Foundations of Robotics Project Report 1

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Question 1: States to implement the kinematic inversion with inverse and Jacobian transpose along the given trajectory. Along with visualization plots of the same. The DH parameters of the model are as follows :

ϴ	α	a	d
ϴ1	0	a1	d0
ϴ2	0	a2	0
0	0	0	-d3
ϴ4	0	0	0

To solve for the Jacobian, the equation for the homogeneous transformations are as follows:

```
T1 = [ cos(t1) -sin(t1)*cos(alpha1)  sin(t1)*sin(alpha1)       a1*cos(t1);
       sin(t1)  cos(t1)*cos(alpha1) -cos(t1)*sin(alpha1)        a1*sin(t1);
       0            sin(alpha1)             cos(alpha1)              d1;
       0            0                       0                        1]

T2 = [ cos(t2) -sin(t2)*cos(alpha2)  sin(t2)*sin(alpha2)       a2*cos(t2);
       sin(t2)  cos(t2)*cos(alpha2) -cos(t2)*sin(alpha2)        a2*sin(t2);
       0            sin(alpha2)             cos(alpha2)              d2;
       0            0                       0                        1]

T3 = [ cos(t3) -sin(t3)*cos(alpha3)  sin(t3)*sin(alpha3)       a3*cos(t3);
       sin(t3)  cos(t3)*cos(alpha3) -cos(t3)*sin(alpha3)        a3*sin(t3);
       0            sin(alpha3)             cos(alpha3)             -d3;
       0            0                       0                        1]

T4 = [ cos(t4) -sin(t4)*cos(alpha4)  sin(t4)*sin(alpha4)       a4*cos(t4);
       sin(t4)  cos(t4)*cos(alpha4) -cos(t4)*sin(alpha4)        a4*sin(t4);
       0            sin(alpha4)             cos(alpha4)              d4;
       0            0                       0                        1]
```

The final Homogeneous matrix, which is the multiplication of all these matrices is :

```
T04 = simplify (Base*T1*T2*T3*T4)
After computation, the final matrix is:

[cos(t1 + t2 + t4), -sin(t1 + t2 + t4), 0, cos(t1 + t2)/2 + cos(t1)/2]
[sin(t1 + t2 + t4),  cos(t1 + t2 + t4), 0, sin(t1 + t2)/2 + sin(t1)/2]
[                0,                  0, 1,                     1 - d3]
[                0,                  0, 0,                          1]
```

After calculating the P0,P1,P2,P3 and P4 from T1-4, the final Jacobian matrix is calculated using the direct kinematics equation :

$$ve=[pe\omega e]=J(q)(\dot{q})$$

Substituting Values to this equation, the representation is:

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \omega_x\\ \omega_y\\ \omega_z\end{bmatrix}=\begin{bmatrix}-a_1s_1-a_2s_{12} & -a_2s_{12} & 0 & 0\\ a_1c_1+a_2c_{12} & a_2c_{12} & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 1 & 1 & 0 & 1\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ -\dot{d}_3\\ \dot{\theta}_4\end{bmatrix}$$

Let $a_1=a_2=0.5$

$$\frac{-s_{12}\dot{\theta}_2}{2}-\frac{(s_1+s_{12})\dot{\theta}_1}{2}=\dot{\boldsymbol{x}}$$

$$\frac{c_{12}\dot{\theta}_2}{2}+\frac{(c_1+c_{12})\dot{\theta}_1}{2}=\dot{\boldsymbol{y}}$$

$$-\dot{d}_3=\dot{\boldsymbol{z}}$$

$$\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_4=\dot{\boldsymbol{\omega}}_{\boldsymbol{z}}$$

The final Jacobian matrix is:

```
[- sin(t1 + t2)/2 - sin(t1)/2, -sin(t1 + t2)/2, 0, 0]
[  cos(t1 + t2)/2 + cos(t1)/2,  cos(t1 + t2)/2, 0, 0]
[                           0,               0, 1, 0]
[                           1,               1, 0, 1]
```

In this matrix, the fourth and fifth rows were entirely consisting of zeroes and hence they were removed to avoid redundancy.

Simulating the inverse and transpose on Simulink, the scope images are as follows:
The gains of each simulation were selected by trial and error.

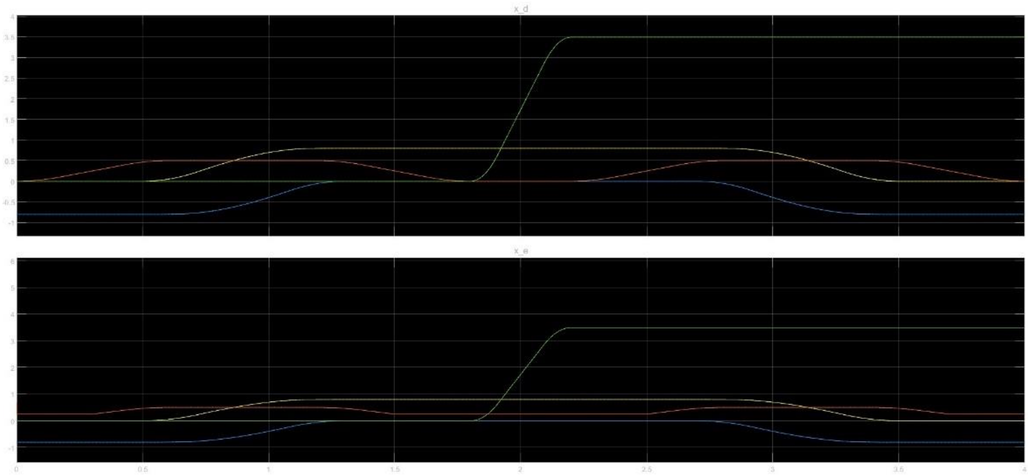


Figure: Scope of inverse simulation

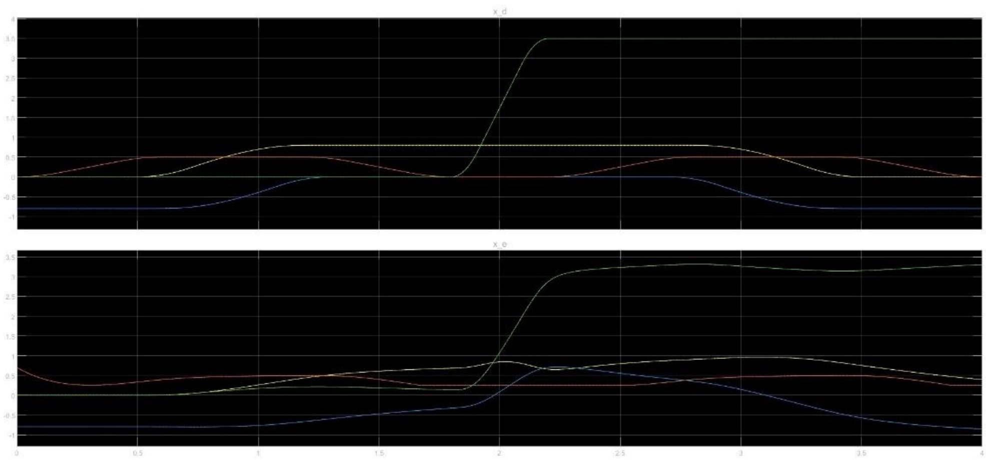
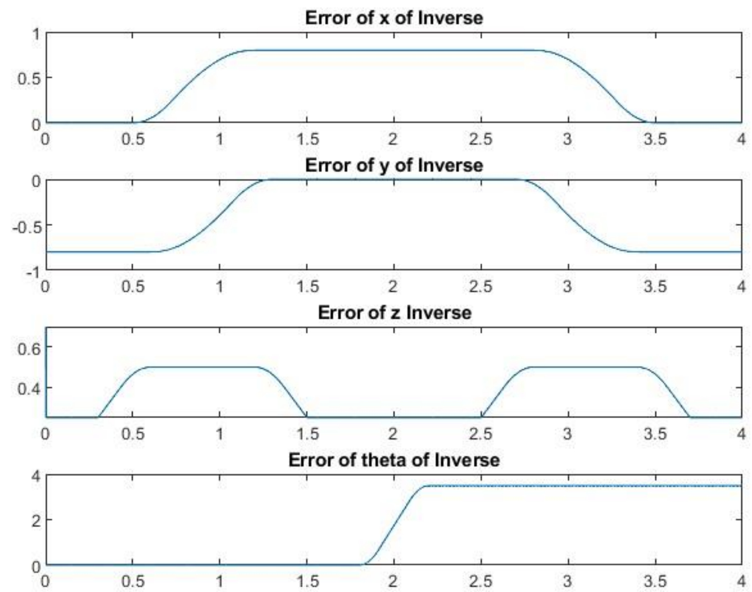


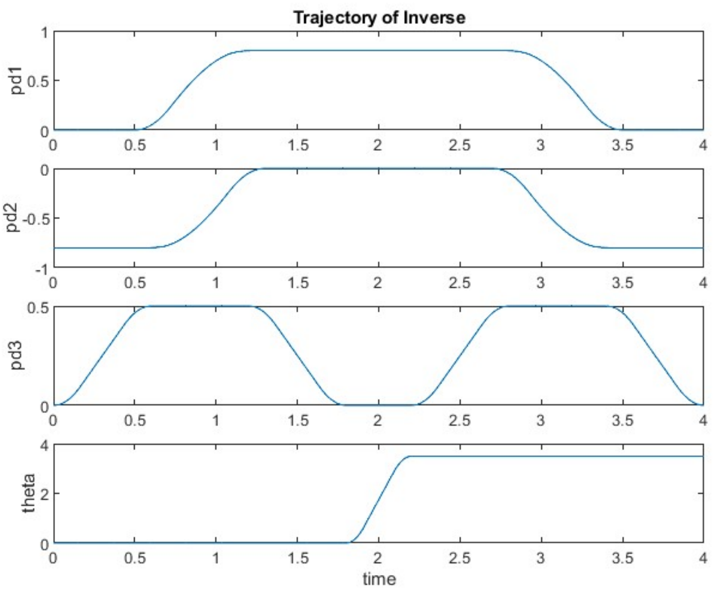
Figure: Scope of Transpose Simulation

After simulating the model on Simulink using the data from kinematics.mat the output graphs are as follows :

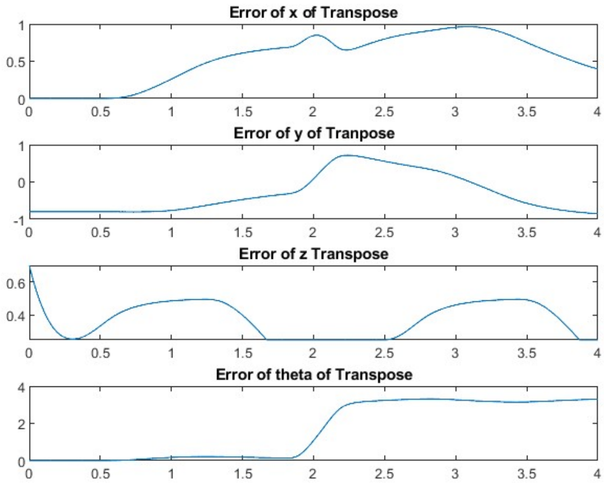
Inverse: Error Graph:



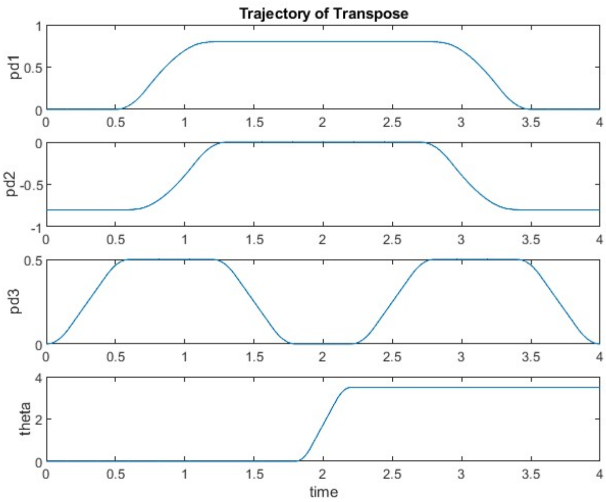
Inverse: Trajectory Graph:



Transpose: Error Graph:



Transpose: Trajectory Graph:



In Question 2 the phi component is asked to be relaxed. Theta 1, 2, and 4 are three rotations that make up the phi. Since theta 1, 2, and 4 together make up the angular velocity of the Z axis, when we try to relax the phi component, we essentially understand that there is no rotation about the Z axis. Therefore, the Analytical Jacobian's fourth row is removed, giving us a 3 by 4 matrix.

Since this new matrix cannot be inversed the pseudo-inverse formula is.

$$J^{\dagger}=J^{T}(JJ^{T})^{-1}$$

Hence the new Jacobian matrix is :

J = [-a2*s12-a1*s1 -a2*s12 0 0;

a2*c12+a1*c1 a2*c12 0 0;

0 0 1 0;

1 1 0 1];

When phi is relaxed the fourth row of the matrix is removed and hence the new matrix is:

J = [-a2*s12-a1*s1 -a2*s12 0 0;

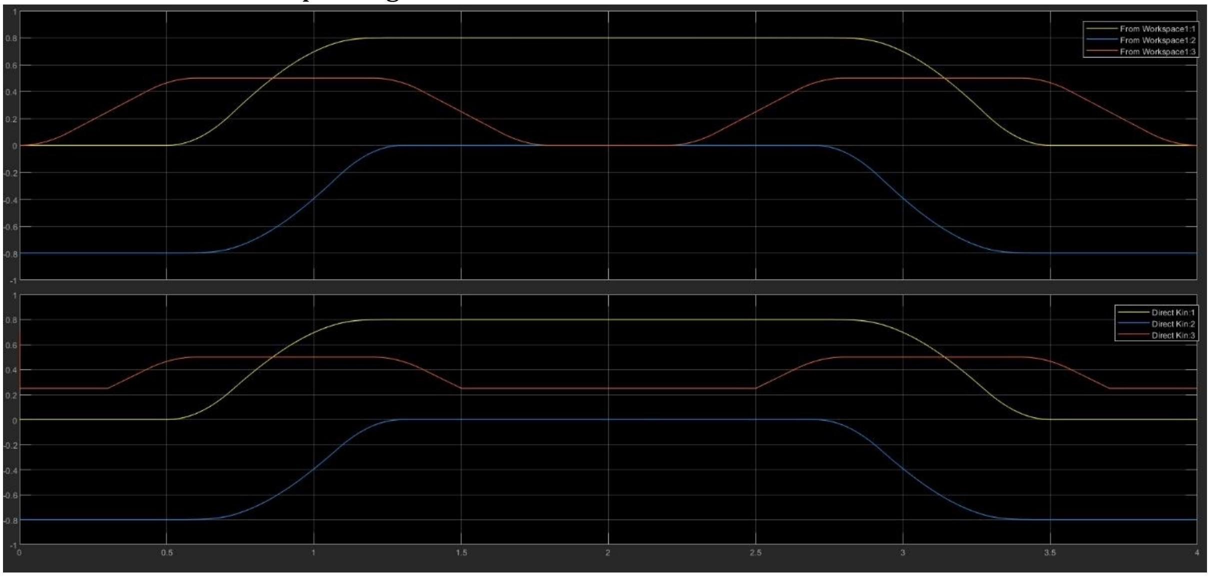
a2*c12+a1*c1 a2*c12 0 0;

0 0 1 0]

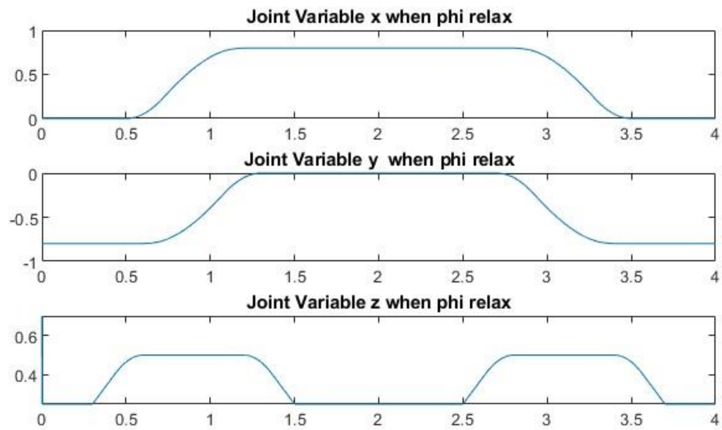
Here, the distance w, is given by the formula:

$w(q)=-18[q12\Pi2+q2.2+(\pi|8)2-q2\Pi/89\pi216+q4216\Pi2+q322\cdot25.$

After simulation the scope image is :



The graphs are :



References :

1 PPT

2 Dundee Matlab