Foundations of Robotics Project 3 Report

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Question 1:

The given data of the robot is as follows:

$$d_0 = 1\ m,\ a_1 = a_2 = 0.5\ m,\ l_1 = l_2 = 0.25\ m$$

$$\theta_{1_{min}} = -\pi/2\ rad,\ \theta_{1_{max}} = \pi/2\ rad,\ \theta_{2_{min}} = -\pi/2\ rad,\ \theta_{2_{max}} = \pi/4\ rad$$

$$m_{l1} = m_{l2} = 25\ kg,\ m_{l3} = 10\ kg, Il1 = I_{l2} = 5\ kgm^2,\ I_{l4} = 1\ kgm^2$$

$$k_{r1} = k_{r2} = 1,\ k_{r3} = 50\ rad/m,\ k_{r4} = 20,$$

$$I_{m1} = I_{m2} = 0.0001\ kgm^2,\ I_{m3} = 0.01\ kgm^2, I_{m4} = 0.005\ kgm^2$$

$$F_{m1} = F_{m2} = 0.0001\ N \cdot m \cdot s/rad, F_{m3} = 0.01\ N \cdot m \cdot s/rad, F_{m4} =\ 0.005\ N \cdot m \cdot s/rad$$

$$d_{3_{min}} = 0.25\ m,\ d_{3_{max}} = 1\ m,\ \theta_{4_{min}} = -2\pi\ rad,\ \theta_{4_{max}} = 2\pi\ rad$$

As done for projects 1 and 2, the frames are depicted into the figure and the DH parameters are

	d_i	α_i	θ_i	a_i
Link 1	0	0	θ_1	a_1
Link 2	0	0	θ_2	a_2
Link 3	d_3	0	0	0
Link 4	0	0	θ_4	0

We need to create a trajectory in the robot's operational space that takes 4 seconds to complete and has a trapezoidal velocity profile for each segment. The trajectory must pass through the following waypoints:

- p0 = [0, -0.80, 0] at time t0 = 0.0
- p1 = [0, -0.80, 0.5] at time t1 = 0.6
- p2 = [0.5, -0.6, 0.5] at time t2 = 2.0
- p3 = [0.8, 0.0, 0.5] at time t3 = 3.4
- p4 = [0.8, 0.0, 0] at time t4 = 4.0.

Here the anticipation time for each segment is 0.2 seconds and the sampling time is 0.001.

The acceleration of this trajectory is defined by the formula:

Acceleration = 4* (Final Position – Initial Position)/ (Final Time – Initial Time)2 − 4 ∗ Initial Time ∗ Final Time

Here the length travelled can be calculated with the following constraints:

$$q(t) = \begin{cases} q_i + \frac{1}{2} * \ddot{q}_c * t^2 & 0 < t < tc \\ q_i + \ddot{q}_c * t_c\left(t - \frac{t_c}{2}\right) & t_c < t < t_f - t_c \\ q_f - \frac{1}{2} * \ddot{q}_c * (t_f - t_c)^2 & t_f - t_c < t < t_f \end{cases}$$

Here "qi" is the initial position, Tc is the time segment in the graph where the graph flattens in the trapezoidal profile, "qf" is the final position and the double differentiation of "qc" is the acceleration during traversal.

The time segment where the graph flattens in the trapezoidal profile, which is denoted as "tc" from the above-given image and GFT in the code is given by the following formula:

$$t_c = \frac{(t_f + t_i)}{2} - \frac{1}{2} * \sqrt{\frac{t_f^2 * q_c - 4 * \ddot{(}q_f - q_\iota)}{\ddot{q_c}}}$$

The variables in the image here follow the same naming rubrics as before.

The path of the operational space is computed by the following formula:

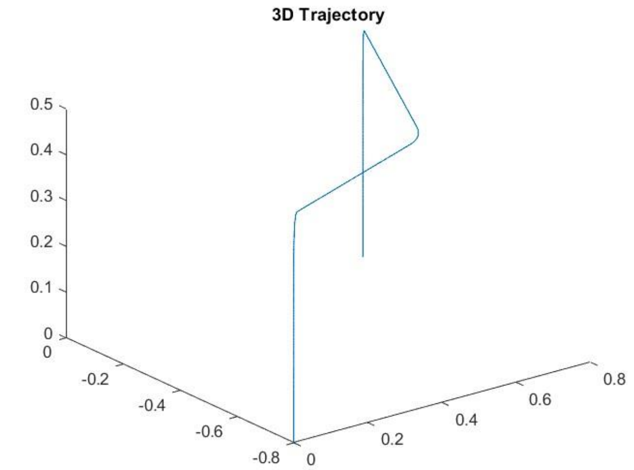
$$\boldsymbol{s_j(t)} = \begin{cases} \boldsymbol{0} & \boldsymbol{0 \le t \le t_{j-1} - \Delta t_j} \\ \boldsymbol{s'_j(t + \Delta t_j)} & \boldsymbol{t_{j-1} - \Delta t_j < t < \ t_j - \Delta t} \\ \boldsymbol{\|p_j - p_{j-1}\|} & \boldsymbol{t_j - \Delta t \le t \le t_f - \Delta t_N} \end{cases}$$

The naming rubrics for each variable here are the same as before, the variable p is calculated by the following formula for J= 1-N :

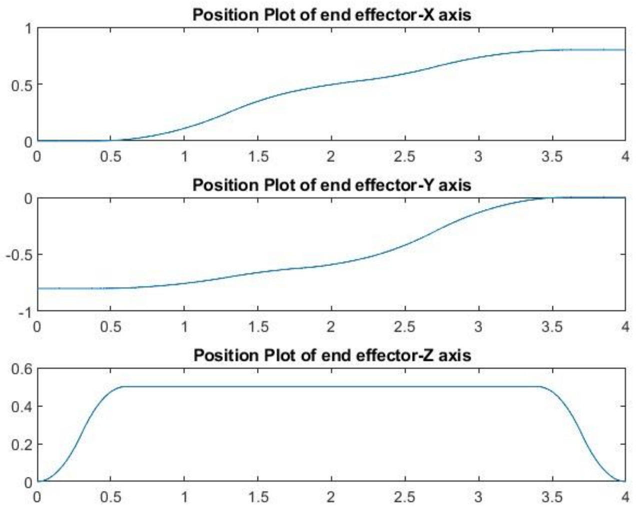
$$\boldsymbol{p_e} = \boldsymbol{p_0} + \sum_{\boldsymbol{j=1}}^{\boldsymbol{N}} \frac{\boldsymbol{s_j}}{\left\|\boldsymbol{p_j - p_{j-1}}\right\|} * (\boldsymbol{p_j - p_{j-1}})$$

Plugging all these values into Matlab, the output graphs are as follows:

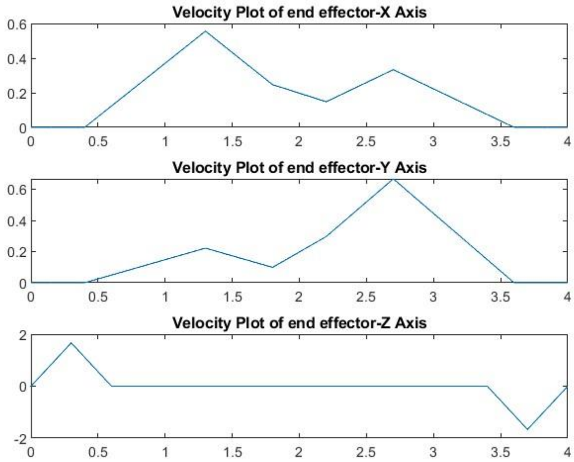
1:3D Trajectory Graph:



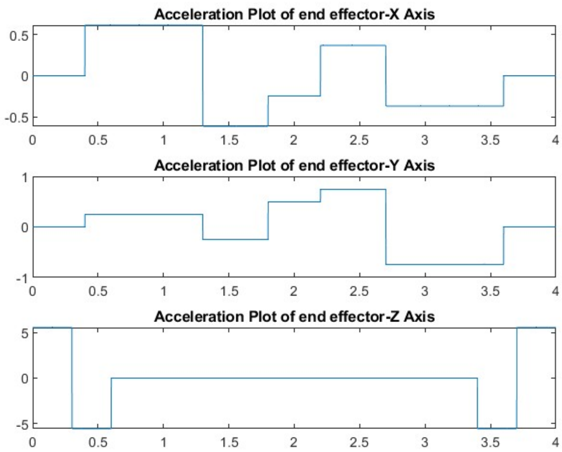
2: Position at end Effector:



3: Velocity at end Effector:



4: Acceleration at end Effector:



Question 2

B cap is calculated using the formula:

$$\boxed{b_{11} = l_1^2 * m_{l1} + (0.25 + l_2^2 + l_2 * c_2) * m_{l2} + (0.5 + 0.5c_2) * (m_{l3} + m_{l4}) + I_{l1} + I_{l2} + I_{l4} + I_{m1} * k_{r1}^2 + I_{m2} + I_{m3} + I_{m4};}$$

$$b_{12} = b_{21} = (0.5 * l_2 * c_2 + l_2^2) * m_{l2} + (0.25 + 0.25 * c_2) * (m_{l3} + m_{l4}) + I_{l2} + I_{l4} + I_{m2} * k_{r2} + I_{m3} + I_{m4};$$

$$b_{13} = b_{31} = -k_{r3} * I_{m3};$$

$$b_{14} = b_{41} = I_{l4} + k_{r4} * I_{m4};$$

$$b_{22} = l_2^2 * m_{l2} + 0.25 * (m_{l3} + m_{l4}) + I_{l2} + I_{l4} + I_{m2} * k_{r2}^2 + I_{m3} + I_{m4};$$

$$b_{23} = b_{32} = -k_{r3} * I_{m3};$$

$$b_{24} = b_{42} = I_{l4} + k_{r4} * I_{m4};$$

$$b_{33} = m_{l3} + I_{m3} * k_{r3}^2;$$

$$b_{34} = b_{43} = 0;$$

$$b_{44} = I_{l4} + k_{r4}^2 * I_{m4};$$

The B cap matrix is calculated using:

$$B\ cap(q) = \begin{bmatrix} b11 & b11 & b13 & b14 \\ b21 & b22 & b23 & b24 \\ b31 & b32 & b33 & b34 \\ b41 & b42 & b43 & b44 \\ & & & \end{bmatrix}$$

$$y = \ddot{q}d + KD * q + KP * q + \dot{\omega}$$

The n-cap matrix is calculated using the formula:

$$n\ (q,\dot{q})\ \tilde{}\ n(q,\dot{q}) = C(q,\dot{q}) * \dot{q} + Fv * \dot{q} + g(q)$$

$$c_{11} = \frac{1}{2} * \frac{\partial b_{11}}{\partial q_2} * \dot{\theta}_2;$$

$$c_{11} = \frac{-l_2 * m_{l2} * s_2 - 0.5 * (m_{l3} + m_{l4}) * s_2}{2} * \dot{\theta}_2;$$

$$c_{12} = \frac{\partial b_{12}}{\partial q_2} * \dot{\theta}_2 + \frac{1}{2} * \frac{\partial b_{11}}{\partial q_2} * \dot{\theta}_1;$$

$$c_{12} = \frac{-l_2 * m_{l2} * s_2 - 0.5 * (m_{l3} + m_{l4}) * s_2}{2} * (\dot{\theta}_1 + \dot{\theta}_2);$$

$$c_{21} = -\frac{1}{2} * \frac{\partial b_{11}}{\partial q_2} * \dot{\theta}_1;$$

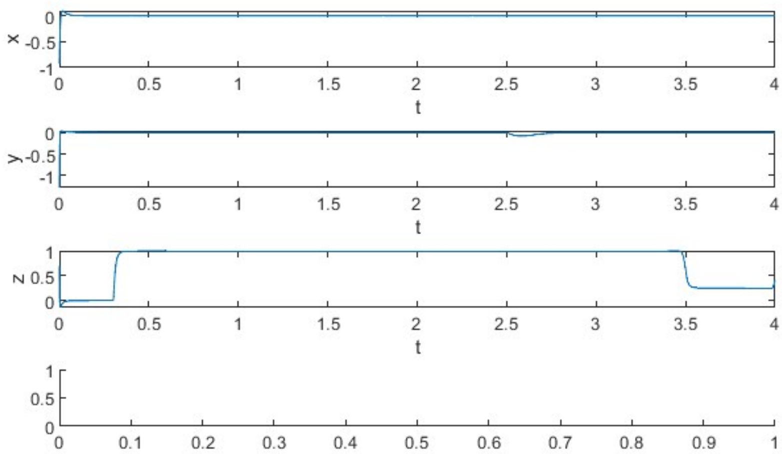
$$c_{21} = \frac{l_2 * m_{l2} * s_2 + 0.5 * (m_{l3} + m_{l4}) * s_2}{2} * \dot{\theta}_1;$$

$$C(q,\dot{q}) = \begin{bmatrix} c_{11} & c_{12} & 0 & 0 \\ c_{21} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

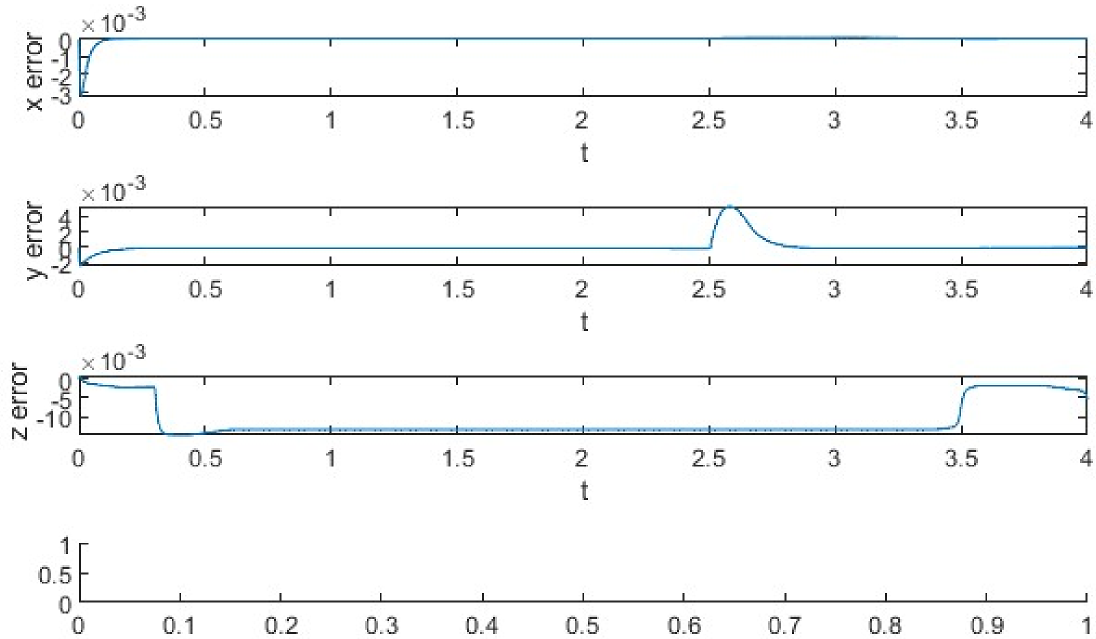
$$G = \begin{bmatrix} 0 \\ 0 \\ -(m_{l3} + m_{l4}) * g \\ 0 \end{bmatrix}$$

The result of the project is as follows :

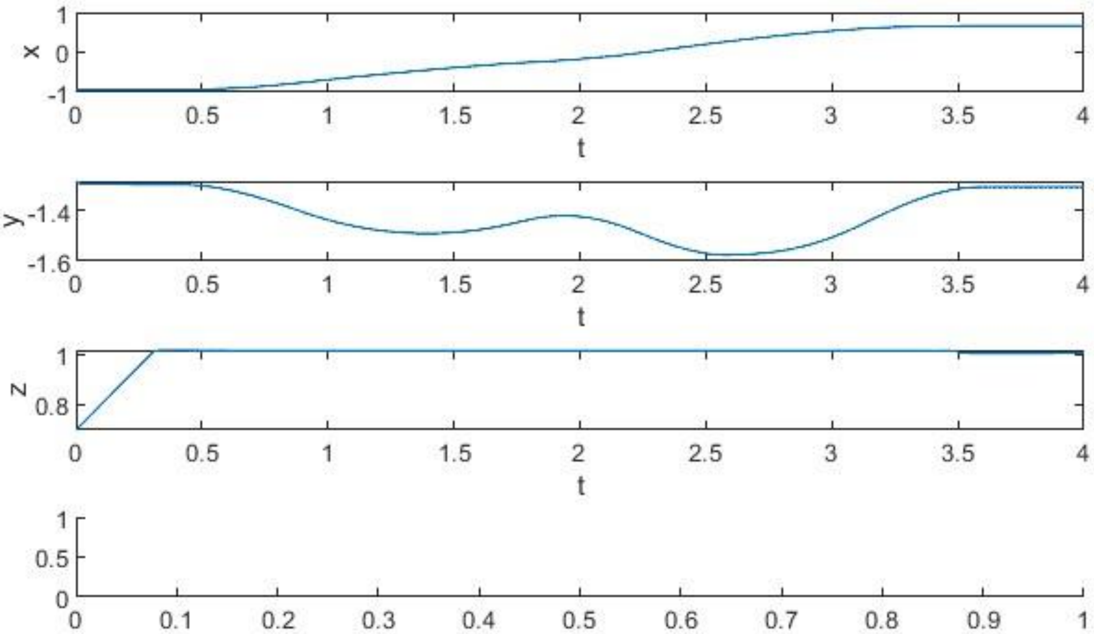
1 : Edot vs time



2 Error vs time



3 q vs time



References:

PPT

Matlab handbook.