MATH 137 Fall 2020: Practice Assignment MT (Practice Assignment Midterm)

Q01. You leave for Montreal on Friday morning at 10am. You eventually make it to your destination at 6pm. After a long weekend of partying you leave the next Monday morning at 10am taking the exactsame route and arriving back in Waterloo at 6pm. Show that there is at least one time between 10am and 6pm where you will be at the same spot on the road on Friday and Monday.

Proof. $\square$

Q02. Consider the polynomial $p(x) = x^6 + x - 1$.

(a) Prove that the polynomial has a root between $x = 0$ and $x = 1$.

(b) Find an interval of length $\frac{1}{2}$ that contains a root of $p(x)$.

Q03. For each function below, determine if the EVT applies on the given domain. If it does, find the points where f achieves its global max or global min. If the EVT does not apply, then determine if f achieves its global max/min or not.

(a) $f(x) = \begin{cases} x^2 & 0 \leq x \leq 2 \\ (x-4)^2 & 2 < x \leq 3 \end{cases}$

(b) $f(x) = \begin{cases} e^x & x < 0 \\ e^{-x} & x \geq 0 \end{cases}$

(c) $f(x) = 4$ for $x \in [1, 6]$

(d) $f(x) = \begin{cases} \sin x & 0 \leq x < \frac{\pi}{2} \\ \cos\left(x - \frac{\pi}{2}\right) & \frac{\pi}{2} < x \leq \pi \end{cases}$

(e) $f(x) = \dfrac{|x|}{x}$ for $x \in [-2, 2]$

Q04. Let f and g be functions that are continuous on $[a, b]$. For each statement below argue why it is true or provide a counterexample.

(a) If $h = f + g$ then h achieves its global max and min on $[a, b]$.

(b) If $h = fg$ then h achieves its global max and min on $[a, b]$.

(c) If $h = \dfrac{f}{g}$ then h achieves its global max and min on $[a, b]$.

(d) If $h = g \circ f$ then h achieves its global max and min on $[a, b]$.

Q05. For the displacement function, $s(t) = 9.8t^2 + 30t + 12$, determine the instantaneous velocity at $t = 1$.

Q06. For each of the following, determine the value of $f'(a)$ using the definition of the derivative.

(a) $f(x) = a^4$, $a = 2$

(b) $f(x) = \sqrt{x-2}$, $a = 9$

(c) $f(x) = \dfrac{3}{x^2+7}$, $a = -3$

(d) $f(x) = \dfrac{1}{\sqrt{x-3}+\sqrt{x-2}}$, $a = 7$ (bonus)

Q07. Determine the equation of the tangent line to the graph of $f(x) = -4x^3 + 3x - 1$ at $x = 2$ using the definition of the derivative.

Q08. Show that $f(x) = |\sin x|$ is continuous but not differentiable for any $x \in \{k\pi : k \in \mathbb{Z}\}$.