Assignement 1

1(a) Lagrange polynomial

$$L_j(x) = \prod_{\substack{i=0 \\ j \neq i}}^{n} \frac{(x - x_i)}{(x_j - x_i)}$$

x_i	y_i
-1	2
0	6
2	4
3	30

$$L_0(x) = \frac{(x-0)(x-2)(x-3)}{(-1-0)(-1-2)(-1-3)}$$

$$= (x^3 - 5x^2 + 6x)\left(\frac{1}{-12}\right)$$

$$L_1(x) = \frac{(x-(-1))(x-2)(x-3)}{(0-(-1))(2-(-1))(3-(-1))}$$

$$= (x^3 - 4x^2 + x + 6)\left(\frac{1}{6}\right)$$

$$L_2(x) = \frac{(x-(-1))(x-0)(x-3)}{(2-(-1))(2-0)(2-3)}$$

$$= (x^3 - 2x^2 - 3x)\left(-\frac{1}{6}\right)$$

$$L_3(x) = \frac{(x-(-1))(x-0)(x-2)}{(3-(-1))(3-0)(3-2)}$$

$$= (x^3 - x^2 - 3x)\left(\frac{1}{12}\right)$$

Scanned by CamScanner

$$P(x) = \sum_{\substack{j=0 \\ i \neq j}}^{n} y_j \cdot L_j(x_i)$$

$$P(x) = \frac{8}{3}x^3 - \frac{13}{3}x^2 - 3x + 6$$

1 (b) Newton method

x_i	y_i
-1	2
0	6
2	4
3	30

$$\frac{6-2}{1} = 4 \qquad \frac{(-1)-4}{2+1} = -\frac{5}{3} \qquad \frac{9+\frac{5}{3}}{4} = \frac{32}{12} = \frac{8}{3}$$

$$\frac{4-6}{2} = -1 \qquad \frac{26+1}{3} = 9$$

$$\frac{30-4}{3-2} = 26$$

$$p(x) = 2 + 4(x+1) - \frac{5}{3}(x+1)(x) + \frac{8}{3}(x+1)(x)(x-2)$$

$$= \frac{8}{3}x^3 - \frac{13}{3}x^2 - 3x + 6$$

2. Proof by contradiction

$P_n(x_i) = Q_n(x_i)$ for $i = 0, \dots, n$

Setting $R_n(x_i) = P_n(x_i) - Q_n(x_i) = 0$ $\quad i = 0, \dots, n$

then the n^{th} order $R_n(x)$ has $n+1$ zeros only
a polynomial idenfical to the zero polynomial
identical to the zero polynomial can have more zeros
then its order

$\therefore$ As $R_n(x) = 0$ then $P_n(x) = Q_n(x)$