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Software Engineering and Programming Languages
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Introduction to Logic Programming – WS 2023
Solutions for Exercise Sheet 3

1 Exercises

Exercise 2 (Simplification of Propositional Formulae)
Simplify the following two propositional formulae by rewriting implications to disjunctions and moving negations inwards to the literals.

1.
$(a \Rightarrow (b \wedge \neg a)) \Rightarrow b$
$\equiv (\neg a \vee (b \wedge \neg a)) \Rightarrow b$
$\equiv \neg(\neg a \vee (b \wedge \neg a)) \vee b$
$\equiv (a \wedge \neg(b \wedge \neg a)) \vee b$
$\equiv (a \wedge (\neg b \vee a)) \vee b$ (task completed)
$\equiv (a \wedge \neg b) \vee (a \wedge a) \vee b$
$\equiv (a \wedge \neg b) \vee a \vee b$
$\equiv ((a \vee a) \wedge (\neg b \vee a)) \vee b$
$\equiv (a \wedge (\neg b \vee a)) \vee b$
$\equiv (a \vee b) \wedge \underbrace{(\neg b \vee a \vee b)}_{\top}$
$\equiv (a \vee b)$

2.
$(\neg(a \vee b)) \wedge (\neg b \Rightarrow a)$
$\equiv (\neg a \wedge \neg b) \wedge (\neg b \Rightarrow a)$
$\equiv (\neg a \wedge \neg b) \wedge (b \vee a)$ (task completed)
$\equiv ((\neg a \wedge (b \vee a)) \wedge (\neg b \wedge (b \vee a))$
$\equiv ((\neg a \wedge b) \vee \underbrace{(\neg a \wedge a)}_{\bot}) \wedge (\neg b \wedge (b \vee a))$
$\equiv (\neg a \wedge b) \wedge (\neg b \wedge (b \vee a))$
$\equiv (\neg a \wedge b) \wedge (\underbrace{(\neg b \wedge b)}_{\bot} \vee (\neg b \wedge a))$
$\equiv (\neg a \wedge b) \wedge (\neg b \wedge a)$
$\equiv (\neg a \wedge b) \wedge (a \wedge \neg b)$
$\equiv \bot$