B1:

Derivation of Differe:

Q-

$$\frac{dT}{dt} = 0.25(75-T), \quad T(0)=50$$

$T_g = T_p + T_h$

T_h: $\frac{dT}{dt} = -\frac{t}{4}$

$T = Ce^{-\frac{t}{4}}$

$T_p = ve^{-\frac{t}{4}}$

$v'e^{-\frac{t}{4}} = \frac{25}{4}$

$v = \int \frac{25}{4} e^{\frac{t}{4}}$

$v = 75e^{\frac{t}{4}}$

$T_p = 75$

$T_g = 75 + Ce^{-\frac{t}{4}}$

$T(0) = 50 = 75 + C$

$C = -25$

$T_g = 75 - 25e^{-\frac{t}{4}}$

C1: Solving Summer diffeq

$$\frac{dT}{dt} = k(M(t)-T(t)),$$
$$M(t) = M_0 - 12\cos\left[\frac{\pi(t-5)}{12}\right], \quad M_0 = 75, \ T_0 = 65$$

T_h: $\frac{dT}{dt} = -kT$
$$T = Ce^{-kt}$$

T_p: $v' e^{-kt} = k \cdot \left(75 - 12\cos\left(\frac{\pi(t-5)}{12}\right)\right)$

$$v = \int 75e^{kt} - 12k\int \cos\left(\frac{\pi(t-5)}{12}\right) e^{kt}$$

$k\int 75e^{kt} = 75e^{kt}$

$12k\int \cos\left(\frac{\pi(t-5)}{12}\right) e^{kt}$? subbing in x for simplicity

$dx = \frac{\pi}{12}$ $\quad \int x = \frac{12}{\pi}$

$$= \cos(x) e^{kt}$$

$u = \cos(x)$ $\quad du = \frac{\pi \sin(x)}{12}$

$dv = e^{kt}$ $\quad v = \frac{e^{kt}}{k}$

$$= 12\cos(x) e^{kt} + \int \pi \sin(x) e^{kt}$$

$\pi \int -\sin(x) e^{kt}$ $\quad u = -\sin x$ $\quad du = -\frac{\pi}{12}\cos x$

$dv = e^{kt}$ $\quad v = \frac{e^{kt}}{k}$

$$= -\pi \sin x e^{kt} + \frac{\pi^2}{12k}\int \cos x e^{kt}$$

$$12k\int \cos(x) e^{kt} = 12\cos(x) e^{kt} + \pi \sin x e^{kt} - \frac{\pi^2}{12k}\int \cos x e^{kt}$$

$$\left(12k+\frac{\pi^2}{12k}\right)\int \cos(x)e^{kt} = 12\cos(x)e^{kt}+\pi\sin x\, e^{kt}$$

$$=\left(\frac{12k}{144k^2+\pi^2}\right)\cdot 12\cos(x)e^{kt}+\pi\sin x\, e^{kt}$$

$$\int \cos(x)e^{kt}=\frac{144k\cos(x)e^{kt}+12\pi\sin x\, e^{kt}}{144k^2+\pi^2}$$

$\cdot 12k$ $\qquad\qquad\qquad\qquad$ $\cdot 12k$

$$=\frac{(k12\cos(x)+\pi\sin(x))ke^{kt}}{k^2+\frac{\pi^2}{144}}$$

$$=\frac{\left(k12\cos\left(\frac{\pi(t-5)}{12}\right)+\pi\sin\left(\frac{\pi(t-5)}{12}\right)\right)ke^{kt}}{k^2+\frac{\pi^2}{144}}$$

$$V=\left(75-\frac{\left(k12\cos\left(\frac{\pi(t-5)}{12}\right)+\pi\sin\left(\frac{\pi(t-5)}{12}\right)\right)k}{k^2+\frac{\pi^2}{144}}\right)e^{kt}$$

$$T_p=75-\frac{\left(k12\cos\left(\frac{\pi(t-5)}{12}\right)+\pi\sin\left(\frac{\pi(t-5)}{12}\right)\right)k}{k^2+\frac{\pi^2}{144}}$$

$$T_g=75+Ce^{-kt}-\left(\frac{\left(k12\cos\left(\frac{\pi(t-5)}{12}\right)+\pi\sin\left(\frac{\pi(t-5)}{12}\right)\right)k}{k^2+\frac{\pi^2}{144}}\right)$$

$k=0.25$:

$$T_g=75+Ce^{-\frac{t}{4}}-36\left(\frac{3\cos\left(\frac{\pi(t-5)}{12}\right)+\pi\sin\left(\frac{\pi(t-5)}{12}\right)}{9+\pi^2}\right)$$

$$T_0 = 65 = 75 + C - 36\left(\frac{3\cos\left(-\frac{5\pi}{12}\right) + \pi\sin\left(-\frac{5\pi}{12}\right)}{9+\pi^2}\right)$$

$$C = -10 - 4.308 = -14.3$$

$$T_g = 75 - 14.3\, e^{-\frac{t}{4}} - 36\left(\frac{3\cos\left(\frac{\pi(t-5)}{12}\right) + \pi\sin\left(\frac{\pi(t-5)}{12}\right)}{9+\pi^2}\right)$$

$$\boxed{T_g \text{ simplified} = 75 - 14.3\, e^{-\frac{t}{4}} - 1.407\left(3\cos(\phi) + \pi\sin(\phi)\right) \quad \left[\phi = \frac{\pi(t-5)}{12}\right]}$$

C_2: Solving Winter Diffeq

$$T_g = \overset{35}{\cancel{75}} + Ce^{-\frac{t}{4}} - 86\left(\frac{3\cos\left(\frac{\pi(t-5)}{12}\right) + \pi\sin\left(\frac{\pi(t-5)}{12}\right)}{9+\pi^2}\right)$$

Resolving for C_1:

unchanged by M_0

$$65 = M_0 + C + 4.308$$

So we can make a function of C:

$$C = 65 - M_0 - 4.308$$

And use matlab

And now we just sub in C_1 and M_0 2 for the original eq:

Solving by hand:	Matlab:
$C_2 = 25.692$ ✓	$C_2 = 25.692$ ✓

We are good to go!