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Software Engineering and Programming Languages
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Introduction to Logic Programming – WS 2023
Solutions for Exercise Sheet 4

1 Exercises

Exercise 2 (Proof by Contradiction, Resolution)
Let `K := {{A,C},{¬C,B},{¬B,¬C,A}}` be a set of clauses. Prove by contradiction and resolution that the statement A holds.

- Resolution: $(\alpha \vee \beta) \wedge (\neg\beta \vee \gamma) \xRightarrow{resolution} (\alpha \vee \gamma)$
- K is a propositional formula in conjunctive normalform represented as a set
- a clause is a disjunction of literals, a conjunctive normalform is a conjunction of clauses
- $(A \vee C) \wedge (\neg C \vee B) \wedge (\neg B \vee \neg C \vee A)$
- assume negated goal: `K = {{A,C},{¬C,B},{¬B,¬C,A},{¬A}}`
- deduce new propositional formulae by resolution:
  - $(A \vee C) \wedge \neg A \xRightarrow{resolution} \mathbf{C}$

    `K = {{A,C},{¬C,B},{¬B,¬C,A},{¬A},{C}}`
  - $(\neg B \vee \neg C \vee A) \wedge \neg A \xRightarrow{resolution} (\neg\mathbf{B} \vee \neg\mathbf{C})$

    `K = {{A,C},{¬C,B},{¬B,¬C,A},{¬A},{C},{¬B,¬C}}`
  - $(\neg B \vee \neg C) \wedge C \xRightarrow{resolution} \neg\mathbf{B}$

    `K = {{A,C},{¬C,B},{¬B,¬C,A},{¬A},{C},{¬B,¬C},{¬B}}`
  - $(\neg C \vee B) \wedge \neg B \xRightarrow{resolution} \neg\mathbf{C}$

    `K = {{A,C},{¬C,B},{¬B,¬C,A},{¬A},{C},{¬B,¬C},{¬B},{¬C}}`
  - $\neg C \wedge C \equiv \perp$
- contradiction found, we have thus proven that the statement `A` holds