SHARP: Environment and Person Independent Activity Recognition with Commodity IEEE 802.11ac Access Points

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Appendix A
OFDM model for the Wi-Fi channel

Next, we detail the main blocks of a Wi-Fi transmission chain, deriving the expression for the CFR in Eq. (2).

A.1 Transmitted signal model

In this subsection, we summarize how orthogonal frequency-division multiplexing (OFDM) is implemented by an IEEE 802.11 transmitter. OFDM uses a large number of closely spaced orthogonal sub-channels, transmitted in parallel. Each sub-channel is modulated with a conventional digital modulation scheme (such as QPSK, 16QAM, etc.).

The input bits are grouped and mapped onto source data symbols, which are complex numbers representing the modulation constellation points. Such constellation points are further grouped into K elements each, i.e., the OFDM symbols. OFDM symbols are then fed to an IFFT block that transforms the data prior to transmitting it over the wireless channel, in parallel, over K sub-channels with carriers spaced apart by $\Delta f = 1/T$ Hz (T is the OFDM symbol time).

The duration of an OFDM symbol is $\bar{T} = T + T_{\rm CP}$, where $T_{\rm CP}$ is the duration of the cyclic prefix, added to mitigate inter-symbol interference. Specifically, for IEEE 802.11ac the transmission bandwidth is 80 MHz, the samples are clocked out at 80 Msps, the number of sub-channels is $K = 256$, $T = 1/\Delta f = 3.2$ μs (i.e., $\Delta f = 312.5$ kHz), $T_{\rm CP} = 0.8$ μs, and, in turn, $\bar{T} = 4$ μs.

Let $\mathbf{a}_m = [a_{m,-K/2}, \dots, a_{m,K/2-1}]$ be m-th OFDM symbol, where $a_{m,k}$ is the k-th OFDM sample. After digital to analog conversion, the baseband OFDM signal for the m-th symbol is

$$x_m(t) = \sum_{k=-K/2}^{K/2-1} a_{m,k} e^{j2\pi kt/T}, \tag{1}$$

where $k \in \{-K/2, \dots, K/2-1\}$ is the sub-channel index. Considering M subsequent blocks, the baseband signal is

$$x(t) = \sum_{m=0}^{M-1} x_m(t)\xi(t - m\bar{T}), \tag{2}$$

with

$$\xi(t) = \begin{cases} 1 \text{ if } t \in [-T_{\rm CP} - TK/2, TK/2] \\ 0 \text{ otherwise} \end{cases}, \tag{3}$$

and the signal transmitted over the Wi-Fi channel is obtained by upconverting $x(t)$ to the carrier frequency f_c,

$$s_{\rm tx}(t) = e^{j2\pi f_c t} x(t). \tag{4}$$

A.2 Received signal model

At each receiver antenna, P signal copies are collected, due to the scatterers that the signal $s_{\rm tx}(t)$ encounters (multi-path propagation). Each path p is characterized by an attenuation $A_p(t)$ and a delay $\tau_p(t)$. Neglecting the additive white Gaussian noise, the received signal $s_{\rm rx}(t)$ is written as

$$\begin{aligned} s_{\rm rx}(t) &= \sum_{p=0}^{P-1} A_p(t) s_{\rm tx}(t - \tau_p(t)) \\ &= e^{j2\pi f_c t} \sum_{p=0}^{P-1} A_p(t) e^{-j2\pi f_c \tau_p(t)} x(t - \tau_p(t)), \end{aligned} \tag{5}$$

and its baseband representation $y(t)$ is expressed as,

$$y(t) = s_{\rm rx}(t) e^{-j2\pi f_c t}. \tag{6}$$

A rectangular window $[m\bar{T}, m\bar{T} + T]$ is used at the receiver to collect and decode the information carried by an OFDM symbol at a time. Without loss of generality, we assume $m = 0$ and hence we omit such index from the following equations. The transmitted symbol a_k is recovered

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by computing the Fourier transform of the signal in the received window:

$$\begin{aligned}\hat{a}_k &= \int_{\bar{T}}^{\bar{T}+T} y(t)e^{-j2\pi kt/T}dt \\ &= \sum_{p=0}^{P-1} A_p e^{-j2\pi f_c \tau_p} \sum_{b=-K/2}^{K/2} a_b e^{-j2\pi b\tau_p/T} \times \\ &\quad \times \int_{\bar{T}}^{\bar{T}+T} e^{j2\pi(b-k)t/T}dt \\ &= a_k T \sum_{p=0}^{P-1} A_p e^{-j2\pi(f_c+k/T)\tau_p},\end{aligned} \tag{7}$$

where we consider A_p and τ_p constant over an integration interval T. The sum in the last line of Eq. (7) corresponds to the frequency response of the Wi-Fi channel,

$$H_k = \sum_{p=0}^{P-1} A_p e^{-j2\pi(f_c+k/T)\tau_p}, \tag{8}$$

that is estimated based on the known preamble symbols. In Eq. (7), we consider that the path attenuation and delay remain constant over each window, i.e., $A_p(t) = A_p$ and $\tau_p(t) = \tau_p$. Also, exchanging the order of integration and summation is legitimate as we deal with finite quantities, and we used $\int_{\bar{T}}^{\bar{T}+T} e^{j2\pi(b-k)t/T}dt = 0$ if $k \neq b$.

A.3 Phase offsets in the Wi-Fi channel estimates

Hardware artifacts make the CFR gathered from Wi-Fi devices slightly deviate from the model in Eq. (8). These artifacts introduce offsets (rotation errors) in the phase information, among which the most significant are [1], [2]:

- *carrier frequency offset (CFO)*, due to the difference between the carrier frequency of the transmitted signal and the one measured at the receiver. The CFO is only partially compensated for at the receiver [3].
- *sampling frequency offset (SFO)*, due to the imperfect synchronization of the clocks between transmitter and receiver.
- *packet detection delay (PDD)*, due to the time required to recover the transmitted modulated symbols from the received signal [3].
- *phase-locked loop phase offset (PPO)*, due to the phase-locked loop (PLL), the entity responsible for randomly generating the initial phase at the transmitter.
- *phase ambiguity (PA)*, due to the phase difference (multiples of π) between the antennas, that in static conditions should remain constant.

Considering these contributions, the complete expression for the phase of the p-th path in the received signal is

$$\begin{aligned}\bar{\phi}_{p,k} = &-2\pi(f_c + k/T)\tau_p + \phi_{\mathrm{CFO}} \\ &- 2\pi k(\tau_{\mathrm{SFO}} + \tau_{\mathrm{PDD}})/T \\ &+ \phi_{\mathrm{PPO}} + \phi_{\mathrm{PA}}.\end{aligned} \tag{9}$$

Note that, while the CFO, SFO and PDD contributions take the same value across different antennas, the initial PLL phase (PPO) and PA are antenna specific [4], [5].

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