Calculator Equations

Reuben

1 equations

Quadrupole moment (eb?) from Matrix Element (eb$^{\lambda/2}$) :

$$Q_s = M.E. \left(\frac{16\pi}{5} \frac{J(2J-1)}{(2J+1)(2J+3)(J+1)} \right)^{\frac{1}{2}} \tag{1}$$

Where Q_s is the spectroscopic quadrupole moment, $M.E.$ represents the matrix element $\langle J|\hat{M}|J\rangle$ for the cases I know work it is $\langle 2_1^+|E2|2_1^+\rangle$ (so $\lambda = 2$), J is the spin of the state (sometimes noted as I) [1, 2] probably also seen in [3].

NOTE J_i is the higher energy state, J_f is the lower energy state.

B(Eλ)$\downarrow$ in W.u. to B(Eλ)$\downarrow$ in e^{2}fm$^{2\lambda}$:

$$B(E\lambda) \downarrow_{e^2fm^{2\lambda}} = B(E\lambda) \downarrow_{W.u.} \frac{(1.2)^{2\lambda}}{4\pi} \frac{3}{(\lambda+3)^2} A^{2\lambda/3} \tag{2}$$

Where A is the mass number of the nucleus, λ is the multipolarity of the transition [4] and in `http://web-docs.gsi.de/~wolle/BUCH/HTML/MODEL/weisskopf.pdf` .

Matrix Element (eb$^{\lambda/2}$) to B(Eλ)$\downarrow$ (e^{2}b$^\lambda$) :

$$B(E\lambda) \downarrow = \frac{(M.E.)^2}{2J_i+1} \tag{3}$$

[1].

B($^E_M\lambda$)$\downarrow$ to B($^E_M\lambda$)$\uparrow$ (unit to same unit, need to convert units before or after if wanting) works for both E and M transitions :

$$B(^E_M\lambda) \uparrow = B(^E_M\lambda) \downarrow \frac{2J_i+1}{2J_f+1} \tag{4}$$

DOUBLE CHECK THE BELOW (they match what I found in text books, but isnt exactly what I expect?)

B(Mλ)$\downarrow$ (W.u.) to B(Mλ)$\downarrow$ (μ_N^2fm$^{(2\lambda-2)}$) :

$$B(M\lambda)\downarrow_{\mu_N^2 fm^{(2\lambda-2)}} = B(M\lambda)\downarrow_{W.u.} 1.2^{2\lambda-2}\frac{10}{3}\frac{3}{\lambda+3}^{2} A^{(2\lambda-2)/3} \quad (5)$$

[4] and in `http://web-docs.gsi.de/~wolle/BUCH/HTML/MODEL/weisskopf.pdf`
.

B(Mλ)$\downarrow$ (μ_N^2fm$^{(2\lambda-2)}$) from Matrix Element ($\mu_N$fm$^{(2\lambda-2)/2}$) :

$$B(M\lambda)\downarrow = \frac{(M.E.)^2}{2J_i+1} \quad (6)$$

[1].

Axial Rotor Limits and link to Triaxiality:

$$Q_s^{rot}(2_1^+) = -\frac{2}{7}\sqrt{\frac{16\pi}{5}B(E2;0_1^+\rightarrow 2_1^+} \quad (7)$$

AND

$$cos(3\gamma) \approx \frac{Q_s(2_1^+)}{Q_s^{rot}(2_1^+)} \quad (8)$$

Where Q_s^{rot} is the axial rotor limit (deformation it would be if exactly oblate or prolate without any triaxial effects), γ is the degree of triaxiality [5, 6]. Can also link the $cos(3\gamma)$ to work by Kumer and/or Cline in the 1960s.

Deformation parameter β in terms of B(E2) (e^2fm^4):

$$\beta = \frac{4\pi}{3ZR_0^2}\sqrt{\frac{B(E2;0_1^+\rightarrow 2_1^+)}{e^2}} \quad (9)$$

Where:

$$R_0 = 1.2A^{\frac{1}{3}}\ \text{fm} \quad (10)$$

[5, 7, 8]

Relation of β (sometimes called β_2) to other quadrupole deformation parameters:
Intrinsic quadrupole moment, Q_0:

$$Q_0 = \sqrt{\frac{16\pi}{5}}\frac{3}{4\pi}R_0^2 Ze\beta \quad (11)$$

Deformation parameter ε (sometimes called ε_2)linked to the elongation of the nucleus:

$$\varepsilon \approx 0.95\beta \tag{12}$$

and

$$\beta = \sqrt{\frac{\pi}{5}}\left(\frac{4}{3}\varepsilon + \frac{4}{9}\varepsilon^2 + \frac{4}{27}\varepsilon^3 + \frac{4}{81}\varepsilon^4 + ...\right) \tag{13}$$

[7–10]

1.1 Unit Converters:

Main thing to remember is that 1 barn = 100 fm^2
Matrix Element (eb$^{\lambda/2}$) to Matrix Element (efm$^\lambda$) : (check this)

$$M.E.efm^{2\lambda/2} = M.E.eb^{\lambda/2}100^{\lambda/2} \tag{14}$$

B(Eλ)$\downarrow$ (e^{2}b$^\lambda$) to B(Eλ)$\downarrow$ (e^{2}fm$^{2\lambda}$) : (check this)

$$B(E\lambda) \downarrow e^2fm^{2\lambda} = B(E\lambda) \downarrow e^2b^\lambda 100^\lambda \tag{15}$$

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