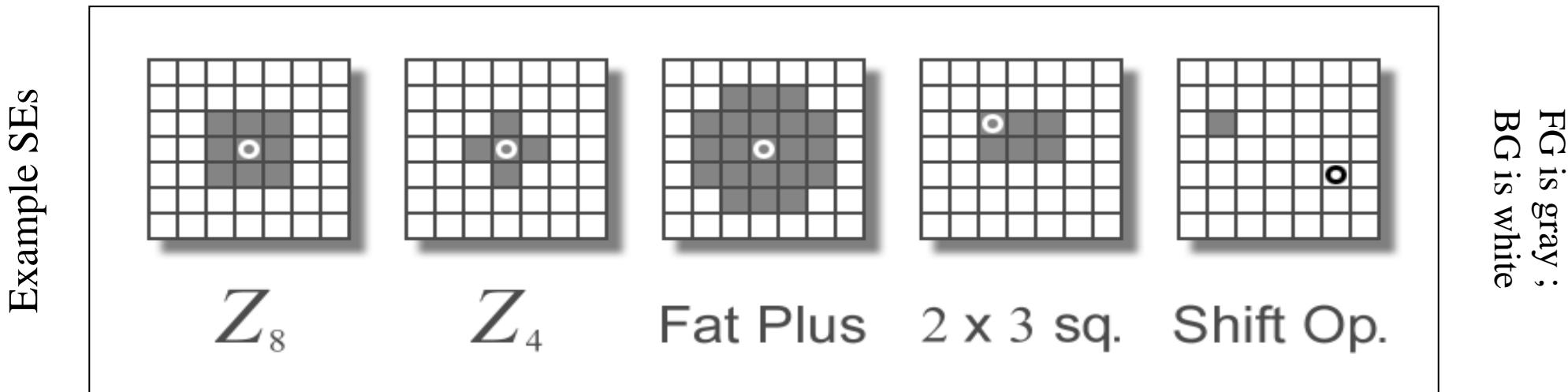
| | | |
|---|----------|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

| | | |
|---|----------|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

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|---|---|----------|---|---|
| 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |

Structuring Element (SE)

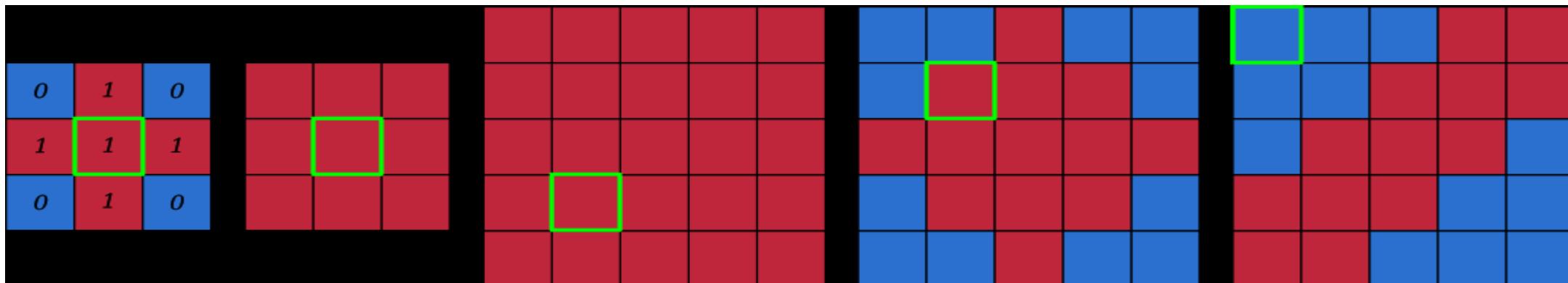
A structuring element is a small image – used as a moving window – whose support delineates pixel neighborhoods in the image plane.



It can be of any shape, size, or connectivity (more than 1 piece, have holes). In the figure the circles mark the location of the structuring element's origin which can be placed anywhere relative to its support.

Structuring Elements

- A *structuring element* in morphology is used to determine the acting range of the operations
- It is typically defined as a binary matrix where pixels valued 0 are not acting, and pixels valued 1 are acting.
- We need to determine an *origin*.
- This origin denotes the pixel that (possibly) changes value in the result image.
- The origin *may* lay outside the structuring element.
- The origin should be highlighted, e.g. with a drawn square.
- We will assume the origin to be at the center pixel of the structuring element, and not specify the location unless this is the case.



Structuring Elements in MATLAB

A structuring elements (SE) in MATLAB

Syntax

[SE = strel\(nhood\)](#)

[SE = strel\('arbitrary',nhood\)](#)

[SE = strel\('diamond',r\)](#)

[SE = strel\('disk',r,n\)](#)

[SE = strel\('octagon',r\)](#)

[SE = strel\('line',len,deg\)](#)

[SE = strel\('rectangle',\[m n\]\)](#)

[SE = strel\('square',w\)](#)

[SE = strel\('cube',w\)](#)

[SE = strel\('cuboid',\[m n p\]\)](#)

[SE = strel\('sphere',r\)](#)

How to use SE?

SE = strel('disk',r,n) creates a disk-shaped structuring element, where r specifies the radius and n specifies the number of line structuring elements used to approximate the disk shape. Morphological operations using disk approximations run much faster when the structuring element uses approximations.

SE3 = strel('disk', 15)

figure imshow(SE3.Neighborhood)

Fundamental Operations

- Fundamentally morphological image processing is very like spatial filtering
- The structuring element is moved across every pixel in the original image to give a pixel in a new processed image
- The value of this new pixel depends on the operation performed
- **There are two basic morphological operations:**
 - Erosion
 - Dilation

Dilation

Dilation

- Dilation of image f by structuring element s is given by $f \oplus s$
- The structuring element s is positioned with its origin at (x, y) and the new pixel value is determined using the rule:

Place the s such that its origin overlaps with f , then

$$g(x, y) = \begin{cases} 1 & \text{if } s \text{ hits } f \\ 0 & \text{otherwise} \end{cases}$$

Dilation of Binary Images

There are a number of equivalent definitions of dilation. Three of them that apply to binary images are:

$$\mathbf{I} \oplus \mathbf{Z} = \left\{ \mathbf{p} \in S_p \mid \left[(\check{\mathbf{Z}} + \mathbf{p}) \cap \text{fg}\{\mathbf{I}\} \right] \neq \emptyset \right\}.$$

The set of all pixel locations, \mathbf{p} , in the image plane where the intersection of $\check{\mathbf{Z}} + \mathbf{p}$ with the foreground of \mathbf{I} is not empty.

$$\mathbf{I} \oplus \mathbf{Z} = \bigcup_{\mathbf{p} \in \text{fg}\{\mathbf{I}\}} (\mathbf{Z} + \mathbf{p}).$$

The union of copies of the SE, one translated to each pixel location in the foreground of the image.

$$\mathbf{I} \oplus \mathbf{Z} = \bigcup_{\mathbf{p} \in \text{fg}\{\mathbf{Z}\}} (\mathbf{I} + \mathbf{p}).$$

The union of copies of the image, one translated to each pixel location in the foreground of the SE.

Dilation of Binary Images

In Dilation, we have input image and structuring element.

- During structure elements window moving on the main input image if we got 1, replace structure element pattern (number of foreground pixels or object pixels) on that place with neighboring pixels. If we got 0, no need to place the structure element pattern.

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 |

A

Take the seed point of window or structure elements: for odd window seed point normally the center

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

B

| | | | |
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| 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 |

Output of Dilation

We have 1 at the window center, place structure elements(patterns of ones at that place)

Dilation

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

A

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|---|---|
| 1 | 1 |
| 1 | 1 |

B

Dilation

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|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 1 |
| 1 | 1 |

Take the seed point of window(green color box)

| | | | |
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| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Receive 1 at the input image (green), replace with structure element pattern(shown in green)

Dilation

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| 1 | 1 |
| 1 | 1 |

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Dilation

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Dilation

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| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Dilation

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Dilation

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 |

Receive 0 at the input image (green),
Don't replace with structure
element pattern(shown in green),
we retain the previous pattern

Dilation

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| 1 | 1 |
| 1 | 1 |

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Dilation

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Dilation

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Dilation

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Dilation

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| 1 | 1 |
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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Dilation

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

3

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

6

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Dilation

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
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| 1 | 1 | 1 | 1 |

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| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Dilation

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

A

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|---|---|
| 1 | 1 |
| 1 | 1 |

B

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

$A \oplus B$

Dilation verification with MATLAB code

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

A

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|---|---|
| 1 | 1 |
| 1 | 1 |

B

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

$A \oplus B$

```
% Input main image matrix  
% We have 4x4 input image matrix  
A=ones(4,4)  
A(2:3,2:3)=0
```

```
% structure element, In this example is [ 1 1 1 1 ]  
B=ones(2,2)
```

```
% Image dilation command in matlab  
Imdilatation=imdilate(A,B)
```

Imdilatation =

| | | | |
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| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Dilation

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|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

A

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| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

B

Dilation

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| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

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|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Dilation

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| 0 | 1 | 0 |
| 1 | 1 | 1 |
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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Dilation

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|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

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|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Dilation

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|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Dilation

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|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

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|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

Dilation

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

Dilation

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

Dilation

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

Dilation

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

Dilation

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

Dilation

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

Dilation

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

A

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

B

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

$A \oplus B$

Dilation verification with MATLAB code

% Generate input image

```
Img2=zeros(8,8)
```

```
Img2(2:7,2:7)=1
```

```
Img2(4:5,4:5)=0
```

% structure element

```
B1=[0 1 0; 1 1 1;0 1 0]
```

| Img2 = | | | | | | | | |
|--------|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

% Image dilation command in matlab

```
Imdilatation=imdilate(Img2,B1)
```

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

$A \oplus B$

| B1 = | | |
|------|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

Imdilatation =

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

¹⁸⁶

Effect of Dilation

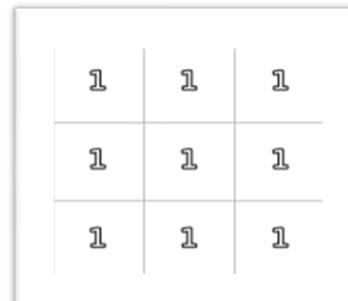
- Dilation *enlarges* foreground objects.
- Dilation fills holes in the foreground (if the structuring element is large enough).
- The result is dependent on the structuring element.
- Larger structuring element gives larger dilation effect.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

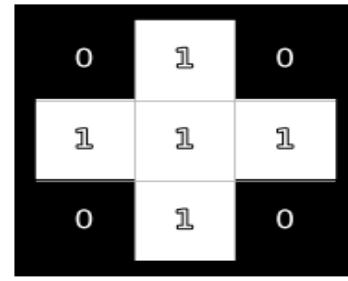
(a) f

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

(a) f



(b) $s1$



(b) $s2$

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |

(c) $f \oplus s1$

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |

(c) $f \oplus s2$

Dilation Example 1



Original image



Dilation by 3×3
square structuring
element



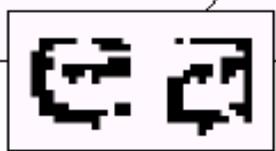
Dilation by 5×5 square
structuring element

Watch out: In these examples a 1 refers to a black pixel!

Dilation Example 2

Original image

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



After dilation

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

Structuring element

Dilation Example 3

314159
950288.
459230



| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |



314159
950288.
459230



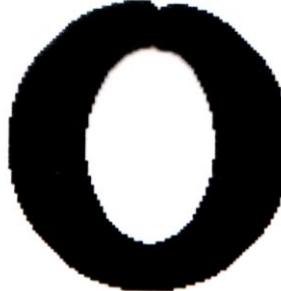
| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |



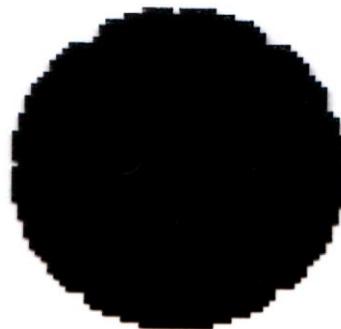
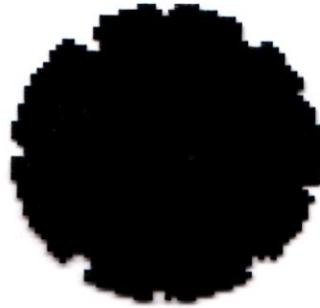
314159
950288.
459230

What is Dilation for?

- Dilation can repair breaks



- Dilation can repair intrusions



- **Watch out:** Dilation enlarges objects

Erosion

Erosion

Erosion of image f by structuring element s is given by $f \ominus s$. The structuring element s is positioned with its origin at (x, y) and the new pixel value is determined using the rule:

Place the s such that its origin overlaps with f , then

$$g(x, y) = \begin{cases} 1 & \text{if } s \text{ fits } f \\ 0 & \text{otherwise} \end{cases}$$

Erosion of Binary Images

There are a number of equivalent definitions of erosion. Three of them that apply to binary images are:

$$\mathbf{I} \ominus \mathbf{Z} = \left\{ \mathbf{p} \in S_p \mid (\mathbf{Z} + \mathbf{p}) \in fg\{\mathbf{I}\} \right\}.$$

The set of all pixel locations, \mathbf{p} , in the image plane where $\mathbf{Z} + \mathbf{p}$ is contained in $fg\{\mathbf{I}\}$.

$$\mathbf{I} \ominus \mathbf{Z} = \bigcap_{\mathbf{p} \in fg\{\mathbf{I}\}} (\mathbf{Z} + \mathbf{p}).$$

The intersection of copies of the refl. SE, one translated to each pixel location in the foreground of the image.

$$\mathbf{I} \ominus \mathbf{Z} = \bigcap_{\mathbf{p} \in fg\{\mathbf{Z}\}} (\mathbf{I} + \mathbf{p}).$$

The intersection of copies of the image, one translated to each pixel location in the foreground of the refl. SE.

Erosion

In erosion, we have input image and structuring elements.

- The structure element or small window move on the main image
- If the pattern (number of foreground pixels or object pixels) of structuring elements are matched with the input image pattern window, Then should place 1 at 0 (If one(1) is there, no need to change), otherwise place zero
- Repeat this process using sliding window on the complete image.
- In this example A is the input image and B is the structure element

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 |

A

Pattern of SE is not match with A image window,
place 0 at the image (center of window)

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

B

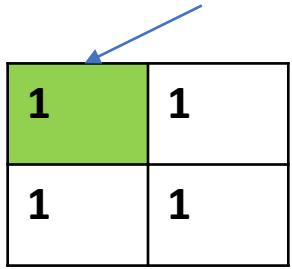
Take the seed point of window or structure elements: for odd window seed
point normally the center

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 |

Erosion

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

A



B

Take the seed point of window or structure elements(green box)

Erosion

| | |
|---|---|
| 1 | 1 |
| 1 | 1 |

| | |
|---|---|
| 1 | 1 |
| 1 | 0 |

No pattern matching with structure element, input window pattern [1 1;1 0] and SE pattern [1 1;1 1]

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|--|--|--|
| 0 | | | |
| | | | |
| | | | |
| | | | |
| | | | |

No pattern matching with structure element, place 0

Erosion

| | |
|---|---|
| 1 | 1 |
| 1 | 1 |

| | |
|---|---|
| 1 | 1 |
| 0 | 0 |

No pattern matching with structure element, input window pattern [1 1;0 0] and SE pattern [1 1;1 1]

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|--|--|
| 0 | 0 | | |
| | | | |
| | | | |
| | | | |

No pattern matching with structure element, place 0 (shown in green box)

Erosion

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | |
|---|---|
| 1 | 1 |
| 1 | 1 |

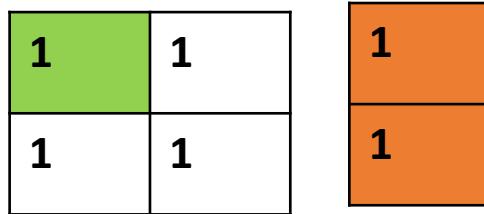
| | |
|---|---|
| 1 | 1 |
| 0 | 1 |

No pattern matching with structure element, input window pattern [1 1;0 1] and SE pattern [1 1;1 1]

| | | | |
|---|---|---|--|
| 0 | 0 | 0 | |
| | | | |
| | | | |
| | | | |

No pattern matching with structure element, place 0 (shown in green box)

Erosion



pattern matching with structure element, input window pattern[1 1; 1 1] and SE pattern [1 1;1 1]

SE element at the edges are don't care and we place 1 1?

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | |
| 1 | 1 | 1 | 1 | |

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| | | | |
| | | | |
| | | | |

pattern matching with structure element,
place 1(shown in green box)

Erosion

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | |
|---|---|
| 1 | 1 |
| 1 | 1 |

| | |
|---|---|
| 1 | 0 |
| 1 | 0 |

No pattern matching with structure element, input window pattern [1 1;0 0] and SE pattern [1 1;1 1]

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | | | |
| | | | |
| | | | |

Erosion

| | |
|---|---|
| 1 | 1 |
| 1 | 1 |

| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

No pattern matching with structure element, input window pattern[0 0;0 0] and SE pattern [1 1;1 1]

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | | |
| | | | |
| | | | |

If no pattern matching with structure element, place 0

Erosion

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

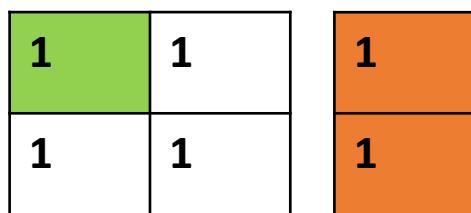
| | |
|---|---|
| 1 | 1 |
| 1 | 1 |

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|---|---|
| 0 | 1 |
| 0 | 1 |

No pattern matching with structure element, input window pattern[0 1;0 1] and SE pattern [1 1;1 1]

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | |
| | | | |
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Erosion



pattern matching with structure element, input window pattern[1 1; 1 1] and SE pattern [1 1;1 1]

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| | | | |
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Erosion

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|---|---|
| 1 | 1 | | |
| 1 | 1 | | |
| 1 | 0 | 1 | 1 |

No pattern matching with structure element, input window pattern[1 0; 1 1] and SE pattern [1 1;1 1]

| | | | |
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| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | | | |
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Erosion

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|---|---|
| 1 | 1 |
| 1 | 1 |

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|---|---|
| 0 | 0 |
| 1 | 1 |

No pattern matching with structure element, input window pattern[0 0; 1 1] and SE pattern [1 1;1 1]

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | | |
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Erosion

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| 1 | 1 |
| 1 | 1 |

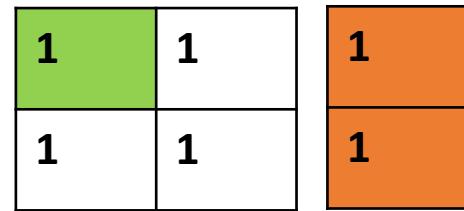
| | |
|---|---|
| 0 | 1 |
| 1 | 1 |

No pattern matching with structure element, input window pattern [0 1; 1 1] and SE pattern [1 1;1 1]

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | |
| | | | |

Erosion



pattern matching with structure element, input window
pattern[1 1; 1 1] and SE pattern [1 1;1 1]

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | |
| 1 | 0 | 0 | 1 | |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

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| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
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Erosion

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| 1 | 1 |
| 1 | 1 |



pattern matching with structure element, input window pattern[1 1; 1 1] and SE pattern [1 1;1 1]

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | | |

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| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
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| 1 | 1 |

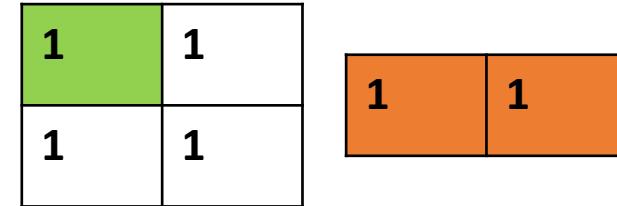


pattern matching with structure element, input window
pattern[1 1; 1 1] and SE pattern [1 1;1 1]

| | | | |
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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | |

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| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
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Erosion

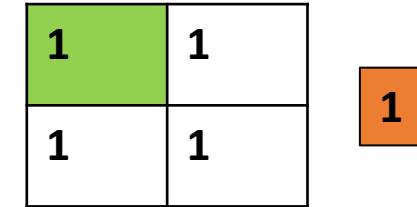


pattern matching with structure element, input window pattern[1 1; 1 1] and SE pattern [1 1;1 1]

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |

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| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | |

Erosion



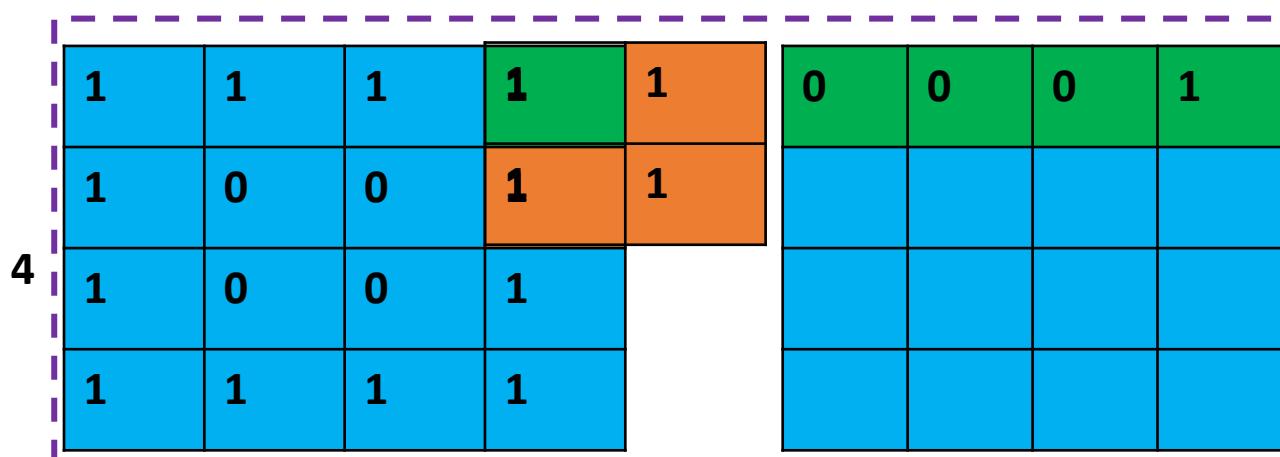
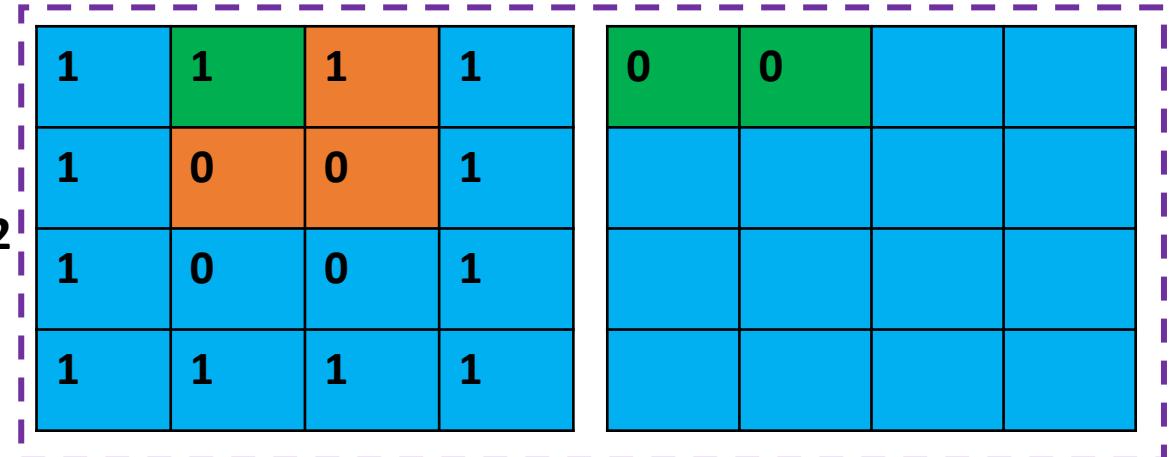
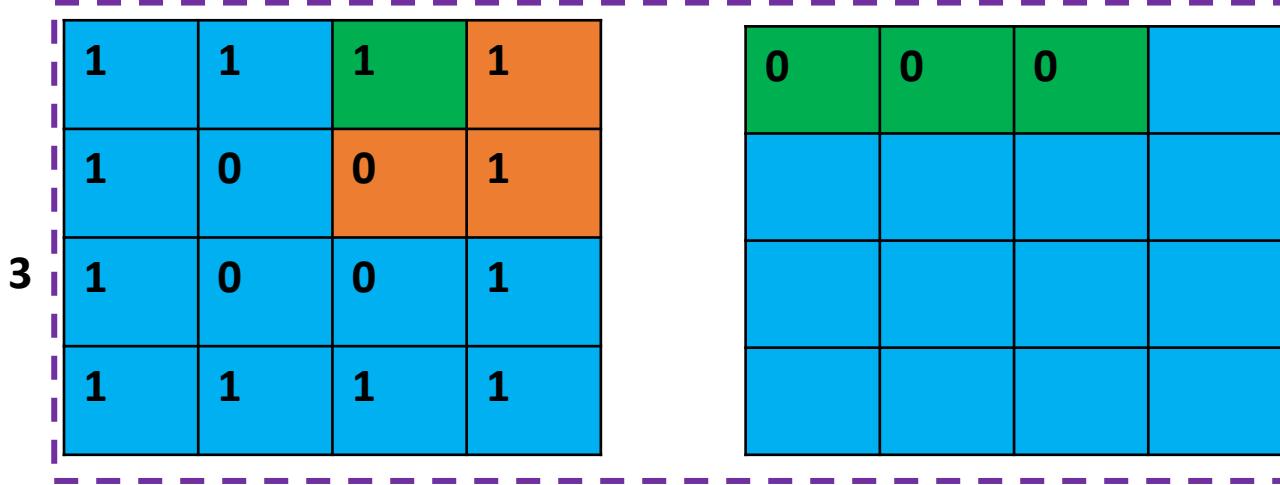
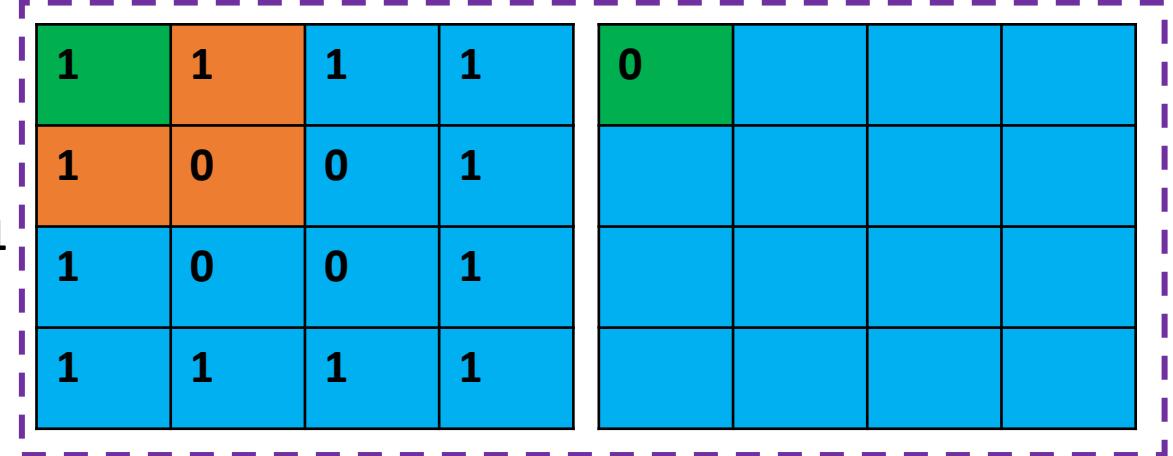
pattern matching with structure element, input window pattern[1 1 ; 1 1] and SE pattern [1 1;1 1]

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| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |
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| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 1 |
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Erosion

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 0 | 0 | 0 | 1 |
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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | |
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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | |
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Erosion

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | | | |

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | |

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | | |

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

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|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | |

Erosion

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| 1 | 1 |
| 1 | 1 |

13

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|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |

14

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

15

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|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |

16

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |

17

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|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

18

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|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |

Erosion

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| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

A

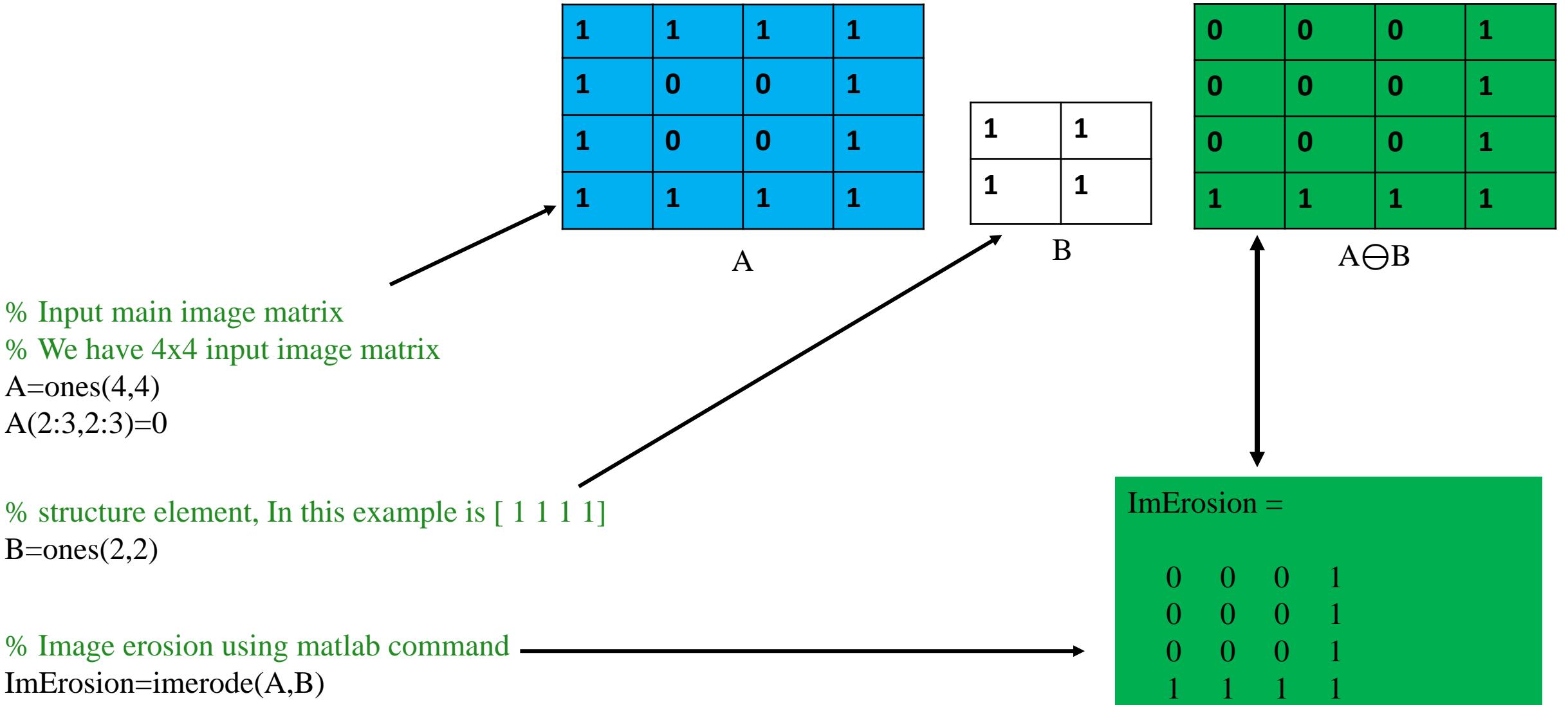
| | |
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| 1 | 1 |
| 1 | 1 |

B

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

$A \ominus B$

Erosion verification with MATLAB code



Erosion

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Erosion

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|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

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|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Erosion

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|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Erosion

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|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Erosion

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| 0 | 1 | 0 |
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| 0 | 1 | 0 |

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Erosion

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| 0 | 1 | 0 |
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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Erosion

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| 0 | 1 | 0 |
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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Erosion

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
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| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

Erosion

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| 0 | 1 | 0 |

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Erosion

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Erosion

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

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|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Erosion

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Erosion

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

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|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Erosion

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|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

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|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

Erosion

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|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

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|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 |

Erosion

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|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

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|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |

Erosion

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

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|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Erosion

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| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |

Erosion

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|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Erosion

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Erosion

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

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|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Erosion

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|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

A

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

B

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$A \ominus B$

Erosion verification with MATLAB code

% Generate input image

```
Img2=zeros(8,8)
```

```
Img2(2:7,2:7)=1
```

```
Img2(4:5,4:5)=0
```



| Img2 = | | | | | | | |
|--------|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$A \ominus B$



Imerosion =

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

% structure element

```
B1=[0 1 0; 1 1 1;0 1 0]
```



| B1 = | | |
|------|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

% Image erosion using matlab command

```
Imerosion=imerode(Img2,B1)
```



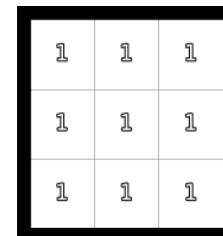
1620

Effects of Erosion

- Erosion *shrinks* foreground objects.
- Foreground holes are enlarged.
- Small (relative to the structuring element size) foreground "peaks" are removed.
- The result is dependent on the structuring element.
- Larger structuring elements means more erosion.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |

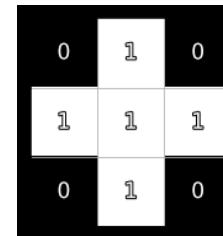
A



B1

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |

A



B2

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |

A \ominus B1

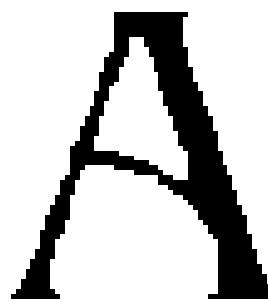
| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

A \ominus B2

Erosion Example 1



Original image



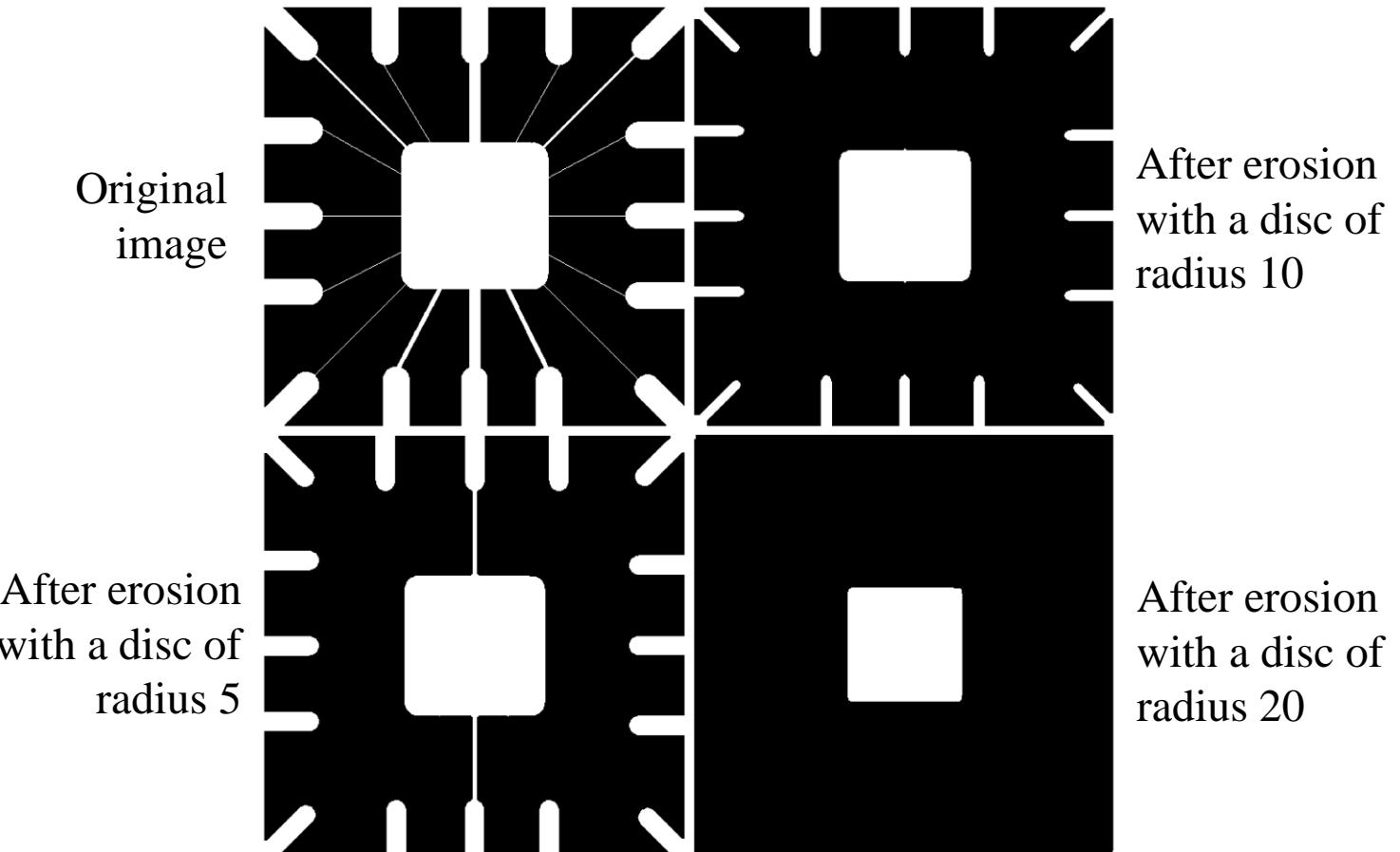
Erosion by 3*3
square structuring
element



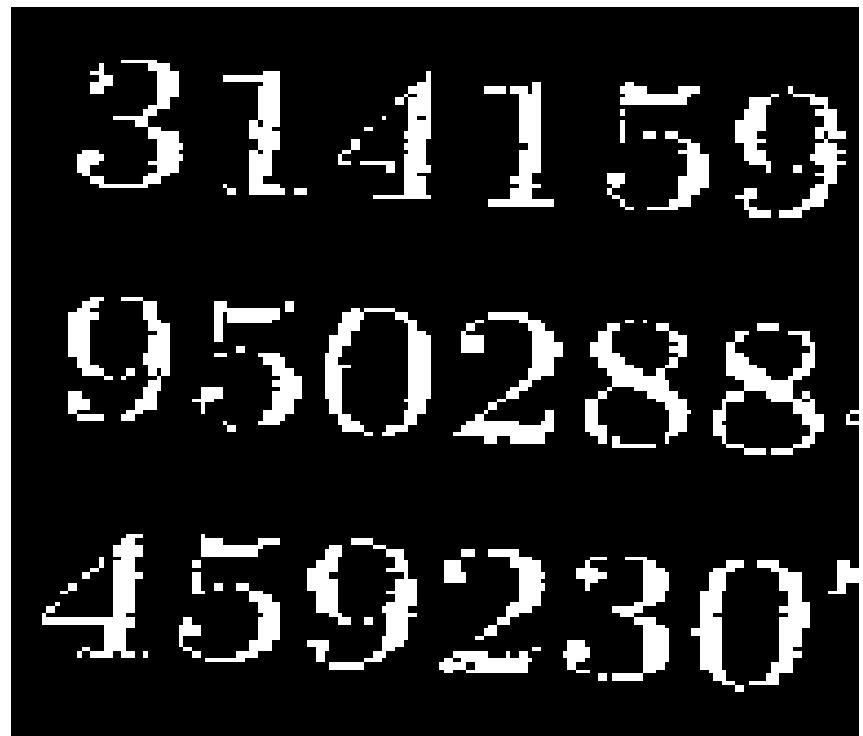
Erosion by 5*5 square
structuring element

Watch out: In these examples a 1 refers to a black pixel!

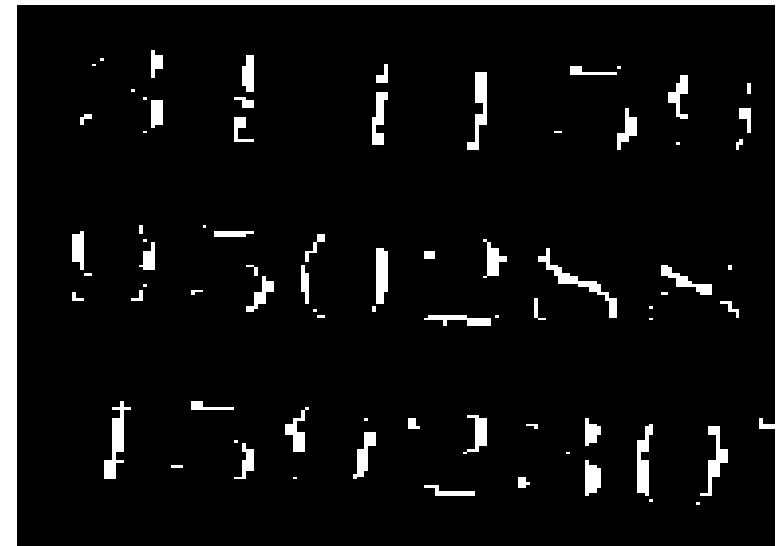
Erosion Example 2



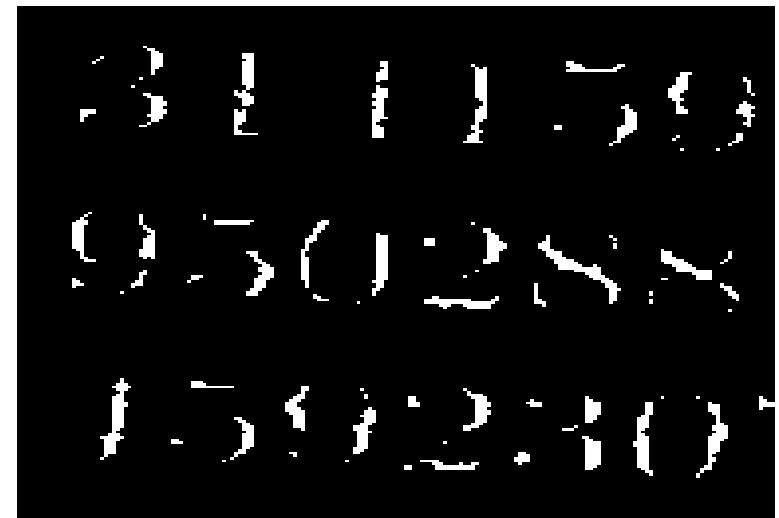
Erosion Example 3



| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

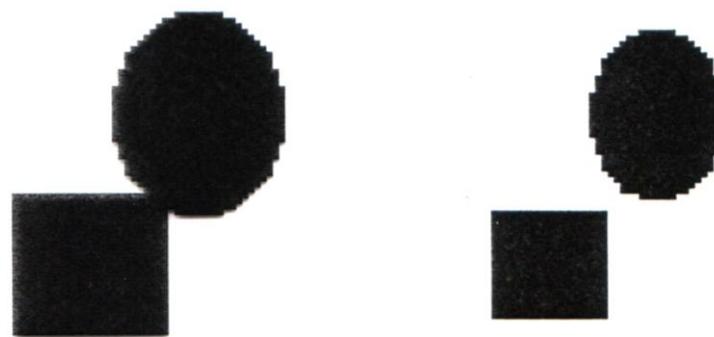


| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

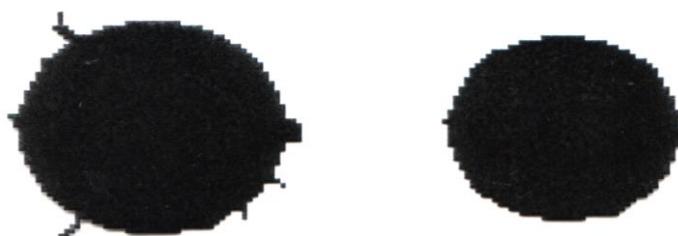


What is Erosion For?

- Erosion can split apart joined objects



- Erosion can strip away extrusions



- **Watch out:** Erosion shrinks objects

Compound Operations

- More interesting morphological operations can be performed by performing combinations of erosions and dilations
- The most widely used of these *compound operations* are:
 - Opening
 - Closing

Opening

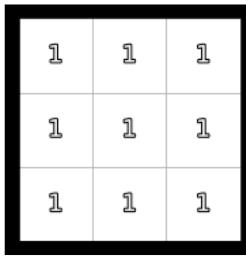
Morphological opening

- Erosion of an image removes all regions that cannot fit the structuring element, and shrinks all other regions.
- We can then dilate the result of the erosion, with this
 - The regions that where shrinked are (approximately) restored.
 - The regions too small to survive an erosion, are not restored.
- This is morphological opening
- $A \circ B = (A \ominus B) \oplus B$
- The name stems from that this operation can provide an opening (a space) between regions that are connected through thin "bridges", almost without affecting the original shape of the larger regions.
- Using erosion only will also open these bridges, but the shape of the larger regions is also altered.
- The size and shape of the structuring element is vital.

Morphological opening

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

A



B

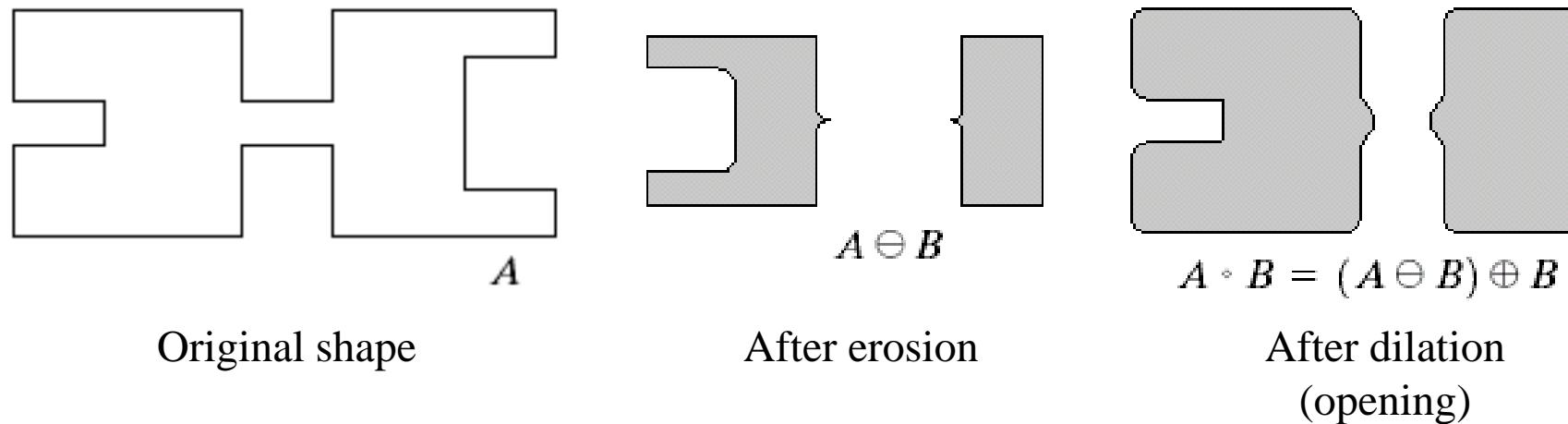
| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$$A \circ B = (A \ominus B) \oplus B$$

Morphological opening

The opening of image A by structuring element B , denoted $A \circ B$ is simply an erosion followed by a dilation

$$A \circ B = (A \ominus B) \oplus B$$



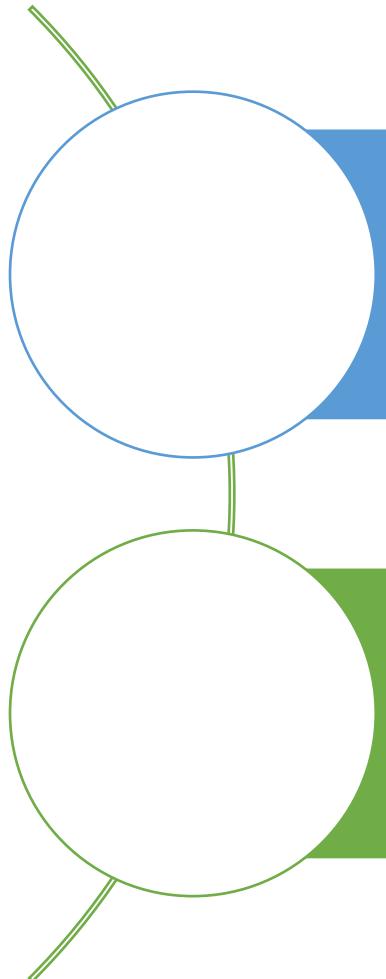
Original shape

After erosion

After dilation
(opening)

Note a disc shaped structuring element is used

Opening and Closing

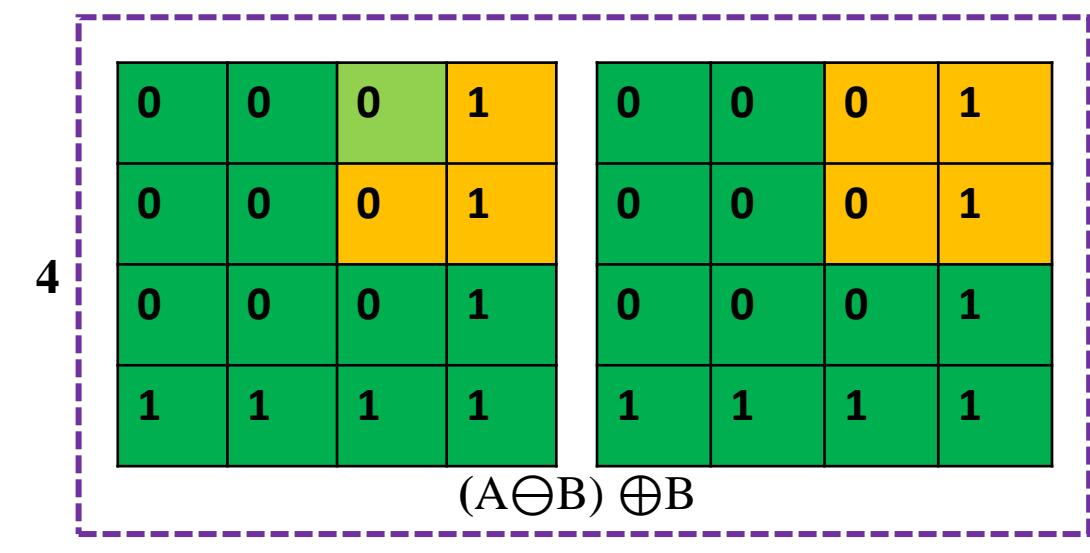
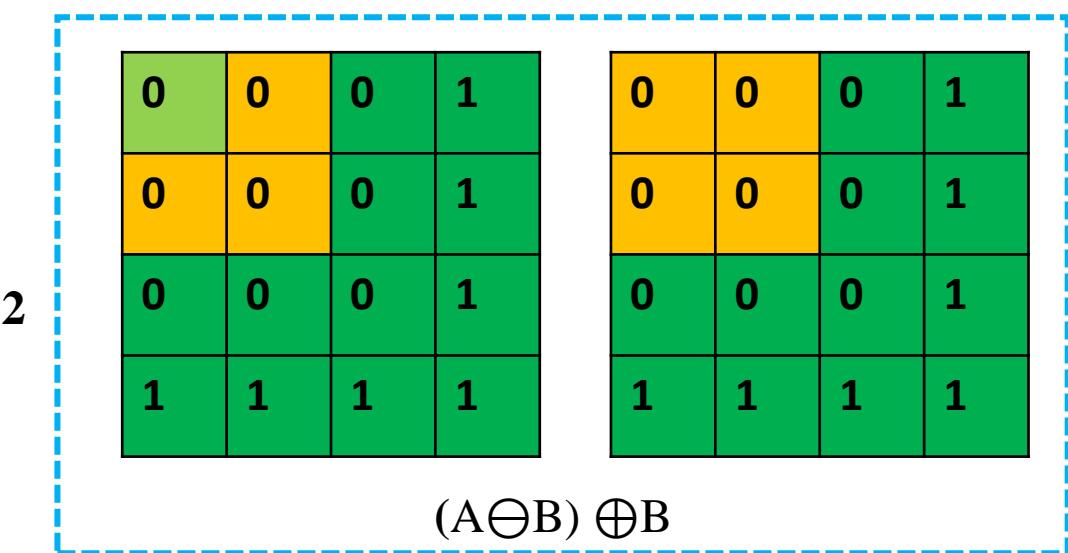
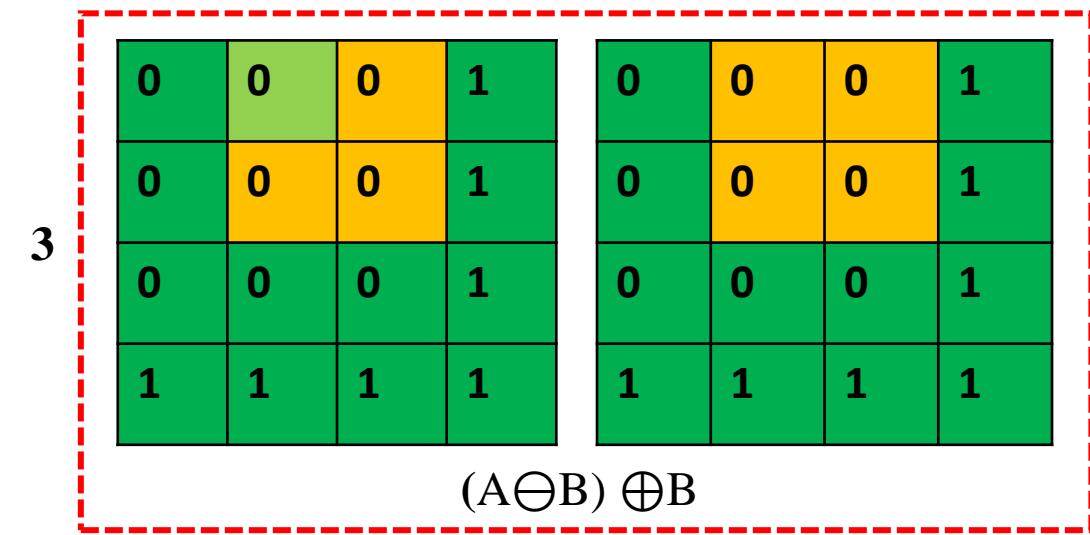
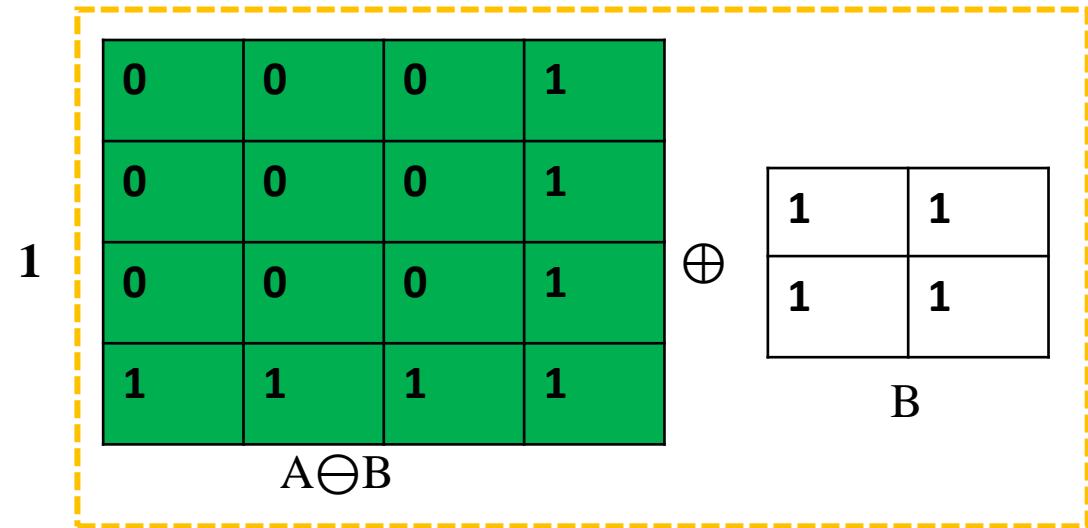


**Opening is erosion followed by
dilation=(A ⊖ B) ⊕ B**

**closing is dilation followed by
erosion=(A ⊕ B) ⊖ B**

Opening

Opening is erosion followed by dilation= $(A \ominus B) \oplus B$



Opening

Opening is erosion followed by dilation= $(A \ominus B) \oplus B$

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Opening

Opening is erosion followed by dilation= $(A \ominus B) \oplus B$

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Opening

Opening is erosion followed by dilation= $(A \ominus B) \oplus B$

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

$A \ominus B$

| | |
|---|---|
| 1 | 1 |
| 1 | 1 |

B

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

$(A \ominus B) \oplus B$

Opening verification with MATLAB code

```
% Input main image matrix  
% We have 4x4 input image matrix  
A=ones(4,4)  
A(2:3,2:3)=0
```

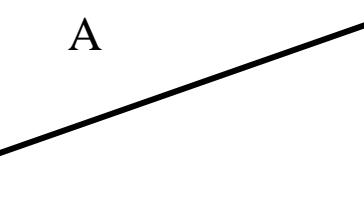
| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | |
|---|---|
| 1 | 1 |
| 1 | 1 |

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

```
% structure element, In this example is [ 1 1 1 1 ]  
B=ones(2,2)
```

% either you can use imopen function in matlab or you can
implement simple
% image open function
opening=imopen(A,B)



| | | | |
|-----------|---|---|---|
| opening = | | | |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Opening verification with MATLAB code

% Input main image matrix

% We have 4x4 input image matrix

A=ones(4,4)

A(2:3,2:3)=0

% structure element, In this example is [1 1 1 1]

B=ones(2,2)

% image opening function

openingf=imdilate(imerode(A,B),B)

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | |
|---|---|
| 1 | 1 |
| 1 | 1 |

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

A

B

$(A \ominus B) \oplus B$

openingf =

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Opening Example 1

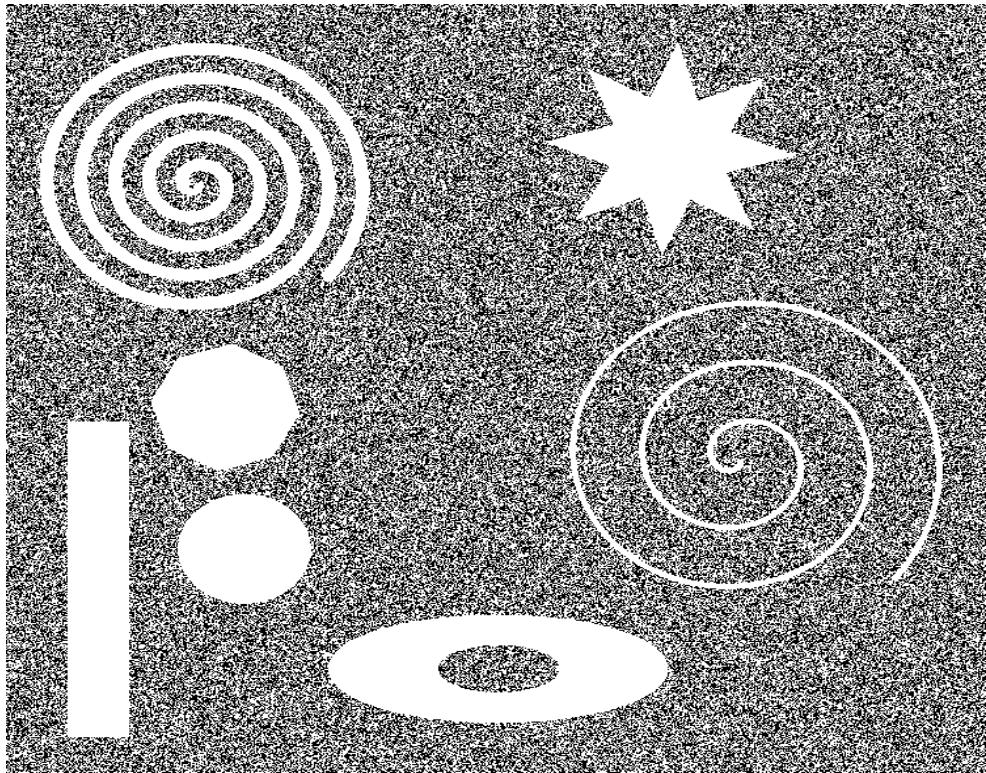
Original
Image



Image After
Opening



Noise Removal by Opening



Original Image

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

SE

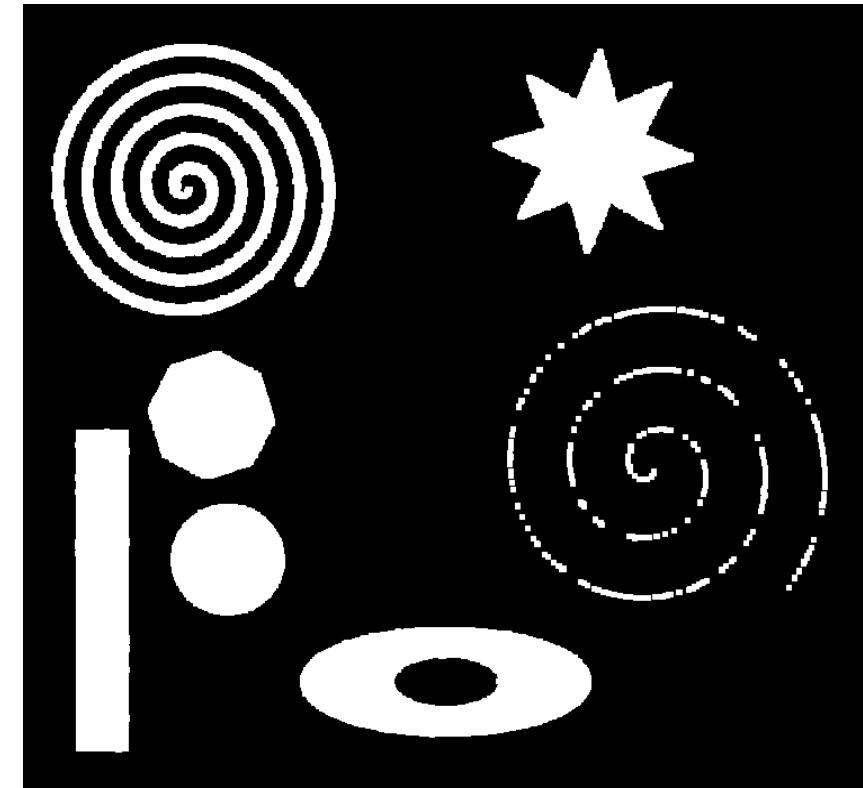


Image After Opening

Closing

Morphological Closing

- Dilation of an image expands foreground regions and fills small (relative to the structuring element) holes in the foreground.
- We can then erode the result of the dilation, then
 - The regions that were expanded are (approximately) restored.
 - The holes that were filled, are not opened again.
- This is morphological closing
- $A \bullet B = (A \oplus B) \ominus B$
- The name stems from that this operation can close small gaps between foreground regions, without altering the larger foreground shapes too much.
- Using dilation only will also close these gaps, but the shape of the larger regions is also altered.
- The size and shape of the structuring element is vital.

Morphological Closing

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

A

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

B

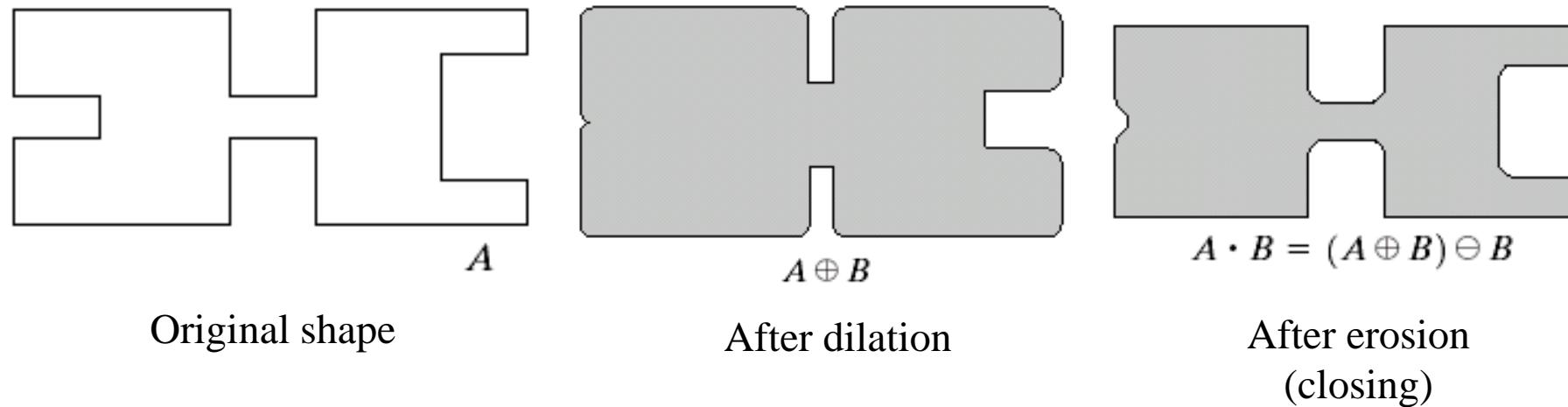
| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |

$$A \bullet B = (A \oplus B) \ominus B$$

Closing

The closing of image A by structuring element B , denoted $A \bullet B$ is simply a dilation followed by an erosion

$$A \bullet B = (A \oplus B) \ominus B$$



Note a disc shaped structuring element is used

Closing

closing is dilation followed by erosion = $(A \oplus B) \ominus B$

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

$A \oplus B$

| | |
|---|---|
| 1 | 1 |
| 1 | 1 |

B

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

$(A \oplus B) \ominus B$

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

2

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

3

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

4

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Closing

closing is dilation followed by erosion= $(A \oplus B) \ominus B$

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Pattern is not matched with SE, place 0

Closing

closing is dilation followed by erosion=(A \oplus B) \ominus B

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

9

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

11

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

12

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

10

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

12

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

10

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

12

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Pattern is not matched with SE, place 0

Closing

closing is dilation followed by erosion= $(A \oplus B) \ominus B$

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|----|--|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|--|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|--|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|--|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 13 | <table border="1"><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr></table> | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | <table border="1"><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr></table> | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | <table border="1"><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr></table> | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | <table border="1"><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr></table> | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 0 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 0 | 0 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 0 | 0 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 0 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 0 | 0 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 0 | 0 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 14 | <table border="1"><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr></table> | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | <table border="1"><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr></table> | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | <table border="1"><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr></table> | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | <table border="1"><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr></table> | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 0 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 0 | 0 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 0 | 0 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 0 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 0 | 0 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 0 | 0 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 15 | <table border="1"><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr></table> | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | <table border="1"><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr></table> | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | <table border="1"><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr></table> | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | <table border="1"><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr></table> | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 0 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 0 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 0 | 0 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 0 | 0 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 16 | <table border="1"><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr></table> | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | <table border="1"><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr></table> | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | <table border="1"><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr></table> | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | <table border="1"><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr></table> | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 0 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 0 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 0 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 0 | 0 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 0 | 0 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

Closing

Closing is dilation followed by erosion=(A \oplus B) \ominus B

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

A \oplus B

| | |
|---|---|
| 1 | 1 |
| 1 | 1 |

B

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

(A \oplus B) \ominus B

Closing verification with MATLAB code

```
% Input main image matrix  
% We have 4x4 input image matrix  
A=ones(4,4)  
A(2:3,2:3)=0
```



| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

A

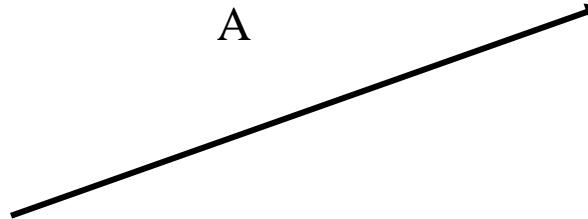
| | |
|---|---|
| 1 | 1 |
| 1 | 1 |

B

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

$(A \oplus B) \ominus B$

```
% structure element, In this example is [ 1 1 1 1 ]  
B=ones(2,2)
```



```
% image closing function in matlab  
closing=imclose(A,B)
```



| |
|-----------|
| closing = |
| 1 1 1 1 |
| 1 0 0 1 |
| 1 0 0 1 |
| 1 1 1 1 |

Closing verification with MATLAB code

```
% Input main image matrix  
% We have 4x4 input image matrix  
A=ones(4,4)  
A(2:3,2:3)=0
```



| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

A

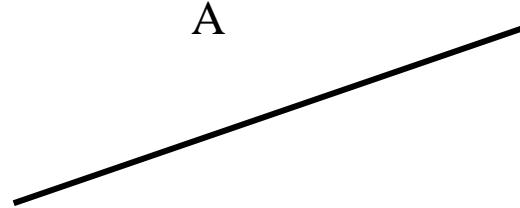
| | |
|---|---|
| 1 | 1 |
| 1 | 1 |

B

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

$(A \oplus B) \ominus B$

```
% structure element, In this example is [ 1 1 1 1 ]  
B=ones(2,2)
```



```
% image closing function in matlab  
closingf=imerode(imdilate(A,B),B)
```



closingf =

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Closing Example

Original
Image

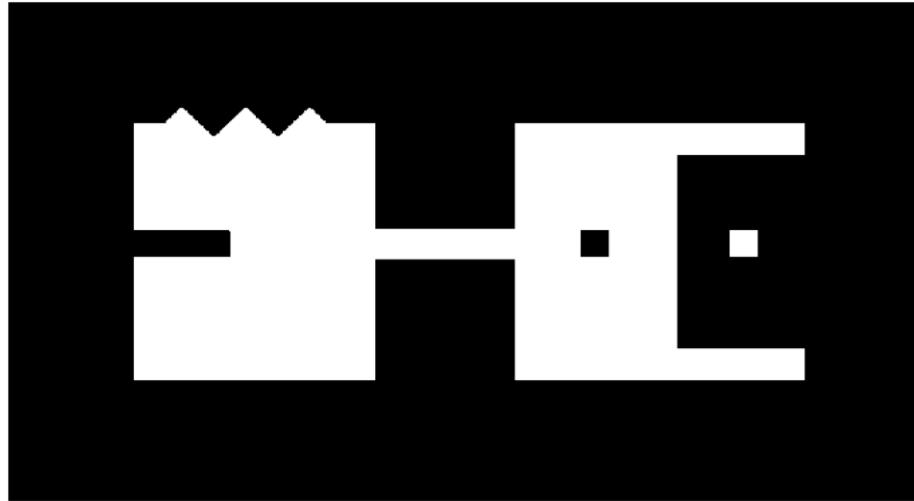


Image After
Closing



Morphological Algorithms

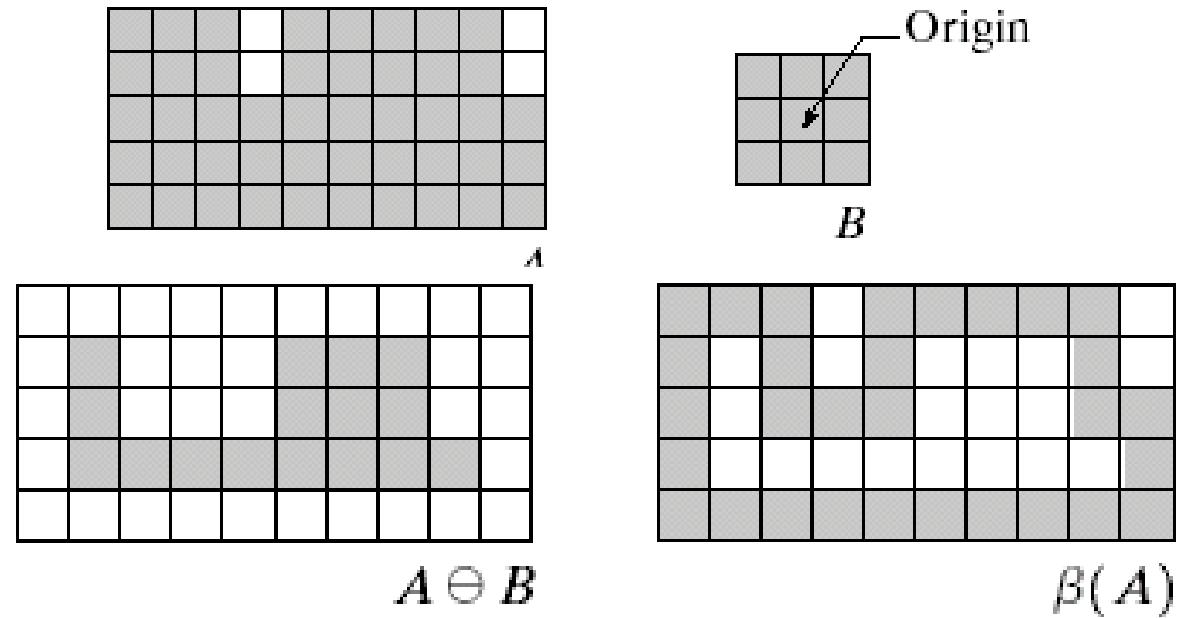
- Using the simple technique we have looked at so far we can begin to consider some more interesting morphological algorithms
- We will look at:
 - Boundary extraction
 - Region filling
 - Extraction of connected components
 - Skeletonisation

Boundary Extraction

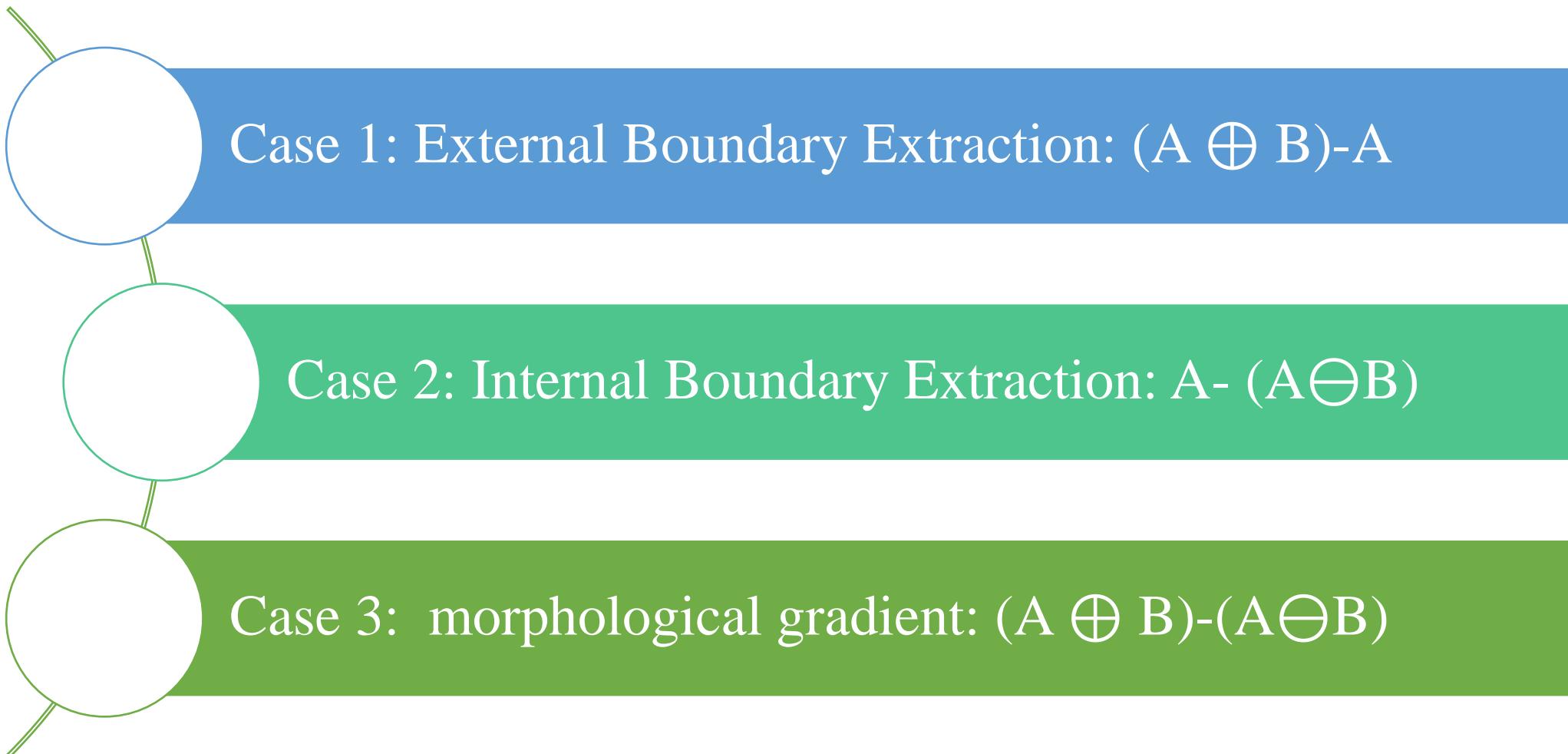
Boundary Extraction

Extracting the boundary (or outline) of an object is often extremely useful
The boundary can be given simply as

$$\beta(A) = A - (A \ominus B)$$



Boundary Extraction

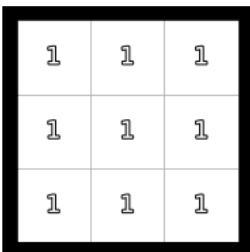


Edge detection with dilation

- Dilation adds pixels along the boundary of the foreground object.
- We can locate the edges by subtracting the original image from the dilated image $g = (A \oplus B) - A$
- The shape of the structuring element determines the connectivity of the contour.
- That is, if the contour is connected using a 4 or 8 connected neighbourhood.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

A



B1

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 |

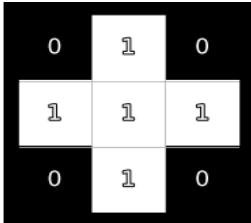
(A \oplus B1)

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |

(A \oplus B1)-A

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

A



B2

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |

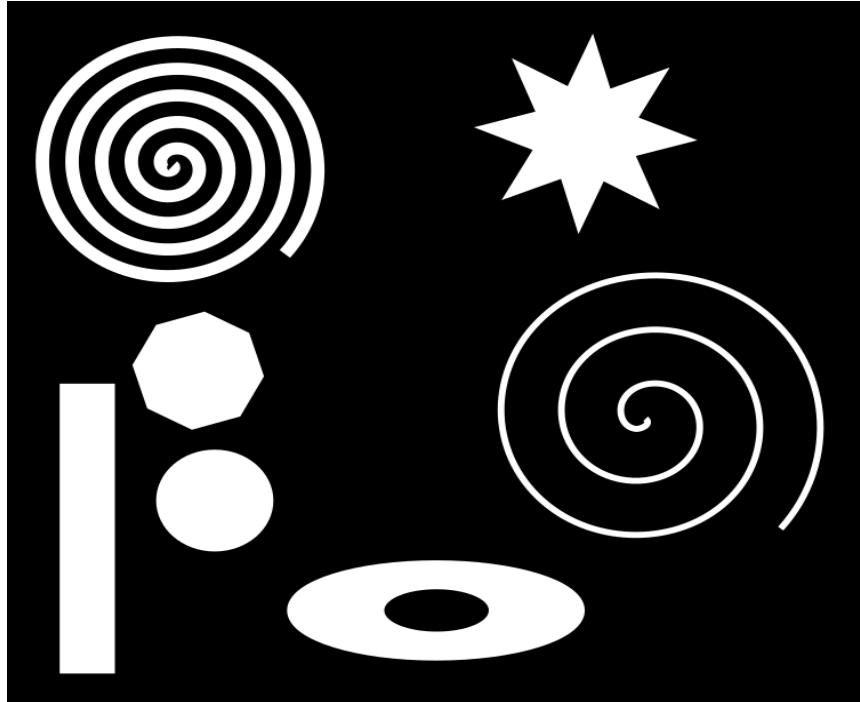
(A \oplus B2)

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |

(A \oplus B2)-A

Edge detection with dilation

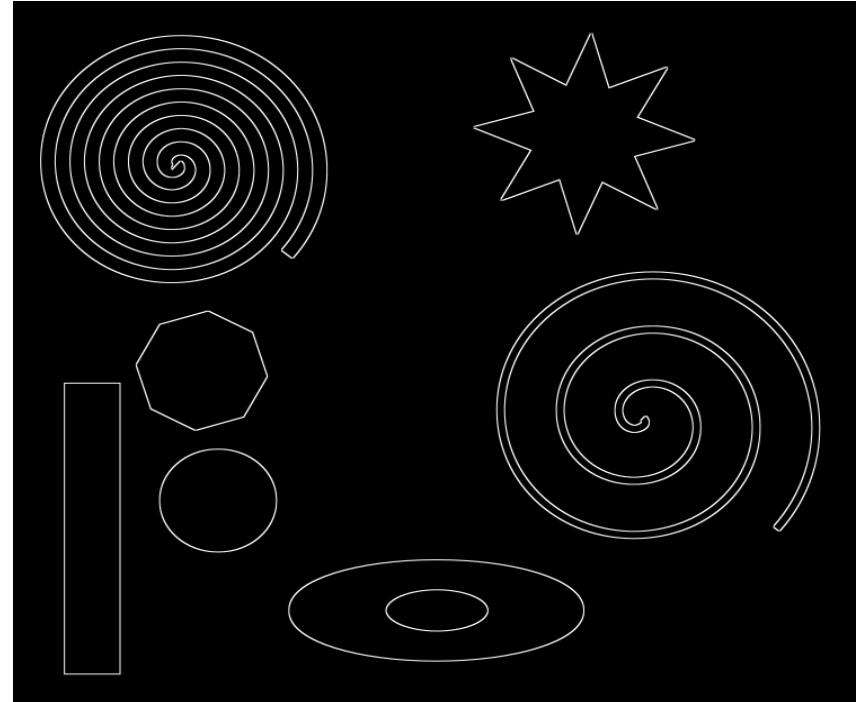
External Boundary Extraction: $(A \oplus B) - A$



A

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$

B



$(A \oplus B) - A$

Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

External Boundary Extraction : $(A \oplus B) - A$

First we need $(A \oplus B)$ and then we can subtract A(input image) from $(A \oplus B)$

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 0 | 1 | | | | |
| 0 | 1 | 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 | 0 |
| | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 | |
| 0 | 0 | 1 | 1 | 0 | 0 | |

A

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$(A \oplus B)$

Boundary Extraction

External Boundary Extraction

| | | | | | |
|---|---|---|---|---|---|
| 1 | 0 | 1 | | | |
| 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | | | | |
|---|---|---|---|---|---|
| | 1 | 0 | 1 | | |
| 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |

A

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 | |
| 0 | 0 | 1 | 1 | 0 | 0 | |

A

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | | | | |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | | | | |
|---|---|---|---|---|---|
| 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | |
| 0 | 1 | 1 | 1 | 1 | 0 | |
| 0 | 0 | 1 | 1 | 0 | 0 | |

A

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | | | | | |
|---|---|---|---|---|---|---|
| | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 |

A

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$(A \oplus B)$

Boundary Extraction

External Boundary Extraction

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 | 0 |

A

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 | 0 |

A

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|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 0 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 |

A

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| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

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|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

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|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 |

A

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 1 |

A

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| 1 | 0 | 1 |
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B

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|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 | 0 |

A

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | | | |

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| 1 | 0 | 1 |
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B

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| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

$(A \oplus B)$

Boundary Extraction

External Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

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|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | | |

A

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|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

$(A \oplus B)$

Boundary Extraction

External Boundary Extraction

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 0 |

A

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

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|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

$(A \oplus B)$

Boundary Extraction

External Boundary Extraction

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 |

A

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|---|---|---|
| 1 | 0 | 1 |
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| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

$(A \oplus B)$

Boundary Extraction

External Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

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| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 | 0 |
| | | | 1 | 0 | 1 |

A

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

$(A \oplus B)$

Boundary Extraction

External Boundary Extraction

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 1 |
| | | | | 1 | 0 |

A

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|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

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|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

$(A \oplus B)$

Boundary Extraction

External Boundary Extraction

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

(A \oplus B)

Boundary Extraction

External Boundary Extraction: $(A \oplus B) - A$

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

$(A \oplus B)$

| | | | | | |
|---|--|--|--|--|--|
| 1 | | | | | |
| 1 | | | | | |
| 0 | | | | | |
| 0 | | | | | |
| 1 | | | | | |
| 1 | | | | | |

A

$(A \oplus B) - A$

Take the difference between images: $(A \oplus B) - A$
If $1 == 1$ or $0 == 0$ place zero(0)
If $1 == 0$ or $0 == 1$ place one(1)

Boundary Extraction

External Boundary Extraction: $(A \oplus B) - A$

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$(A \oplus B)$

| | | | | | |
|---|---|--|--|--|--|
| 1 | 1 | | | | |
| 1 | 0 | | | | |
| 0 | 0 | | | | |
| 0 | 0 | | | | |
| 1 | 0 | | | | |
| 1 | 1 | | | | |

A

$(A \oplus B) - A$

Take the difference between images: $(A \oplus B) - A$
If $1==1$ or $0==0$ place zero(0)
If $1==0$ or $0==1$ place one(1)

Boundary Extraction

External Boundary Extraction: $(A \oplus B) - A$

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

$(A \oplus B)$

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

B

A

$(A \oplus B) - A$

| | | | | | |
|---|---|---|--|--|--|
| 1 | 1 | 0 | | | |
| 1 | 0 | 0 | | | |
| 0 | 0 | 0 | | | |
| 0 | 0 | 0 | | | |
| 1 | 0 | 0 | | | |
| 1 | 1 | 0 | | | |

Take the difference between images: $(A \oplus B) - A$
If $1 == 1$ or $0 == 0$ place zero(0)
If $1 == 0$ or $0 == 1$ place one(1)

Boundary Extraction

External Boundary Extraction: $(A \oplus B) - A$

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

$(A \oplus B)$

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | | | | |
|---|---|---|---|--|--|
| 1 | 1 | 0 | 0 | | |
| 1 | 0 | 0 | 0 | | |
| 0 | 0 | 0 | 0 | | |
| 0 | 0 | 0 | 0 | | |
| 1 | 0 | 0 | 0 | | |
| 1 | 1 | 0 | 0 | | |

$(A \oplus B) - A$

Take the difference between images: $(A \oplus B) - A$
If $1 == 1$ or $0 == 0$ place zero(0)
If $1 == 0$ or $0 == 1$ place one(1)

Boundary Extraction

External Boundary Extraction: $(A \oplus B) - A$

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

$(A \oplus B)$

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

| | | | | | |
|---|---|---|---|---|--|
| 1 | 1 | 0 | 0 | 1 | |
| 1 | 0 | 0 | 0 | 0 | |
| 0 | 0 | 0 | 0 | 0 | |
| 0 | 0 | 0 | 0 | 0 | |
| 1 | 0 | 0 | 0 | 0 | |
| 1 | 1 | 0 | 0 | 1 | |

A

$(A \oplus B) - A$

Take the difference between images: $(A \oplus B) - A$
If $1 == 1$ or $0 == 0$ place zero(0)
If $1 == 0$ or $0 == 1$ place one(1)

Boundary Extraction

External Boundary Extraction: $(A \oplus B) - A$

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

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|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$(A \oplus B)$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 |

A

$(A \oplus B) - A$

Take the difference between images: $(A \oplus B) - A$
If $1==1$ or $0==0$ place zero(0)
If $1==0$ or $0==1$ place one(1)

Boundary Extraction

External Boundary Extraction: $(A \oplus B) - A$

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

-

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$(A \oplus B)$

B

A

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|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 |

$(A \oplus B) - A$

Boundary Extraction (MATLAB)

External Boundary Extraction: $(A \oplus B) - A$

```
% Construct the input image for  
boundary extraction
```

```
Imb=zeros(6,6)
```

```
Imb(1,3:4)=1
```

```
Imb(2,2:5)=1
```

```
Imb(3:4,:)=1
```

```
Imb(5,2:5)=1
```

```
Imb(6,3:4)=1
```

```
Imb =
```

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |

```
% structure element(SE) for boundary extraction
```

```
SE=zeros(3,3)
```

```
SE(1,1)=1
```

```
SE(1,3)=1
```

```
SE(2,2)=1
```

```
SE(3,1)=1
```

```
SE(3,3)=1
```

```
SE =
```

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

```
% First image dilation
```

```
Imdilte=imdilate(Imb,SE)
```

```
Imdilte =
```

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

```
% Subtract input image from image dilation
```

```
BoundaryExternal=Imdilte-Imb
```

```
BoundaryExternal =
```

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 |

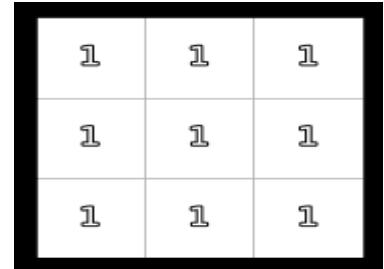
| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 |

Edge detection with erosion

- Erosion removes pixels along the boundary of the foreground object.
- We can locate the edges by subtracting the eroded image from the original $f = A - (A \ominus B)$
- The shape of the structuring element determines the connectivity of the contour.
- That is, if the contour is connected using a 4 or 8 connected neighbourhood.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| o | o | 1 | 1 | 1 | 1 | 1 | o | 1 | 1 | 1 | 1 | o |
| o | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | o |
| o | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | o |
| 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| o | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| o | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | o |
| o | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | o |
| o | o | o | o | o | 1 | 1 | 1 | o | o | o | o | o |

A



B1

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| o | o | o | o | o | o | o | o | o | o | o | o |
| o | o | o | 1 | 1 | o | o | o | 1 | o | o | o |
| o | o | 1 | o | o | o | 1 | 1 | 1 | o | o | o |
| o | o | 1 | o | o | o | 1 | 1 | 1 | o | o | o |
| o | o | 1 | o | o | o | 1 | 1 | 1 | o | o | o |
| o | o | 1 | 1 | 1 | 1 | 1 | 1 | 1 | o | o | o |
| o | o | 1 | 1 | 1 | 1 | 1 | 1 | 1 | o | o | o |
| o | o | o | o | o | o | 1 | o | o | o | o | o |
| o | o | o | o | o | o | o | o | o | o | o | o |

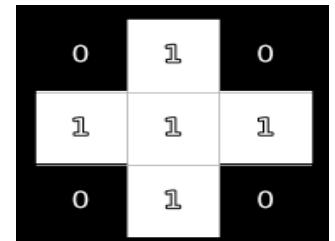
A \ominus B1

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| o | o | 1 | 1 | 1 | 1 | 1 | o | 1 | 1 | 1 | 1 | o |
| o | 1 | 1 | 1 | o | o | o | 1 | 1 | 1 | o | 1 | o |
| o | 1 | o | 1 | 1 | 1 | 1 | o | 1 | 1 | 1 | o | 1 |
| 1 | 1 | o | 1 | 0 | 1 | 1 | 1 | o | 1 | 1 | 1 | 1 |
| o | 1 | o | 1 | 1 | 1 | 1 | 1 | o | 1 | 1 | 1 | 1 |
| o | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | o | 1 | 1 | 1 |
| o | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | o | 1 | 1 | 1 |
| o | o | o | o | o | 1 | 1 | 1 | o | o | o | 1 | o |
| o | o | o | o | o | o | o | o | o | o | o | o | o |

A- (A \ominus B1)

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| o | o | 1 | 1 | 1 | 1 | 1 | o | 1 | 1 | 1 | 1 | o |
| o | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | o |
| o | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | o |
| 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| o | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| o | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | o |
| o | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | o |
| o | o | o | o | o | 1 | 1 | 1 | 1 | o | o | o | o |
| o | o | o | o | o | o | o | o | o | o | o | o | o |

A



B2

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| o | o | o | o | o | o | o | o | o | o | o | o |
| o | o | 1 | 1 | 1 | 1 | o | 1 | 1 | 1 | o | o |
| o | o | 1 | 1 | o | 1 | 1 | 1 | 1 | 1 | o | o |
| o | 1 | 1 | 1 | 0 | o | 1 | 1 | 1 | 1 | 1 | o |
| o | o | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| o | o | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| o | o | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| o | o | o | o | o | 1 | 1 | 1 | 1 | o | o | o |
| o | o | o | o | o | o | o | o | o | o | o | o |

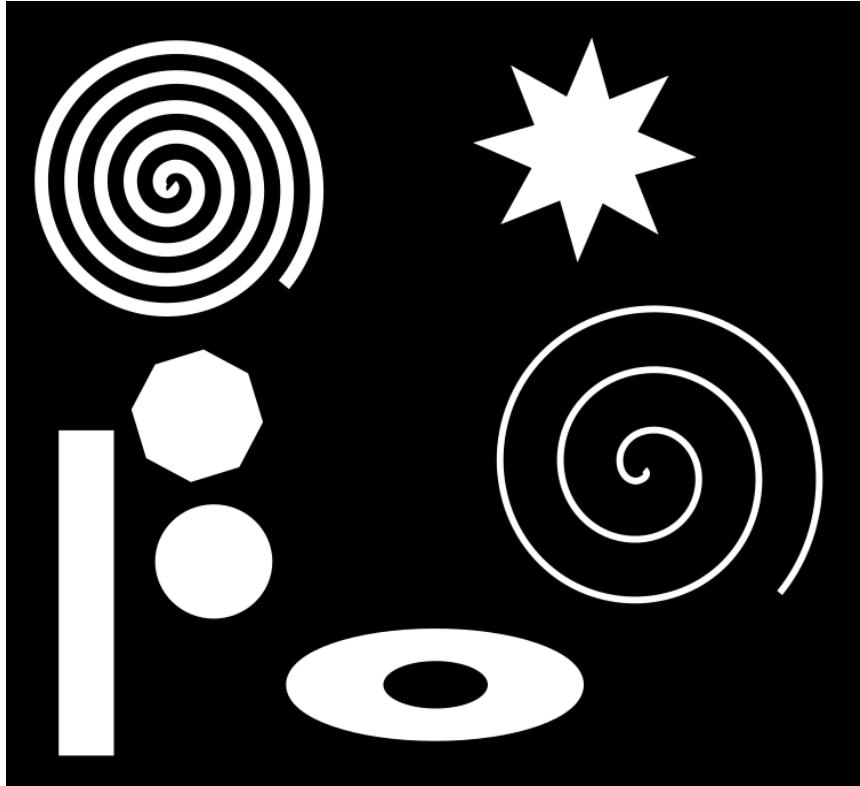
A \ominus B2

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| o | o | 1 | 1 | 1 | 1 | 1 | o | 1 | 1 | 1 | 1 | o |
| o | 1 | 0 | 0 | 0 | 0 | o | 1 | 0 | 0 | 1 | 0 | o |
| o | 1 | 0 | 0 | 0 | 1 | o | 1 | 0 | 0 | 0 | 1 | o |
| 1 | 0 | 0 | 1 | 0 | 1 | o | 1 | 0 | 0 | 0 | 0 | 1 |
| o | 1 | 0 | 0 | 1 | 1 | o | 1 | 0 | 0 | 0 | 0 | 1 |
| o | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| o | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | o |
| o | o | o | o | o | 1 | 1 | 1 | 1 | o | o | o | o |
| o | o | o | o | o | o | o | o | o | o | o | o | o |

A- (A \ominus B2)

Edge detection with erosion

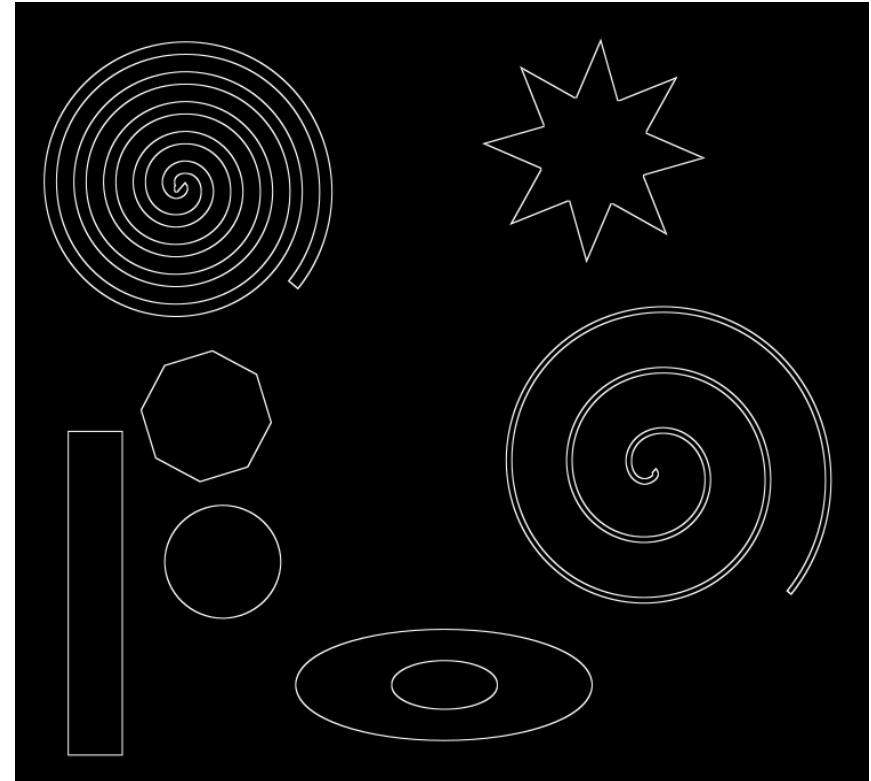
External Boundary Extraction: $(A \oplus B) - A$



A

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$

B



$A - (A \oplus B)$

Boundary Extraction

Internal Boundary Extraction : $A - A \ominus B$

First we need $A \ominus B$ and then we can subtract $A \ominus B$

A (input image) from A

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 0 | 1 | | | | |
| 0 | 1 | 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 | 0 |
| | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 | |
| 0 | 0 | 1 | 1 | 0 | 0 | |

A

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$A \ominus B$

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

Boundary Extraction

Internal Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | | |
|---|---|---|---|---|---|--|
| 1 | 0 | 1 | | | | |
| 0 | 1 | 0 | 1 | 0 | 0 | |
| 1 | 0 | 1 | 1 | 1 | 0 | |
| 1 | 1 | 1 | 1 | 1 | 1 | |
| 1 | 1 | 1 | 1 | 1 | 1 | |
| 0 | 1 | 1 | 1 | 1 | 0 | |
| 0 | 0 | 1 | 1 | 0 | 0 | |

A

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$A \ominus B$

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

Boundary Extraction

Internal Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

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|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$A \ominus B$

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

Boundary Extraction

Internal Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

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|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$A \ominus B$

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

Boundary Extraction

Internal Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

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|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$A \ominus B$

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|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |

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| 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

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|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

Boundary Extraction

Internal Boundary Extraction

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|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

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|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 |

A

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|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$A \ominus B$

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

Boundary Extraction

Internal Boundary Extraction

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| 1 | 0 | 1 |
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| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$A \ominus B$

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|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 | |
| 0 | 0 | 1 | 1 | 0 | 0 | |

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|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

Boundary Extraction

Internal Boundary Extraction

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| 1 | 0 | 1 |
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| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$A \ominus B$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

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| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

Boundary Extraction

Internal Boundary Extraction

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| 1 | 0 | 1 |
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B

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| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$A \ominus B$

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| 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

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| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

Boundary Extraction

Internal Boundary Extraction

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| 1 | 0 | 1 |
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| 1 | 0 | 1 |

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| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$A \ominus B$

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| 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

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| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

Boundary Extraction

Internal Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

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|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$A \ominus B$

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

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|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

Boundary Extraction

Internal Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

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|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$A \ominus B$

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | |
| 0 | 1 | 1 | 1 | 1 | 0 | |
| 0 | 0 | 1 | 1 | 0 | 0 | |

A

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

Boundary Extraction

Internal Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

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|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$A \ominus B$

| | | | | | | |
|---|---|---|---|---|---|---|
| | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 | |
| 0 | 0 | 1 | 1 | 0 | 0 | |

A

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

Boundary Extraction

Internal Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$A \ominus B$

Boundary Extraction

Internal Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
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| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$A \ominus B$

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 1 |
| | | | 1 | 0 | 1 |

A

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

Boundary Extraction

Internal Boundary Extraction

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$A \ominus B$

Boundary Extraction

Internal Boundary Extraction: $A - (A \ominus B)$

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$A \ominus B$

$A - (A \ominus B)$ Take the difference between images: $A - (A \ominus B)$
If $1 == 1$ or $0 == 0$ place zero(0)
If $1 == 0$ or $0 == 1$ place one(1)

Boundary Extraction

Internal Boundary Extraction: $A - (A \ominus B)$

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$A \ominus B$

$A - (A \ominus B)$

Take the difference between images: $A - (A \ominus B)$
If $1 == 1$ or $0 == 0$ place zero(0)
If $1 == 0$ or $0 == 1$ place one(1)

Boundary Extraction

Internal Boundary Extraction: $A - (A \ominus B)$

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 |

$A \ominus B$

$A - (A \ominus B)$

Take the difference between images: $A - (A \ominus B)$
If $1 == 1$ or $0 == 0$ place zero(0)
If $1 == 0$ or $0 == 1$ place one(1)

Boundary Extraction

Internal Boundary Extraction: $A - (A \ominus B)$

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |

$A \ominus B$

$A - (A \ominus B)$

Take the difference between images: $A - (A \ominus B)$
If $1 == 1$ or $0 == 0$ place zero(0)
If $1 == 0$ or $0 == 1$ place one(1)

Boundary Extraction

Internal Boundary Extraction: $A - (A \ominus B)$

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

B

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |

$A \ominus B$

$A - (A \ominus B)$

Take the difference between images: $A - (A \ominus B)$
If $1 == 1$ or $0 == 0$ place zero(0)
If $1 == 0$ or $0 == 1$ place one(1)

Boundary Extraction

Internal Boundary Extraction: $A - (A \ominus B)$

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

A

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |

$A \ominus B$

$A - (A \ominus B)$

Take the difference between images: $A - (A \ominus B)$
If $1 == 1$ or $0 == 0$ place zero(0)
If $1 == 0$ or $0 == 1$ place one(1)

Boundary Extraction (MATLAB)

Internal Boundary Extraction: $A - (A \ominus B)$

% Construct the input image for boundary extraction

```
Imb=zeros(6,6)
```

```
Imb(1,3:4)=1
```

```
Imb(2,2:5)=1
```

```
Imb(3:4,:)=1
```

```
Imb(5,2:5)=1
```

```
Imb(6,3:4)=1
```

Imb =

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |

% Image erosion matlab function

```
Imderode=imerode(Imb,SE) —————→
```

Imderode =

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

% structure element(SE) for boundary extraction

```
SE=zeros(3,3)
```

```
SE(1,1)=1
```

```
SE(1,3)=1
```

```
SE(2,2)=1
```

```
SE(3,1)=1
```

```
SE(3,3)=1
```

SE =

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |

BoundaryExternal =

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |

Boundary Extraction

Morphological gradient: $(A \oplus B) - (A \ominus B)$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

$(A \oplus B)$

| | | | | | |
|---|--|--|--|--|--|
| 1 | | | | | |
| 1 | | | | | |
| 0 | | | | | |
| 0 | | | | | |
| 1 | | | | | |
| 1 | | | | | |

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$(A \ominus B)$

$(A \oplus B) - (A \ominus B)$

Take the difference between images: $(A \oplus B) - (A \ominus B)$

If $1 == 1$ or $0 == 0$ place zero(0)

If $1 == 0$ or $0 == 1$ place one(1)

Boundary Extraction

Morphological gradient: $(A \oplus B) - (A \ominus B)$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$(A \oplus B)$

| | | | | | |
|---|---|--|--|--|--|
| 1 | 1 | | | | |
| 1 | 1 | | | | |
| 0 | 1 | | | | |
| 0 | 1 | | | | |
| 1 | 1 | | | | |
| 1 | 1 | | | | |

$(A \ominus B)$

$(A \oplus B) - (A \ominus B)$

Take the difference between images: $(A \oplus B) - (A \ominus B)$

If $1 == 1$ or $0 == 0$ place zero(0)

If $1 == 0$ or $0 == 1$ place one(1)

Boundary Extraction

Morphological gradient: $(A \oplus B) - (A \ominus B)$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$(A \oplus B)$

| | | | | | |
|---|---|---|--|--|--|
| 1 | 1 | 0 | | | |
| 1 | 1 | 1 | | | |
| 0 | 1 | 0 | | | |
| 0 | 1 | 0 | | | |
| 1 | 1 | 1 | | | |
| 1 | 1 | 0 | | | |

$(A \ominus B)$

$(A \oplus B) - (A \ominus B)$

Take the difference between images: $(A \oplus B) - (A \ominus B)$

If $1 == 1$ or $0 == 0$ place zero(0)

If $1 == 0$ or $0 == 1$ place one(1)

Boundary Extraction

Morphological gradient: $(A \oplus B) - (A \ominus B)$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$(A \oplus B)$

| | | | | | |
|---|---|---|---|--|--|
| 1 | 1 | 0 | 0 | | |
| 1 | 1 | 1 | 1 | | |
| 0 | 1 | 0 | 0 | | |
| 0 | 1 | 0 | 0 | | |
| 1 | 1 | 1 | 1 | | |
| 1 | 1 | 0 | 0 | | |

$(A \ominus B)$

$(A \oplus B) - (A \ominus B)$

Take the difference between images: $(A \oplus B) - (A \ominus B)$

If $1 == 1$ or $0 == 0$ place zero(0)

If $1 == 0$ or $0 == 1$ place one(1)

Boundary Extraction

Morphological gradient: $(A \oplus B) - (A \ominus B)$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$(A \oplus B)$

| | | | | | |
|---|---|---|---|---|--|
| 1 | 1 | 0 | 0 | 1 | |
| 1 | 1 | 1 | 1 | 1 | |
| 0 | 1 | 0 | 0 | 1 | |
| 0 | 1 | 0 | 0 | 1 | |
| 1 | 1 | 1 | 1 | 1 | |
| 1 | 1 | 0 | 0 | 1 | |

$(A \ominus B)$

$(A \oplus B) - (A \ominus B)$

Take the difference between images: $(A \oplus B) - (A \ominus B)$

If $1 == 1$ or $0 == 0$ place zero(0)

If $1 == 0$ or $0 == 1$ place one(1)

Boundary Extraction

Morphological gradient: $(A \oplus B) - (A \ominus B)$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$(A \oplus B)$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 |

$(A \ominus B)$

$(A \oplus B) - (A \ominus B)$

Take the difference between images: $(A \oplus B) - (A \ominus B)$

If $1 == 1$ or $0 == 0$ place zero(0)

If $1 == 0$ or $0 == 1$ place one(1)

Boundary Extraction (MATLAB)

Morphological gradient: $(A \oplus B) - (A \ominus B)$

% Construct the input image
for boundary extraction

Imb=zeros(6,6)

Imb(1,3:4)=1

Imb(2,2:5)=1

Imb(3:4,:)=1

Imb(5,2:5)=1

Imb(6,3:4)=1

Imb =

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |

% First image dilation

Imdilte=imdilate(Imb,SE) →

Imdilte =

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

% Image erosion matlab function

Imderode=imerode(Imb,SE) →

Imderode =

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 |

% structure element(SE) for boundary extraction

SE=zeros(3,3)

SE(1,1)=1

SE(1,3)=1

SE(2,2)=1

SE(3,1)=1

SE(3,3)=1

SE =

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |

% subtract image erosion from dilation

morphologicalgradient=Imdilte-Imderode

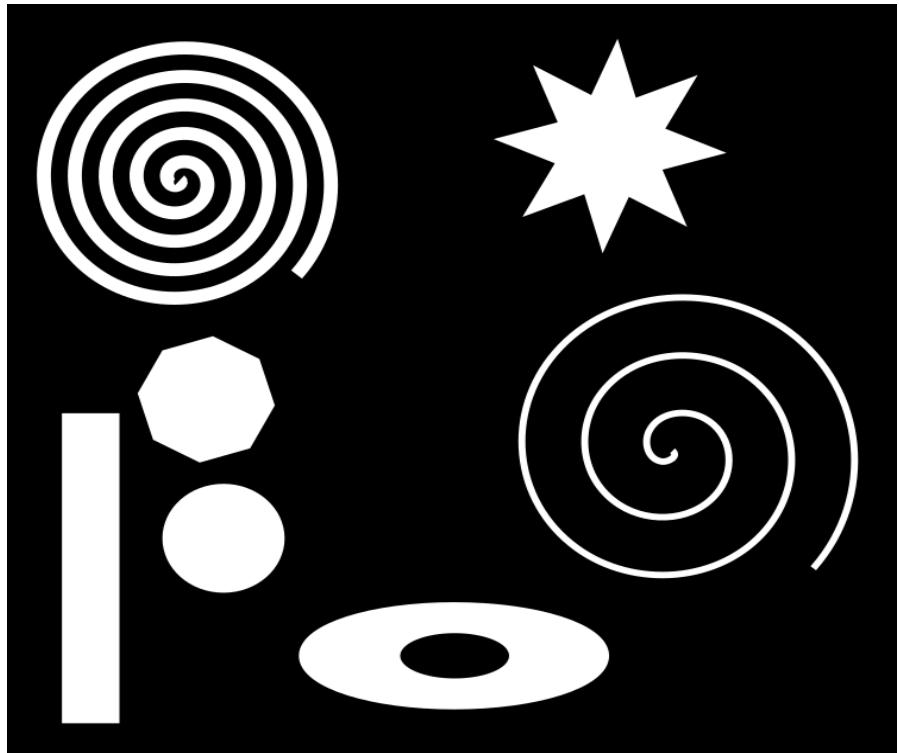
| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 |

morphologicalgradient =

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 |

Edge detection with Morphological gradient

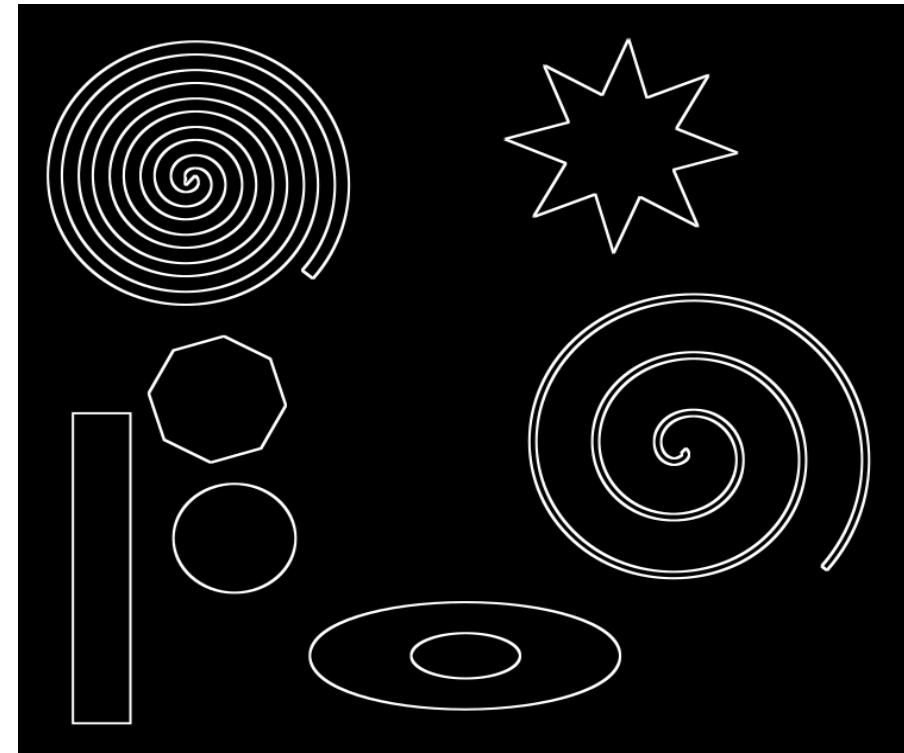
Morphological gradient : $(A \oplus B) - (A \ominus B)$



A

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$

B



$A - (A \ominus B)$

Boundary Extraction Example

A simple image and the result of performing boundary extraction using a square 3*3 structuring element

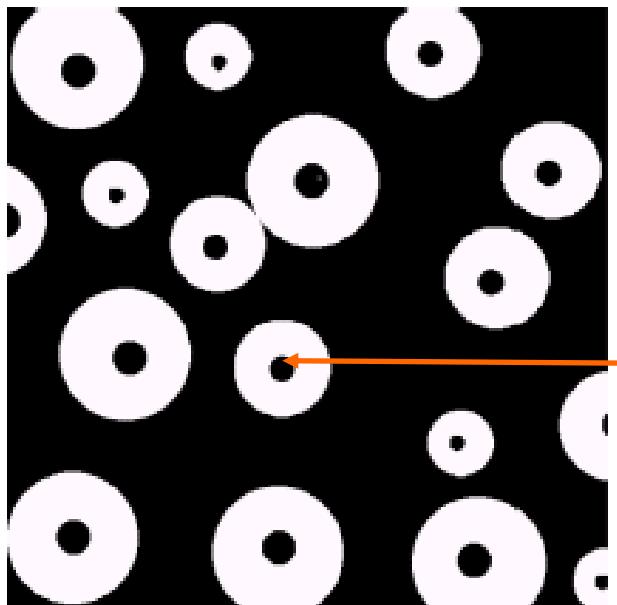


Region Filling

Region Filling

Given a pixel inside a boundary, *region filling* attempts to fill that boundary with object pixels (1s)

- Region filling is a simple application of dilation.
- We can use it to extract connected foreground components.
- Conversely, it can be used to extract holes in foreground regions. This can then be used to fill said holes.



Given a point inside here, can we fill the whole circle?

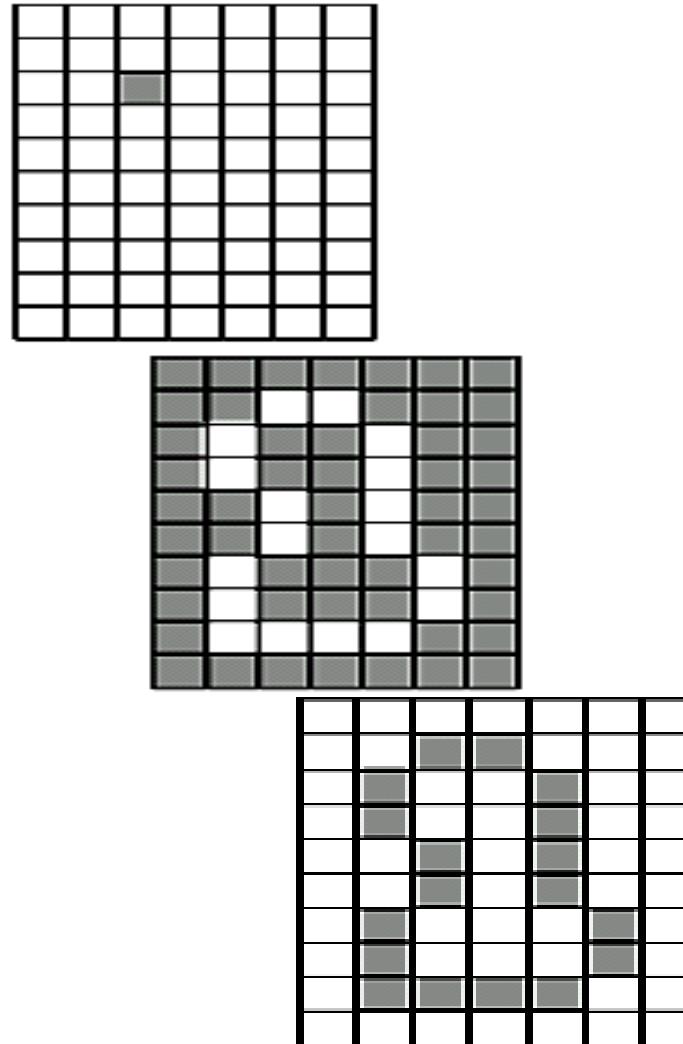
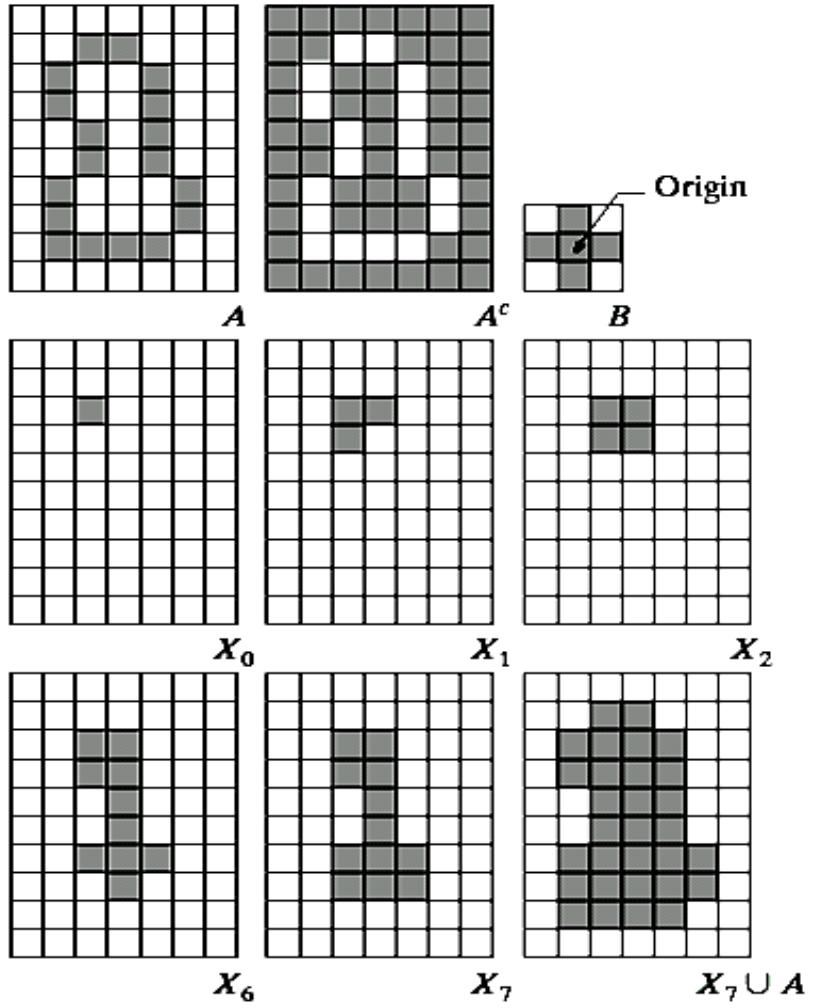
Region Filling

- The key equation for region filling is

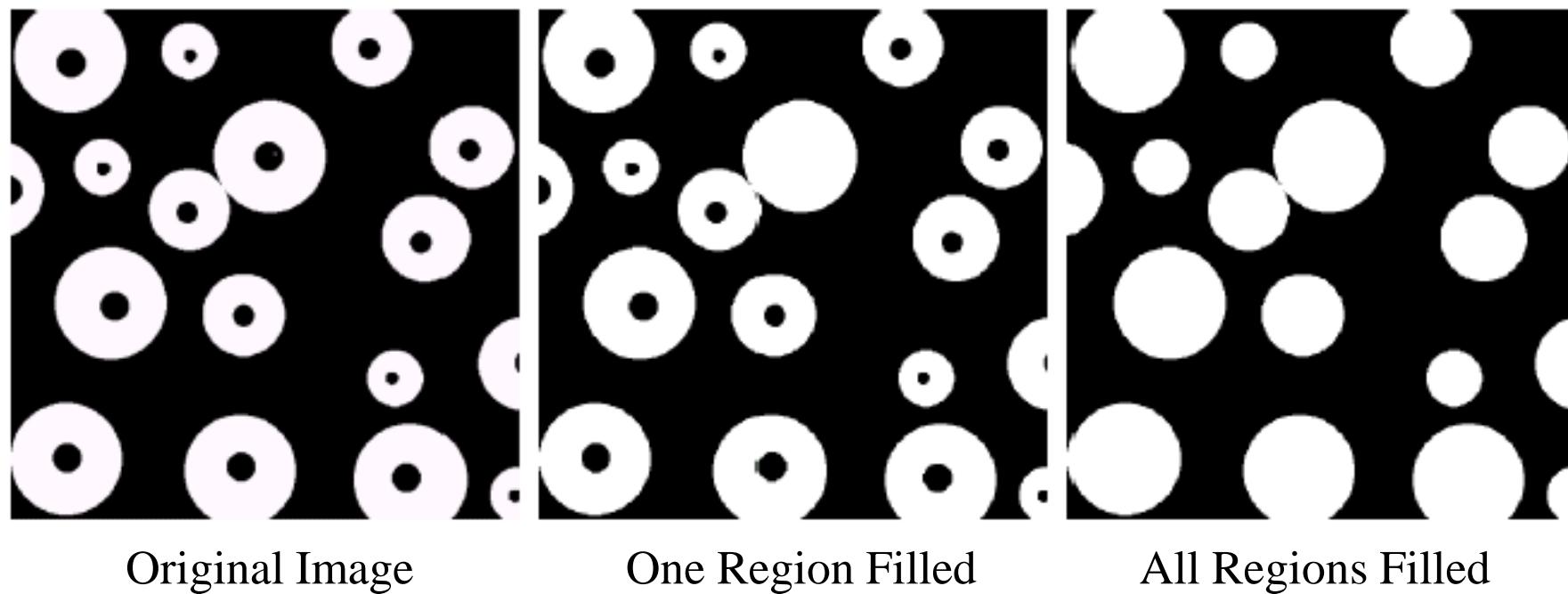
$$X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1, 2, 3, \dots$$

- Where X_0 is simply the starting point inside the boundary, B is a simple structuring element and A^c is the complement of A
- This equation is applied repeatedly until X_k is equal to X_{k-1}
- Finally the result is unioned with the original boundary

Region Filling Step By Step



Region Filling Example 1

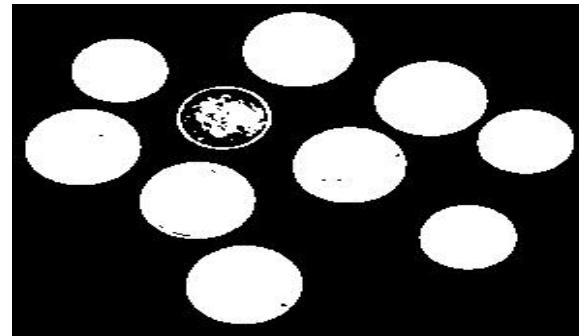


Region Filling Example 2

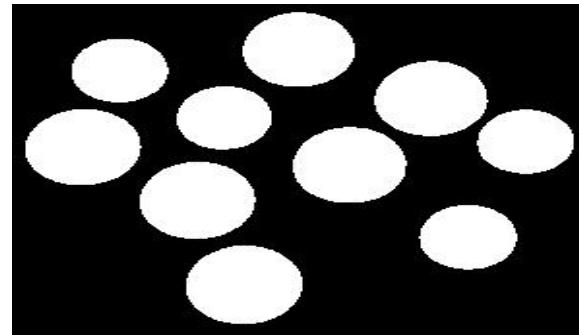
```
%% Region filling  
%reading into workspace  
I = imread('coins.png');  
figure  
imshow(I)  
title('Original Image')
```



```
%Convert image to binary image  
BW = imbinarize(I);  
figure  
imshow(BW)  
title('Original Image Converted to Binary Image')
```



```
%Fill holes in the binary image and display the result.  
BW2 = imfill(BW,'holes');  
figure  
imshow(BW2)  
title('Filled Image')
```

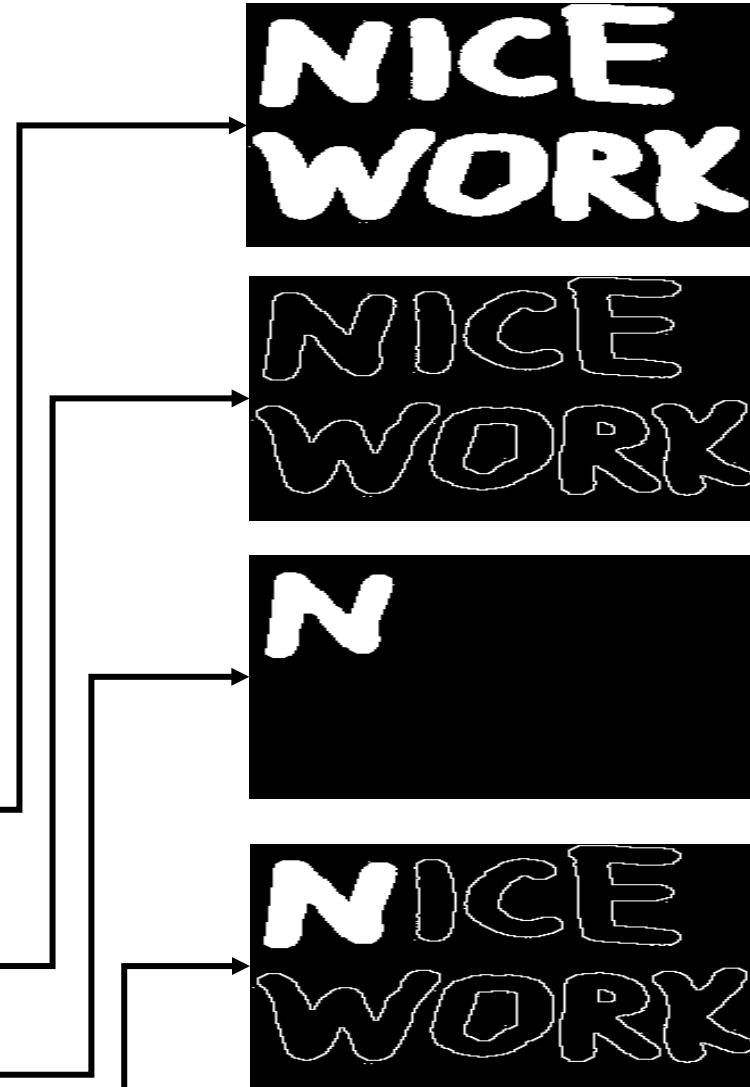


Region Filling MATLAB function

```
function out=regfill(im,pos,kernel)
% REGFILL(IM,POS,KERNEL) performs region filling of binary image IMAGE,
% with kernel KERNEL, starting at point with coordinates given by POS.
%
% Example:
% n=imread('nicework.tif');
% nb=n&~imerode(n,ones(3,3));
% nr=regfill(nb,[74,52],ones(3,3));
%
current=zeros(size(im));
last=zeros(size(im));
last(pos(1),pos(2))=1;
current=imdilate(last,kernel)&~im;
while any(current(:)~=last(:)),
last=current;
current=imdilate(last,kernel)&~im;
end;
out=current;

%% Region filling
%test example
n=imread('nice.jpeg');
% convert into binary
n1= im2bw(n, 0.3); ——————
% structure element
sq=ones(3,3) ——————
%Extract boundary using erosion or dilation
nb=n1-imerode(n1,sq); ——————
ndi=imdilate(n1,sq)-n1;
%Image filling algorithm to fill the pattern at certain position
nf=regfill(nb,[74,52],sq); ——————
ndi=regfill(ndi,[74,52],sq);
add=nf|nb ——————
```

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$



Connected Component

Extracting connected foreground components

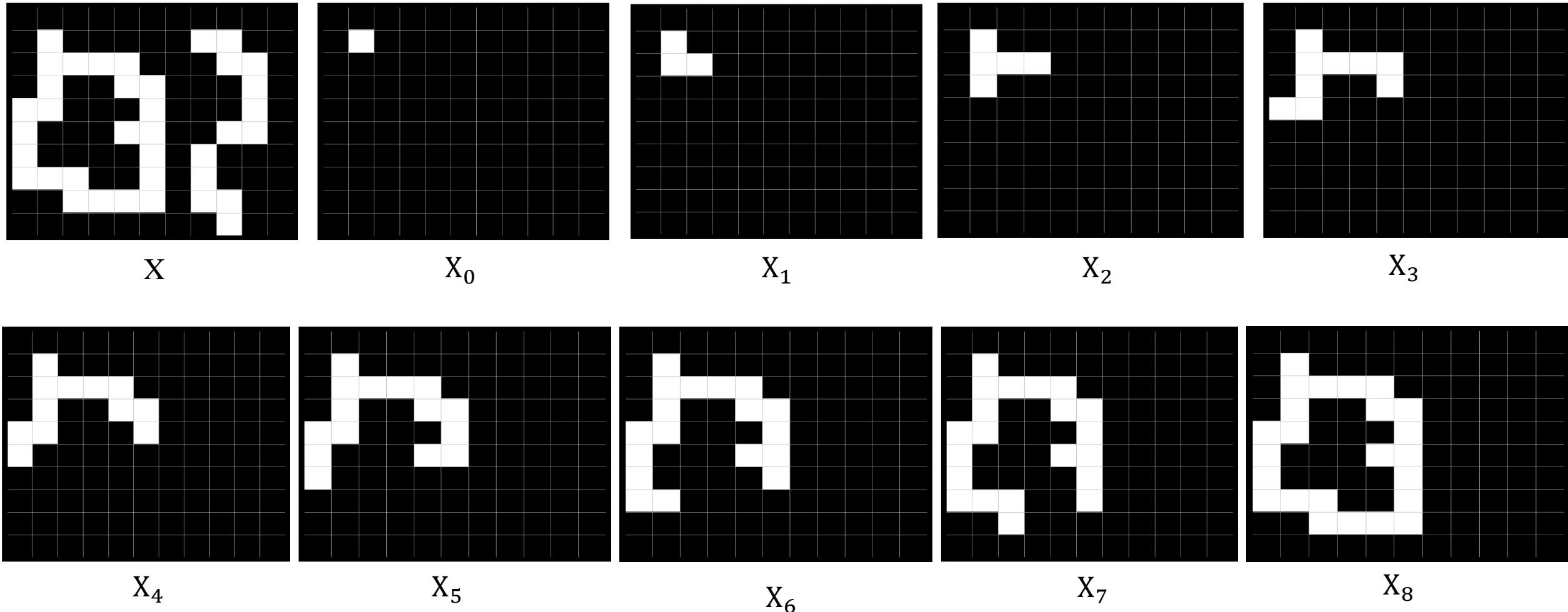
Assume you have a binary image A and a binary, square structuring element B.

- To extract all connected components (with 8-connectivity)
- Select a 2D point A on the structure.
- Let X_0 be the same shape as A, and initialize it with 0 in every element, except at X, where it gets value 1.
- Then, let

$$X_k = (X_{k-1} \oplus B) \cap A \quad k=1, \dots, K$$

- Continue until no change is observed, that is, when $X_k = X_{k-1}$

Extracting connected foreground components

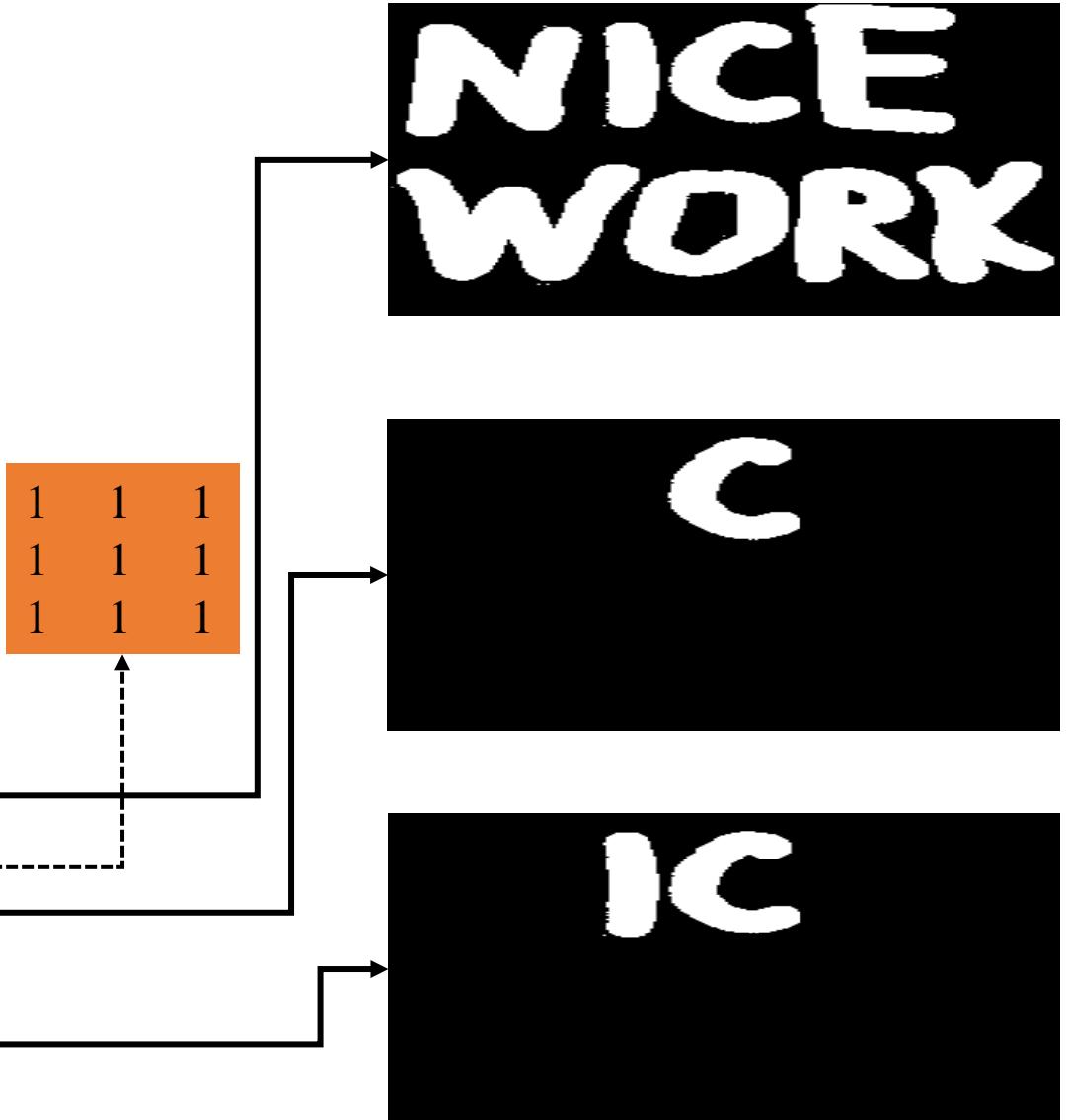


Region Filling MATLAB function

```
function out=components(im,pos,kernel)
% COMPONENTS(IM,POS,KERNEL) produces the connected component of binary image
% IMAGE which includes the point with coordinates given by POS, using
% kernel KERNEL.
%
% Example:
% n=imread('nicework.tif');
% nc=components(nb,[74,52],ones(3,3));
%
current=zeros(size(im));
last=zeros(size(im));
last(pos(1),pos(2))=1;
current=imdilate(last,kernel)&im;
while any(current(:)~=last(:)),
last=current;
current=imdilate(last,kernel)&im;
end;
out=current;

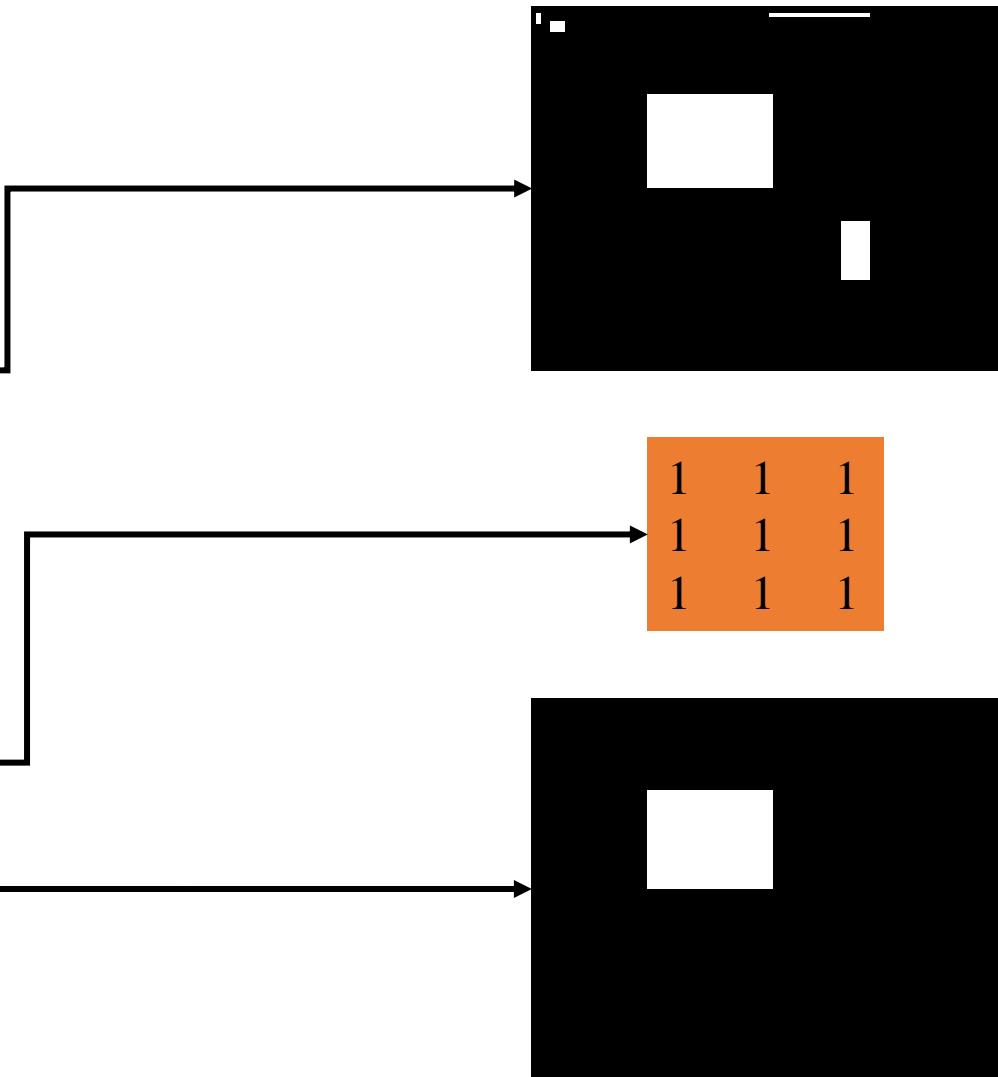
%test example
n=imread('nice.jpeg');
% convert into binary
n1= im2bw(n, 0.3);
% structure element
sq=ones(3,3)
nc=components(n1,[57,97],sq);
figure,imshow(nc)

% bigger strucrure element
sq2=ones(11,11);
nc2=components(n1,[57,97],sq2);
figure,imshow(nc2)
```



Connected Component MATLAB function

```
% construct binary image  
Image=zeros(100,100);  
Image(3:5,2:2)=1  
Image(5:7,5:7)=1  
Image(5:7,5:7)=1  
Image(7:8,3:2)=1  
Image(3,50:70)=1  
Image(25:50,25:50)=1  
Image(60:75,65:70)=1  
figure,imshow(Image)  
%imwrite(Image,'bwimage.png')  
% structure element  
sq=ones(3,3)  
% connected component function  
nc=components(Image,[25,30],sq);  
figure,imshow(nc)
```



Skeletonization

Skeletonization

Skeleton of an object can be defined by the medial axis transform; we may imagine fires burning in along all edges of the object. The places where the lines of fire meet form the skeleton. The skeleton may be produced by morphological methods.

| Erosions | Openings | Set differences |
|---|--------------------------|---|
| A | $A \circ B$ | $A - (A \circ B)$ |
| $(A \ominus B) \ominus B = (A \ominus 2B)$ | $(A \ominus B) \circ B$ | $(A \ominus B) - ((A \ominus B) \circ B)$ |
| $(A \ominus 2B) \ominus B = (A \ominus 3B)$ | $(A \ominus 2B) \circ B$ | $(A \ominus 2B) - ((A \ominus 2B) \circ B)$ |
| $(A \ominus 3B) \ominus B = (A \ominus 4B)$ | $(A \ominus 3B) \circ B$ | $(A \ominus 3B) - ((A \ominus 3B) \circ B)$ |
| | | |
| $(A \ominus kB)$ | $(A \ominus kB) \circ B$ | $(A \ominus kB) - ((A \ominus kB) \circ B)$ |

Operations used to construct the skeleton

Skeletonization

```
function skel = imskel(image,str)
% IMSKEL(IMAGE,STR) - Calculates the skeleton of binary image IMAGE
using
% structuring element STR. This function uses Lanteljou's algorithm.
%
skel=zeros(size(image));
e=image;
while (any(e(:))),
o=imopen(e,str);
skel=skel | (e&~o);
e=imerode(e,str);
end

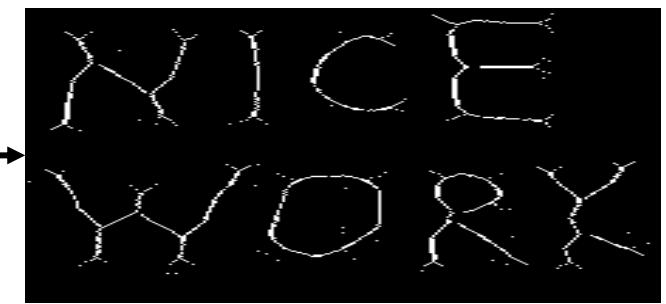
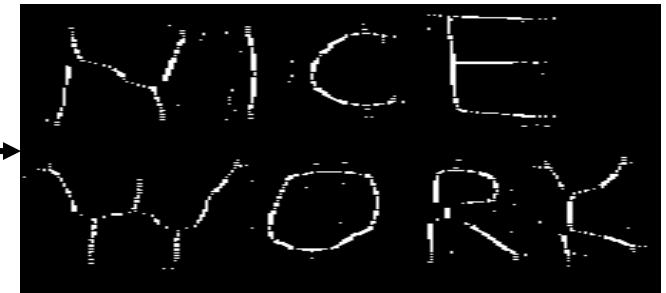
n=imread('nice.jpeg');
% convert into binary
n1= im2bw(n, 0.3);  
-----  
% structure element
sq=ones(3,3)  
-----  
figure,imshow(n1)
% Skeletonization
nk=imskel(n1,sq);  
-----  
figure,imshow(nk)
%imwrite(nk,'skelization.png')

% cross structure element
cr=[0 1 0;1 1 1;0 1 0]  
-----  
figure,imshow(n1)
% Skeletonization
nk=imskel(n1,cr);  
-----  
figure,imshow(nk)
```

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$



$$\begin{matrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{matrix}$$



Summary

The purpose of morphological processing is primarily to remove imperfections added during segmentation

Using the basic operations we can perform *dilation* and *erosion*

Using the basic operations we can perform *opening* and *closing*

More advanced morphological operation can then be implemented using combinations of all of these



Q&A