Portfolio Optimization Using Genetic ALgorithm

CL-643 Assignment

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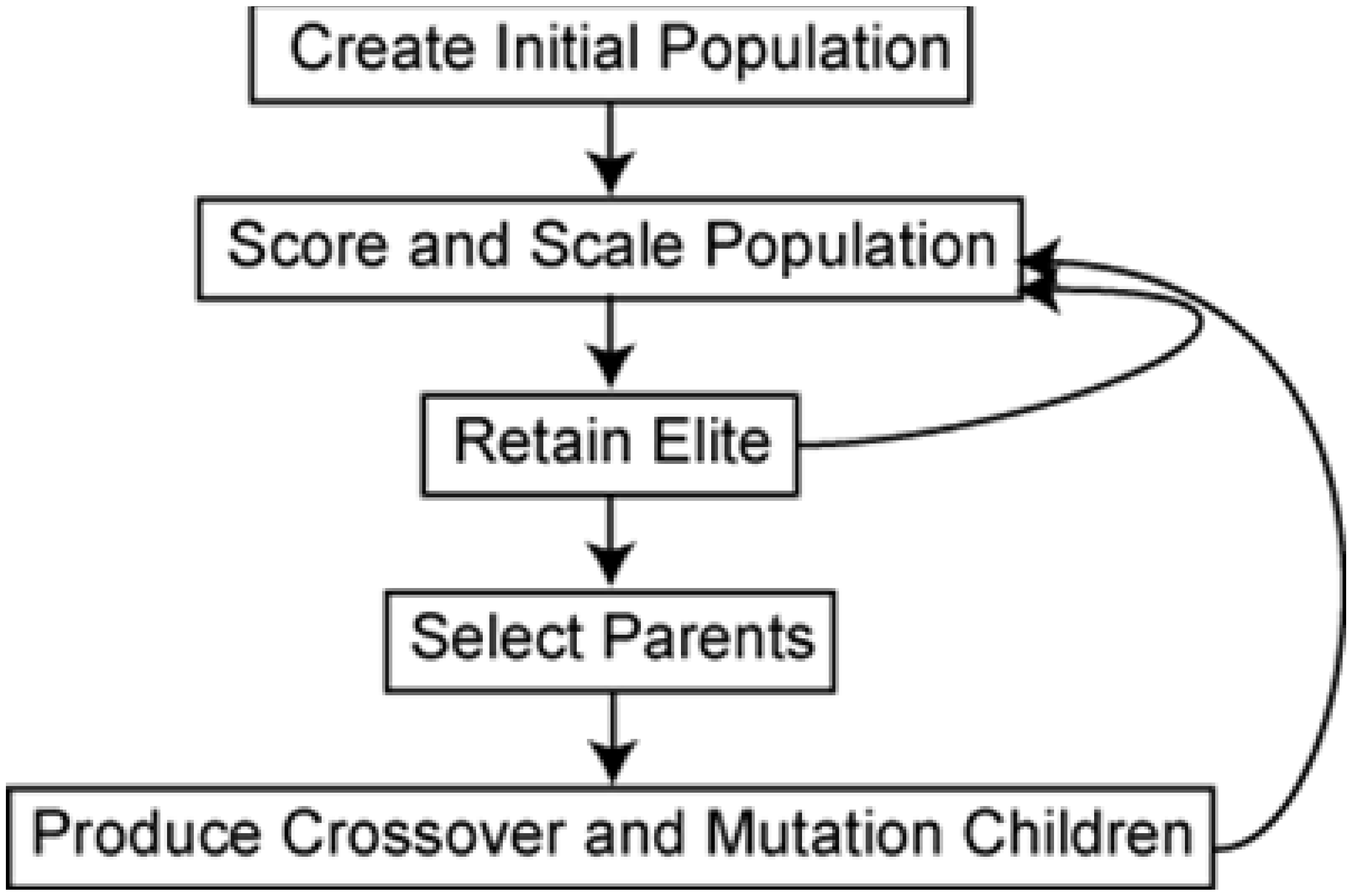
Problem Statement

Suppose we have identified N financial assets we wish to invest in, such as stocks, funds, bonds, or ETFs. Each asset has a historical return, which refers to the relative price difference between two periods: days, weeks, or months.

Our objective is to construct an investment portfolio comprising several assets and allocate a fraction x of total capital to each asset, known as the weight. The aim of portfolio optimization is to determine the optimal weights for each asset that maximizes returns and minimizes risk while adhering to certain constraints and then compare it to standard MPT.

Solving Approach



Our approach involves utilizing the Genetic Algorithm to identify the optimal weights for maximizing returns and minimizing risk. The Sharpe Ratio will serve as the fitness measure to assess the effectiveness of the approach

Why GA, not PSO ?

The effectiveness of Particle Swarm Optimization (PSO) in unconstrained portfolio optimization has been demonstrated in various studies. One such study conducted by Kamali in 2014 revealed that PSO outperformed Genetic Algorithms (GA) in terms of obtaining higher-quality portfolios in a shorter amount of time.
In another study in 2011, PSO was found to be a superior algorithm compared to a GA and an industry tool for portfolio optimization.
Additionally, PSO was shown to achieve better consistency and faster convergence towards an optimal solution for a behavioral portfolio model than a GA.

Hence, we will focus on GA for this paper and will compare the Genetic Algorithm method to the classical Modern Portfolio Method.

Detailed Process

- Combine the data into a single table and calculate the historical returns for different time periods for each stock.

- A gene refers to the portion of the overall capital allocated to a particular stock, while a chromosome is a collection of genes that signify the proportions of the total capital assigned to each stock. It is a prerequisite for the summation of all genes in a chromosome to be equal to 1.

Detailed Process

- To ensure that the sum of six random numbers is equal to one, one can first generate the random numbers and then calculate a factor by dividing one by the sum of the random numbers. The generated random numbers can then be multiplied by this factor, which will result in the sum of the random numbers being equal to one.

- Generate an initial population of chromosomes randomly. The Sharpe ratio is a fitness function that measures the portfolio's performance by maximizing returns and minimizing risk simultaneously. It is calculated as:

S = (Mean Portfolio Return - Risk-free Rate) / Standard Deviation of Portfolio Return.

$$\mathbf{S = (\mu - r) / \sigma}$$

Detailed Process

- Select the elite population by filtering the chromosomes with the highest returns based on the fitness function. The mutation is a function that randomly selects two elements in a chromosome between 0 and 5 and swaps them.

- Heuristic crossover or Blend Crossover is a method that uses the fitness values of the two parent chromosomes to determine the direction of the search.

Detailed Process

- β is a random number between 0 and 1, the next generation is a function that creates a new generation of chromosomes by performing mutation, mating, or crossover based on probability.

Offspring A = Best Parent + β * (Best Parent - Worst Parent)

Offspring B = Worst Parent - β * (Best Parent - Worst Parent)

- Iterate the entire process until there is no further improvement in the maximum returns or for a fixed number of iterations.

Portfolio Selection Model

We are examining a portfolio comprising of n weights (W1,W2, …., Wn), where Wi represents the proportion of total invested wealth by the investors in asset i, ri is a random variable that indicates the expected return of asset i, and n denotes the total number of assets in the portfolio. The portfolio's return, rp, is determined by the following equation:

$$rp = W1r1 + W2r2 + \cdots + Wnrn = \sum_i^n Wiri$$

$$E(rp) = E[\sum_i^n Wiri] = \sum_i^n WiE(ri)$$

We let, $E(rp) = R \; and \; E(ri) = \mu i$

$$R = \sum_i^n Wi\mu i$$

$$\sigma^2 p = E[(rp - \mu p)^2]$$

$$\sigma^2 p = \sum_i^n Wi^2\sigma^2 + 2\sum_i^{n-1}\sum_{j=i+1}^n WiWj\sigma ij$$

But, var $(ri) = \sigma ii$

$$Var(P) = \sum_{i=1}^n \sum_{j=1}^n WiWj\sigma ij$$

Portfolio Selection Model

The Sharpe Ratio serves as the fitness measure to assess the effectiveness of the approach as discussed in the detailed process section, where the constraints include budget constraint, cardinality constraint, and floor and ceiling constraints. The budget constraint ensures that the investor allocates the entire budget to a portfolio. The cardinality constraint ensures that a specific number of assets, K, are included in the portfolio. The floor constraint, which is a lower-bound constraint, helps avoid administrative costs for very small holdings. In contrast, the ceiling constraint, an upper-bound constraint, avoids excessive exposure to a particular asset.

$$\sum_i^n Wi = 1 \qquad\qquad 0 < Wi < \{0.25, 0.35, 0.45\}$$

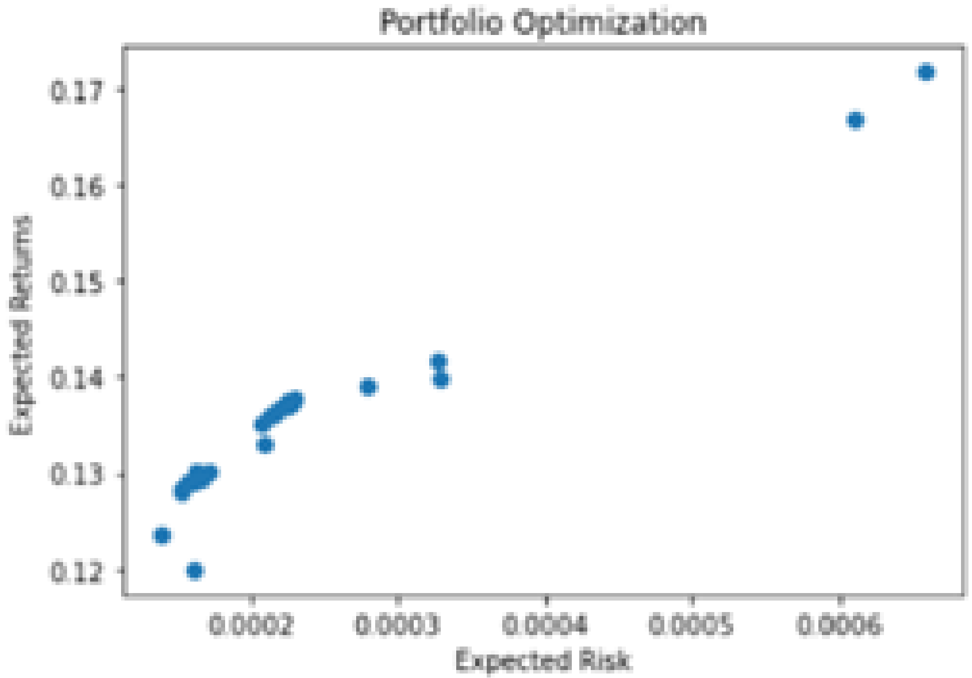
Here, Wi is the weight assigned to ith asset.

$$\sum Zi = K$$

Here, K is the sum of the total number of assets available. In our case, the value of K is 6.

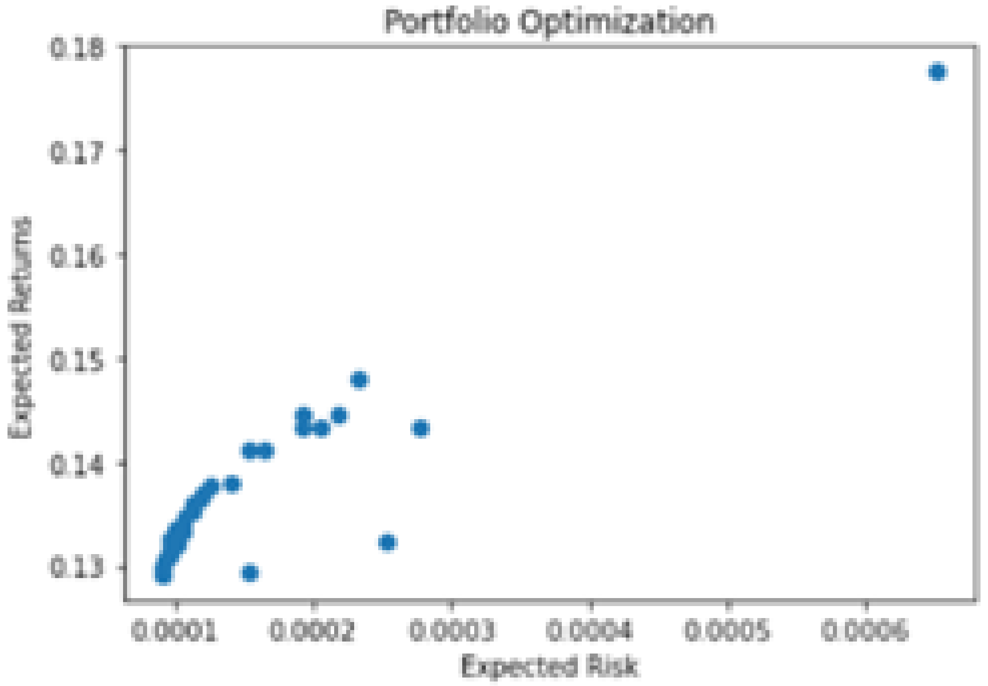
Assessing results for GA

25% Upper Bound



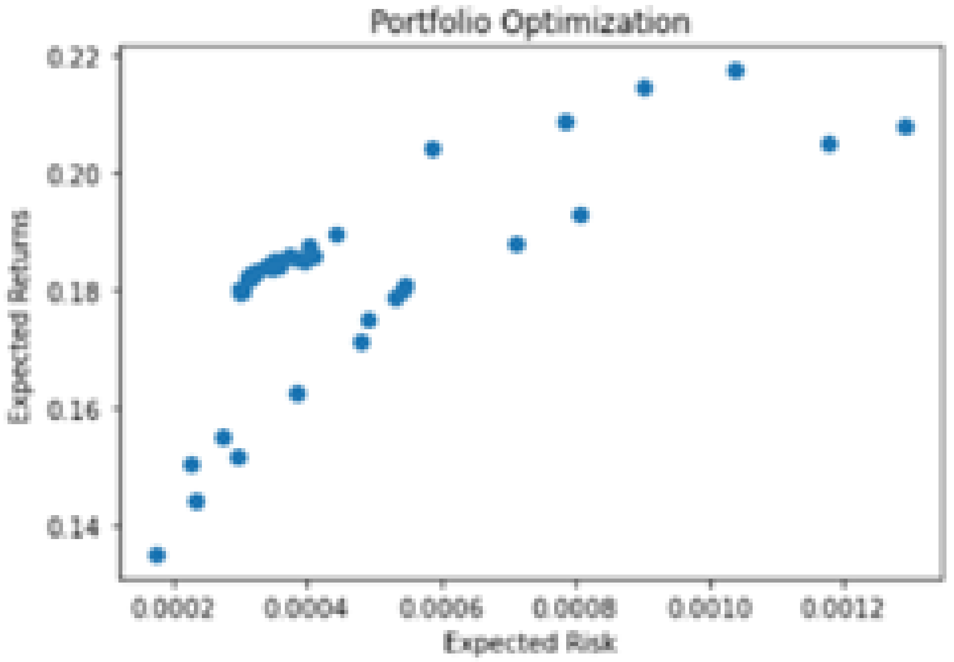
Expected returns - 13.74636 %
risk - 0.0002274

35% Upper Bound



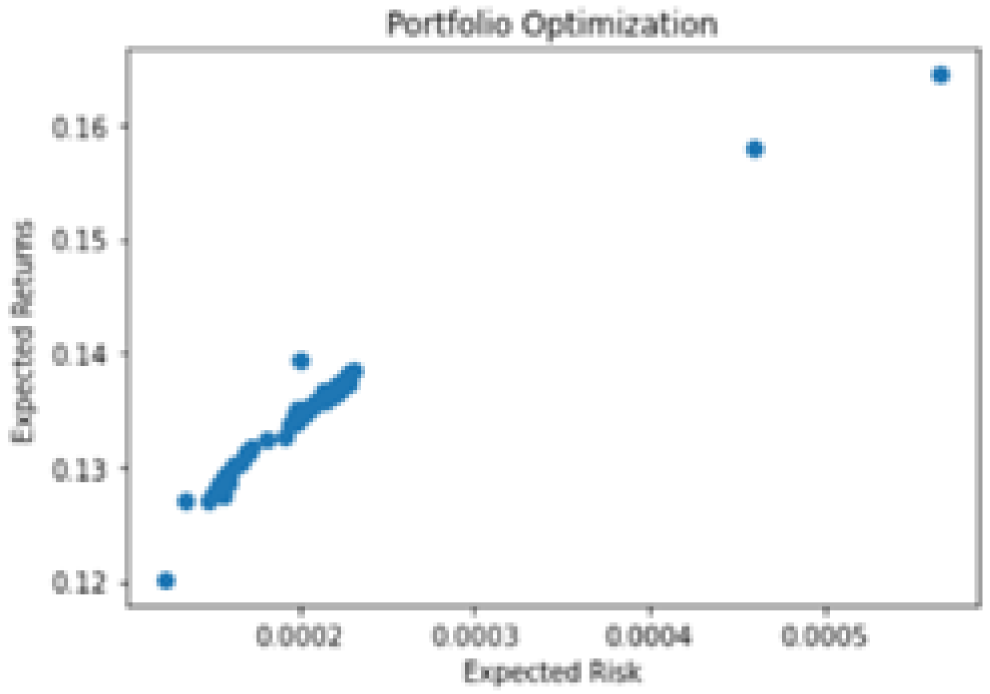
Expected returns - 13.22523 %
risk - 9.573475e-05

45% Upper Bound



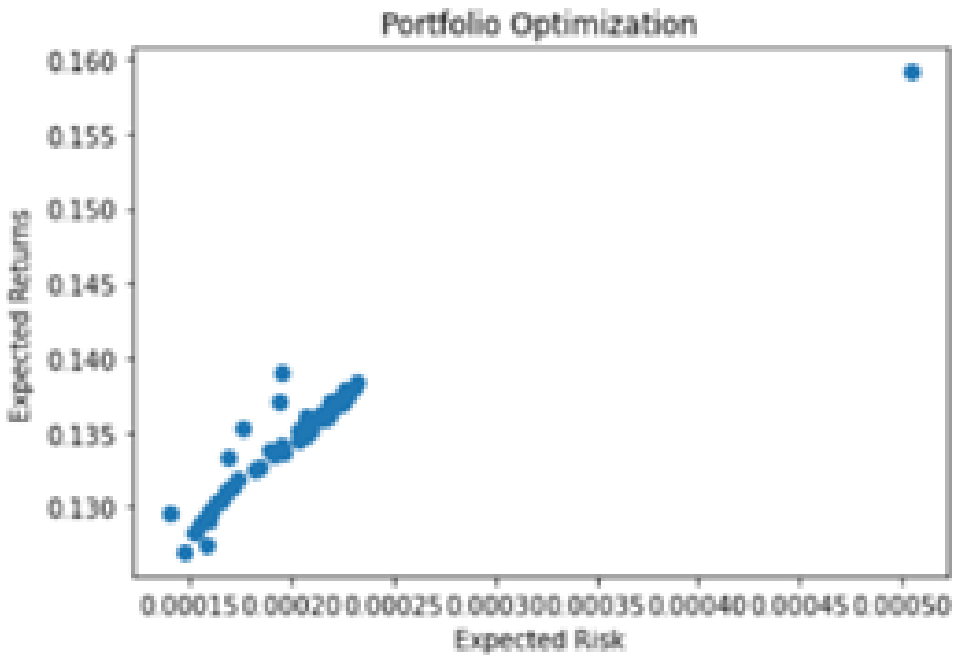
Expected returns - 17.99140 %
risk - 0.0003015

- Population Size = 200



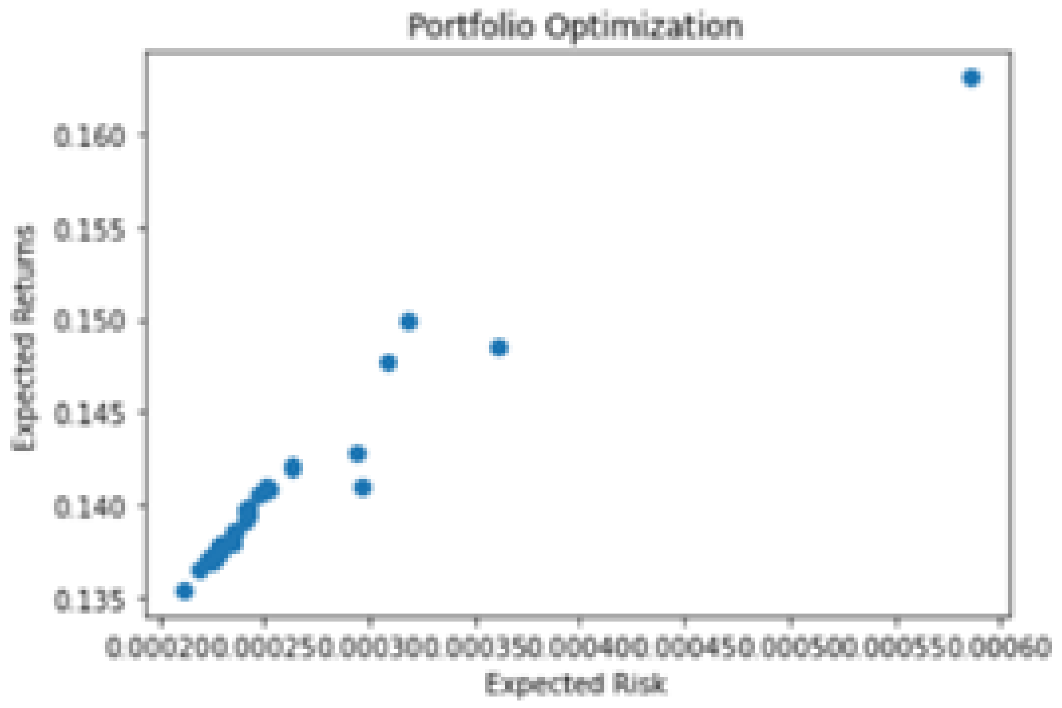
Expected returns - 13.74572 %
risk - 0.0002273

- Population SIze = 300



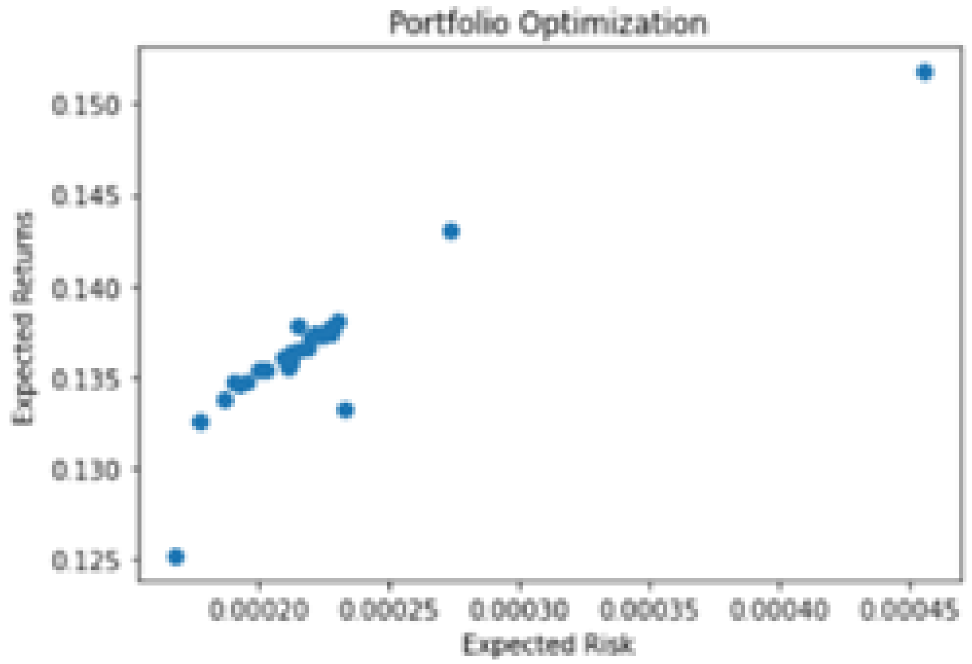
Expected returns - 13.74740 %
risk - 0.0002272

* Mutation probability = 0.3 and Crossover probability = 0.7



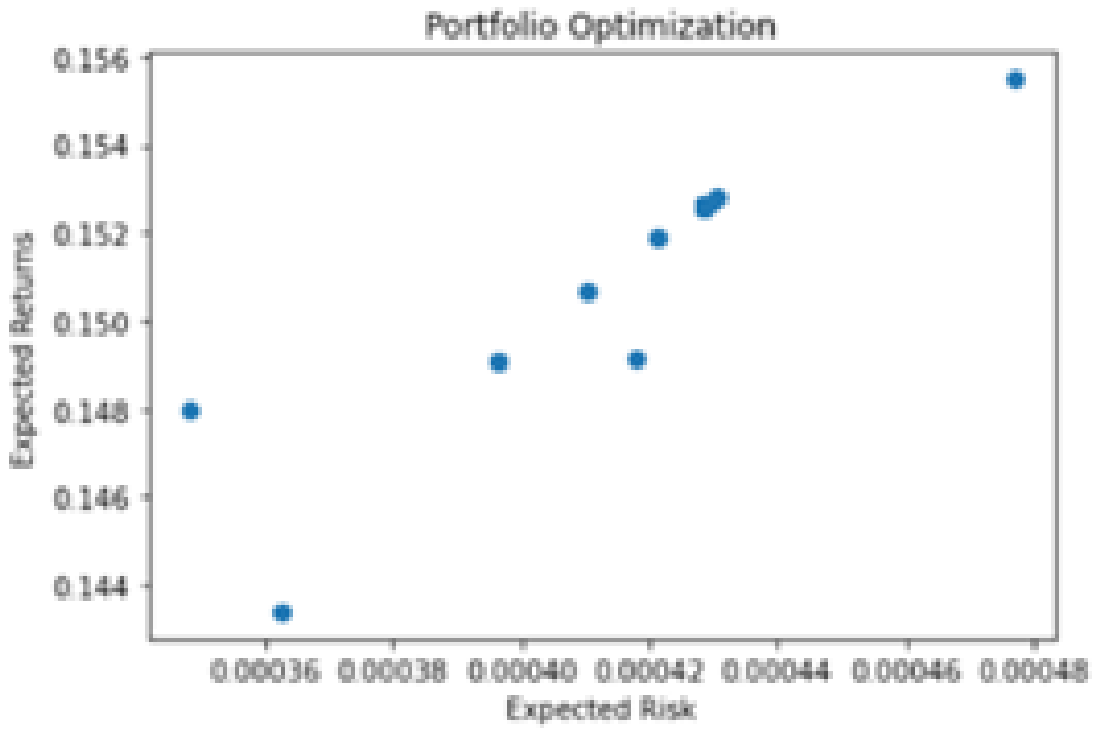
Expected returns - 13.77101 %
risk - 0.0002291

* Mutation probability = 0.5 and Crossover probability = 0.5



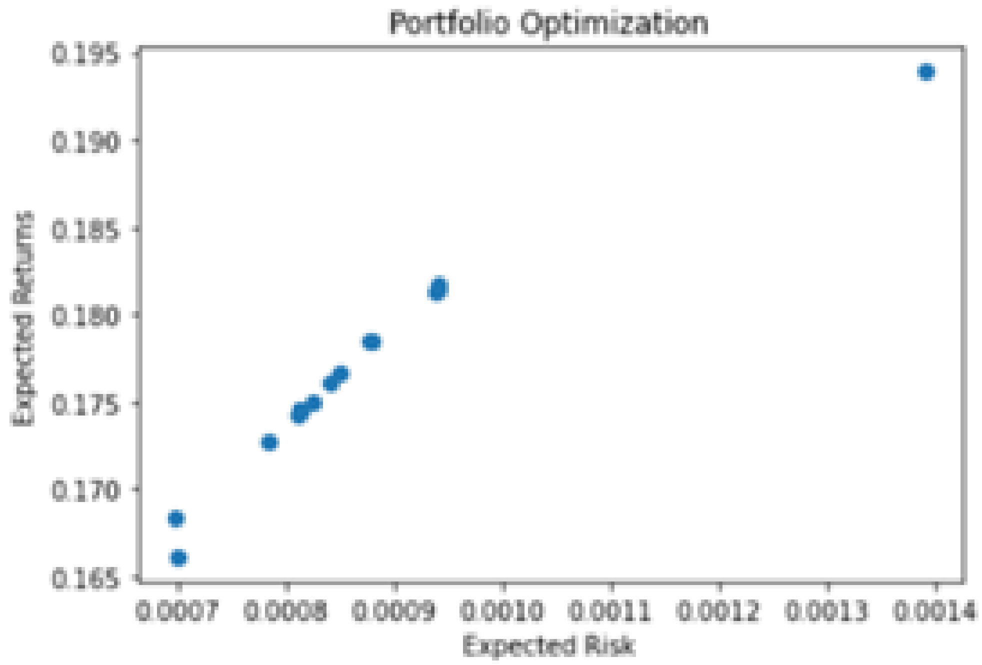
Expected returns - 13.74656 %
risk - 0.0002274

- Mutation probability = 0.4 and Crossover probability = 0.6



Expected returns - 15.26039 %
risk - 0.0004285

- Mutation probability = 0.6 and Crossover probability = 0.4



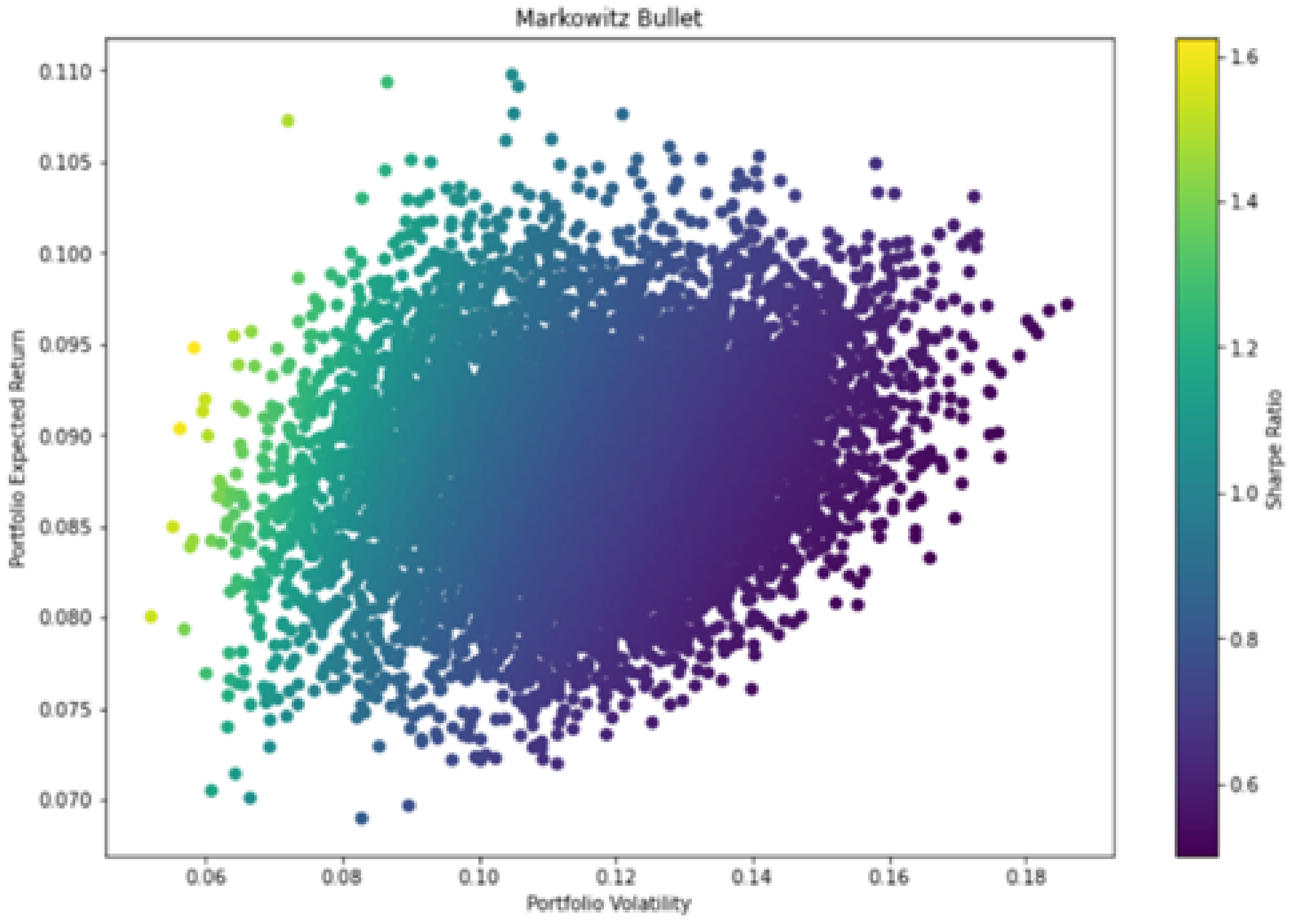
Expected returns - 17.45709 %
risk - 0.0008149

Comparing Returns

The results show that the GA is a better method to work with a complex portfolio optimization problem because of the constraints involved.

It is because GA can incorporate additional factors such as weight constraint, objective function constraint, and other limitations discussed in the previous sections. In terms of expected returns, both methods aim to maximize the portfolio's expected return. However, GA may be more effective in generating higher expected returns since it can incorporate additional constraints and objective functions that may impact expected returns.

Markowitz Bullet is a graphical representation of the set of efficient portfolios that can be generated through MVPO. Efficient portfolios offer the highest expected return for a given level of risk or the lowest risk for a given level of expected return.



Portfolio Expected Return: 8.833333333333333%
Portfolio Variance: 0.0004946111111111108

The code uses around 10000 iterations to generate the bullet-shaped figure with all the possible portfolios. The covariance matrix was provided to the code directly from the generated one in the genetic algorithm code.

We can see that even the mean-variance portfolio optimization generates a portfolio with around 12 to 13 % returns, like the arithmetic genetic algorithm, which shows it is a possible portfolio. Still, both methods use different approaches to optimize a portfolio leading to other choices for the optimal portfolio

Conclusion

we compared the performance of a Genetic Algorithm (GA) with Heuristic and Arithmetic Crossover and Mean-Variance Optimization (MVO) for a Portfolio Optimization Problem. The aim was to find an optimal portfolio of stocks with maximum return and minimum risk using historical financial data.

Our experiments revealed that Arithmetic Crossover performed the best among the tested algorithms in generating more efficient portfolios with higher returns and lower risk. However, the results suggest that GA with Heuristic Crossover and MVO are promising approaches for solving portfolio optimization problems.

Arithmetic Crossover outperformed the other algorithms in this specific dataset, it is essential to note that the performance of the algorithms may vary in different universes or datasets. Further investigation is required to evaluate these algorithms' robustness and performance on various datasets.

Thank You