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Software Engineering and Programming Languages
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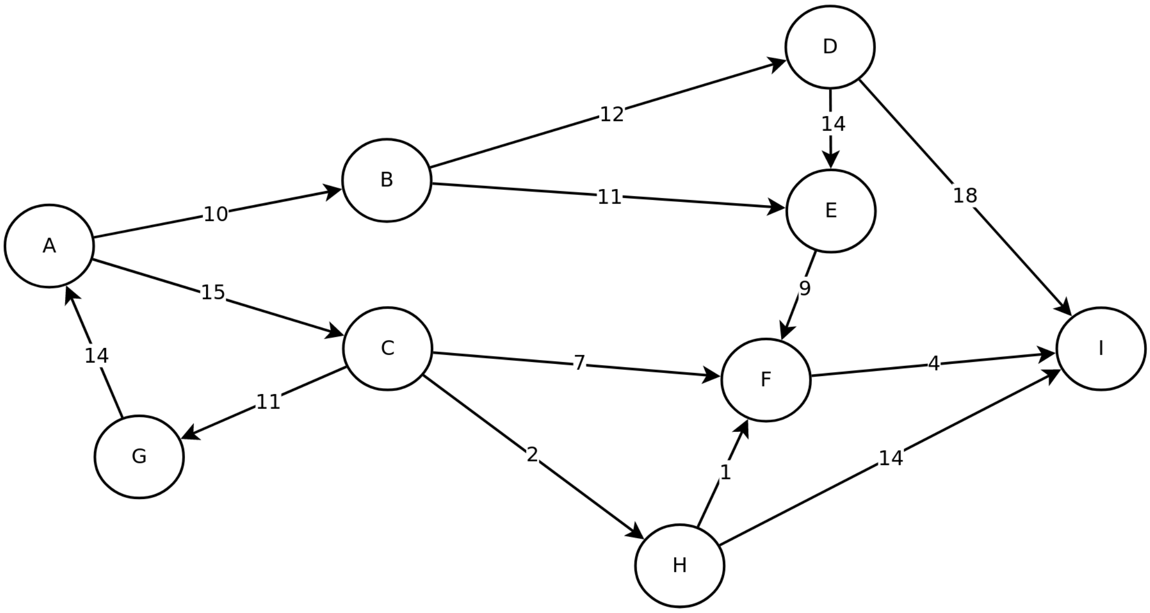
Introduction to Logic Programming – WS 2023
Solutions for Exercise Sheet 9

1 Exercises

Exercise 3 (Informed Search - A*)
The following figure shows a directed graph with weighted edges describing the actual costs between two nodes.



The heuristic function `h` is defined as follows:
`h(A) = 22, h(B) = 20, h(C) = 3, h(D) = 14, h(E) = 9, h(F) = 1, h(G) = 32, h(H) = 2, h(I) = 0`

Find the shortest path from the node `A` to the node `I` by applying the A* algorithm. State all computed f-values and the queue in each step.

An heuristic function is admissable if it does not overestimate the actual costs.

An heuristic function is monotone if it is admissible and for all nodes k, k' where k' is a neighbor of k the formula $h(k) \leq c(k, k') + h(k')$ is true.

Once a node has been visited and a monotone heuristic function is used, we know that we have found the shortest path to this node. We thus do not have to visit this node again. Yet, this is just a performance improvement and does not affect the actual result.

We compute so called f-values for each neighbor node using $f(x) = g(x) + h(x)$ which are stored in a queue. We visit the node with the smallest f-value next. The nodes which have not been selected are still in the queue.

- $Q_0 = [(22, A)]$, initial node is A
  - $f(B) = \underbrace{g(A) + c(A, B)}_{\text{g(B)}} + h(B) = 0 + 10 + 20 = 30$
  - $f(C) = \underbrace{g(A) + c(A, C)}_{\text{g(C)}} + h(C) = 0 + 15 + 3 = 18$
  - $Q_1 := [\mathbf{(18,C)}, (30, B)]$
- visit node C
  - $f(G) = g(C) + c(C, G) + h(G) = 15 + 11 + 32 = 58$
  - $f(F) = g(C) + c(C, F) + h(F) = 15 + 7 + 1 = 23$
  - $f(H) = g(C) + c(C, H) + h(H) = 15 + 2 + 2 = 19$
  - $Q_2 := [\mathbf{(19,H)}, (23, F), (30, B), (58, G)]$
- visit node H
  - $f(F) = g(H) + c(H, F) + h(F) = 17 + 1 + 1 = 19$
  - $f(I) = g(H) + c(H, I) + h(I) = 17 + 14 + 0 = 31$
  - $Q_3 := [\mathbf{(19,F)}, (30, B), (31, I), (58, G)]$ (the value for node F has been updated)
- visit node F
  - $f(I) = g(F) + c(F, I) + h(I) = 18 + 4 + 0 = 22$
  - $Q_4 := [\mathbf{(22,I)}, (30, B), (58, G)]$ (the value for node I has been updated)
- visit node I
  - finished, the shortest path from node A to I is $A \to C \to H \to F \to I$