Introduction and Logic

Discrete Mathematic

Anders Kalhauge

Fall 2017

Outline

Introduction
- Who am I
- Plan
- Book

Logic

Introduction

- Who am I
- Plan
- Book

Logic

Anders Kalhauge

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- 21 years experience as IT consultant in the private sector
- 16 years teaching computer science for students and private companies
- Main interests
  - Programming and programming languages
  - Development of large scale systems
  - Software architecture

Overview of the course

Discrete mathematics is the foundation of computing. All aspects depends on the other, but I have tried to do some separation anyway.

Introduction **2 sessions** This part includes introduction to discrete mathematics, logic, and predicates.

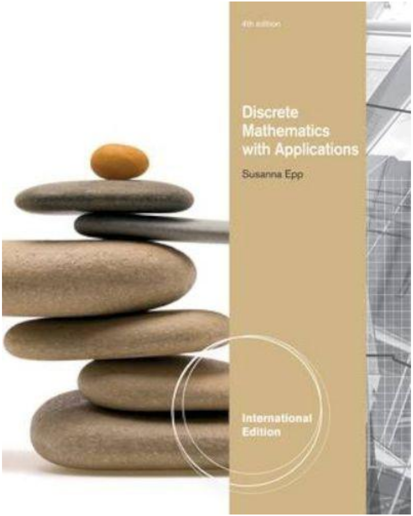
Dynamic Analysis **2 sessions** This part deals with more advanced stuff as set theory. Also regular expressions and finite state automata is introduced, we will use them to analyse the runtime behaviour of systems.

Static Analysis **4 sessions** This part focuses on the parts not covered above, **and** especially math used to do static analysis. We will also look at a practical example of static analysis implemented in code.

Overall plan

Details are subject to change

Date	Subject	Assignment
01/09	Intro Logic	Implementing logic
05/09	Predicates	A small PROLOG program
26/09	Set theory	A set manipulation framework in Kotlin, C#, or Java
03/10	Regular languages	A finite state automaton for dynamic analysis of a log file
07/11	Sequences	TBD
14/11	Relations	TBD
21/11	Static analysis	TBD
28/11	Design by Contract	A C# code contract application



Also used at ITU

Introduction

Who am I

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Logic

Statements

- A **statement** (or **proposition**) is a sentence that is true or false but not both.
- A **compound statement** is build up from simpler ones using logical connectives and paranthesis
- $\neg a$ read "not a" is the **negation** of a
- $a \wedge b$ read "a and b" is the **conjunction** of a and b
- $a \vee b$ read "a or b" is the **disjunction** of a and b

Negation - not

Means not the case that a, as in "it is not the case that the house is pink".

Boolean Algebra

$$\bar{a}$$

Math

$$\neg a \text{ or } \sim a$$

Java, C#, ...

```
!a
```

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Java, C#, ...

```
!a
```

a	$\neg a$
f	t
t	f

Conjunction - and

Means both a and b, as in “he has a car and a bicycle”.

Boolean Algebra

$$a \cdot b$$

Math

$$a \wedge b$$

Java, C#, ...

```
a && b
```

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Boolean Algebra

$$a \cdot b$$

Math

$$a \wedge b$$

Java, C#, ...

```
a && b
```

a	b	$a \wedge b$
f	f	f
f	t	f
t	f	f
t	t	t

Disjunction - inclusive or

Means either a or b or both, as in “do you want sugar or cream”.

Boolean Algebra

$$a + b$$

Math

$$a \vee b$$

Java, C#, ...

```
a || b
```

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Means either a or b or both, as in "do you want sugar or cream".

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```
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```

a	b	$a \vee b$
f	f	f
f	t	t
t	f	t
t	t	t

Using truth tables

a	b	c	$a \wedge b$	$\neg c$	$(a \wedge b) \vee \neg c$
f	f	f			
f	f	t			
f	t	f			
f	t	t			
t	f	f			
t	f	t			
t	t	f			
t	t	t			

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a	b	c	$a \wedge b$	$\neg c$	$(a \wedge b) \vee \neg c$
f	f	f	f		
f	f	t	f		
f	t	f	f		
f	t	t	f		
t	f	f	f		
t	f	t	f		
t	t	f	t		
t	t	t	t		

a	b	c	$a \wedge b$	$\neg c$	$(a \wedge b) \vee \neg c$
f	f	f	f	t	
f	f	t	f	f	
f	t	f	f	t	
f	t	t	f	f	
t	f	f	f	t	
t	f	t	f	f	
t	t	f	t	t	
t	t	t	t	f	

Using truth tables

a	b	c	$a \wedge b$	$\neg c$	$(a \wedge b) \vee \neg c$
f	f	f	f	t	t
f	f	t	f	f	
f	t	f	f	t	
f	t	t	f	f	
t	f	f	f	t	
t	f	t	f	f	
t	t	f	t	t	
t	t	t	t	f	

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a	b	c	$a \wedge b$	$\neg c$	$(a \wedge b) \vee \neg c$
f	f	f	f	t	t
f	f	t	f	f	f
f	t	f	f	t	
f	t	t	f	f	
t	f	f	f	t	
t	f	t	f	f	
t	t	f	t	t	
t	t	t	t	f	

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a	b	c	$a \wedge b$	$\neg c$	$(a \wedge b) \vee \neg c$
f	f	f	f	t	t
f	f	t	f	f	f
f	t	f	f	t	t
f	t	t	f	f	f
t	f	f	f	t	t
t	f	t	f	f	f
t	t	f	t	t	
t	t	t	t	f	

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f	f	f	f	t	t
f	f	t	f	f	f
f	t	f	f	t	t
f	t	t	f	f	f
t	f	f	f	t	t
t	f	t	f	f	f
t	t	f	t	t	t
t	t	t	t	f	t

Means either a or b, but not both, as in "do you want beer or wine".

Boolean Algebra

$$a \oplus b$$

Math

$$(a \vee b) \wedge \neg(a \wedge b)$$

Java, C#, ...

```
a ^ b
```

Exclusive or

Means either a or b, but not both, as in "do you want beer or wine".

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$$a \oplus b$$

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$$(a \vee b) \wedge \neg(a \wedge b)$$

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```
a ^ b
```

a	b	$(a \vee b) \wedge \neg(a \wedge b)$
f	f	f
f	t	t
t	f	t
t	t	f

Exclusive or - how?

a	b	$a \vee b$	$a \wedge b$	$\neg(a \wedge b)$	$(a \vee b) \wedge \neg(a \wedge b)$
f	f	f	f	t	f
f	t	t	f	t	t
t	f	t	f	t	t
t	t	t	t	f	f

Tautologies and Contradictions

- Something that is always true is called a **tautology**.
  tautologies are written: **t**.
  $a = a$ is a tautology.
- Something that can never be true is called a **contradiction**.
  contradictions are written: **c**.
  $a \neq a$ is a contradiction.

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Law(s)		
Commutative	$a \wedge b \equiv b \wedge a$	$a \vee b \equiv b \vee a$
Associative	$(a \wedge b) \wedge c \equiv a \wedge (b \wedge c)$	$(a \vee b) \vee c \equiv a \vee (b \vee c)$
Distributive	$a \wedge (b \vee c) \equiv (a \wedge b) \vee (a \wedge c)$	$a \vee (b \wedge c) \equiv (a \vee b) \wedge (a \vee c)$
Identity	$a \wedge \mathbf{t} \equiv a$	$a \vee \mathbf{c} \equiv a$
Negation	$a \vee \neg a \equiv \mathbf{t}$	$a \wedge \neg a \equiv \mathbf{c}$
	$\neg(\neg a) \equiv a$	
	$\neg\,\mathbf{t} \equiv \mathbf{c}$	$\neg\,\mathbf{c} \equiv \mathbf{t}$
Idempotent	$a \wedge a \equiv a$	$a \vee a \equiv a$
Universal bounds	$a \vee \mathbf{t} \equiv \mathbf{t}$	$a \wedge \mathbf{c} \equiv \mathbf{c}$
De Morgan's	$\neg(a \wedge b) \equiv \neg a \vee \neg b$	$\neg(a \vee b) \equiv \neg a \wedge \neg b$
Absorption	$a \vee (a \wedge b) \equiv a$	$a \wedge (a \vee b) \equiv a$

Assignment

Show, using a truth table, that the distributive laws are valid.

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a	b	c	a	$b \vee c$	$a \wedge (b \vee c)$	$a \wedge b$	$a \wedge c$	$(a \wedge b) \vee (a \wedge c)$
f	f	f	f					
f	f	t	f					
f	t	f	f					
f	t	t	f					
t	f	f	t					
t	f	t	t					
t	t	f	t					
t	t	t	t					

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f	f	f	f	f				
f	f	t	f	t				
f	t	f	f	t				
f	t	t	f	t				
t	f	f	t	f				
t	f	t	t	t				
t	t	f	t	t				
t	t	t	t	t				

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f	f	f	f	f	f			
f	f	t	f	t	f			
f	t	f	f	t	f			
f	t	t	f	t	f			
t	f	f	t	f	f			
t	f	t	t	t	t			
t	t	f	t	t	t			
t	t	t	t	t	t			

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f	f	f	f	f	f	f		
f	f	t	f	t	f	f		
f	t	f	f	t	f	f		
f	t	t	f	t	f	f		
t	f	f	t	f	f	f		
t	f	t	t	t	t	f		
t	t	f	t	t	t	t		
t	t	t	t	t	t	t		

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f	f	f	f	f	f	f	f	
f	f	t	f	t	f	f	f	
f	t	f	f	t	f	f	f	
f	t	t	f	t	f	f	f	
t	f	f	t	f	f	f	f	
t	f	t	t	t	t	f	t	
t	t	f	t	t	t	t	f	
t	t	t	t	t	t	t	t	

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f	f	f	f	f	f	f	f	f
f	f	t	f	t	f	f	f	f
f	t	f	f	t	f	f	f	f
f	t	t	f	t	f	f	f	f
t	f	f	t	f	f	f	f	f
t	f	t	t	t	t	f	t	t
t	t	f	t	t	t	t	f	t
t	t	t	t	t	t	t	t	t

`if` *hypothesis* `then` *conclusion*

- The thuth of the conclusion is **conditioned** to the truth of the hypothesis
- `if` 4686 if divisible by 6 `then` 4686 is divisible by 3
- $4686 \mod 6 = 0 \rightarrow 4686 \mod 3 = 0$
- `if` 0 = 1 `then` 1 = 2 or $(0 = 1) \rightarrow (1 = 2)$ is **true!**

a	b	$a \rightarrow b$
f	f	t
f	t	t
t	f	f
t	t	t

Conditional statement as or

$$a \rightarrow b \equiv \neg a \vee b$$

- $a \rightarrow b =$ **if** you do not get to work on time **then** you are fired
- $a =$ you do not get to work on time
- $b =$ you are fired
- $\neg a =$ you do get to work on time
- $\neg a \vee b =$ you get to work on time **or** you are fired

a	b	$a \rightarrow b$	$\neg a$	$\neg a \vee b$
f	f	t	t	t
f	t	t	t	t
t	f	f	f	f
t	t	t	f	t

The negation of a conditional statement is as follows

$$\neg(a \to b) \equiv a \wedge \neg b$$

Use De Morgan's law to show that this is really the case. Tip: use the "or" equivalent of the conditional statement.

The contrapositive

of a conditional statement

If you are not fired then you got to work on time.

$$a \to b \equiv \neg b \to \neg a$$

Note that a and b swap positions

a	b	$a \to b$	$\neg b$	$\neg a$	$\neg b \to \neg a$
f	f	t	t	t	t
f	t	t	f	t	t
t	f	f	t	f	f
t	t	t	f	f	t

Look at the conditional statement:

`if` you do not clean your room, `then` you will not get cake.

- Create a truth table for the above statement!
- What should you do to get cake?

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of a conditional statement

Given the conditional statement

$$a \rightarrow b$$

Then the converse is

$$b \rightarrow a$$

And the inverse is

$$\neg a \rightarrow \neg b$$

Note that **neither** the converse **nor** the inverse are equivalent to the original conditional statement

The biconditional - only if

John will break the world's record for the mile run **only if** he runs the mile in under four minutes.

$$a \leftrightarrow b$$

- $a =$ John will break the world's record for the mile run
- $b =$ he runs the mile in under four minutes
- $a \leftrightarrow b \equiv b \leftrightarrow a$

John runs the mile in under four minutes **only if** he will break the world's record for the mile run.

a	b	$a \leftrightarrow b$
f	f	t
f	t	f
t	f	f
t	t	t

Fill in the x'es

a	b	x	x	x	x	x	x	x	x
f	f	f	t	f	t	f	t	f	t
f	t	f	f	t	t	f	f	t	t
t	f	f	f	f	f	t	t	t	t
t	t	f	f	f	f	f	f	f	f

a	b	x	x	x	x	x	x	x	x
f	f	f	t	f	t	f	t	f	t
f	t	f	f	t	t	f	f	t	t
t	f	f	f	f	f	t	t	t	t
t	t	t	t	t	t	t	t	t	t

All dyadic boolean functions

a	b	**c**	$\neg(a \vee b)$	$\neg(b \rightarrow a)$	$\neg a$	$\neg(a \rightarrow b)$	$\neg b$	$\neg(a \leftrightarrow b)$	$\neg(a \wedge b)$
f	f	f	t	f	t	f	t	f	t
f	t	f	f	t	t	f	f	t	t
t	f	f	f	f	f	t	t	t	t
t	t	f	f	f	f	f	f	f	f

a	b	$a \wedge b$	$a \leftrightarrow b$	b	$a \rightarrow b$	a	$b \rightarrow a$	$a \vee b$	**t**
f	f	f	t	f	t	f	t	f	t
f	t	f	f	t	t	f	f	t	t
t	f	f	f	f	f	t	t	t	t
t	t	t	t	t	t	t	t	t	t

Arguments

- An argument is a sequence of statements
- An argument form is a series of statement forms.
- All statements (or statement forms) in an argument (or argument form), except for the final one, are called **premises** (or assumptions or hypotheses)
- The final statement (or statement form) is called the **conclusion**.
- The symbol $\therefore$ read "therefore" is placed before the conclusion.

$$\begin{aligned} &\text{Socrates is a man} \rightarrow \text{Socrates is mortal} \\ &\text{Socrates is a man} \\ \therefore\, &\text{Socrates is mortal.} \end{aligned}$$

$$
\begin{aligned}
&a \to b \\
&a \\
\therefore\ &b
\end{aligned}
$$

Argument forms - Modus Tollens

$$\begin{aligned} &a \rightarrow b \\ &\neg b \\ \therefore\ &\neg a \end{aligned}$$

$$a$$
$$\therefore a \lor b$$

$$b$$
$$\therefore a \lor b$$

Argument forms - Specialization

$$\begin{aligned} & a \wedge b \\ \therefore\ & a \end{aligned}$$

$$\begin{aligned} & a \wedge b \\ \therefore\ & b \end{aligned}$$

Argument forms - Conjunction

$$a$$

$$b$$

$$\therefore a \wedge b$$

Argument forms - Elimination

$$
\begin{array}{rl}
 & a \vee b \\
 & \neg b \\
\therefore & a
\end{array}
$$

$$
\begin{array}{rl}
 & a \vee b \\
 & \neg a \\
\therefore & b
\end{array}
$$

Argument forms - Transivity

$$
\begin{aligned}
a &\to b \\
b &\to c \\
\therefore a &\to c
\end{aligned}
$$

Argument forms - Division into cases

$$
\begin{aligned}
& a \vee b \\
& a \rightarrow c \\
& b \rightarrow c \\
\therefore\ & c
\end{aligned}
$$

Argument forms - Contradiction

$$\neg a \to \mathbf{c}$$
$$\therefore a$$