# Introduction to Logic Programming – WS 2023
# Solutions for Exercise Sheet 6

## 1 Exercises

**Exercise 2 (Unification)**
Find the most general unificator (mgu) $\sigma$ for each of the following pairs of terms if they can be unified. Do *not* use a Prolog interpreter to solve the tasks. It is sufficient to state the results for $\sigma$.

a) `[]` and `[]`

$\sigma = \emptyset$

b) `[H|T]` and `[1,2,[3]]`

$\sigma = \{\texttt{H/1,T/[2,[3]]}\}$

Unification algorithm by Robinson:

1. $\Theta_0 := \emptyset$
2. $\texttt{[H|T]}\Theta_0 = \texttt{[H|T]} \neq \texttt{[1,2,[3]]} = \texttt{[1,2,[3]]}\Theta_0$
   first disagreement tuple: $<a_0, b_0> = <\texttt{H}, \texttt{1}>$
3. $a_0 = \texttt{H}$ is a variable which does not occur in $b_0 = \texttt{1}$
   $\Theta_1 := \{a_0/b_0\} = \Theta_0\{\texttt{H/1}\} = \{\texttt{H/1}\}$
2. $\texttt{[H|T]}\Theta_1 = \texttt{[1|T]} \neq \texttt{[1,2,[3]]} = \texttt{[1,2,[3]]}\Theta_1$
   second disagreement tuple: $<a_1, b_1> = <\texttt{T}, \texttt{[2,[3]]}>$
3. $a_1 = \texttt{T}$ is a variable which does not occur in $b_1 = \texttt{[2,[3]]}$
   $\Theta_2 := \Theta_1\{a_1/b_1\} = \{\texttt{H/1}, \texttt{T/[2,[3]]}\}$
2. $\texttt{[H|T]}\Theta_2 = \texttt{[1|[2,[3]]]} = \texttt{[1,2,[3]]} = \texttt{[1,2,[3]]}\Theta_2$

c) `[X,Y]` and `[c|[[a,b]]]`

`[c|[[a,b]]] = [c,[a,b]]`

$\sigma = \{\texttt{X/c,Y/[a,b]}\}$

d) `2` and `1+1`

not unifiable because both terms have different functors

e) `r(a,X)` and `r(Y,r(a,b))`

$\sigma =$ `{Y/a,X/r(a,b)}`

f) `f(X,Y)` and `f(Y,f(X))`

not unifiable with occurs-check (`unify_with_occurs_check/2` in Prolog)

without occurs-check: $\sigma =$ `{X/Y,Y/f(Y)}`

For `=/2` the Prolog interpreter does not apply the occurs-check since it would be too much unnecessary overhead. In Prolog, we usually don't want to unify two terms with circular variable dependencies anyway.

g) `n(a,b)` and `f(X,Y)`

not unifiable because both terms have different functors

h) `f(a,b)` and `f(X,X)`

not unifiable

Unification algorithm by Robinson:

1. $\Theta_0 := \emptyset$
2. $\mathtt{f(a,b)}\Theta_0 = \mathtt{f(a,b)} \neq \mathtt{f(X,X)} = \mathtt{f(X,X)}\Theta_0$
   first disagreement tuple: $<a_0, b_0> = <\mathtt{a}, \mathtt{X}>$
3. $b_0 = \mathtt{X}$ is a variable which does not occur in $a_0 = \mathtt{a}$
   $\Theta_1 := \{b_0/a_0\} = \Theta_0\{\mathtt{X/a}\} = \{\mathtt{X/a}\}$
2. $\mathtt{f(a,b)}\Theta_1 = \mathtt{f(a,b)} \neq \mathtt{f(a,a)} = \mathtt{f(X,X)}\Theta_1$
   second disagreement tuple: $<a_1, b_1> = <\mathtt{b}, \mathtt{a}>$
3. both terms are not unifiable because neither $a_1 = \mathtt{b}$ nor $b_1 = \mathtt{a}$ are variables

i) `[1,2|E]-E` and `[X,Y,F|G]-[a,b,c]`

$\sigma = \texttt{\{X/1,Y/2,E/[a,b,c],F/a,G/[b,c]\}}$

Unification algorithm by Robinson:

1. $\Theta_0 := \emptyset$
2. $\texttt{[1,2|E]-E}\Theta_0 = \texttt{[1,2|E]-E} \neq \texttt{[X,Y,F|G]-[a,b,c]} = \texttt{[X,Y,F|G]-[a,b,c]}\Theta_0$
   first disagreement tuple: $<a_0, b_0> = <\texttt{1}, \texttt{X}>$
3. $b_0 = \texttt{X}$ is a variable which does not occur in $a_0 = \texttt{1}$
   $\Theta_1 := \{b_0/a_0\} = \Theta_0\{\texttt{X/1}\} = \{\texttt{X/1}\}$
2. $\texttt{[1,2|E]-E}\Theta_1 = \texttt{[1,2|E]-E} \neq \texttt{[1,Y,F|G]-[a,b,c]} = \texttt{[X,Y,F|G]-[a,b,c]}\Theta_1$
   second disagreement tuple: $<a_1, b_1> = <\texttt{2}, \texttt{Y}>$
3. $b_1 = \texttt{Y}$ is a variable which does not occur in $a_1 = \texttt{2}$
   $\Theta_2 := \Theta_1\{b_1/a_1\} = \{\texttt{X/1}, \texttt{Y/2}\}$
2. $\texttt{[H|T]}\Theta_2 = \texttt{[1,2|E]-E} \neq \texttt{[1,2,F|G]-[a,b,c]} = \texttt{[X,Y,F|G]-[a,b,c]}\Theta_2$
   third disagreement tuple: $<a_2, b_2> = <\texttt{E}, \texttt{[F|G]}>$
3. $a_2 = \texttt{E}$ is a variable which does not occur in $b_2 = \texttt{[F|G]}$
   $\Theta_3 := \Theta_2\{a_2/b_2\} = \{\texttt{X/1}, \texttt{Y/2}, \texttt{E/[F|G]}\}$
2. $\texttt{[1,2|E]-E}\Theta_3 = \texttt{[1,2|[F|G]]-[F|G]} = \texttt{[1,2,F|G]-[F|G]} \neq$
   $\texttt{[1,2,F|G]-[a,b,c]} = \texttt{[X,Y,F|G]-[a,b,c]}\Theta_3$
   fourth disagreement tuple: $<a_3, b_3> = <\texttt{F}, \texttt{a}>$
3. $a_3 = \texttt{F}$ is a variable which does not occur in $b_3 = \texttt{a}$
   $\Theta_4 := \Theta_3\{a_3/b_3\} = \{\texttt{X/1}, \texttt{Y/2}, \texttt{E/[F|G]}, \texttt{F/a}\} = \{\texttt{X/1}, \texttt{Y/2}, \texttt{E/[a|G]}, \texttt{F/a}\}$
2. $\texttt{[1,2|E]-E}\Theta_4 = \texttt{[1,2,a|G]-[a|G]} \neq \texttt{[1,2,a|G]-[a,b,c]} = \texttt{[X,Y,F|G]-[a,b,c]}\Theta_4$
   fifth disagreement tuple: $<a_4, b_4> = <\texttt{G}, \texttt{[b,c]}>$
3. $a_4 = \texttt{G}$ is a variable which does not occur in $b_4 = \texttt{[b,c]}$
   $\Theta_5 := \Theta_4\{a_4/b_4\} = \{\texttt{X/1}, \texttt{Y/2}, \texttt{E/[a,b,c]}, \texttt{F/a}, \texttt{G/[b,c]}\}$
2. $\texttt{[1,2|E]-E}\Theta_5 = \texttt{[1,2,a,b,c]-[a,b,c]} = \texttt{[X,Y,F|G]-[a,b,c]}\Theta_5$

**Exercise 3 (Unification)**
Decide for each pair of substitutions (unificators) which substitution is more general.

Use the following definition: The substitution $\Theta$ is more general than $\Phi$ if there exists $\sigma$ such that $\Phi = \Theta\sigma$.

a) `{X/a}` and `{X/a,Y/a}`

   `{X/a}` is more general than `{X/a,Y/a}`

   $\Phi =$ `{X/a,Y/a}` $=$ `{X/a}{Y/a}` $= \Theta\sigma$

b) `{X/Y}` and `{X/a,Y/a}`

   `{X/Y}` is more general than `{X/a,Y/a}`

   $\Phi =$ `{X/a,Y/a}` $=$ `{X/Y}{Y/a}` $= \Theta\sigma$

c) `{X/Y,Z/a}` and `{X/a,Y/a,Z/a}`

   `{X/Y,Z/a}` is more general than `{X/a,Y/a,Z/a}`

   $\Phi =$ `{X/a,Y/a,Z/a}` $=$ `{X/Y,Z/a}{Y/a}` $= \Theta\sigma$

d) `{X/1,Y/Z}` and `{X/1,Y/2,Z/3}`

   There exists no $\sigma$ since the substitutions can not be equal.

e) `{X/a}` and `{X/b}`

   There exists no $\sigma$ since the substitutions can not be equal.