MATHEMATICAL FORMULAE HANDBOOK

Class: IX - X

NICE INFOTECH, POLICE BAZAAR, SHILLONG-1

Algebra

1. $(a+b)^2 = a^2 + 2ab + b^2;\ a^2 + b^2 = (a+b)^2 - 2ab$
2. $(a-b)^2 = a^2 - 2ab + b^2;\ a^2 + b^2 = (a-b)^2 + 2ab$
3. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
4. $(a+b)^3 = a^3 + b^3 + 3ab(a+b);\ a^3 + b^3 = (a+b)^3 - 3ab(a+b)$
5. $(a-b)^3 = a^3 - b^3 - 3ab(a-b);\ a^3 - b^3 = (a-b)^3 + 3ab(a-b)$
6. $a^2 - b^2 = (a+b)(a-b)$
7. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
8. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
9. $a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \cdots + b^{n-1})$
10. $a^n = a.a.a \ldots n$ times
11. $a^m.a^n = a^{m+n}$
12. $\dfrac{a^m}{a^n} = a^{m-n}$ if $m > n$

    $= 1 \qquad$ if $m = n$

    $= \dfrac{1}{a^{n-m}}$ if $m < n; a \in R, a \neq 0$
13. $(a^m)^n = a^{mn} = (a^n)^m$
14. $(ab)^n = a^n.b^n$
15. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$
16. $a^0 = 1$ where $a \in R, a \neq 0$
17. $a^{-n} = \dfrac{1}{a^n}, a^n = \dfrac{1}{a^{-n}}$
18. $a^{p/q} = \sqrt[q]{a^p}$
19. If $a^m = a^n$ and $a \neq \pm 1, a \neq 0$ then $m = n$
20. If $a^n = b^n$ where $n \neq 0$, then $a = \pm b$
21. If $\sqrt{x}, \sqrt{y}$ are quadratic surds and if $a + \sqrt{x} = \sqrt{y}$, then $a = 0$ and $x = y$
22. If $\sqrt{x}, \sqrt{y}$ are quadratic surds and if $a + \sqrt{x} = b + \sqrt{y}$ then $a = b$ and $x = y$
23. If a, m, n are positive real numbers and $a \neq 1$, then $\log_a mn = \log_a m + \log_a n$
24. If a, m, n are positive real numbers, $a \neq 1$, then $\log_a \left(\dfrac{m}{n}\right) = \log_a m - \log_a n$
25. If a and m are positive real numbers, $a \neq 1$ then $\log_a m^n = n \log_a m$
26. If a, b and k are positive real numbers, $b \neq 1, k \neq 1$, then $\log_b a = \dfrac{\log_k a}{\log_k b}$
27. $\log_b a = \dfrac{1}{\log_a b}$ where a, b are positive real numbers, $a \neq 1, b \neq 1$
28. if a, m, n are positive real numbers, $a \neq 1$ and if $\log_a m = \log_a n$, then $m = n$

29. if $a+ib=0$ where $i=\sqrt{-1}$, then $a=b=0$

30. if $a+ib=x+iy$, where $i=\sqrt{-1}$, then $a=x$ and $b=y$

31. The roots of the quadratic equation $ax^2+bx+c=0$; $a\neq 0$ are $\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

    The solution set of the equation is $\left\{\dfrac{-b+\sqrt{\Delta}}{2a},\dfrac{-b-\sqrt{\Delta}}{2a}\right\}$
    where $\Delta=$ discriminant $=b^2-4ac$

32. The roots are real and distinct if $\Delta>0$.

33. The roots are real and coincident if $\Delta=0$.

34. The roots are non-real if $\Delta<0$.

35. If α and β are the roots of the equation $ax^2+bx+c=0, a\neq 0$ then
    i) $\alpha+\beta=\dfrac{-b}{a}=-\dfrac{\text{coeff. of } x}{\text{coeff. of } x^2}$
    ii) $\alpha\cdot\beta=\dfrac{c}{a}=\dfrac{\text{constant term}}{\text{coeff. of } x^2}$

36. The quadratic equation whose roots are α and β is $(x-\alpha)(x-\beta)=0$
    i.e. $x^2-(\alpha+\beta)x+\alpha\beta=0$
    i.e. $x^2-Sx+P=0$ where $S=$Sum of the roots and $P=$Product of the roots.

37. For an arithmetic progression (A.P.) whose first term is (a) and the common difference is (d).
    i) n^{th} term$=t_n=a+(n-1)d$
    ii) The sum of the first (n) terms $=S_n=\dfrac{n}{2}(a+l)=\dfrac{n}{2}\{2a+(n-1)d\}$
    where $l=$last term$=a+(n-1)d$.

38. For a geometric progression (G.P.) whose first term is (a) and common ratio is (γ),
    i) n^{th} term$=t_n=a\gamma^{n-1}$.
    ii) The sum of the first (n) terms:

$$\begin{aligned} S_n &= \frac{a(1-\gamma^n)}{1-\gamma} \quad \text{if} \gamma<1 \\ &= \frac{a(\gamma^n-1)}{\gamma-1} \quad \text{if } \gamma>1 \\ &= na \quad \text{if } \gamma=1 \end{aligned}.$$

39. For any sequence $\{t_n\}, S_n-S_{n-1}=t_n$ where $S_n=$Sum of the first (n) terms.

40. $\sum\limits_{\gamma=1}^{n}\gamma=1+2+3+\cdots+n=\dfrac{n}{2}(n+1)$.

41. $\sum\limits_{\gamma=1}^{n}\gamma^2=1^2+2^2+3^2+\cdots+n^2=\dfrac{n}{6}(n+1)(2n+1)$.

42. $\sum_{\gamma=1}^{n} \gamma^3 = 1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 = \frac{n^2}{4}(n+1)^2.$

43. $n! = (1).(2).(3).\ldots.(n-1).n.$

44. $n! = n(n-1)! = n(n-1)(n-2)! = \ldots..$

45. $0! = 1.$

46. $(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + \cdots + b^n, n > 1.$

Trigonometry:

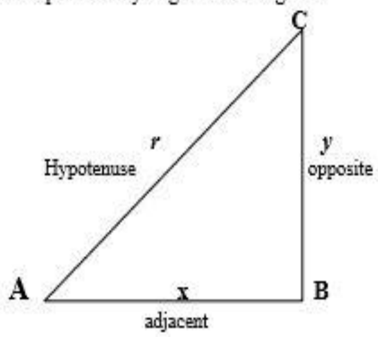
Right Triangle Definitions of Trigonometric Functions

Note: sin & cos are complementary angles, so are tan & cot and sec & cos, and the sum of complementary angles is 90 degrees.

$\sin\theta = \frac{opp}{hyp} = \frac{y}{r}$ $\qquad$ $\csc\theta = \frac{hyp}{opp} = \frac{r}{y}$

$\cos\theta = \frac{adj}{hyp} = \frac{x}{r}$ $\qquad$ $\sec\theta = \frac{hyp}{adj} = \frac{r}{x}$

$\tan\theta = \frac{opp}{adj} = \frac{y}{x}$ $\qquad$ $\cot\theta = \frac{adj}{opp} = \frac{x}{y}$



Adjacent = is the side adjacent to the angle in consideration. So if we are considering Angle A, then the adjacent side is CB

Trigonometric Values of Special Angles

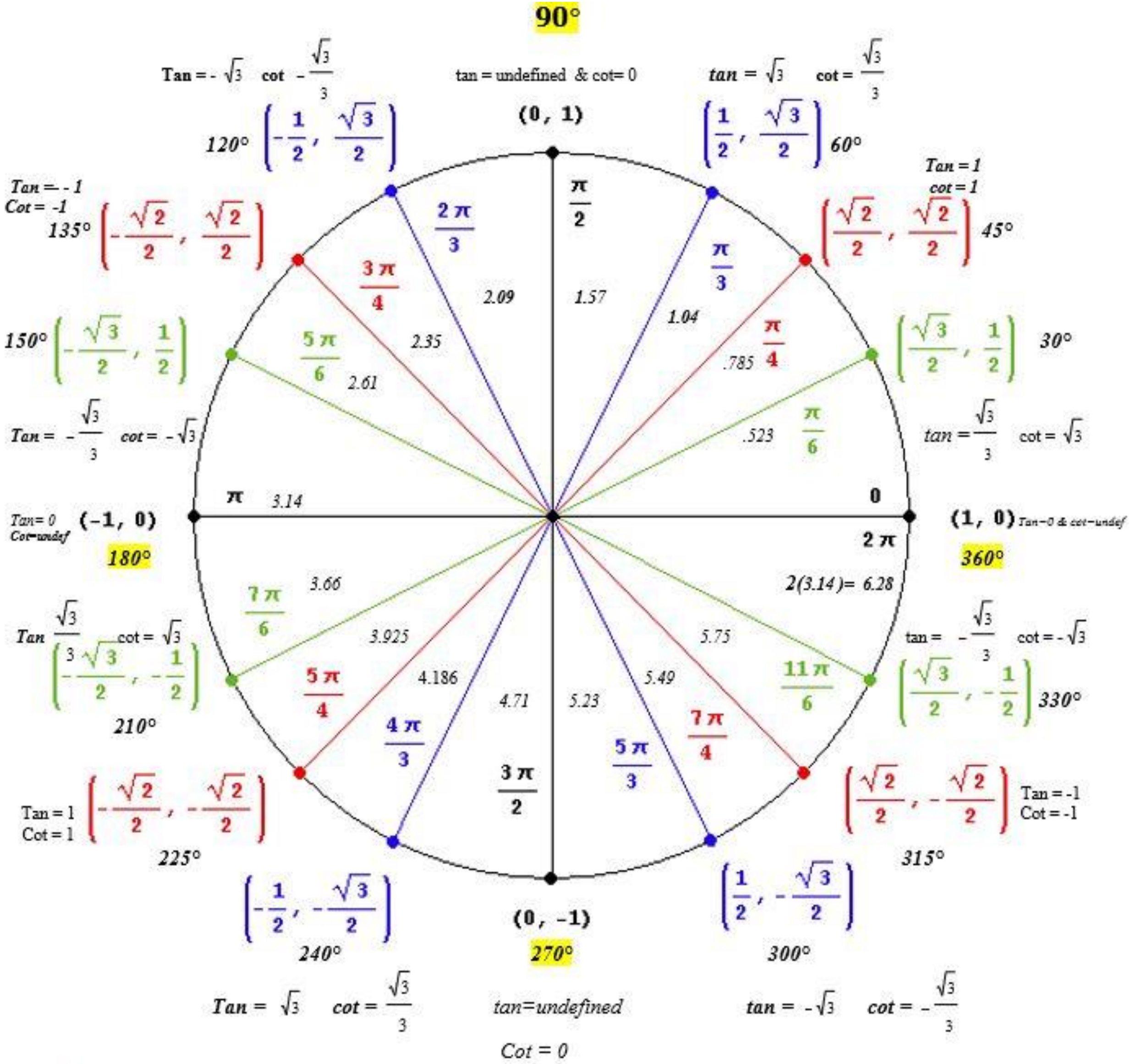
Degrees	0°	30°	45°	60°	90°	180°	270°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
sinθ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
cosθ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
tanθ	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined	0	undefined

To Convert *Degrees to Radians*, Multiply by $\frac{\pi\,\text{rad}}{180\,\text{deg}}$

To Convert *Radians to Degrees*, Multiply by $\frac{180\,\text{deg}}{\pi\,\text{rad}}$

Vocabulary

- Cotangent Angles - are two angles with the same terminal side
- Reference Angle - is an acute angle formed by terminal side of angle(α) with x-axis



Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \qquad \csc x = \frac{1}{\sin x}$$

$$\cos x = \frac{1}{\sec x} \qquad \sec x = \frac{1}{\cos x}$$

$$\tan x = \frac{1}{\cot x} \qquad \cot x = \frac{1}{\tan x}$$

Ratio or Quotient Identities

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

$$\sin x = \cos x \tan x \qquad \cos x = \sin x \cot x$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$

Note: there are only three, basic Pythagorean identities, the other forms are the same three identities, just arranged in a different order.

Pythagorean Identities in Radical Form

$$\sin x = \pm\sqrt{1 - \cos^2 x}$$

$$\tan x = \pm\sqrt{\sec^2 x - 1}$$

$$\cos x = \pm\sqrt{1 - \sin^2 x}$$

Confunction Identities

$$\sin\left(\frac{\pi}{2}-x\right)=\cos x \qquad \cos\left(\frac{\pi}{2}-x\right)=\sin x$$

$$\tan\left(\frac{\pi}{2}-x\right)=\cot x \qquad \cot\left(\frac{\pi}{2}-x\right)=\tan x$$

$$\sec\left(\frac{\pi}{2}-x\right)=\csc x \qquad \csc\left(\frac{\pi}{2}-x\right)=\sec x$$

Odd-Even Identities

Also called negative angle identities

Sin $(-x)$ = $-\sin x$ Csc $(-x)$ = $-\csc x$

Cos $(-x)$ = $\cos x$ Sec $(-x)$ = $\sec x$

Tan $(-x)$ = $-\tan x$ Cot $(-x)$ = $-\cot x$

Phase Shift $= \frac{-c}{b}$

Period $= \frac{2\pi}{b}$

Sum and Difference Formulas/Identities

$$\sin(u+v)=\sin u\cos v+\cos u\sin v$$
$$\sin(u-v)=\sin u\cos v-\cos u\sin v$$

$$\cos(u+v)=\cos u\cos v-\sin u\sin v$$
$$\cos(u-v)=\cos u\cos v+\sin u\sin v$$

$$\tan(u+v)=\frac{\tan u+\tan v}{1-\tan u\tan v}$$

$$\tan(u-v)=\frac{\tan u-\tan v}{1+\tan u\tan v}$$

How to Find Reference Angles

Step 1: Determine which quadrant the angle is in

Step 2: Use the appropriate formula

Quad I	=	is the angle itself		
Quad II	=	$180-\theta$	or	$\pi - \theta$
Quad III	=	$\theta - 180$	or	$\theta - \pi$
Quad IV	=	$360-\theta$	or	$2\pi - \theta$

Double Angle Identities

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos 2A = 2\cos^2 A - 1$$

$$\cos 2A = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

Half Angle Identities

$$\sin\frac{A}{2}=\pm\sqrt{\frac{1-\cos A}{2}}$$

$$\cos\frac{A}{2}=\pm\sqrt{\frac{1+\cos A}{2}}$$

$$\tan\frac{A}{2}=\frac{1-\cos A}{\sin A}$$

$$\tan\frac{A}{2}=\frac{\sin A}{1+\cos A}$$

Power Reducing Formulas

$$\sin^2 u=\frac{1-\cos 2u}{2}$$

$$\cos^2 u=\frac{1+\cos 2u}{2}$$

$$\tan^2 u=\frac{1-\cos 2u}{1+\cos 2u}$$

Product-to-Sum Formulas

$$\sin u\sin v=\frac{1}{2}\left[\cos(u-v)-\cos(u+v)\right]$$

$$\cos u\cos v=\frac{1}{2}\left[\cos(u-v)-\cos(u+v)\right]$$

$$\sin u\cos v=\frac{1}{2}\left[\sin(u+v)+\sin(u-v)\right]$$

$$\cos u\sin v=\frac{1}{2}\left[\sin(u+v)-\sin(u-v)\right]$$

Sum-to-Product Formulas

$$\sin x+\sin y=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x-\sin y=2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos x+\cos y=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos x-\cos y=-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

Law of Sines

Solving Oblique Triangles using sine: AAS, ASA, SSA, SSS, SAS

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines

Cosine: SAS, SSS

Standard Form	Alternative Form
$a^2 = b^2 + c^2 - 2bc\cos A$	$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
$b^2 = a^2 + c^2 - 2ac\cos B$	$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$
$c^2 = b^2 + a^2 - 2ab\cos C$	$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

Finding the Area of non-90degree Triangles

Area of an Oblique Triangle

$$area = \frac{1}{2}bc\sin A = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B$$

Heron's Formula

Step 1: Find "s" $\quad s = \dfrac{(a+b+c)}{2}$

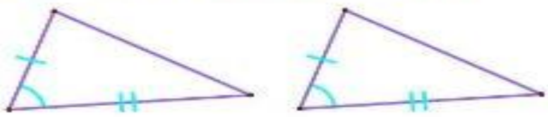
Step 2: Use the formula $\quad area = \sqrt{s(s-a)(s-b)(s-c)}$

Geometry:

1. <u>Congruence of triangles:</u>

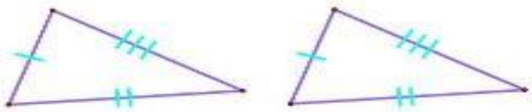
The following theorems present conditions under which triangles are congruent.

Side-Angle-Side (SAS) Congruence



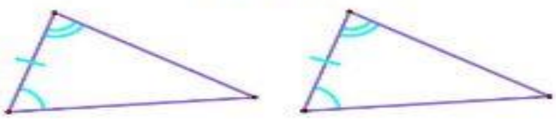
SAS congruence requires the congruence of two sides and the angle between those sides. Note that there is no such thing as SSA congruence; the congruent angle must be between the two congruent sides.

Side-Side-Side (SSS) Congruence



SSS congruence requires the congruence of all three sides. If all of the sides are congruent then all of the angles must be congruent. The converse is not true; there is no such thing as AAA congruence.

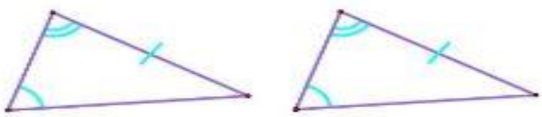
Angle-Side-Angle (ASA) Congruence



ASA congruence requires the congruence of two angles and the side between those angles.

Note: ASA and AAS combine to provide congruence of two triangles whenever any two angles and any one side of the triangles are congruent.

Angle-Angle-Side (AAS) Congruence



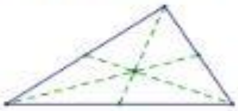
AAS congruence requires the congruence of two angles and a side which is not between those angles.

CPCTC

CPCTC means **"corresponding parts of congruent triangles are congruent."** It is a very powerful tool in geometry proofs and is often used shortly after a step in the proof where a pair of triangles is proved to be congruent.

2. Different Centers of Triangles and inequalities:

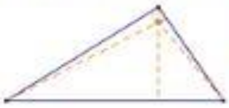
Centroid (Medians)



The centroid is the intersection of the three medians of a triangle. A median is a line segment drawn from a vertex to the midpoint of the line opposite the vertex.

- The centroid is located 2/3 of the way from a vertex to the opposite side. That is, the distance from a vertex to the centroid is double the length from the centroid to the midpoint of the opposite line.
- The medians of a triangle create 6 inner triangles of equal area.

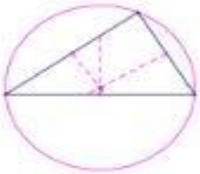
Orthocenter (Altitudes)



The orthocenter is the intersection of the three altitudes of a triangle. An altitude is a line segment drawn from a vertex to a point on the opposite side (extended, if necessary) that is perpendicular to that side.

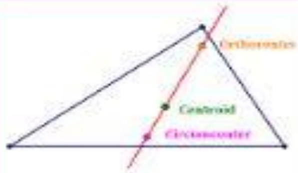
- In an acute triangle, the orthocenter is inside the triangle.
- In a right triangle, the orthocenter is the right angle vertex.
- In an obtuse triangle, the orthocenter is outside the triangle.

Circumcenter (Perpendicular Bisectors)



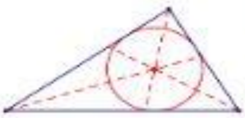
The circumcenter is the intersection of the perpendicular bisectors of the three sides of the triangle. A perpendicular bisector is a line which both bisects the side and is perpendicular to the side. The circumcenter is also the center of the circle circumscribed about the triangle.

- In an acute triangle, the circumcenter is inside the triangle.
- In a right triangle, the circumcenter is the midpoint of the hypotenuse.
- In an obtuse triangle, the circumcenter is outside the triangle.



Euler Line: Interestingly, the centroid, orthocenter and circumcenter of a triangle are collinear (i.e., lie on the same line, which is called the Euler Line).

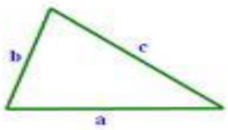
Incenter (Angle Bisectors)



The incenter is the intersection of the angle bisectors of the three angles of the triangle. An angle bisector cuts an angle into two congruent angles, each of which is half the measure of the original angle. The incenter is also the center of the circle inscribed in the triangle.

Triangle Inequality: The sum of the lengths of any two sides of a triangle is greater than the length of the third side. This is a crucial element in deciding whether segments of any 3 lengths can form a triangle.

$$a + b > c \quad and \quad b + c > a \quad and \quad c + a > b$$



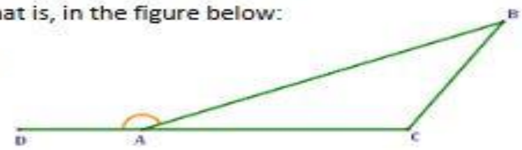
Exterior Angle Inequality: The measure of an external angle is greater than the measure of either of the two non-adjacent interior angles. That is, in the figure below:

$$m\angle DAB > m\angle B \quad and \quad m\angle DAB > m\angle C$$

Note: the Exterior Angle Inequality is much less relevant than the Exterior Angle Equality.

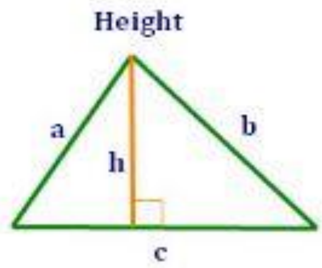
Exterior Angle Equality: The measure of an external angle is equal to the sum of the measures of the two non-adjacent interior angles. That is, in the figure below:

$$m\angle DAB = m\angle B + m\angle C$$



Height

The formula for the length of a height of a triangle is derived from Heron's formula for the area of a triangle:



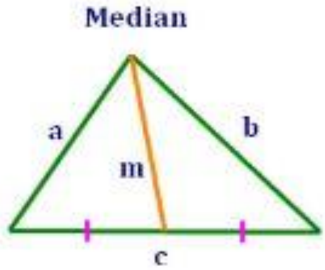
$$h = \frac{2\sqrt{s\,(s-a)\,(s-b)\,(s-c)}}{c}$$

where, $s = \frac{1}{2}(a + b + c)$, and

a, b, c are the lengths of the sides of the triangle.

Median

The formula for the length of a median of a triangle is:

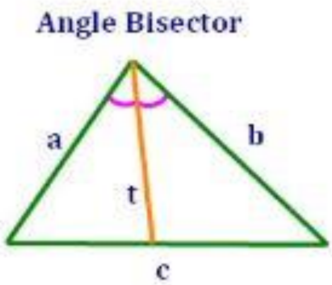


$$m = \frac{1}{2}\sqrt{2a^2 + 2b^2 - c}$$

where, a, b, c are the lengths of the sides of the triangle.

Angle Bisector

The formula for the length of an angle bisector of a triangle is:



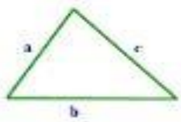
$$t = \sqrt{ab\left(1 - \frac{c^2}{(a+b)^2}\right)}$$

where, a, b, c are the lengths of the sides of the triangle.

Perimeter of a Triangle

The perimeter of a triangle is simply the sum of the measures of the three sides of the triangle.

$$P = a + b + c$$

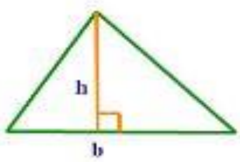


Area of a Triangle

There are two formulas for the area of a triangle, depending on what information about the triangle is available.

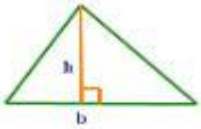
Formula 1: The formula most familiar to the student can be used when the base and height of the triangle are either known or can be determined.

$$A = \frac{1}{2}bh$$

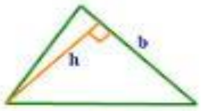


where, b is the length of the base of the triangle.
h is the height of the triangle.

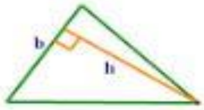
Note: The base can be any side of the triangle. The height is the measure of the altitude of whichever side is selected as the base. So, you can use:



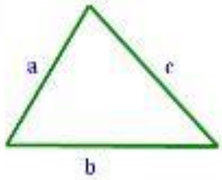
or



or



Formula 2: Heron's formula for the area of a triangle can be used when the lengths of all of the sides are known. Sometimes this formula, though less appealing, can be very useful.



$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where, $s = \frac{1}{2}P = \frac{1}{2}(a+b+c)$. **Note:** s is sometimes called the semi-perimeter of the triangle.

a, b, c are the lengths of the sides of the triangle.

Trigonometric Formulas

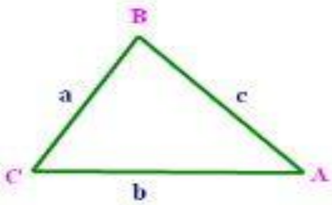
The following formulas for the area of a triangle come from trigonometry. Which one is used depends on the information available:

Two angles and a side:

$$A = \frac{1}{2}\cdot\frac{a^2\cdot\sin B\cdot\sin C}{\sin A} = \frac{1}{2}\cdot\frac{b^2\cdot\sin A\cdot\sin C}{\sin B} = \frac{1}{2}\cdot\frac{c^2\cdot\sin A\cdot\sin B}{\sin C}$$

Two sides and an angle:

$$A = \frac{1}{2}\,ab\,\sin C = \frac{1}{2}\,ac\,\sin B = \frac{1}{2}\,bc\,\sin A$$

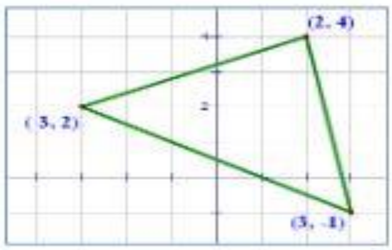


Coordinate Geometry

If the three vertices of a triangle are displayed in a coordinate plane, the formula below, using a determinant, will give the area of a triangle.

Let the three points in the coordinate plane be: (x_1, y_1), (x_2, y_2), (x_3, y_3). Then, the area of the triangle is one half of the absolute value of the determinant below:

$$A = \frac{1}{2}\cdot\left|\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right|$$

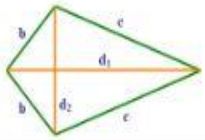
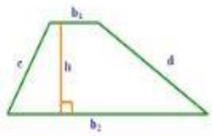
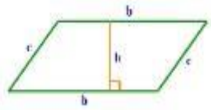
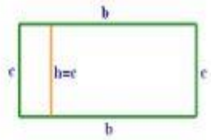
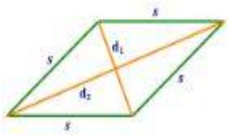
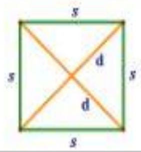


Example: For the triangle in the figure at right, the area is:

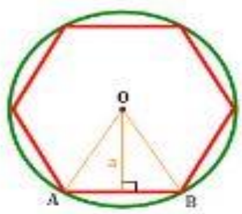
$$A = \frac{1}{2}\cdot\left|\begin{vmatrix} 2 & 4 & 1 \\ -3 & 2 & 1 \\ 3 & -1 & 1 \end{vmatrix}\right|$$

$$= \frac{1}{2}\cdot\left|\left(2\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} - 4\begin{vmatrix} -3 & 1 \\ 3 & 1 \end{vmatrix} + \begin{vmatrix} -3 & 2 \\ 3 & -1 \end{vmatrix}\right)\right| = \frac{1}{2}\cdot 27 = \frac{27}{2}$$

Perimeter and Area of Quadrilaterals

Name	Illustration	Perimeter	Area
Kite		$P = 2b + 2c$	$A = \frac{1}{2}(d_1 d_2)$
Trapezoid		$P = b_1 + b_2 + c + d$	$A = \frac{1}{2}(b_1 + b_2)h$
Parallelogram		$P = 2b + 2c$	$A = bh$
Rectangle		$P = 2b + 2c$	$A = bh$
Rhombus		$P = 4s$	$A = bh = \frac{1}{2}(d_1 d_2)$
Square		$P = 4s$	$A = s^2 = \frac{1}{2}(d^2)$

Definitions – Regular Polygons



- The **center** of a polygon is the center of its circumscribed circle. Point $\boldsymbol{O}$ is the center of the hexagon at right.
- The **radius** of the polygon is the radius of its circumscribed circle. $\overline{\boldsymbol{OA}}$ and $\overline{\boldsymbol{OB}}$ are both radii of the hexagon at right.
- The **apothem** of a polygon is the distance from the center to the midpoint of any of its sides. **a** is the apothem of the hexagon at right.
- The **central angle** of a polygon is an angle whose vertex is the center of the circle and whose sides pass through consecutive vertices of the polygon. In the figure above, $\angle \boldsymbol{AOB}$ is a central angle of the hexagon.

Area of a Regular Polygon

$$\boldsymbol{A = \frac{1}{2} aP}$$ where, a is the apothem of the polygon

P is the perimeter of the polygon

Surface Area and Volume of a Right Cylinder

Surface Area: $SA = Ch + 2B$
$= 2\pi rh + 2\pi r^2$

Lateral SA: $SA = Ch = 2\pi rh$

Volume: $V = Bh = \pi r^2 h$

where, $C = \text{the circumference of the base}$
$h = \text{the height of the cylinder}$
$B = \text{the area of the base}$
$r = \text{the radius of the base}$

Surface Area and Volume of an Oblique Cylinder

Surface Area: $SA = Pl + 2B$

Volume: $V = Bh = \pi r^2 h$

* A **right section** of an oblique cylinder is a cross section perpendicular to the axis of the cylinder.

where, $P = \text{the perimeter of a}$ right section* of the cylinder
$l = \text{the slant height of the cylinder}$
$h = \text{the height of the cylinder}$
$B = \text{the area of the base}$
$r = \text{the radius of the base}$

Surface Area and Volume of a Regular Pyramid

Surface Area: $SA = \frac{1}{2}Ps + B$

Lateral SA: $SA = \frac{1}{2}Ps$

Volume: $V = \frac{1}{3}Bh$

where, $P = \text{the perimeter of the base}$
$s = \text{the slant height of the pyramid}$
$h = \text{the height of the pyramid}$
$B = \text{the area of the base}$

Surface Area and Volume of an Oblique Pyramid

Surface Area: $SA = LSA + B$

Volume: $V = \frac{1}{3}Bh$

where, $LSA = \text{the lateral surface area}$
$h = \text{the height of the pyramid}$
$B = \text{the area of the base}$

The lateral surface area of an oblique pyramid is the sum of the areas of the faces, which must be calculated individually.

Surface Area and Volume of a Right Cone

Surface Area: $SA = \pi rs + \pi r^2$

Lateral SA: $SA = \pi rs$

Volume: $V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$

where, $r = \text{the radius of the base}$
$s = \text{the slant height of the cone}$
$h = \text{the height of the cone}$
$B = \text{the area of the base}$

Surface Area and Volume of an Oblique Cone

Surface Area: $SA = LSA + \pi r^2$

Volume: $V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$

where, $LSA = \text{the lateral surface area}$
$r = \text{the radius of the base}$
$h = \text{the height of the cone}$

There is no easy formula for the lateral surface area of an oblique cone.

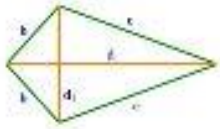
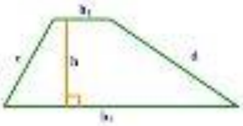
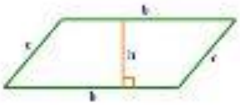
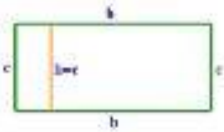
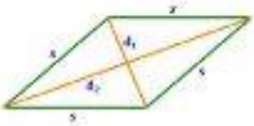
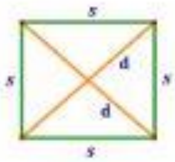
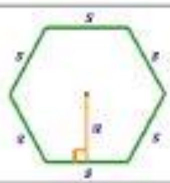
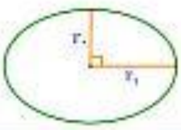
Surface Area and Volume of a Sphere

Surface Area: $SA = 4\pi r^2$

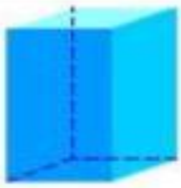
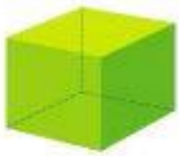
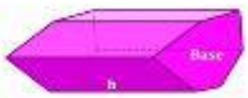
Volume: $V = \frac{4}{3}\pi r^3$

where, $r = \text{the radius of the sphere}$

Summary of 2D shapes:

Shape	Figure	Perimeter	Area
Kite		$P = 2b + 2c$ $b, c = sides$	$A = \frac{1}{2}(d_1 d_2)$ $d_1, d_2 = diagonals$
Trapezoid		$P = b_1 + b_2 + c + d$ $b_1, b_2 = bases$ $c, d = sides$	$\mathrm{A} = \frac{1}{2}(\mathrm{b}_1 + \mathrm{b}_2)\mathrm{h}$ $\mathrm{b}_1, \mathrm{b}_2 = \mathrm{bases}$ $\mathrm{h} = \mathrm{height}$
Parallelogram		$P = 2b + 2c$ $b, c = sides$	$\mathrm{A} = \mathrm{bh}$ $b = base$ $h = height$
Rectangle		$P = 2b + 2c$ $b, c = sides$	$\mathrm{A} = \mathrm{bh}$ $b = base$ $h = height$
Rhombus		$P = 4s$ $s = side$	$A = bh = \frac{1}{2}(d_1 d_2)$ $d_1, d_2 = diagonals$
Square		$P = 4s$ $s = side$	$A = s^2 = \frac{1}{2}(d_1 d_2)$ $d_1, d_2 = diagonals$
Regular Polygon		$P = ns$ $n = number\ of\ sides$ $s = side$	$A = \frac{1}{2}\, a \cdot P$ $a = apothem$ $P = perimeter$
Circle		$C = 2\pi r = \pi d$ $r = radius$ $d = diameter$	$A = \pi r^2$ $r = radius$
Ellipse		$P \approx 2\pi \sqrt{\frac{1}{2}(r_1{}^2 + r_2{}^2)}$ $r_1 = major\ axis\ radius$ $r_2 = minor\ axis\ radius$	$A = \pi r_1 r_2$ $r_1 = major\ axis\ radius$ $r_2 = minor\ axis\ radius$

Summary of 3D shapes:

Shape	Figure	Surface Area	Volume
Sphere		$SA = 4\pi r^2$ $r = radius$	$V = \frac{4}{3}\pi r^3$ $r = radius$
Right Cylinder		$SA = 2\pi rh + 2\pi r^2$ $h = height$ $r = radius\ of\ base$	$V = \pi r^2 h$ $h = height$ $r = radius\ of\ base$
Cone		$SA = \pi rl + \pi r^2$ $l = slant\ height$ $r = radius\ of\ base$	$V = \frac{1}{3}\pi r^2 h$ $h = height$ $r = radius\ of\ base$
Square Pyramid		$SA = 2sl + s^2$ $s = base\ side\ length$ $l = slant\ height$	$V = \frac{1}{3}s^2 h$ $s = base\ side\ length$ $h = height$
Rectangular Prism		$SA = 2 \cdot (lw + lh + wh)$ $l = length$ $w = width$ $h = height$	$V = lwh$ $l = length$ $w = width$ $h = height$
Cube		$SA = 6s^2$ $s = side\ length\ (all\ sides)$	$V = s^3$ $s = side\ length\ (all\ sides)$
General Right Prism		$SA = Ph + 2B$ $P = Perimeter\ of\ Base$ $h = height\ (or\ length)$ $B = area\ of\ Base$	$V = Bh$ $B = area\ of\ Base$ $h = height$

Co Ordinate Geometry:

Distance Formula

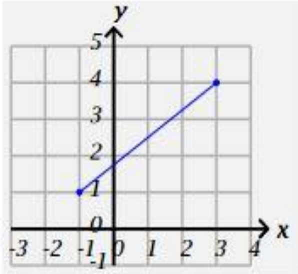
The distance between the points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

In our diagram, we take $P_1 = (x_1, y_1)$ to be $(-1, 1)$ and $P_2 = (x_2, y_2)$ to be $(3, 4)$.

Then $$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} =$$

$$\sqrt{(3 - (-1))^2 + (4 - 1)^2} = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$



Slope

The slope of the line passing through the points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

so in our example $m = \dfrac{4 - 1}{3 - (-1)} = \dfrac{3}{4}$

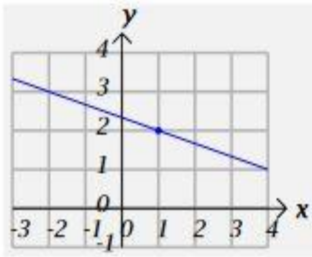
Equations of Lines

These come in many useful forms:

Point-Slope Form

The equation of the line passing through the point $P_1 = (x_1, y_1)$ with slope m is $y - y_1 = m(x - x_1)$

Thus given the point $P_1 = (1, 2)$ and the slope $m = -\dfrac{1}{3}$ the equation of the line is $y - 2 = -\frac{1}{3}(x - 1)$



Point-Point Form

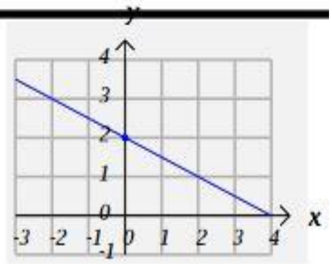
The equation of the line passing through the points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

This just comes from putting the two previous formulas together.

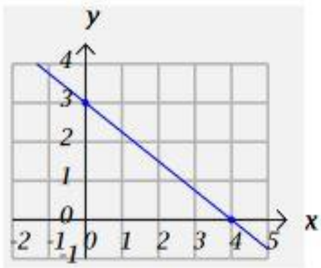
Slope-Intercept Form

The equation of the line passing through the y-axis at the point $(0, b)$ with slope m is $y = mx + b$. For example, if $m = -\dfrac{1}{2}$ and $b = 2$, the equation is $y = -\dfrac{1}{2}x + 2$



Intercept-Intercept Form

The equation of the line passing through the intercepts $(a, 0)$ and $(0, b)$ is $\dfrac{x}{a} + \dfrac{y}{b} = 1$. For example, the equation of the line through $(4, 0)$ and $(0, 3)$ is $\frac{x}{4} + \frac{y}{3} = 1$.



General Form

Every line has infinitely many equations of the form

$$Ax + By + C = 0.$$

For any fixed line, they are non-zero multiples of each other.

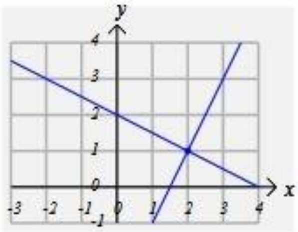
Parallel & Perpendicular Lines

Two lines with slopes m_1 and m_2 are **parallel** if $m_1 = m_2$.

perpendicular if $m_1 m_2 = -1$.

Example: Find the equation of the line through the point $(2,1)$ which is perpendicular to the line $y = -\frac{1}{2}x + 2$.

Solution: The slope of the perpendicular line is $-\frac{1}{-\frac{1}{2}} = 2$, so the equation of the perpendicular line is, using the Point-Slope Form: $y - 1 = 2(x - 2)$



Distance from a Point to a Line

The distance from the point $P_0 = (x_0, y_0)$ to a line ℓ with equation $Ax + By + C = 0$ is

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

Example: Find the distance from the point $(3,4)$ to the line with equation $y = -\frac{1}{2}x + 2$.

Solution: We must rewrite the equation of the line in General form:

$y = -\frac{1}{2}x + 2$. becomes $2y = -x + 2$ or $x + 2y - 2 = 0$, so we apply the Distance Formula with $x_0 = 3$, $y_0 = 4$, $A = 1$, $B = 2$, and $C = -2$:

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}} = \frac{|(1)(3) + (2)(4) + (-2)|}{\sqrt{(1)^2 + (2)^2}} = \frac{|3 + 8 - 2|}{\sqrt{1 + 4}} = \frac{9}{\sqrt{5}}$$