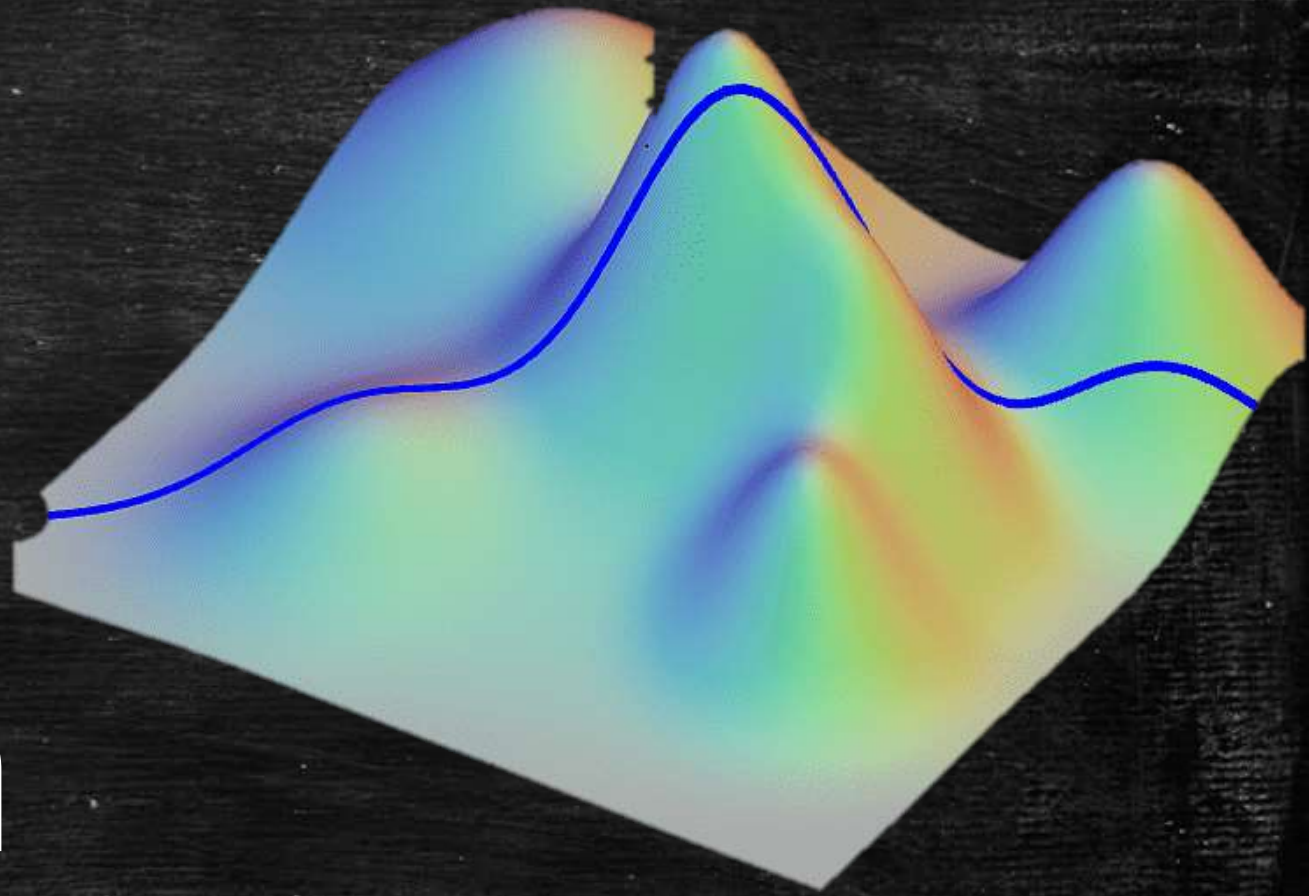


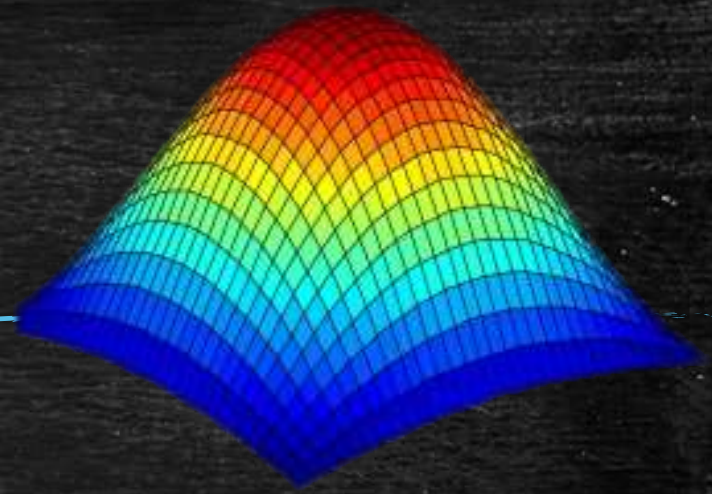
Turunan

Damar W.

Kalkulus

Teknologi Informasi - Universitas Tidar

Turunan

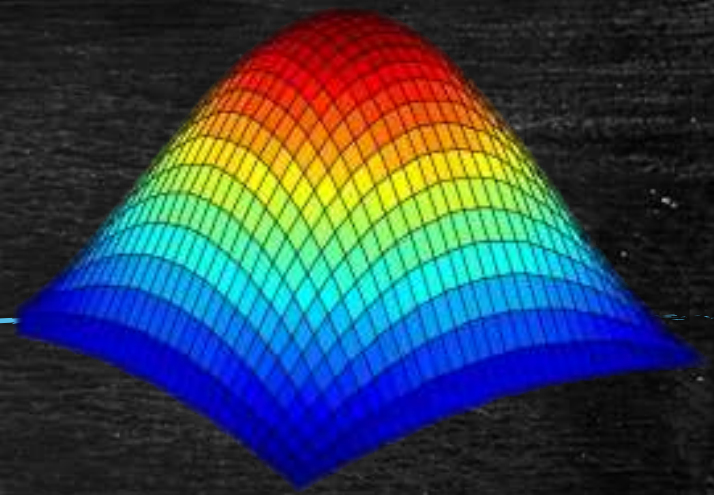


Apa itu Turunan?

Turunan fungsi f adalah fungsi lain f' (dibaca " f aksen") yang nilainya sebarang pada bilangan c adalah

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

Turunan



Contoh, diketahui pers. fgs sbb,

carilah nilai dari f'(4)

$$f(x) = 13x - 6$$

$$f'(4) = ???$$

Penyelesaian :

$$f'(4) = \lim_{h \to 0} \frac{f(4+h) - f(4)}{h}$$

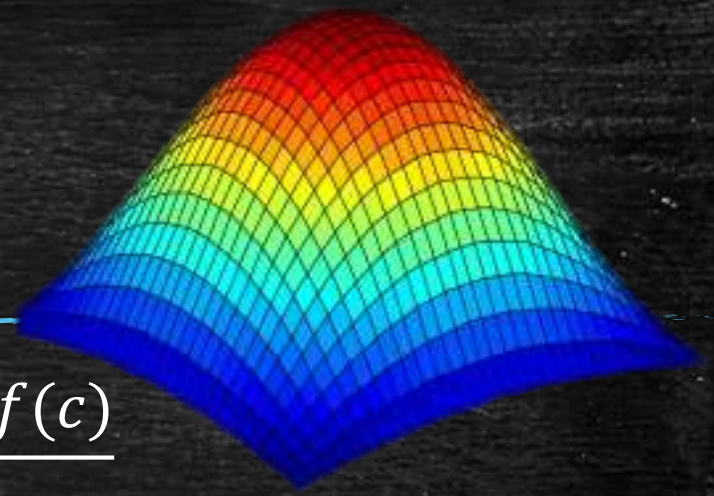
$$\lim_{h \to 0} = \frac{[13(4+h) - 6] - [13(4) - 6]}{h}$$

$$= \lim_{h \to 0} \frac{13h}{h}$$

$$= \lim_{h \to 0} 1\,3$$

$$= 13$$

Turunan



Diketahui pers. fgs sbb:

carilah nilai dari f'(c)

$$f(x) = x^3 + 7x$$
$$f'(c) = ???$$

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

$$= \lim_{h \to 0} \frac{[(c+h)^3 + 7(c+h)] - (c^3 + 7c)}{h}$$

$$= \lim_{h \to 0} \frac{c^3 + 3c^2h + 3ch^2 + h^3 + 7c + 7h - c^3 - 7c}{h}$$

$$= \lim_{h \to 0} \frac{3c^2h + 3ch^2 + h^3 + 7h}{h}$$

$$= \lim_{h \to 0} 3\,c^2 + 3ch + h^2 + 7$$

$$= 3c^2 + 7$$

Turunan

- Latihan Soal:

1 $f(x) = 2x^3$
$f'(5) = \ldots . ?$

2 $f(x) = x^2 + 2x$
$f'(3) = \ldots . ?$

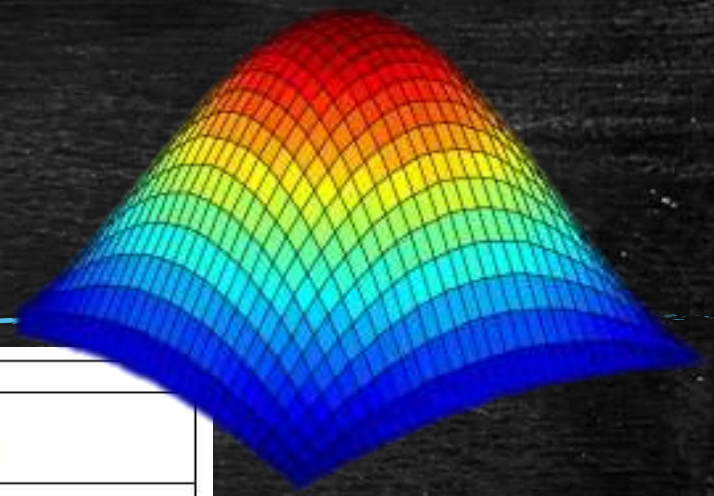
3 $f(x) = x^2$
$f'(1) = \ldots . ?$

4 $f(x) = 2x + 1$
$f'(x) = \ldots . ?$

5 $f(x) = x^2 + 2x$
$f'(x) = \ldots . ?$

6 $f(x) = 3x^2 + 4$
$f'(x) = \ldots . ?$

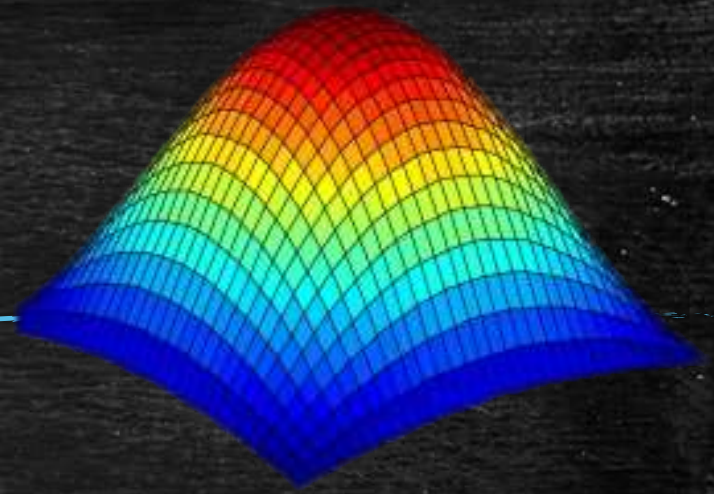
Turunan



Fungsi, $y(x)$	Turunan, y'	Fungsi, $y(x)$	Turunan, y'
Konstanta	0	$\sin^{-1}(ax+b)$	$\frac{a}{\sqrt{1-(ax+b)^2}}$
x^n	nx^{n-1}	$\cos^{-1}(ax+b)$	$\frac{-a}{\sqrt{1-(ax+b)^2}}$
e^x	e^x	$\tan^{-1}(ax+b)$	$\frac{a}{1+(ax+b)^2}$
e^{-x}	$-e^{-x}$	$\sinh(ax+b)$	$a\cosh(ax+b)$
e^{ax}	ae^{ax}	$\cosh(ax+b)$	$a\sinh(ax+b)$
$\ln x$	$\frac{1}{x}$	$\tanh(ax+b)$	$a\,\mathrm{sech}^2(ax+b)$
$\sin x$	$\cos x$	$\mathrm{cosech}(ax+b)$	$-a\,\mathrm{cosech}(ax+b)\coth(ax+b)$
$\cos x$	$-\sin x$	$\mathrm{sech}(ax+b)$	$-a\,\mathrm{sech}(ax+b)\tanh(ax+b)$
$\sin(ax+b)$	$a\cos(ax+b)$	$\coth(ax+b)$	$-a\,\mathrm{cosech}^2(ax+b)$
$\cos(ax+b)$	$-a\sin(ax+b)$	$\sinh^{-1}(ax+b)$	$\frac{a}{\sqrt{(ax+b)^2+1}}$
$\tan(ax+b)$	$a\sec^2(ax+b)$	$\cosh^{-1}(ax+b)$	$\frac{a}{\sqrt{(ax+b)^2-1}}$
$\mathrm{cosec}(ax+b)$	$-a\,\mathrm{cosec}(ax+b)\cot(ax+b)$	$\tanh^{-1}(ax+b)$	$\frac{a}{\sqrt{1-(ax+b)^2}}$
$\sec(ax+b)$	$a\sec(ax+b)\tan(ax+b)$		

Tabel 1.1 Beberapa fungsi yang sering digunakan beserta dengan turunannya

Turunan



Beberapa Aturan pada Operasi Turunan
Jika *u* dan *v* adalah sebuah fungsi,
dan *c* merupakan konstanta, maka:

1. $(u+v)' = u' + v'$

2. $(uv)' = u'v + uv'$

3. $(cu)' = cu'$

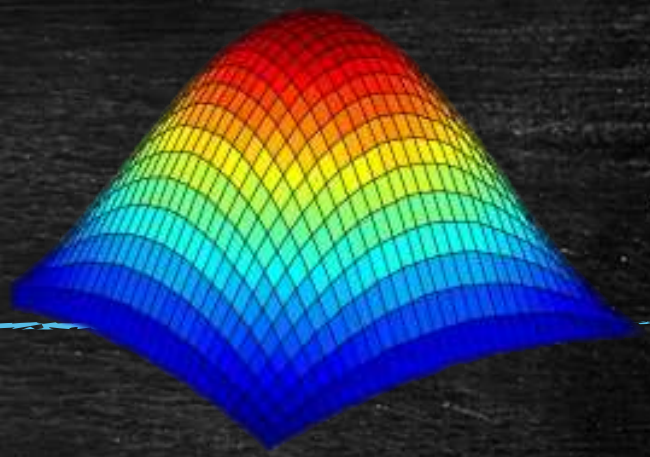
4. $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

5. Jika $y = y(z)$

dan $z = z(x)$

maka $\left(\frac{dy}{dx}\right) = \frac{dy}{dz} * \frac{dz}{dx}$

Latihan



Carilah turunan dari

$5x^2+7x-6$ dan $4x^6+3x^5-10x^2+5x+16$

$D_x(5x^2+7x-6)$

$=D_x(5x^2+7x)-D_x(6)$

$=D_x(5x^2)+D_x(7x)-D_x(6)$

$=5.D_x(x^2)+7.D_x(x)-D_x(6)$

$=5.2x+7.1-0$

$=10x+7$

$\mathrm{Dx}(4x^6+3x^5-10x^2+5x+16)$

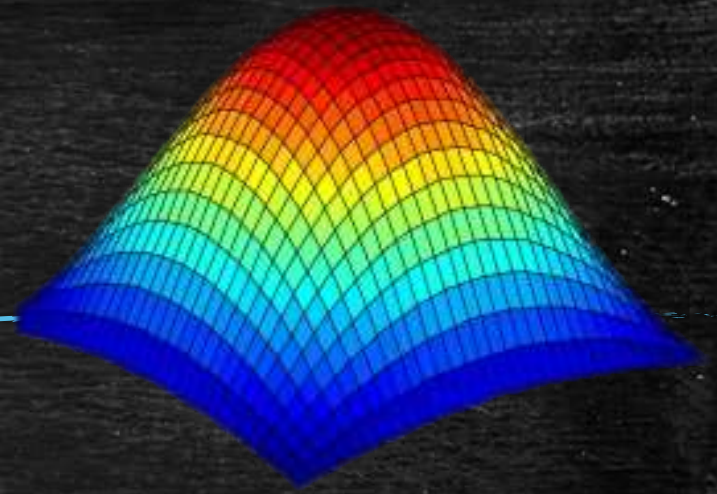
$=D_x(4x^6)+D_x(3x^5)-D_x(10x^2)+D_x(5x)+D_x(16)$

$=4.D_x(x^6)+3.D_x(x^5)-10.D_x(x^2)+5.D_x(x)+D_x(16)$

$=4(6x^5)+3(5x^4)-10(2x)+5(1)+0$

$=24x^5+15x^4-20x+5$

Soal



Carilah turunan dari

$$y = (3x^2 - 5)(2x^4 - x)$$

dan

$$y = \frac{(3x - 5)}{(x^2 + 7)}$$

Jawaban :

$$y = (3x^2 - 5)(2x^4 - x)$$

misal

$u = (3x^2 - 5) \qquad u' = 6x$

$v = (2x^4 - x) \qquad v' = 8x - 1$

maka

$$y' = u'v + uv'$$

$$= (6x)(2x^4 - x) + (3x^2 - 5)(8x^3 - 1)$$

$$= 12x^5 - 6x^2 + 24x^5 - 3x^2 - 40x^3 + 5$$

$$= 36x^5 - 40x^3 - 9x^2 + 5$$

$$y = \frac{(3x - 5)}{(x^2 + 7)}$$

misal

$u = 3x - 5 \qquad u' = 3$

$v = x^2 + 7 \qquad v' = 2x$

maka

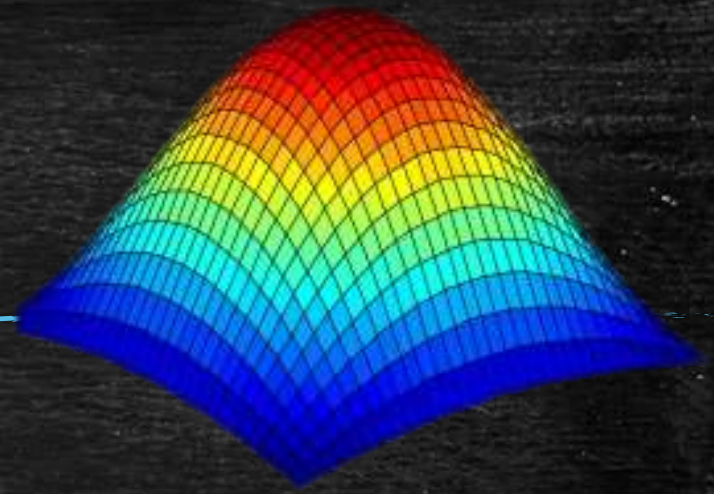
$$y' = \frac{u'v - uv'}{v^2}$$

$$= \frac{(3)(x^2 + 7) - (3x - 5)(2x)}{(x^2 + 7)^2}$$

$$= \frac{3x^2 + 21 - (6x^2 - 10x)}{x^4 + 14x^2 + 49}$$

$$= \frac{-3x^2 + 10x + 21}{x^4 + 14x^2 + 49}$$

Latihan-selesai



Carilah turunan-turunannya dari persamaan berikut:

1. $y = 4x^2 - 5x + 7$
2. $y = (3x + 1)(4x^2 - 2)$