COMP 2211 Exploring Artificial Intelligence
Supplementary Notes: A Proof of Bayes' Theorem
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Bayes' Theorem

- As mentioned in class, Bayes' theorem is defined as:

$$P(B|E) = \frac{P(B)P(E|B)}{P(E)}$$
$$= \frac{P(B)P(E|B)}{P(E|B)P(B) + P(E|NOT\ B)P(NOT\ B)}$$

- In other words, Bayes' theorem gives us the conditional probability of B given that E has occurred as long as we know $P(E|B)$, $P(E|NOT\ B)$, and $P(B) = 1 - P(NOT\ B)$.

A Proof of Bayes' Theorem

- By the definition of conditional probability:

$$P(E|B) = \frac{P(E \text{ AND } B)}{P(B)} \tag{1}$$

$$P(E \text{ AND } B) = P(E|B)P(B) \tag{2}$$

- By the law of total probability:

$$P(E) = P((E \text{ AND } B) \text{ OR } (E \text{ AND } (NOT \text{ } B)))$$
$$= P((E \text{ AND } B) + (E \text{ AND } (NOT \text{ } B))) \tag{3}$$

- By substituting Equation (2) to Equation (3):

$$P(E) = P(E|B)P(B) + P(E|NOT \text{ } B)P(NOT \text{ } B) \tag{4}$$

where on Equation (4), that fact that E AND B and E AND $(NOT \text{ } B)$ are mutually exclusive events was used.

A Proof of Bayes' Theorem

- Since *E AND B = B AND E*, by Equation (1) and (2), we have:

$$P(B|E) = \frac{P(B\ AND\ E)}{P(E)}$$
$$= \frac{P(E|B)P(B)}{P(E)} \tag{5}$$

- By merging Equation (4) to (5), we get the following:

$$P(B|E) = \frac{P(E|B)P(B)}{P(E|B)P(B) + P(E|NOT\ B)P(NOT\ B)} \tag{6}$$

Equation (6) is what we wanted to prove. :)

That's all!

Any questions?