The 7th Report of Graduation Design

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Sichuan - 27th March 2020

Outline

Work of last week

1. Research on the laser frame for GRACE-FO
2. Organise GRACE-FO 1B data
3. Finish KBR error propagation
4. Finish laser frequency noise model
5. Attitude angles . . . on the way

Arrangement of next week

1. Finish LRI error propagation
2. Revise all fortran programme into module-style

Please

Ask questions *at the end* if you want!

Brief recap of last week

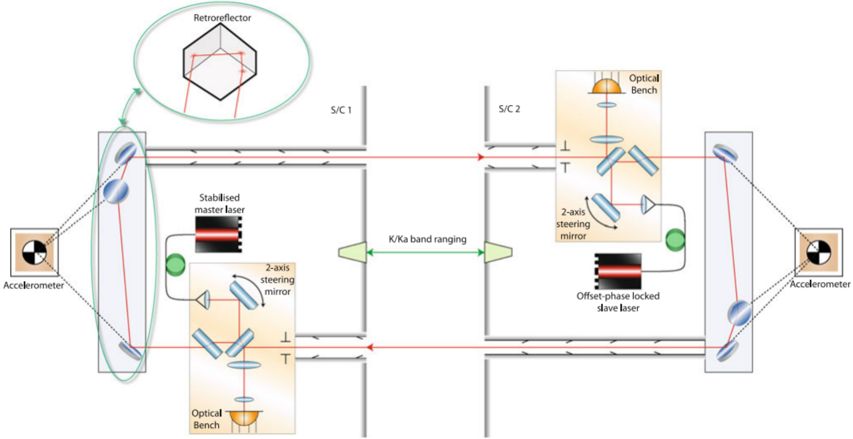
Mentioned this . . .

Better to review the presentation of last week!
Therefore, let's go back to those slide . . .

- The principle of two-way ranging
- The principle of phasemetre

The fundamentals of LRI of GRACE-FO

OK, after the brief recap of last week, let's dive into the fundamentals of LRI ranging of GRACE-FO

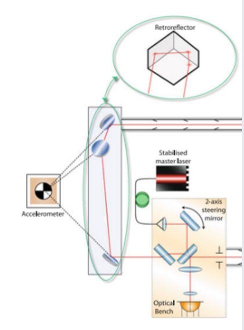


Key elements and TMA

Key words

off-axis, racetrack, three mirror assembly(TMA), Retro-reflector, vertex, transponder

TMA, retro-reflector



$\Longrightarrow$ **Assumptions:**

- Three mirrors perpendicular to one another.
- Vertex of the retro-reflector is outside.
- Pathlength is twice than the distance . . .
- Two beams is **anti-parallel**.
- Lateral offsets are the same.

Transponder

Why the transponder exists???

Since the core of the two-ranging system is the transmitting satellite is the one to receive, why bother to arrange a slave transponder in the tracking satellite???

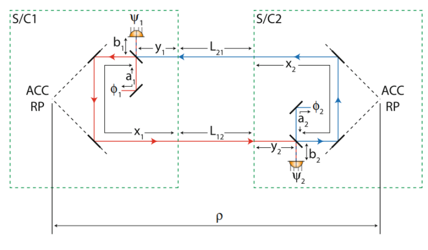
Concerns

According to the fourth assumption of the last slide, the transmitting beam is anti-parallel to the receiving one. But the separation of the two GRACE FO satellites is so large so that this assumption cannot be taken unless the attitude of the two satellites can be sufficiently controlled. Unfortunately, the test illustrates it is not gonna happen.

Two-way ranging for GRACE-FO: Off-axis and racetrack

Alas! The laser axis has been **BLOCKED**!

- Continuing the design style of the GRACE, the KBR phase centre and cold gas tank are placed right in the centre of the front of the satellite.
- The shift offset cannot be too large in case that the error of attitude noise aligned into the range.



$$\rho(t) \approx \frac{1}{2}\left[x_1(t) + L_{12}(T) + y_2(t) + L_{21}(t) + x_2(t) + y_1(t)\right] \quad (1)$$

Noise sources

According to Sheard

- **Two dominant sources:** pointing-induced noise, laser frequency noise
- **Other sources:** USO, shot, laser power, photodetector noise . . .

According to Vitali

- laser frequency noise
- triple mirror assembly pointing jitter coupling
- additional linear and quadratic pointing jitter coupling

"Technical job" . . .

Tools used

Python package: dask, one parallel and efficient package to handle large dataframes.
Python library, documents at `https://docs.dask.org/en/lat`

Results

Input $\Longrightarrow$ the raw data of GRACE-FO 1B for Arbitrary days
Output $\Longrightarrow$ the united dataframe for those days
The same procedure as pre-processing Taiji-01

KBR error propagation

Good lord! Brought it off at last!!!

The error propagation formula

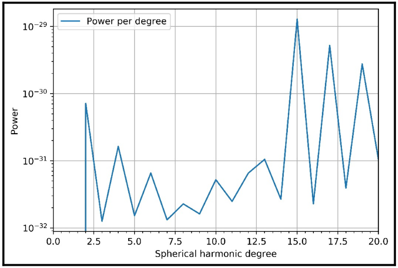
$$\sigma_n^2\left(\delta T\right) = \frac{1}{1 - P_n\left(cos\theta\right)}\frac{R_e}{GM}\left(\frac{r}{R_e}\right)^{2n+1}\sigma_n^2\left(\delta\dot{\rho}\right) \tag{2}$$

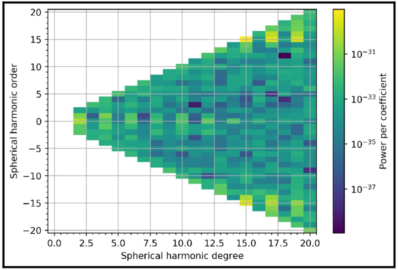
The noise sources of KBR ranging

- USO frequency shift
- System noise
- Take the above two as a whole
- Mutipath noise

Degree error

Range-rate noise caused by USO frequency shift and system noise





A small episode: Range

Since the arbitrary-days-long data is available, the multipath noise went into a different situation.

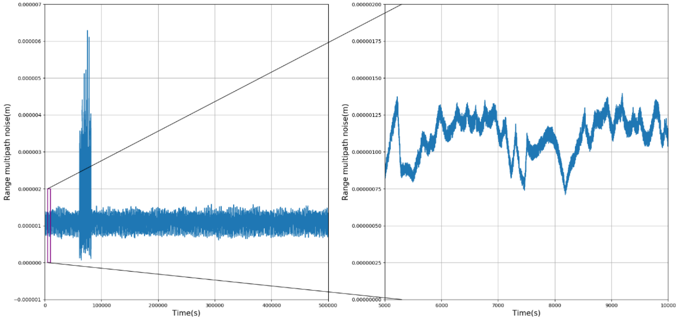


Figure 1: Range noise because of multipath noise (20190101-20190131)

A small episode: Range-rate

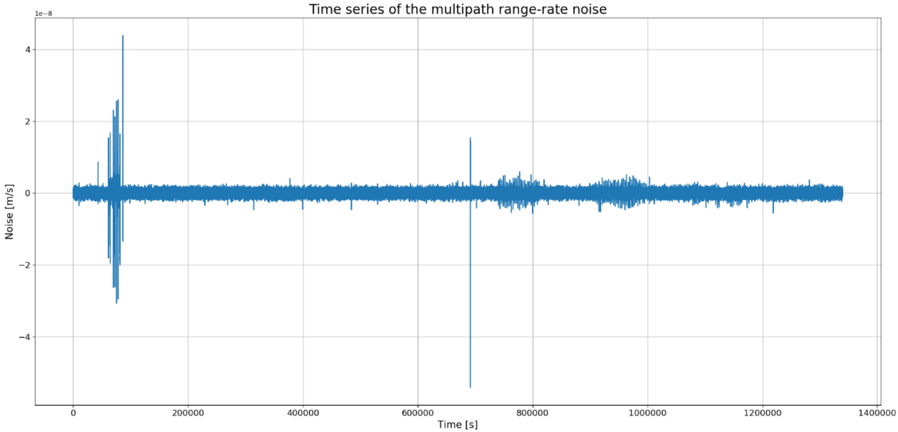


Figure 2: Range-rate noise because of multipath noise (20190101-20190131)

A small episode: PSD

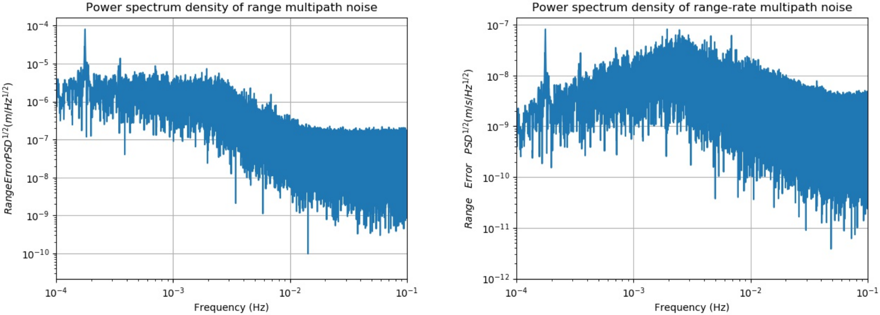
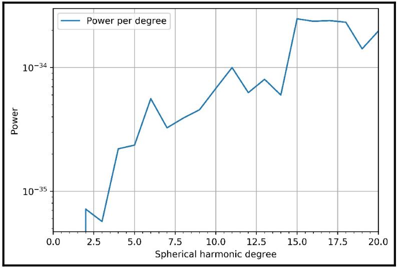
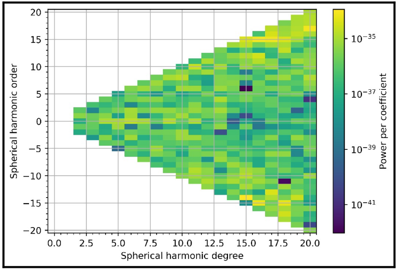


Figure 3: PSD of multipath range and range-rate noise

Degree error

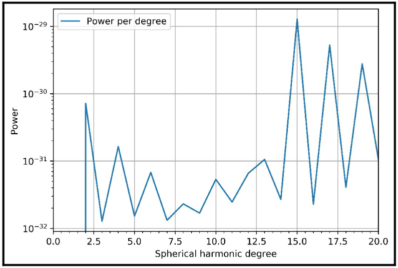
Degree error because of multipath noise.

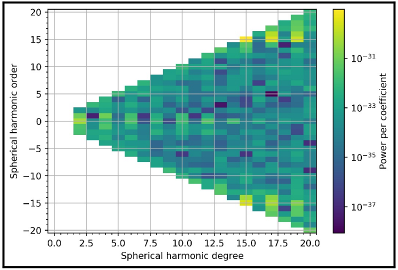




Degree error

Total degree error of KBR ranging





The magnitude of the range-rate noise by USO frequency shift and system noise is much larger than the one caused by multipath noise.

Filter to generate noise

Formula $\Longrightarrow$ ASD

According to the cavity test of JPL, the ASD of the range noise because of laser frequency noise is as follows:

$$\bar{\delta}_{\rho LF}(f) = 5 \times 10^{-9}\sqrt{1 + \left(\frac{0.0182Hz}{f}\right)^2}\frac{m}{\sqrt{Hz}} \tag{3}$$

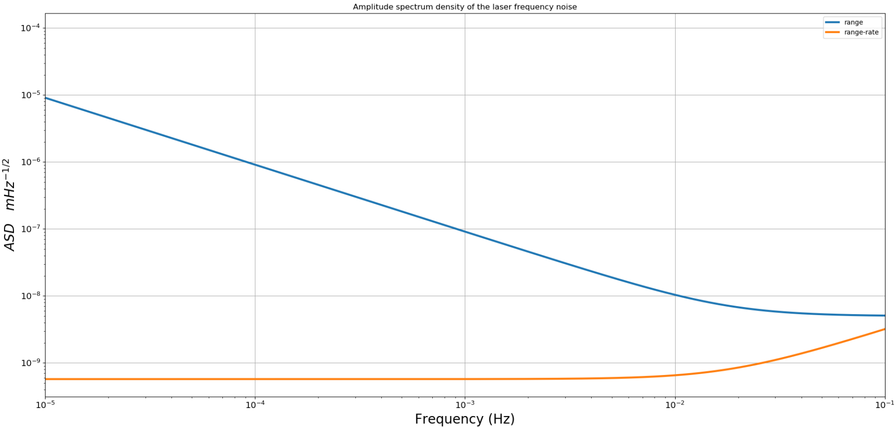
Formula $\Longrightarrow$ Time series

According to Sheard, the coupling of laser frequency noise into the range measurement is proportional to the satellite separation.

$$\delta_{\rho LF}(t) = \frac{\rho}{238} \times \mathcal{F}^{-1}(5 \times 10^{-9}\sqrt{1 + (\frac{0.0182Hz}{f})^2}\frac{m}{\sqrt{Hz}}) \tag{4}$$

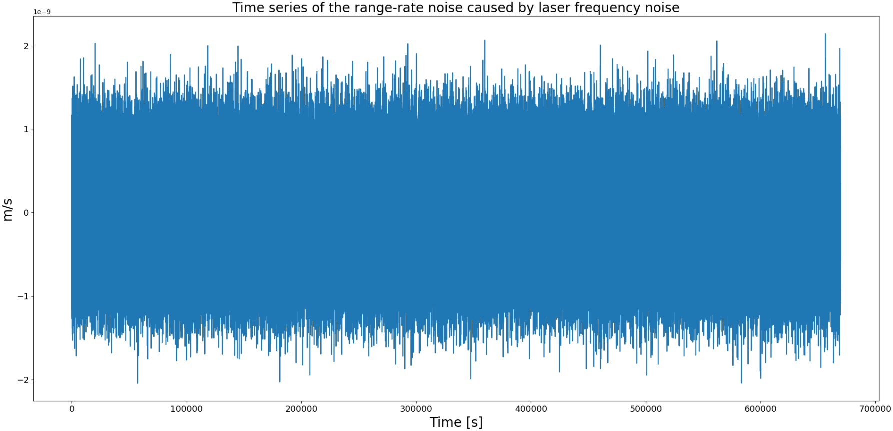
PSD of range and range-rate noise

$$\bar{\delta}_{\dot{\rho}LF} = 2\pi f \bar{\delta}_{\rho LF} \quad (5)$$



Time series of range rate

Filtering approach as always . . .



Another two noise for LRI

Triple mirror assembly poiting jitter coupling

$$\delta\rho_{TMA_A} = -\left(e_{AB}^{ICRF}\right)^T \cdot \mathbf{R}_{SF}^{ICRF} \cdot \boldsymbol{v}_A^{SF} \tag{6}$$

Additional linear and quadratic pointing jitter coupling

$$\delta\rho_{ALQ_A} = \begin{bmatrix} c_{x_A} & c_{y_A} & c_{z_A} \end{bmatrix} \cdot \begin{bmatrix} \theta_{x_A} \\ \theta_{y_A} \\ \theta_{z_A} \end{bmatrix} + \begin{bmatrix} \theta_{x_A} & \theta_{y_A} & \theta_{z_A} \end{bmatrix} \cdot \begin{bmatrix} c_{xx_A} & c_{xy_A} & c_{xz_A} \\ 0 & c_{yy_A} & c_{yz_A} \\ 0 & 0 & c_{zz_A} \end{bmatrix} \cdot \begin{bmatrix} \theta_{x_A} \\ \theta_{y_A} \\ \theta_{z_A} \end{bmatrix} \tag{7}$$

Interpretation

$\mathbf{R}_{SF}^{ICRF}$ is the transformation matrix from SF into ICRF; θ_{x_A} is the roll angle for GRACE-FO C.

Transformation matrix

According to Vilali and spiceypy,

The matrix rotating from SF into ICRF is related to the quaternions by:

$$\mathbf{R}_{SF}^{ICRF} = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2\,(q_1 q_2 - q_0 q_3) & 2\,(q_1 q_3 + q_0 q_2) \\ 2\,(q_1 q_2 + q_0 q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2\,(q_2 q_3 - q_0 q_1) \\ 2\,(q_1 q_3 - q_0 q_2) & 2\,(q_2 q_3 + q_0 q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix} \tag{8}$$

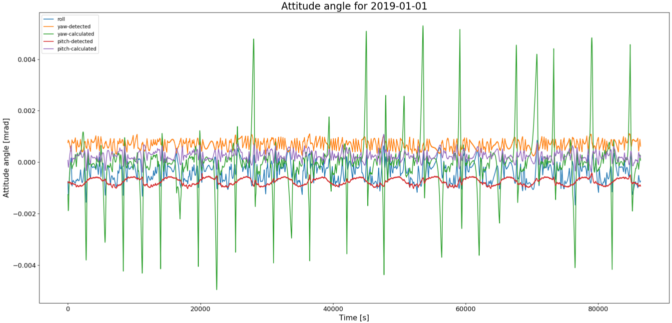
The matrix rotating from ICRF into LOSF is derived from the orbital positions:

$$\mathbf{R}_{ICRF}^{LOSF} = [\boldsymbol{x}_{LOSF}, \boldsymbol{y}_{LOSF}, \boldsymbol{z}_{LOSF}] \tag{9}$$

$$\theta_x = \arctan(\frac{R_{32}}{R_{33}}); \theta_y = -\arcsin(R_{31}); \theta_z = \arctan(\frac{R_{21}}{R_{11}}) \tag{10}$$

$$\boldsymbol{x}_{LOSF_A} = \frac{\boldsymbol{r}_B - \boldsymbol{r}_A}{|\boldsymbol{r}_B - \boldsymbol{r}_A|}; \boldsymbol{y}_{LOSF_A} = \frac{\boldsymbol{x}_{LOSF_A} \times \boldsymbol{r}_A}{\left|\boldsymbol{x}_{LOSF_A} \times \boldsymbol{r}_A\right|}; z_{LOSF_A} = \boldsymbol{x}_{LOSF_A} \times \boldsymbol{y}_{LOSF_A} \tag{11}$$

Bad news . . .



Big difference

The yaw attitude angle is completely different.
Tried several rotating method, it is the one that is most correct.
Still don't know how to fix it.

LRI noise model

Obstacles

- The attitude angle

Wish to accomplish

- Model of LRI noise model
- Error propagation of LRI
- Plotting all diagrams

Tools

- Astropy
- Spiceypy

Module Style

Problems met

Hard to manage hundreds even thousands of variables and dozens of subtoutines

Printing report seems to be a problem

Advantages

- Better encapsulation
- Objective-oriented
- More easy to manage variables and subroutines

Thanks for listening!

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Sichuan - 27th March 2020