COMP2211 Exploring Artificial Intelligence

K-Nearest Neighbor Classifier

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Problem

- Suppose we have the height, weight and T-shirt size of some customers as follows:

Height (in cm)	158	158	158	160	160	163	163	160	163
Weight (in kg)	58	59	63	59	60	60	61	64	64
T-shirt Size	M	M	M	M	M	M	M	L	L

Height (in cm)	165	165	165	168	168	168	170	170	170
Weight (in kg)	61	62	65	62	63	66	63	64	68
T-shirt Size	L	L	L	L	L	L	L	L	L

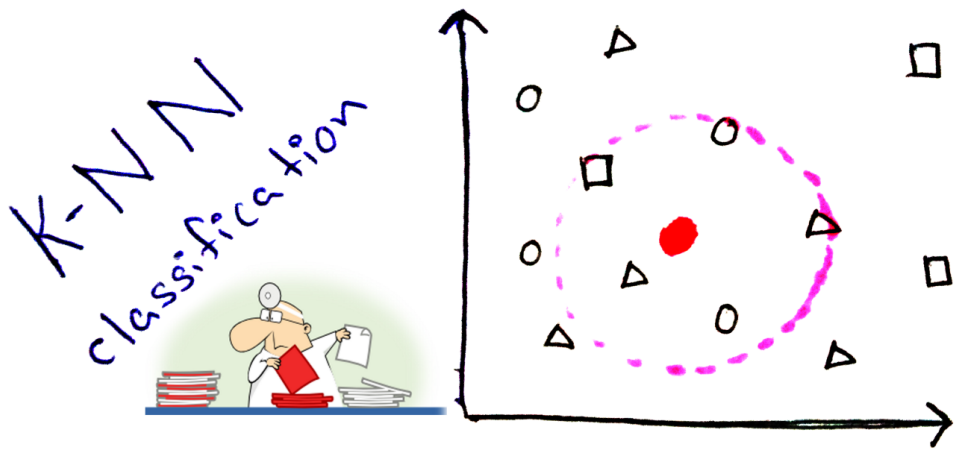
- Can we predict the T-shirt size of a new customer given only the height and weight information we have?

Questions: Can we solve it using Naive Bayes Classifier? How?
What is the problem of Naive Bayes model in this task?

K-Nearest Neighbor Model

K-Nearest Neighbor

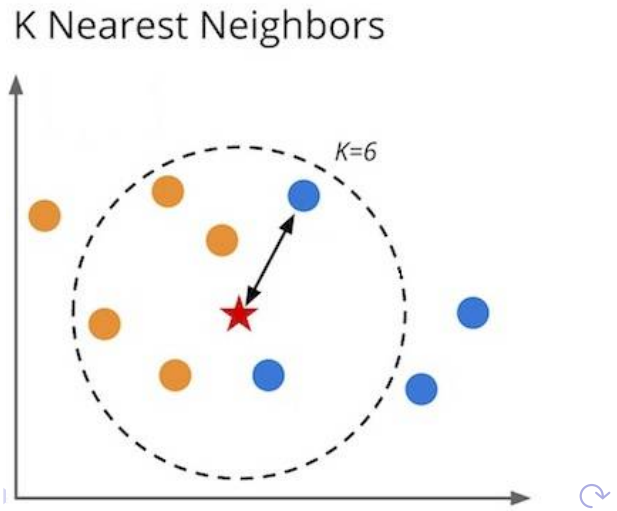
- K-Nearest Neighbor (aka KNN) is one of the most used machine learning algorithms due to its simplicity.
- KNN is a lazy learning (why lazy?), non-parametric algorithm, as it does not make any assumptions on the data being studied (e.g. the distribution of the data).
- It uses data with several classes to predict the classification of the new sample point. In other words, it captures information of all training data and classifies the new data based on



Steps

1. Prepare training data and test data.
2. Select a value K.
3. Determine which distance function is to be used.
4. Compute the distance of the new data to its n training samples.
5. Sort the distances obtained and take the K-nearest data samples.
6. Assign the test sample to the class based on the majority vote of its K nearest neighbors.

Question: Why do we call KNN lazy learning? Which of the above steps are learning (training)? What steps are testing?



Parametric Model vs. Non-parametric Model

	Parametric Model	**Non-parametric Model**
Assumption	Relies on specific assumptions about the underlying distribution of the population being studied. Typically assume that the data follows a known probability distribution, and estimate the parameters of this distribution using the available data.	Does not rely on specific assumptions about the underlying distribution of the population being studied.
Basic idea	There is a set of fixed parameters that are used to determine a probability model that is used in Machine Learning as well.	No need to make any assumption of parameters for the given population or the population we are studying. There is no fixed set of parameters are available.
Example	Naïve Bayes: classifies data points based on Bayes' theorem and assuming conditional independence between features.	KNN: classifies data points based on the k nearest neighbours.

Step 1: Prepare Training Data and Test Data & Step 2: Select a Value K

- Training data

Height (in cm)	158	158	158	160	160	163	163	160	163
Weight (in kg)	58	59	63	59	60	60	61	64	64
T-shirt Size	M	M	M	M	M	M	M	L	L

Height (in cm)	165	165	165	168	168	168	170	170	170
Weight (in kg)	61	62	65	62	63	66	63	64	68
T-shirt Size	L	L	L	L	L	L	L	L	L

- Testing data

Height: 161cm and weight: 61kg

- Let's pick $K = 5$

Step 3: Determine Which Distance Function to Use

- Let $\mathbf{x^{Train}}$ be a training data, which has n attributes:

$$\mathbf{x^{Train}} = \{x_1^{Train}, x_2^{Train}, \ldots, x_n^{Train}\}$$

and $\mathbf{x^{Test}}$ be a testing data, which also has n attributes:

$$\mathbf{x^{Test}} = \{x_1^{Test}, x_2^{Test}, \ldots, x_n^{Test}\}$$

- Assume Euclidean distance is used

$$distance(\mathbf{x^{Train}}, \mathbf{x^{Test}}) = \sqrt{\sum_{i=1}^{n}(x_i^{Train} - x_i^{Test})^2}$$

Step 4: Compute the Distance of the New Data to Its n Training Samples

Testing data:
Height: 161cm and weight: 61kg

Height (in cm)	158	158	158	160	160	163	163	160	163
Weight (in kg)	58	59	63	59	60	60	61	64	64
T-shirt Size	M	M	M	M	M	M	M	L	L
Distance between the training data and testing data	4.24	3.61	3.61	2.24	1.41	2.24	2.00	3.16	3.61

Height (in cm)	165	165	165	168	168	168	170	170	170
Weight (in kg)	61	62	65	62	63	66	63	64	68
T-shirt Size	L	L	L	L	L	L	L	L	L
Distance between the training data and testing data	4.00	4.12	5.66	7.07	7.28	8.60	9.22	9.49	11.40

Step 5: Sort the Distances Obtained and Take the K-nearest Data Samples

$$K = 5$$

Height (in cm)	160	163	160	163	160	158	158	163	165
Weight (in kg)	60	61	59	60	64	59	63	64	61
T-shirt Size	M	M	M	M	L	M	M	L	L
Distance between the training data and testing data	1.41	2.00	2.24	2.24	3.16	3.61	3.61	3.61	4.00

Height (in cm)	165	158	165	168	168	168	170	170	170
Weight (in kg)	62	58	65	62	63	66	63	64	68
T-shirt Size	L	M	L	L	L	L	L	L	L
Distance between the training data and testing data	4.12	4.24	5.66	7.07	7.28	8.60	9.22	9.49	11.40

Step 6: Assign the Test Sample to the Class Based On The Majority Vote of Its K Nearest Neighbors

Height (in cm)	160	163	160	163	160
Weight (in kg)	60	61	59	60	64
T-shirt Size	M	M	M	M	L
Distance between the training data and testing data	1.41	2.00	2.24	2.24	3.16

- Among the 5 customers, 4 of them had "Medium T-shirt Sizes" and 1 had "Large T-shirt Size".
- So, we classify the test data to "Medium T-shirt", i.e., the best guess for the customer with height = 161cm and weight = 61kg is "Medium T-shirt Size".

KNN Implementation using Scikit-Learn

```
from sklearn.preprocessing import LabelEncoder      # Import LabelEncoder function
from sklearn.neighbors import KNeighborsClassifier # Import KNeighborsClassifier function
import numpy as np  # Import NumPy

# Assign features and label variables
height = np.array([158, 158, 158, 160, 160, 163, 163, 160, 163, 165, 165, 165, 168, 168,
weight = np.array([58, 59, 63, 59, 60, 60, 61, 64, 64, 61, 62, 65, 62, 63, 66, 63, 64, 68
size = np.array(['M','M','M','M','M','M','M','L','L','L','L','L','L','L','L','L','L','L']

# Create LabelEncoder that used to convert string labels to numbers
encoder = LabelEncoder()
# Convert string labels into numbers
label = encoder.fit_transform(size)
# Combine height and weight into single list of tuples
features = list(zip(height,weight))

# Create a KNN classifier and set K=5
model = KNeighborsClassifier(n_neighbors=5)
# Train the model using the training sets
model.fit(features, label)

# Predict output
predicted = model.predict([[161,61]]) # height = 161, weight = 61

# Convert number to string label
predicted = encoder.inverse_transform(predicted)
print(predicted)
```

Standardization

- When the training data are measured in different units, it is important to standardize variables (i.e. the attributes) before calculating distance.
- For example, if one attribute is based on height in cm, and the other is based on weight in kg, then height will influence more on the distance calculation.
- In order to make the attributes comparable, we need to standardize them which can be done by the following method:

$$X_{new} = \frac{X - mean}{standard\ deviation}$$

where X is the attribute value, X_{new} is the standardized attribute value, $mean$ is the mean attribute value of all training data, and $standard\ deviation$ is the standard deviation of the attribute value of all the training data.

- Mean of height = 164, Standard deviation of height = 4.33
- Mean of Weight = 62.33, Standard deviation of weight = 2.63
- After standardization using $X_{new} = \frac{X - mean}{standard\ deviation}$, the 5th closest value got changed as height was dominating earlier before standardization.

Testing data: Height: 161cm and weight: 61kg

Height (in cm)	160	163	160	163	160	158	158	163	165
Weight (in kg)	60	61	59	60	64	59	63	64	61
T-shirt Size	M	M	M	M	L	M	M	L	L
Distance between the training data and testing data	1.41	2.00	2.24	2.24	3.16	3.61	3.61	3.61	4.00
Distance between the training data and testing data (standardization)	0.44	0.46	0.60	0.79	1.16	1.03	1.03	1.23	0.92

Height (in cm)	165	158	165	168	168	168	170	170	170
Weight (in kg)	62	58	65	62	63	66	63	64	68
T-shirt Size	L	M	L	L	L	L	L	L	L
Distance between the training data and testing data	4.12	4.24	5.66	7.07	7.28	8.60	9.22	9.49	11.40
Distance between the training data and testing data (standardization)	1.00	1.33	1.78	1.66	1.79	2.49	2.22	2.37	3.37

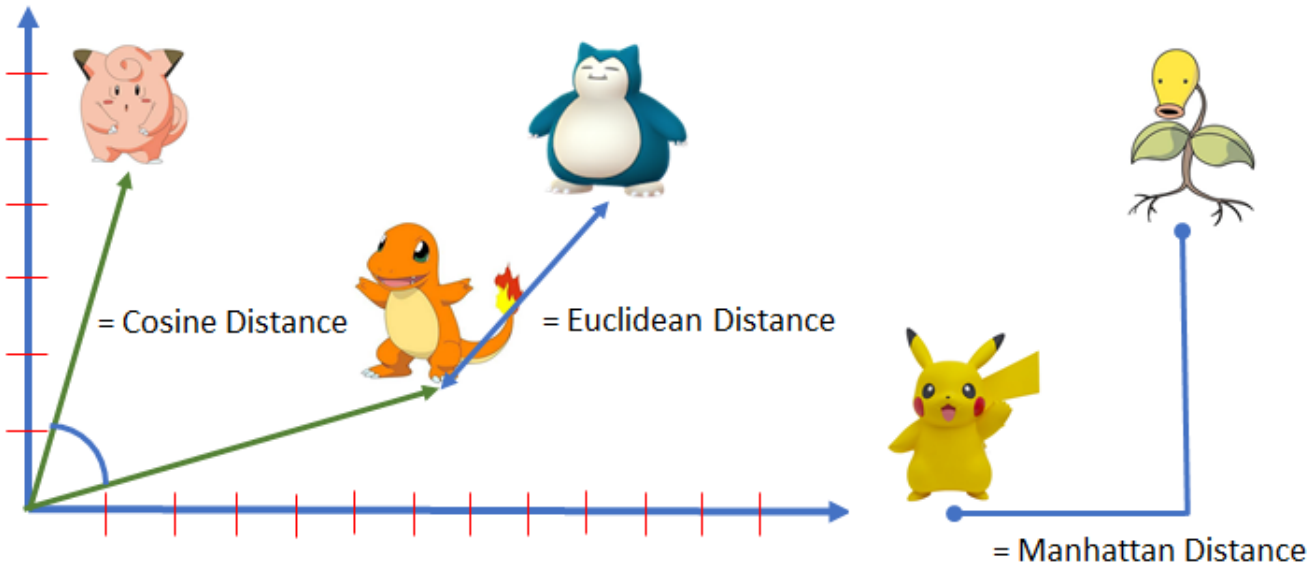
KNN Implementation using Scikit-Learn (Standardization)

```
from sklearn.preprocessing import LabelEncoder      # Import LabelEncoder function
from sklearn.neighbors import KNeighborsClassifier # Import KNeighborsClassifier function
import numpy as np                                  # Import NumPy

# Assign features and label variables
height = np.array([158, 158, 158, 160, 160, 163, 163, 160, 163, 165, 165, 165, 168, 168,
h_mean = np.mean(height); h_std = np.std(height,ddof=1) # ddof=1 is to make the divisor t
height = (height - h_mean)/h_std
weight = np.array([58, 59, 63, 59, 60, 60, 61, 64, 64, 61, 62, 65, 62, 63, 66, 63, 64, 68
w_mean = np.mean(weight); w_std = np.std(weight,ddof=1) # ddof=1 is to make the divisor t
weight = (weight - w_mean)/w_std
size = ['M','M','M','M','M','M','M','L','L','L','L','L','L','L','L','L','L','L']

# Create LabelEncoder that used to convert string labels to numbers
encoder = LabelEncoder()
# Convert string labels into numbers
label = encoder.fit_transform(size)
# Combine height and weight into single list of tuples
features = list(zip(height,weight))
# Create a KNN classifier and set K=5
model = KNeighborsClassifier(n_neighbors=5)
# Train the model using the training sets
model.fit(features, label)

# Predict output
predicted = model.predict([[(161-h_mean)/h_std,(61-w_mean)/w_std]]) # height = 161, weigh
# Convert number to string label
predicted = encoder.inverse_transform(predicted)
```

How to Select Distance Functions?

Distance Functions

- There are a lot of different distance functions available.
- Some common ones are:
  - Euclidean distance (The one we used earlier)
  - Manhattan distance (aka City block distance)
  - Cosine distance
  - Hamming distance



Manhattan Distance

Manhattan distance (City-block distance)

The sum of the absolute differences between points across all the dimensions.

$$distance(\mathbf{x^{Train}}, \mathbf{x^{Test}}) = \sum_{i=1}^{n} |x_i^{Train} - x_i^{Test}|$$

where $|\cdot|$ denote absolute value.

- Example:
  Training data: (Height, Weight) = (160, 60), Testing data: (Height, Weight) = (163, 61)

$$distance = |160 - 163| + |60 - 61| = 4$$

Question: When Manhattan distance is preferred over Euclidean distance?

Cosine Distance

$$cos\theta = \frac{\mathbf{x^{Train}} \cdot \mathbf{x^{Test}}}{|\mathbf{x^{Train}}| \times |\mathbf{x^{Test}}|} = \frac{\sum_{i=1}^{n}(x_i^{Train} \times x_i^{Test})}{\sqrt{\sum_{i=1}^{n}(x_i^{Train})^2}\sqrt{\sum_{i=1}^{n}(x_i^{Test})^2}}$$

$$Distance = 1 - cos\theta$$

- Example:
  Training data: (Height, Weight) = (160, 60), Testing data: (Height, Weight) = (163, 61)

$$cos\theta = \frac{(160 \times 163) + (60 \times 61)}{\sqrt{160^2 + 60^2}\sqrt{163^2 + 61^2}} = 0.99999977$$

$$Distance = 1 - 0.99999977 = 2.26 \times 10^{-7}$$

Remark

Using Cosine distance, we get values between 0 and 2, where 0 means the training data and test data are 100% similar to each other, 2 means they are absolutely different, and values between 0 and 2 mean varying degrees of dissimilarity.

Hamming Distance

- Hamming distance is a way for comparing two binary data strings.
- While comparing two binary strings of equal length, Hamming distance is the number of bit positions in which the two bits are different.
- Example: Suppose there are two strings 1101 1001 and 1001 1101, then the hamming distance between the two strings is 2.

1	1	0	1	1	0	0	1
1	0	0	1	1	1	0	1
	+1				+1		

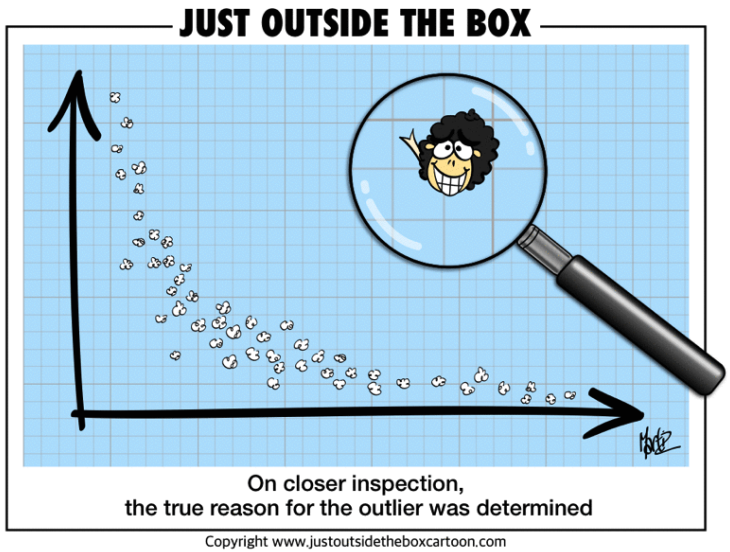
Hamming distance is for categorical data.

You will encounter bit strings when you one-hot encode categorical columns of data. More details will be given in a later topic.

How to Select K?

Outlier

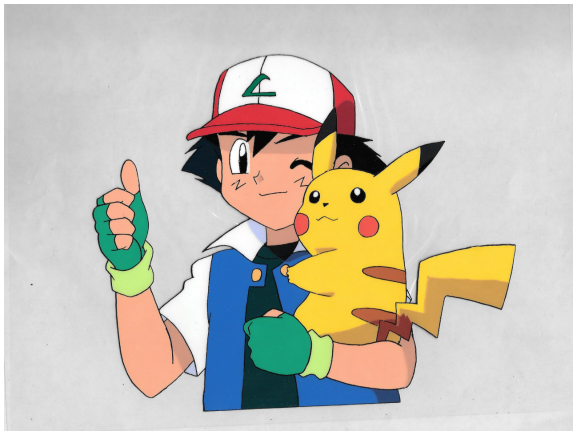
- A Low K-value is sensitive to outliers and a higher K-value is more resilient to outliers as it considers more voters to decide prediction.



Good Value of K

- To break a tie,
  - Decrease K by 1 until we have broken the tie.
  - Put more weight for the nearest points than the farther points.
- Should not be too small or too large.

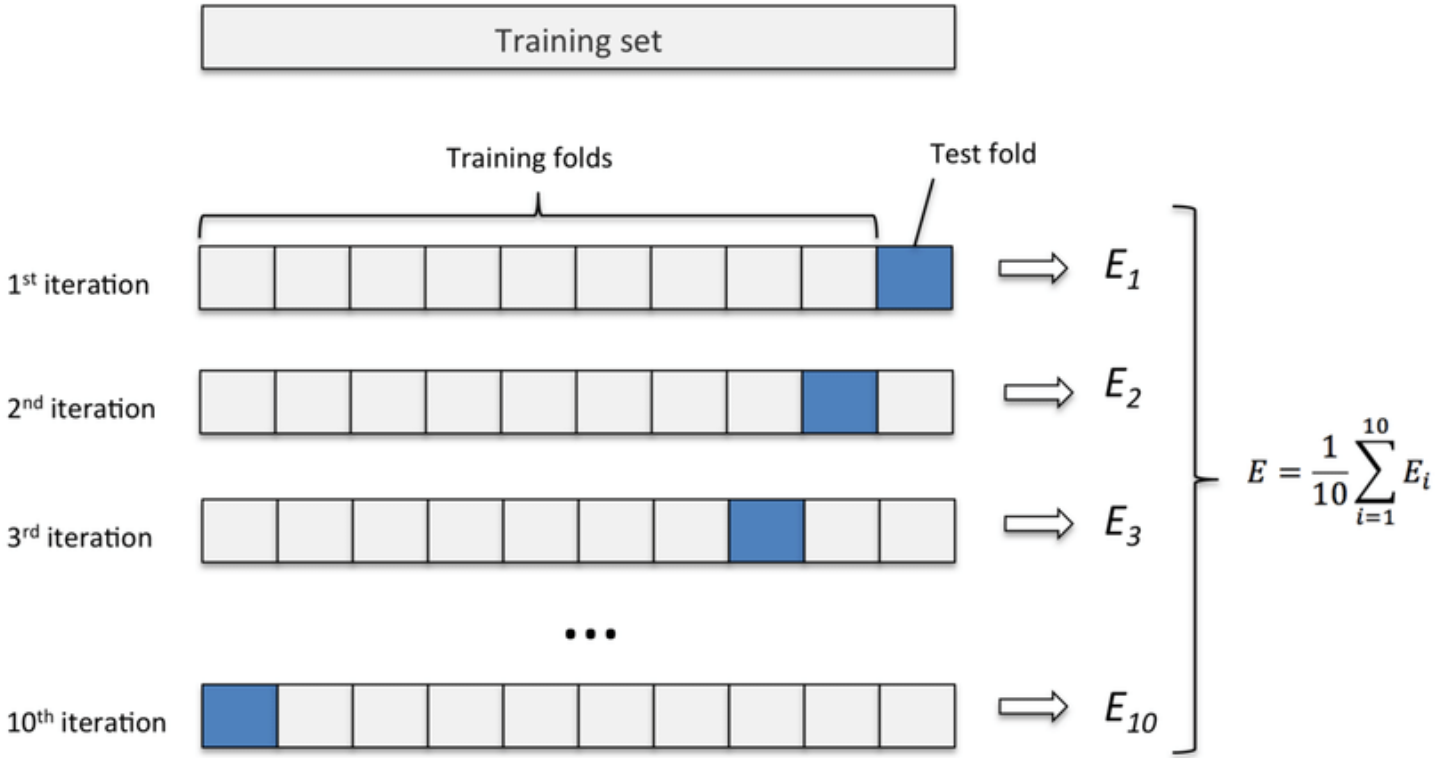
Rule of thumb: Set $K = \sqrt{N}$, where N is the number of training examples in the dataset.
(An explanation will be given in a supplementary notes.)



Using Cross-validation to Estimate K

- Suppose we select K using d-fold cross validation algorithm.
- Input of d-fold cross-validation algorithm:
  - A KNN model
  - A list of candidate K values: $[K_1, K_2, \ldots, K_q]$
  - A training dataset consisting of N data samples
  - The number of folds: d
- Output of d-fold cross-validation algorithm: the best K value among $[K_1, K_2, \ldots, K_q]$
- Steps:
  1. Split the training data into d groups, or folds, of approximately equal size.
  2. Hold the first group. This is called the validation set.
  3. Train your classifier on the remaining data.
  4. For each value of K
     - Classify each data in the validation set, using its K-nearest neighbors in the training set.
     - Record the error.
  5. Repeat steps 2-4 for all d choices of the validation set.
  6. For each choice of K, find the average error across validation sets. Choose the value of K with the lowest error.

Using Cross-validation to Estimate K

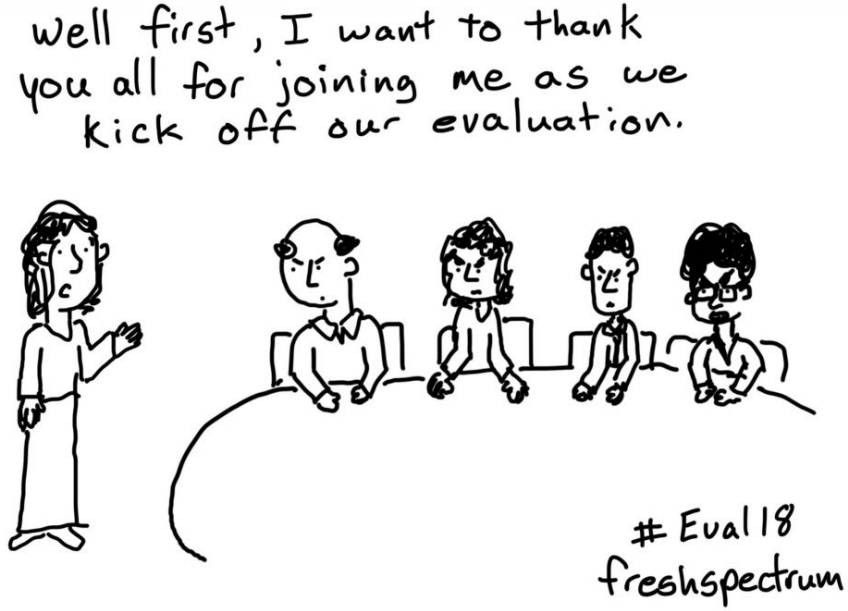


Question: How many errors are computed during the cross-validation process? How to evaluate the model, i.e., measure the error?

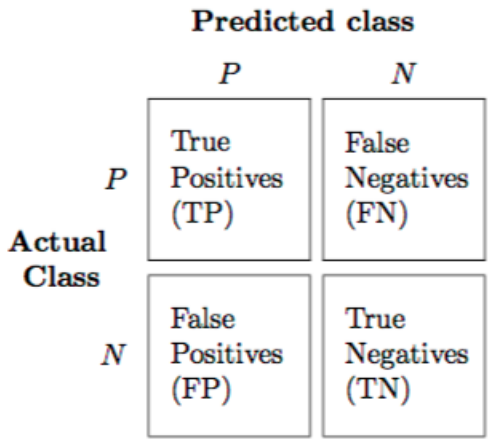
Model Evaluation

Model Evaluation for Classification Problems

- Confusion Matrix
- Precision, Recall, and F1 Score
- Matthew's Correlation Coefficient (MCC)



2×2 Confusion Matrix for Binary Classification



- $Error = \frac{FP+FN}{FP+FN+TP+TN} = 1 - Accuracy$
- $Accuracy = \frac{TP+TN}{FP+FN+TP+TN} = 1 - Error$

- True Positive (TP): It refers to the number of predictions where the classifier correctly predicts the positive class as positive.
- True Negative (TN): It refers to the number of predictions where the classifier correctly predicts the negative class as negative.
- False Positive (FP): It refers to the number of predictions where the classifier incorrectly predicts the negative class as positive.
- False Negative (FN): It refers to the number of predictions where the classifier incorrectly predicts the positive class as negative.

Precision and Recall

- Precision: It tells us what fraction of predicts as a positive class were actually positive. To calculate precision, use the following formula:

$$Precision = \frac{TP}{TP + FP}$$

- Recall: It tells us what fraction of positive samples were correctly predicted as positive by the classifier. To calculate precision, use the following formula:

$$Recall = \frac{TP}{TP + FN}$$

Precision and Recall

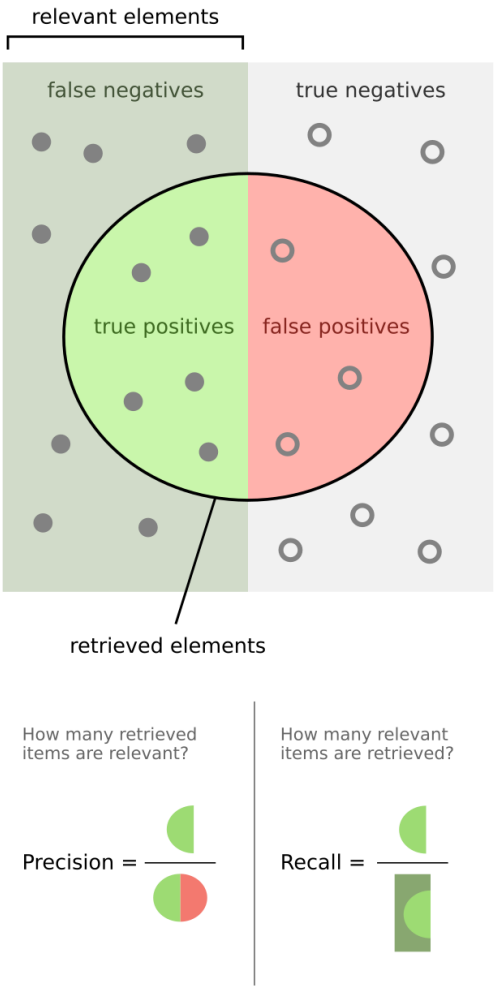


Figure source: Wikipedia

- Precision tells us how much we can trust the model when it predicts an individual sample as Positive.

$$Precision = \frac{TP}{TP + FP}$$

- Recall measures the ability of the model to find all the Positive samples in the dataset.

$$Recall = \frac{TP}{TP + FN}$$

Note: The model may have high precision while low recall, or high recall while low precision. Imagine that we simply classify all data to positive, what will be the recall? How about the precision?

F1-score

- F1-score: It combines precision and recall into a single measure.
- Mathematically, it is the harmonic mean of precision and recall.

$$F1 - score = 2 \times \frac{Precision \times Recall}{Precision + Recall} = \frac{2TP}{2TP + FP + FN}$$

- In a perfect world, we want a model that has a precision of 1 and a recall of 1. That means a F1-score of 1, i.e., a 100% accuracy which is often not the case for a machine learning model.

Matthew's Correlation Coefficient Formula

- Matthew's Correlation Coefficient Formula (aka MCC) is a best single-value classification metric which helps to summarize the confusion matrix or an error matrix.
- MCC is calculated by the formula

$$MCC = \frac{TN \times TP - FN \times FP}{\sqrt{(TP + FP)(TP + FN)(TN + FP)(TN + FN)}}$$

- If the prediction returns good rates for all 4 of these entities (i.e., TP, TN, FP, FN), it is said to be a reliable measure producing high scores.
- To suit most correlation coefficients, MCC also ranges between $+1$ and -1 as:
  - $+1$ is the best agreement between the predicted and actual values.
  - 0 is no agreement. Meaning that prediction is random according to the actuals.

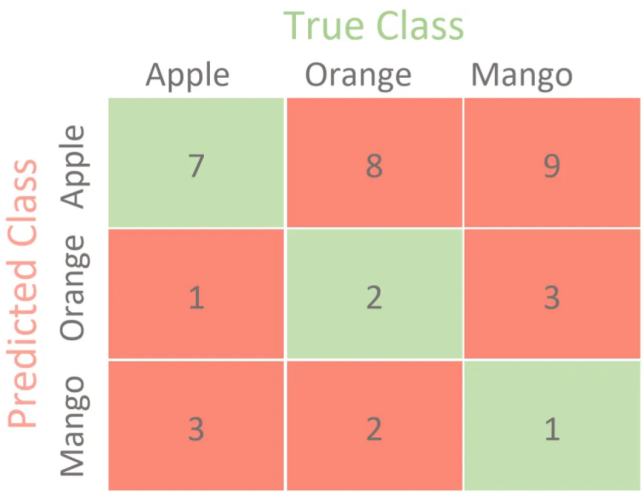
Example of MCC

- Confusion matrix with entries:
  - TP $= 90$
  - FP $= 4$
  - TN $= 1$
  - FN $= 5$
- When we substitute these values in the formula:

$$MCC = \frac{1 \times 90 - 5 \times 4}{\sqrt{(90+4)(90+4)(1+4)(1+5)}}$$
$$= 0.14$$

- 0.14 means the classifier is very close to a random guess classifier.
- Hence, it seems that the MCC helps us to identify the ineffectiveness of the classifier in classifying especially the negative class samples.

Confusion Matrix for Multi-class Classification



- Unlike binary classification, there are no positive or negative class.

- To find TP, TN, FP and FN for each individual class, let's take class Apple as an example.
  - TP means both actual and predicted values are apple. So, TP $= 7$.
  - TN means both the actual and predicted values are non-apple. So, TN $= 1 + 2 + 2 + 3 = 8$.
  - FP means the predicted value is apple, but the actual value is non-apple. So, $8 + 9 = 17$.
  - FN means the predicted value is non-apple, but the actual value is apple. So, $1 + 3 = 4$.
- Precision $= \frac{TP}{TP+FP} = \frac{7}{7+17} = 0.29$
- Recall $= \frac{TP}{TP+FN} = \frac{7}{7+4} = 0.64$
- F1-score $= \frac{2TP}{2TP+FP+FN} = \frac{2(7)}{2(7)+17+4} = 0.4$

Confusion Matrix for Multi-class Classification

- Similarly, we can calculate the measures for the other classes.

Class	Precision	Recall	F1-score
Apple	0.29	0.64	0.40
Orange	0.33	0.17	0.22
Manago	0.17	0.08	0.11

Question: How to combine the F1-score of each class to have a single measure for the whole model?

- Micro F1
- Macro F1
- Weighted F1

Micro F1

- Micro F1 is also called micro-averaged F1-score.
- It is calculated by considering the total TP, total FP and total FN of the model.
- It does not consider each class individually, it calculates the metrics globally.
- For our example:
  - Total TP = 7+2+1 = 10
  - Total FP = (8+9) + (1 + 3) + (3 + 2) = 26
  - Total FN = (1+3) + (8 + 2) + (9+3) = 26
  - Precision = 10/(10 + 26) = 0.28
  - Recall = 10/(10+26) = 0.28
  - Micro F1 = $\frac{2TP}{2TP+FP+FN} = \frac{2(10)}{2(10)+26+26} = 0.28$
- Now, you can see that we are calculating the metrics globally, all the measures become equal, i.e.,
  Precision = Recall = Micro F1 = Accuracy.

Macro F1

- Macro F1 is also called macro-averaged F1-score.
- It calculates metrics for each class individually and then take unweighted mean of the measures.
- According to the data in the table shown earlier:
  - Apple's F1-score $= 0.40$
  - Orange's F1-score $= 0.22$
  - Mango's F1-score $= 0.11$
- Hence, macro F1 $= (0.40 + 0.22 + 0.11)/3 = 0.24$

Weighted F1

- Weighted F1 is also called weighted-averaged F1-score.
- Unlike Macro F1, it takes a weighted mean of the measures.
- The weights for each class are the total number of samples of that class.
- Since we had 11 Apples, 12 Oranges, and 13 Mangoes, so
  Weighted F1 $= \frac{((0.4*11)_(0.22*12)+(0.11*13))}{(11+12+13)} = 0.24$

Error Measurement for Regression Problems

- There are many ways to measure errors. Here are a few examples:
  - Mean Absolute Error (MAE)

$$MAE = \frac{\sum_{i=1}^{m} |a_i - p_i|}{m}$$

  - Mean Square Error (MSE)

$$MSE = \frac{\sum_{i=1}^{m} (a_i - p_i)^2}{m}$$

  - Mean Absolute Percentage Error (MAPE)

$$MAPE = \frac{\sum_{i=1}^{m} \left| \frac{a_i - p_i}{a_i} \right|}{m}$$

where $(a_1, a_2, \ldots, a_m)$ are the actual labels and $(p_1, p_2, \ldots, p_m)$ are the predicted labels.

Analysis on KNN

Pros and Cons

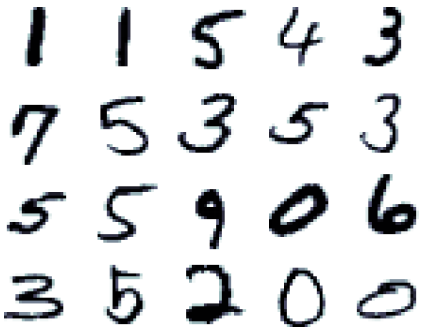
- Pros:
  - Easy to understand.
  - No assumptions about data.
  - Can be applied to both classification and regression.
    Note: For KNN regression, the resulting value can be calculated by taking the average of the output value for the top K nearest neighbors.
  - Works easily on multi-class problems.
- Cons:
  - Memory intensive/Computationally expensive.
  - Sensitive to scale of data.
  - Struggle when high number of attributes.
  - Does not work well with categorical features since it is difficult to find the distance between dimensions with categorical features.

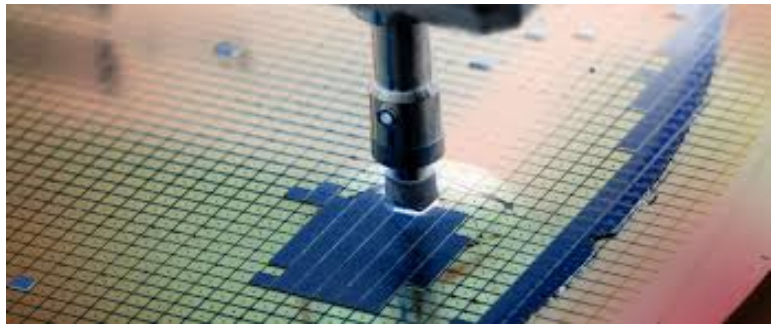
Speed Up KNN

- Reduce the dimension of training data (e.g., using Principle-Component Analysis).
- Use good data structures to store the data, e.g., KD-tree.
- Parallelizing the distance computations.

KNN Applications

- Handwritten character classification
- Fast content-based image retrieval
- Intrusion detection
- Fault detection for semiconductor manufacturing processes
- ...





Practice Problem

- Suppose we have age and salary of some customers, as well as whether they purchased certain item or not.

Age	19	35	26	27	19	27	27	32	25	35	26
Salary (k)	19	20	43	57	76	58	84	150	33	65	80
Purchased	No	No	No	No	No	No	No	Yes	No	No	No

Age	26	20	32	18	29	47	45	46	48	45	47
Salary (K)	52	86	18	82	80	25	26	28	29	22	49
Purchased	No	No	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes

- Can we predict whether a new customer will purchase the item given only the age and salary of the customer?

Step 1: Prepare Training Data and Test Data & Step 2: Select a Value K

- Training data

Age	19	35	26	27	19	27	27	32	25	35	26
Salary (k)	19	20	43	57	76	58	84	150	33	65	80
Purchased	No	No	No	No	No	No	No	Yes	No	No	No

Age	26	20	32	18	29	47	45	46	48	45	47
Salary (K)	52	86	18	82	80	25	26	28	29	22	49
Purchased	No	No	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes

- Testing data

Age: 20 and Salary (K): 38

- Let's pick $K = 5$

Step 3: Determine Which Distance Function to Use

- Let $\mathbf{x^{Train}}$ be a training data, which has n attributes:

$$\mathbf{x^{Train}} = \{x_1^{Train}, x_2^{Train}, \ldots, x_n^{Train}\}$$

and $\mathbf{x^{Test}}$ be a testing data, which also has n attributes:

$$\mathbf{x^{Test}} = \{x_1^{Test}, x_2^{Test}, \ldots, x_n^{Test}\}$$

- Assume Euclidean distance is used

$$distance(\mathbf{x^{Train}}, \mathbf{x^{Test}}) = \sqrt{\sum_{i=1}^{n}(x_i^{Train} - x_i^{Test})^2}$$

Step 4: Compute the Distance of the New Data to Its n Training Samples

Testing data:
Age: 20 and Salary (in k): 38

Age	19	35	26	27	19	27	27	32	25	35
Salary (k)	19	20	43	57	76	58	84	150	33	65
Purchased	No	No	No	No	No	No	No	Yes	No	No
Distance between the training data and testing data	19.03	23.43	7.81	20.25	38.01	21.19	46.53	112.64	7.07	30.8

Age	26	20	32	18	29	47	45	46	48	45
Salary (k)	52	86	18	82	80	25	26	28	29	22
Purchased	No	No	No	No	No	Yes	Yes	Yes	Yes	Yes
Distance between the training data and testing data	15.23	48.00	23.32	44.05	42.95	29.97	27.73	27.86	29.41	29.6

Step 5: Sort the Distances Obtained and Take the K-nearest Data Samples

$$K = 5$$

Age	25	26	26	19	27	27	32	35	45	46
Salary (k)	33	43	52	19	57	58	18	20	26	28
Purchased	No	No	No	No	No	No	No	No	Yes	Yes
Distance between the training data and testing data	7.07	7.81	15.23	19.03	20.25	21.19	23.32	23.43	27.73	27.86

Age	48	45	47	35	19	26	29	18	27	20
Salary (k)	29	22	25	65	76	80	80	82	84	86
Purchased	Yes	Yes	No	No	No	No	No	No	No	No
Distance between the training data and testing data	29.41	29.68	29.97	30.89	38.01	42.43	42.95	44.05	46.53	48.00

Step 6: Assign the Test Sample to the Class Based On The Majority Vote of Its K Nearest Neighbors

Age	25	26	26	19	27
Salary (k)	33	43	52	19	57
Purchased	No	No	No	No	No
Distance between the training data and testing data	7.07	7.81	15.23	19.03	20.25

- Among the 5 customers, 5 of them did not purchase the item.
- So, we classify the test data to "No", i.e., the best guess for the customer with age $= 20$ and salary $= 38$k will not purchase the item.

KNN Implementation using Scikit-Learn

```
from sklearn.preprocessing import LabelEncoder      # Import LabelEncoder function
from sklearn.neighbors import KNeighborsClassifier # Import KNeighborsClassifier function
import numpy as np  # Import NumPy

# Assign features and label variables
age = np.array([19, 35, 26, 27, 19, 27, 27, 32, 25, 35, 26, 26, 20, 32, 18, 29, 47, 45, 4
salary = np.array([19, 20, 43, 57, 76, 58, 84, 150, 33, 65, 80, 52, 86, 18, 82, 80, 25, 2
purchased = np.array(['No', 'No', 'No', 'No', 'No', 'No', 'No', 'Yes', 'No', 'No', 'No',

# Create LabelEncoder that used to convert string labels to numbers
encoder = LabelEncoder()
# Convert string labels into numbers
label = encoder.fit_transform(purchased)
# Combine height and weight into single list of tuples
features = list(zip(age,salary))

# Create a KNN classifier and set K=5
model = KNeighborsClassifier(n_neighbors=5)
# Train the model using the training sets
model.fit(features, label)

# Predict output
predicted = model.predict([[20,38]]) # age = 20, salary = 38

# Convert number to string label
predicted = encoder.inverse_transform(predicted)
print(predicted)
```

- Mean of age = 31.86, Standard deviation of age = 10.18
- Mean of salary = 53.73, Standard deviation of salary = 32.46
- After standardization using $X_{new} = \frac{X - mean}{standard\ deviation}$, the 5 closest values remain unchanged.

Testing data: Age: 20 and salary(k): 38

Age	25	26	26	19	27	27	32	35	45	
Salary (k)	33	43	52	19	57	58	18	20	26	
Purchased	No	No	No	No	No	No	No	No	Yes	
Distance between the training data and testing data	7.07	7.81	15.23	19.03	20.25	21.19	23.32	23.43	27.73	2
Distance between the training data and testing data (standardization)	0.51	0.61	0.73	0.59	0.90	0.92	1.33	1.57	2.48	

Age	48	45	47	35	19	26	29	18	27	
Salary (k)	29	22	25	65	76	80	80	82	84	
Purchased	Yes	Yes	No	No	No	No	No	No	No	
Distance between the training data and testing data	29.41	29.68	29.97	30.89	38.01	42.43	42.95	44.05	46.53	4
Distance between the training data and testing data (standardization)	2.76	2.50	2.68	1.69	1.17	1.42	1.57	1.37	1.58	

KNN Implementation using Scikit-Learn (Standardization)

```
from sklearn.preprocessing import LabelEncoder      # Import LabelEncoder function
from sklearn.neighbors import KNeighborsClassifier # Import KNeighborsClassifier function
import numpy as np                                  # Import NumPy

# Assign features and label variables
age = np.array([19, 35, 26, 27, 19, 27, 27, 32, 25, 35, 26, 26, 20, 32, 18, 29, 47, 45, 4
a_mean = np.mean(age)
a_std = np.std(age,ddof=1) # ddof=1 is to make the divisor to n-1, i.e., sample mean
age = (age - a_mean)/a_std
salary = np.array([19, 20, 43, 57, 76, 58, 84, 150, 33, 65, 80, 52, 86, 18, 82, 80, 25, 2
s_mean = np.mean(salary)
s_std = np.std(salary,ddof=1) # ddof=1 is to make the divisor to n-1, i.e., sample mean
salary = (salary - s_mean)/s_std
purchased = np.array(['No', 'No', 'No', 'No', 'No', 'No', 'No', 'Yes', 'No', 'No', 'No',
                      'No', 'No', 'No', 'No', 'No', 'Yes', 'Yes', 'Yes', 'Yes', 'Yes', 'Y

# Create LabelEncoder that used to convert string labels to numbers
encoder = LabelEncoder()
# Convert string labels into numbers
label = encoder.fit_transform(purchased)
# Combine height and weight into single list of tuples
features = list(zip(age,salary))
# Create a KNN classifier and set K=5
model = KNeighborsClassifier(n_neighbors=5)
# Train the model using the training sets
model.fit(features, label)

# Predict output
```

That's all!

Any question?

Acknowledgments

- The lecture notes are developed based on Dr. Desmond Tsoi's lecture slides.