Calculator Equations

Reuben

1 equations

Quadrupole moment (eb?) from Matrix Element ($\mathrm{eb}^{\lambda/2}$) :

$$Q_s = M.E. \left(\frac{16\pi}{5} \frac{J(2J-1)}{(2J+1)(2J+3)(J+1)} \right)^{\frac{1}{2}} \tag{1}$$

Where Q_s is the spectroscopic quadrupole moment, $M.E.$ represents the matrix element $\langle J|\hat{M}|J\rangle$ for the cases I know work it is $\langle 2_1^+|E2|2_1^+\rangle$ (so $\lambda = 2$), J is the spin of the state (sometimes noted as I) [1, 2] probably also seen in [3].

NOTE J_i is the higher energy state, J_f is the lower energy state.

B(Eλ)$\downarrow$ in W.u. to B(Eλ)$\downarrow$ in $\mathrm{e}^2\mathrm{fm}^{2\lambda}$:

$$B(E\lambda)\downarrow_{e^2fm^{2\lambda}} = B(E\lambda)\downarrow_{W.u.} \frac{(1.2)^{2\lambda}}{4\pi} \frac{3}{(\lambda+3)^2} A^{2\lambda/3} \tag{2}$$

Where A is the mass number of the nucleus, λ is the multipolarity of the transition [4, 5] and in `http://web-docs.gsi.de/~wolle/BUCH/HTML/MODEL/weisskopf.pdf`
.

Matrix Element ($\mathrm{eb}^{\lambda/2}$) to B(Eλ)$\downarrow$ ($\mathrm{e}^2\mathrm{b}^\lambda$) :

$$B(E\lambda)\downarrow = \frac{(M.E.)^2}{2J_i+1} \tag{3}$$

[1].

B($^E_M\lambda$)$\downarrow$ to B($^E_M\lambda$)$\uparrow$ (unit to same unit, need to convert units before or after if wanting) works for both E and M transitions :

$$B(^E_M\lambda)\uparrow = B(^E_M\lambda)\downarrow \frac{2J_i+1}{2J_f+1} \tag{4}$$

DOUBLE CHECK THE BELOW (they match what I found in text books, but isnt exactly what I expect?)

B(Mλ)$\downarrow$ (W.u.) to B(Mλ)$\downarrow$ ($\mu_N^2\mathrm{fm}^{(2\lambda-2)}$) :

$$B(M\lambda)\downarrow_{\mu_N^2 fm^{(2\lambda-2)}} = B(M\lambda)\downarrow_{W.u.} 1.2^{2\lambda-2}\frac{10}{\pi}{\frac{3}{\lambda+3}}^{2} A^{(2\lambda-2)/3} \tag{5}$$

π written as 3 in some text books. [4] and in `http://web-docs.gsi.de/~wolle/BUCH/HTML/MODEL/weisskopf.`
.

B(Mλ)$\downarrow$ ($\mu_N^2\mathrm{fm}^{(2\lambda-2)}$) from Matrix Element ($\mu_N\mathrm{fm}^{(2\lambda-2)/2}$) :

$$B(M\lambda)\downarrow = \frac{(M.E.)^2}{2J_i+1} \tag{6}$$

[1].

Axial Rotor Limits and link to Triaxiality:

$$Q_s^{rot}(2_1^+) = -\frac{2}{7}\sqrt{\frac{16\pi}{5}B(E2;0_1^+\rightarrow 2_1^+} \tag{7}$$

AND

$$cos(3\gamma) \approx \frac{Q_s(2_1^+)}{Q_s^{rot}(2_1^+)} \tag{8}$$

Where Q_s^{rot} is the axial rotor limit (deformation it would be if exactly oblate or prolate without any triaxial effects), γ is the degree of triaxiality [6, 7]. Can also link the $cos(3\gamma)$ to work by Kumer and/or Cline in the 1960s.

Deformation parameter β in terms of B(E2) (e^2fm^4):

$$\beta = \frac{4\pi}{3ZR_0^2}\sqrt{\frac{B(E2;0_1^+\rightarrow 2_1^+)}{e^2}} \tag{9}$$

Where:

$$R_0 = 1.2A^{\frac{1}{3}}\ \mathrm{fm} \tag{10}$$

[6, 8, 9]

Relation of β (sometimes called β_2) to other quadrupole deformation parameters:
Intrinsic quadrupole moment, Q_0:

$$Q_0 = \sqrt{\frac{16\pi}{5}}\frac{3}{4\pi}R_0^2Ze\beta \tag{11}$$

Deformation parameter ε (sometimes called ε_2) linked to the elongation of the nucleus:

$$\varepsilon \approx 0.95\beta \tag{12}$$

and

$$\beta = \sqrt{\frac{\pi}{5}}\left(\frac{4}{3}\varepsilon + \frac{4}{9}\varepsilon^2 + \frac{4}{27}\varepsilon^3 + \frac{4}{81}\varepsilon^4 + ...\right) \tag{13}$$

[8–11]

Minimum distance between centres of nuclei, assuming the nucleus radius is $\sim 1.2A^{1/3}$:

$$d_{min} = 1.25(A_1^{1/3} + A_2^{1/3}) + s \tag{14}$$

Where s is the separation between the edges of the nuclei (to satisfy Cline's criterion s = 5 fm) [12–14].

Coulomb barrier potential [15]:

$$V_C = \frac{Z_1 Z_2 e^2}{R} \approx 1.44\frac{Z_1 Z_2}{R}(\text{MeV}) \tag{15}$$

Coulomb barrier pot is used with the minimum distance to find the beam energy at a set distance AND as such the limit for the beam energy for safe coulex:

$$E_{bomb} = 0.72\left(1 + \frac{1}{sin(\frac{\theta}{2})}\right)\frac{A_1 + A_2}{A_2}\frac{Z_1 Z_2}{1.25(A_1^{1/3} + A_2^{1/3}) + s} \tag{16}$$

$$E_{max} = 1.44\frac{A_1 + A_2}{A_2}\frac{Z_1 Z_2}{1.25(A_1^{1/3} + A_2^{1/3}) + 5} \tag{17}$$

[12–15]

1.1 Unit Converters:

Main thing to remember is that 1 barn = 100 fm^2
Matrix Element (eb$^{\lambda/2}$) to Matrix Element (efm$^{\lambda}$) : (check this)

$$M.E.efm^{2\lambda/2} = M.E.eb^{\lambda/2}100^{\lambda/2} \tag{18}$$

B(Eλ)$\downarrow$ (e^{2}b$^{\lambda}$) to B(Eλ)$\downarrow$ (e^{2}fm$^{2\lambda}$) : (check this)

$$B(E\lambda)\downarrow e^2 fm^{2\lambda} = B(E\lambda)\downarrow e^2 b^{\lambda} 100^{\lambda} \tag{19}$$

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