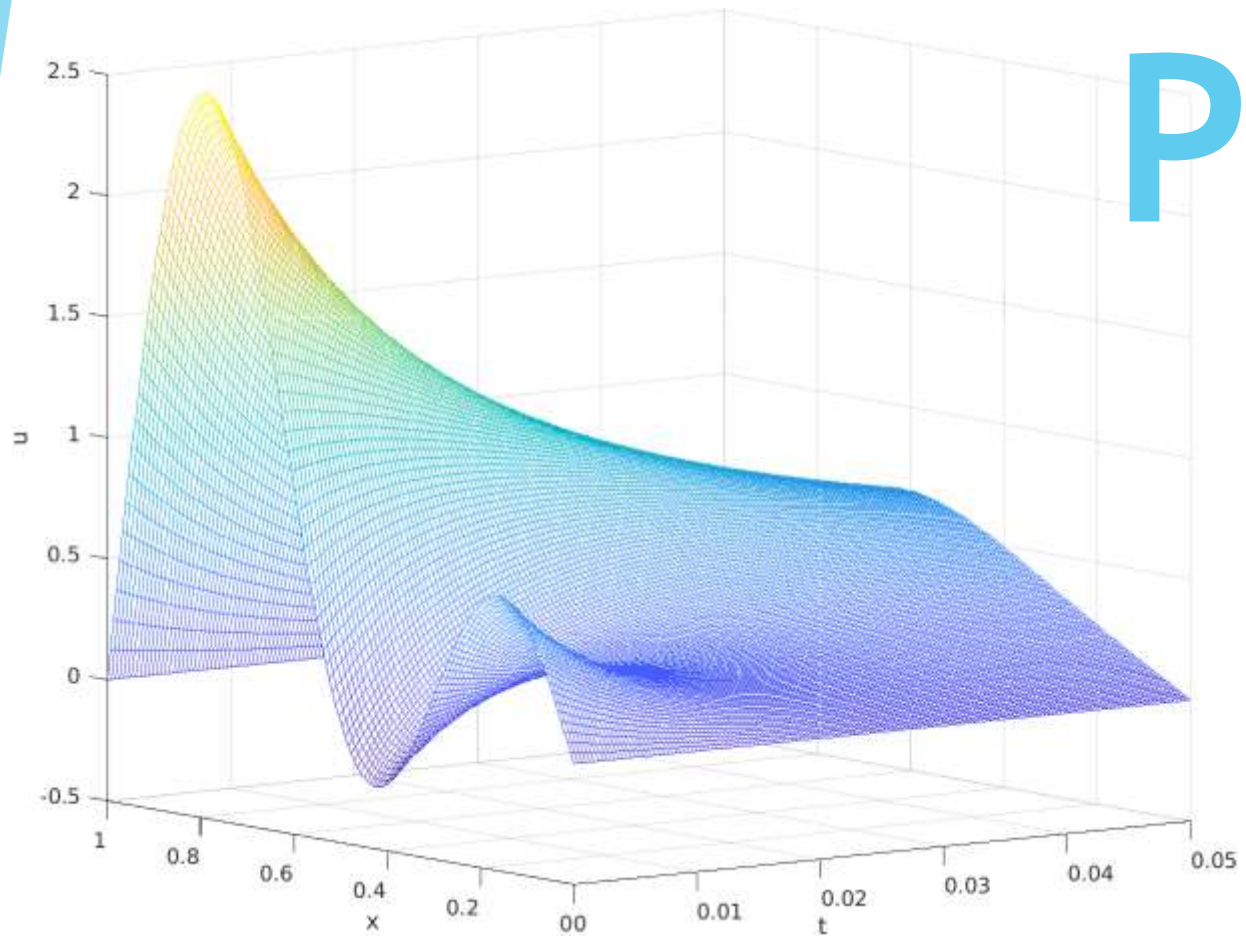
Turunan Parsial



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Turunan Parsial

Apabila terdapat sebuah fungsi $f = R^2 \to R$ dengan daerah definisi Df

Dianggap f terdiferensialkan pada $A \subset D_f$

Maka turunan parsial f terhadap x di $y = y_0$ adalah

$$f_x(x_0, y_0) = \lim_{h \to 0} \frac{f(x+h, y_0) - f(x, y_0)}{h}$$

Turunan parsial f terhadap y di $x = x_0$ adalah

$$f_y(x_0, y_0) = \lim_{h \to 0} \frac{f(x_0, y+h) - f(x_0, y)}{h}$$

Misal $z = f(x, y)$

$$\frac{\partial z}{\partial x} = f_x(x, y) = \frac{\partial f(x, y)}{\partial x}$$

$$\frac{\partial z}{\partial y} = f_y(x, y) = \frac{\partial f(x, y)}{\partial y}$$

Turunan Parsial

Turunan Parsial Tingkat Tinggi

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2}$$

$$f_{yy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2}$$

$$f_{xy} = (f_x)_y = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x}$$

$$f_{yx} = (f_y)_x = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x \partial y}$$

Turunan Parsial

Contoh. Tentukanlah

$$f_{xx}, f_{xy}, f_{yy}, f_{yx} \quad \text{dari} \quad f(x,y) = 2x^2y^3 - x^3y^5$$

Jawab

$$f_x(x,y) = 2x^2y^3 - x^3y^5 = 4xy^3 - 3x^2y^5$$
$$f_{xx}(x,y) = 4y^3 - 6xy^5$$
$$f_{xy}(x,y) = 12xy^2 - 15x^2y^4$$

$$f_y(x,y) = 2x^2y^3 - x^3y^5 = 6x^2y^2 - 5x^3y^4$$
$$f_{yy}(x,y) = 12x^2y - 20x^3y^3$$
$$f_{yx}(x,y) = 12xy^2 - 15x^2y^4$$

Turunan Parsial

Latihan

Tentukan semua turunan parsial order 2 dari $w = x^3y^2 - xy^5$.