Foundations of Robotics Project 2 Report

Name: Reuben O Jacob

NYU ID: N16440385

Question 1:

The given DH parameters are:

θ	α	a	d
$\theta 1$	0	a1	d0
$\theta 2$	0	a2	0
0	0	0	-d3
$\theta 4$	0	0	0

The kinematic matrices for the given parameters are solved as follows :

```
 T1 = [ cos(t1) -sin(t1)*cos(alpha1) sin(t1)*sin(alpha1)    a1*cos(t1);
        sin(t1) cos(t1)*cos(alpha1) -cos(t1)*sin(alpha1)    a1*sin(t1);
        0       sin(alpha1)          cos(alpha1)            d1;
        0          0                    0                   1]

T2 = [ cos(t2) -sin(t2)*cos(alpha2) sin(t2)*sin(alpha2)  a2*cos(t2);
       sin(t2) cos(t2)*cos(alpha2) -cos(t2)*sin(alpha2) a2*sin(t2);
        0      sin(alpha2)          cos(alpha2)         d2;
        0        0                     0                 1]

T3 = [ cos(t3) -sin(t3)*cos(alpha3) sin(t3)*sin(alpha3)  a3*cos(t3);
       sin(t3) cos(t3)*cos(alpha3) -cos(t3)*sin(alpha3) a3*sin(t3);
        0          sin(alpha3)      cos(alpha3)        -d3;
        0            0               0                   1]

T4 = [  cos(t4) -sin(t4)*cos(alpha4) sin(t4)*sin(alpha4) a4*cos(t4);
       sin(t4) cos(t4)*cos(alpha4) -cos(t4)*sin(alpha4) a4*sin(t4);
       0        sin(alpha4)         cos(alpha4)           d4;
       0         0                   0                    1]
```

The final Homogeneous matrix, which is the multiplication of all these matrices is :
T04 = simplify (Base*T1*T2*T3*T4)

```
After computation, the final matrix is:
[cos(t1 + t2 + t4), -sin(t1 + t2 + t4), 0, cos(t1 + t2)/2 + cos(t1)/2]
[sin(t1 + t2 + t4), cos(t1 + t2 + t4), 0, sin(t1 + t2)/2 + sin(t1)/2]
[ 0, 0, 1, 1 - d3]
[ 0, 0, 0, 1]
```

After calculating the P0,P1, P2,P3 and P4 from T1-4, the final Jacobian matrix is calculated using the direct kinematics equation :

$$ve=[pe\omega e]=J(q)(\dot{q})$$

The final Jacobian matrix is:

```
[- sin(t1 + t2)/2 - sin(t1)/2, -sin(t1 + t2)/2, 0,      0]
[ cos(t1 + t2)/2 + cos(t1)/2,   cos(t1 + t2)/2, 0,      0]
[ 0,                                0,          1,      0]
[ 1,                                1,          0,      1]
```

In my code, I have written the Jacobian values directly using values from the Kinematic trajectory file to reduce processing time.

This Jacobian is fed into a Matlab Simulink file to attain the graphs. The gain values were assumed using trial and error. The structure of this file was attained from the lecture slides.

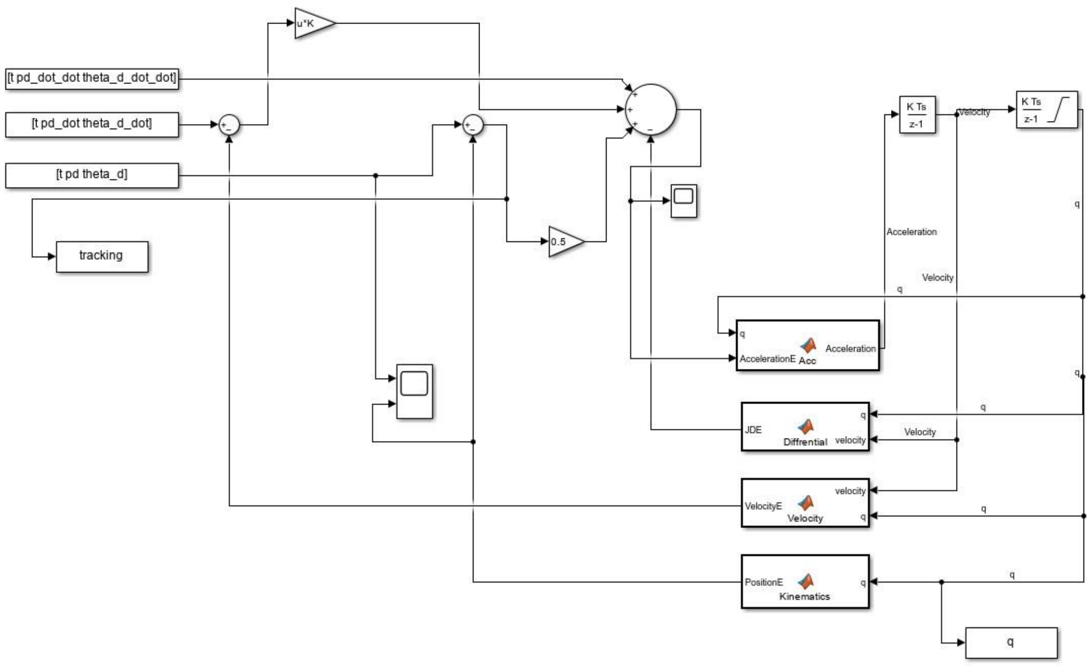


Fig1: Matlab Simulink Diagram

The Scope Diagram of the Simulink file is as follows:

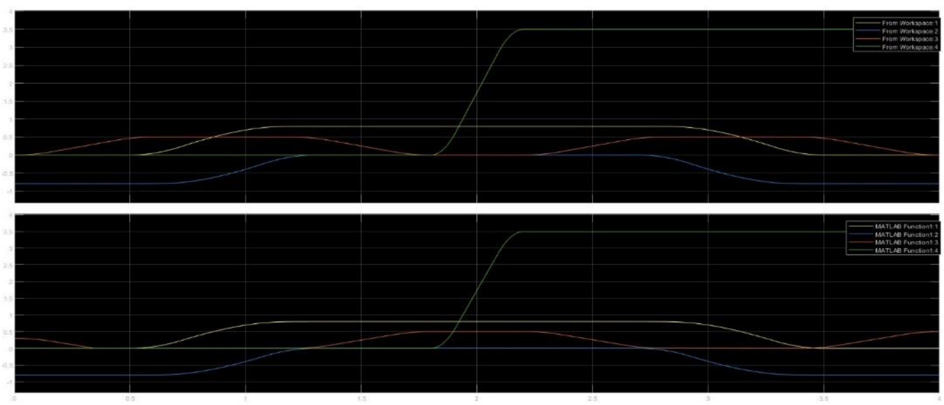


Fig2: Scope Diagram of Question 1

The graphs of the joint variables and errors are as follows:

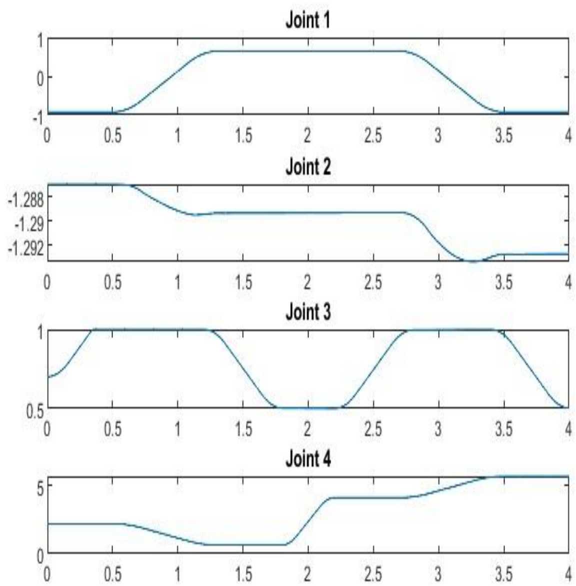


Fig3: Graph of Joint Variables

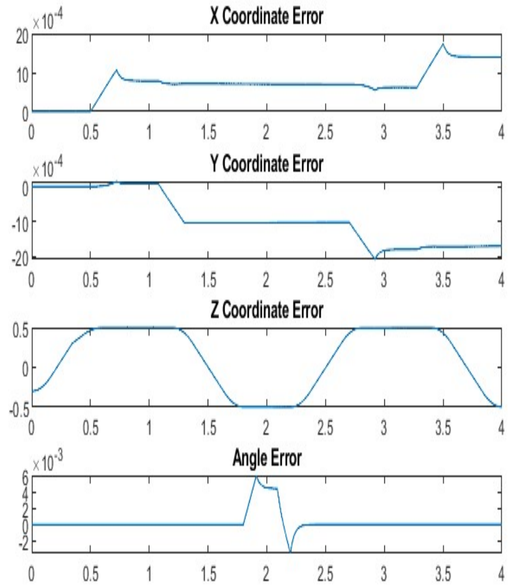


Fig4: Operational Space Errors

<u>Question 2:</u>

In this question, we are supposed to relax the z component and implement a Matlab/Simulink second-order algorithm for kinematic inversion along the given trajectory maximizing the distance from a given obstacle along the path.

In order to relax the z component, we have to delete the third row of the Jacobian. Since the number of rows and columns of this matrix do not match, the pseudo-inverse method is used to calculate the inverse.

The formula of the pseudo inverse is as follows:

$$J^{\dagger} = JT(JJT)-1$$

The formula used to find q dot dot i.e. acceleration is:

$$\ddot{\boldsymbol{q}} = \boldsymbol{J}_A^{\dagger}\left(\ddot{\boldsymbol{x}}_d + \boldsymbol{K}_D\dot{\boldsymbol{e}} + \boldsymbol{K}_P\boldsymbol{e} - \dot{\boldsymbol{J}}_A(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}\right) + (\boldsymbol{I}_n - \boldsymbol{J}_A^{\dagger}\boldsymbol{J}_A)\ddot{\boldsymbol{q}}_0$$

I have written the distance from the obstacle w(q) as

```
wq =[(cos(q(2))/2)-0.2;sin(q(2)/2)+0.9;q(3)-0.3;0];
```

This was calculated using the formula:

$$w(\boldsymbol{q}) = \min_{\boldsymbol{p},\boldsymbol{o}} \|\boldsymbol{p}(\boldsymbol{q}) - \boldsymbol{o}\|$$

Here O is the centre point of the obstacle and p is the position vector along the obstacle structure.

In matrix form, it is written as:

$$w(q) = \begin{bmatrix} c_{124} & -s_{124} & 0 & a_1c_1 + a_2c_{12} \\ s_{124} & c_{124} & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 0\cdot 4 \\ -0.7 \\ 0\cdot 5 \\ 1 \end{bmatrix} - \begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \\ 1 \end{bmatrix}$$

The following values were used as input in a Matlab Simulink file displayed below which was created using the structure provided in the lecture slides.

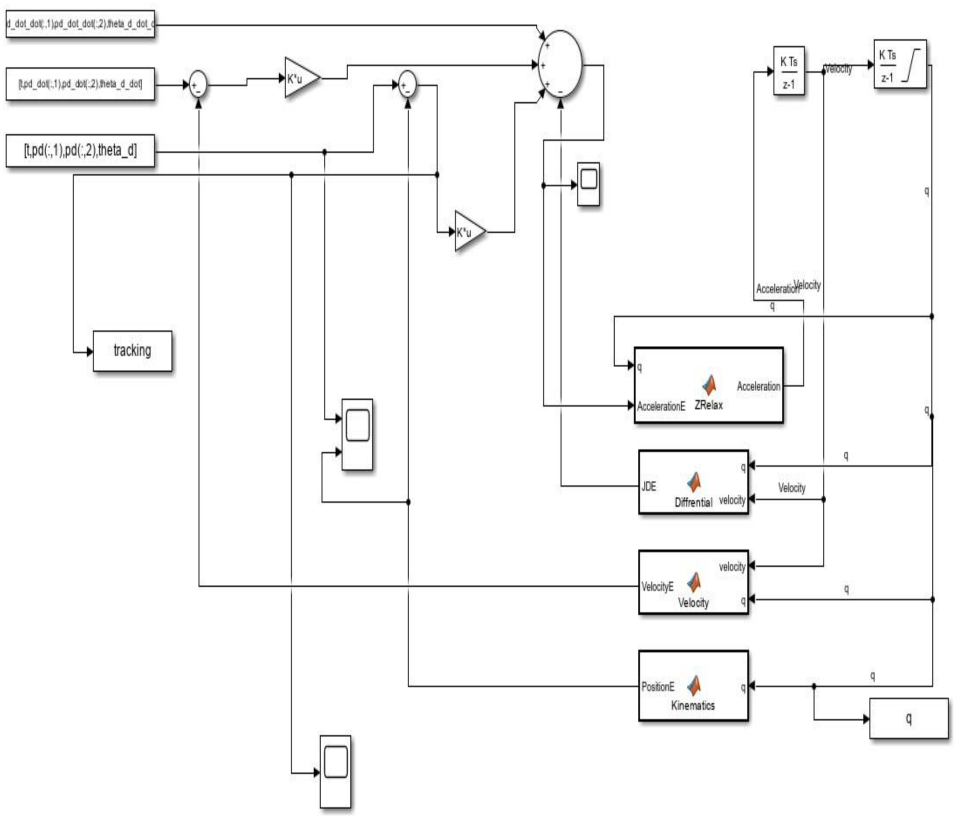


Fig5: Matlab Simulink Diagram of Question 2

The Scope Diagram of the Simulink file is as follows:

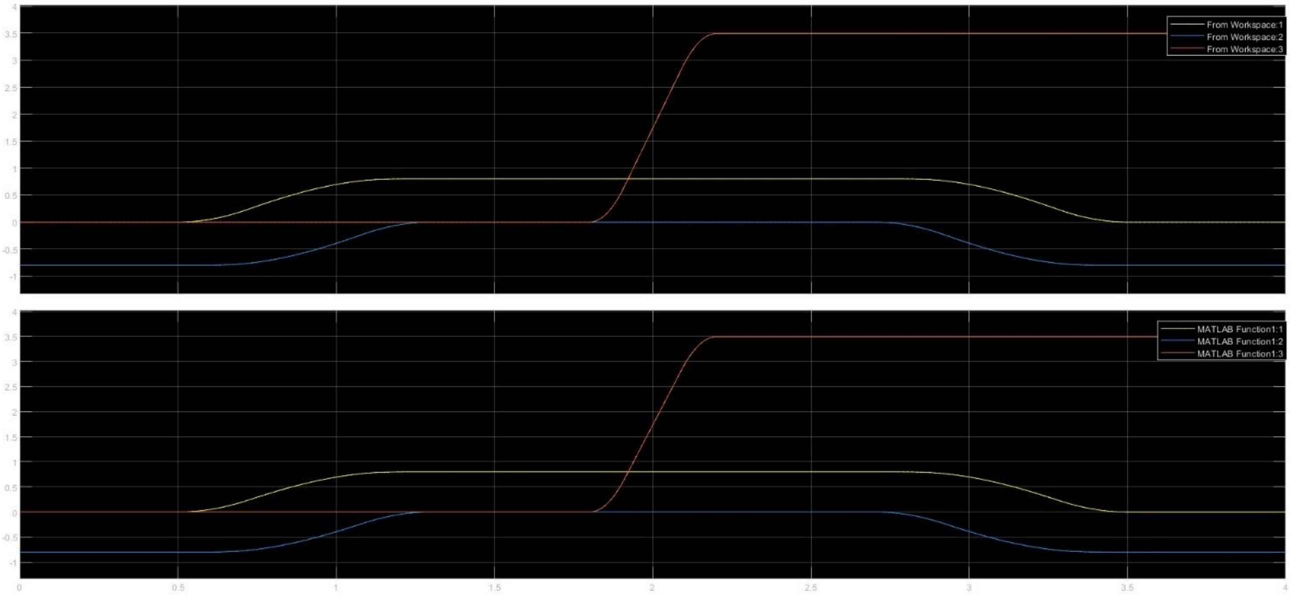
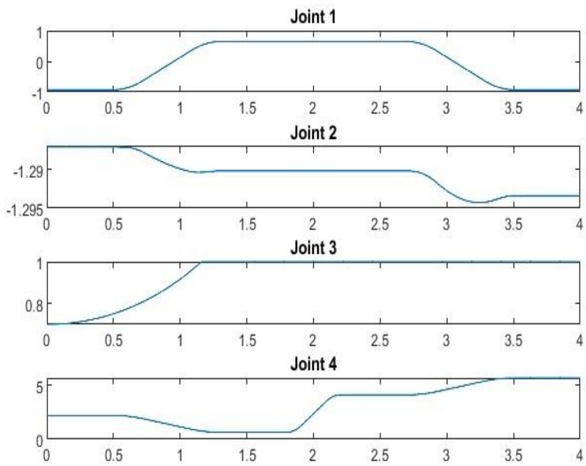


Fig6: Scope Diagram of Question 2

The graphs of the joint variables and errors are as follows:



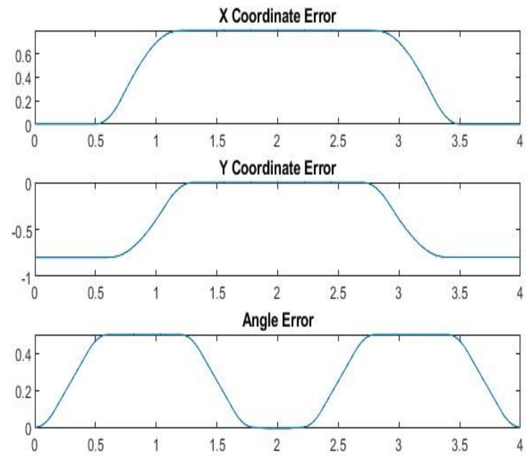


Fig7: Graph of Joint Variables

Fig8: Operational Space Errors

References:

- B. Siciliano, L. Sciavicco, L. Villani, and G. Oriolo, Robotics: Modelling, Planning and Control. New York, NY, USA: Springer, 2009
- Lecture Slides