Analogical Model of Merging Process in Urban Transportation System by Application of Oblique Shockwave Theory

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***Abstract*—Congestion in the merging roadways cannot be neglect, and it is necessary to build merging roadways in more massive scale by addressing it. In this paper, We compare the oblique shockwave in aerodynamic with the dynamic process in the merging roadways. Besides, We present the formation of the oblique shock wave in the turning section and merging section. Based on the similar mechanics between airflow and traffic flow, a traffic oblique shockwave model is proposed to explain the mechanics between the merging stream and the stream from the upstream. With this analogical model, we can obtain more regularities of the process of merging roadways.**

***Index Terms*—Oblique Shockwave, Traffic Flow, Merging Roadways, Analogical Model**

I. Introduction

In the existing traffic models, the macroscopic traffic model is a mathematical model that combines traffic vehicles with fluid mechanics, which is used to describe the dynamic process of the whole system. Under the assumption that traffic flow is fluid, researchers apply the physics mechanics into the traffic system, and analysis the dynamic characteristics of the macroscopic traffic system. The Lighthill and Whitham give the earliest start to model the traffic by the traffic flow in 1955 [1], then the field of traffic research can be combined with fluid mechanics, and the shockwave theory was introduced in 1956 [2]. On this basis, these macroscopic traffic models can be applied to different traffic conditions. The three-phase theory which explains the mechanics of the breakdown and congestion is the conceptual innovation in traffic flow [3], compared to classical traffic flow theory. The proposal of the synchronized flow reveals the breakdown of the traffic flow in bottleneck is the natural transition from freeway flow to synchronized flow [4]. The fundamental macroscopic diagram involved the parameters of traffic flow also sufficiently used to predict the capability of a road system [5] [6]. The macroscopic traffic flow model of shockwave share the same structure with the compressed wave in fluid mechanics, the traffic flow in highways is a close analogy to the straight shock wave in aerodynamics [7]. The shockwave model proposed can be utilized to mitigate the congestion on highways. However, many congested roads emerged in the merging area. The current traffic flows based on the straight shock wave are not describe the physical mechanics in merging area effectively. For the merging road section, cell automata(CA) is usually considered as the effective method to describe the dynamic process [9]. Almost all the traffic system can apply CA for simulation since every single vehicle can be treated as a unit cell [10]. In larger traffic system, the evolution of cellular automata's state is only related to its current state and the state of its local neighborhood. For the future-oriented intelligent transportation system, the merging roadways will be the major bottleneck for the mass traffic flow possible. The study of the traffic flow model in merging roadways is significant. The same equations have the same solutions. Combined with aerodynamics, we apply the mechanics of oblique shockwave into the merging area.

In this paper, an analogical model based on the theory of oblique shock wave is proposed, which may be used to describe the dynamic process in merging roadways and design the passing pattern for future autonomous vehicles environment. In section II, we summarize the major characteristics of an oblique shock wave in aerodynamics and introduce the model of oblique shockwave in aerodynamics. Through these characteristics, we provide two sections to describe the formation of traffic oblique shock wave. In section III, we present an analogical model to simulate the process in the scenarios we provided. We study the merging roadways case of Xian, and the numerical analysis and data calibration are given in section IV, with the comparison of our model result. Finally, the conclusions are given in section V.

II. Oblique Shock Wave(OSW)

A. Fluid Oblique Shock Wave

Shockwave is the compression of the abrupt change of the air stress, density and temperature on the wave surface, and the shock wave is formed by the superposition of the infinite number of weak compression waves. In aerodynamics, an oblique shock wave is generated when the supersonic airflow flows through a sharp wedge as shown in the Fig. 1.

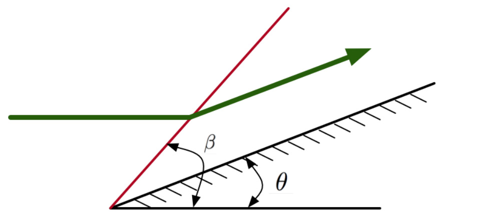

Fig. 1. Oblique Shock Wave

We summarize the basic features of oblique shock wave that similar to the traffic shockwave as follows:

- The wavefront of the oblique shock wave is not perpendicular to the flow
- When the flow passes the oblique shock wave, its point will be diverted.
- The angle of the shock angle is related to the angle of the sharp wedge.

The change of the pressure, density and temperature of the gas before and after the oblique shock is:

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_1^2 \sin^2\beta - 1\right) \tag{1}$$

where M_1 is the velocity of incoming airflow,

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2 \sin^2\beta}{(\gamma-1)M_1^2 \sin^2 +2} \tag{2}$$

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\frac{\rho_1}{\rho_2} \tag{3}$$

The velocity of the airflow after the oblique shockwave is:

$$M_2 = \frac{1}{\sin(\beta-\theta)}\sqrt{\frac{1+\frac{\gamma-1}{2}M_1^2\sin^2\beta}{\gamma M_1^2 \sin^2\beta - \frac{\gamma-1}{2}}} \tag{4}$$

B. Oblique Shock Wave(OSW) Pattern in Traffic

1) In Turning Section: A large number of the turning target vehicles are traveling in the straight lane when the turning target vehicles not in the turn lane. When the turning target vehicles realize the control distance between the intersection and itself are under the circumstance which it has to enter the turning lane, the turning target vehicles choose to turn to the turning lane. Then, the merging flow consists of a large number of the turning target vehicles is formed. At this point, the control distance between turning target vehicles and the intersection is the sharp wedge of the traffic oblique shock wave, and then the turning target vehicles in the edge of the merging flow form the wave array of the traffic oblique shock wave. The formation of the OSW pattern in the turning section is shown in Fig. 2.

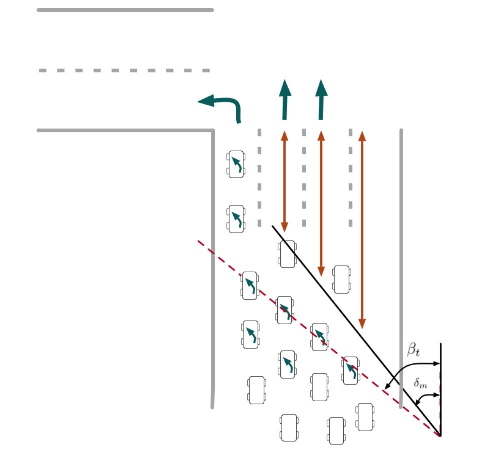

Fig. 2. Oblique Shock Wave(OSW) Pattern in turning section

2) In Merging Section: The traffic oblique shock wave also exists in the road segment of the merging roadways as four lanes to 3 lanes or 3 lanes to 2 lanes. In these road segments, the merging vehicle chooses to merge to the main lane when it appears enough large distance or the appropriate distance they consider. The merging cars form the merging traffic flow. The faster car on the main road will choose to slow down properly or close up to the outer lane for avoidance. The main lane traffic flow change in the velocity and direction because of the evasion. The merging traffic flow forms the sharp wedge of traffic oblique shock wave. The main lane vehicles close to merging cars generate the wavefront of the traffic oblique shock wave. Oblique shock wave shifts to the upstream of traffic flow, the sharp wedge disappears when the merging vehicles enter the main lane completely. Then the slow down vehicles recover their velocity, the wavefront fades, the oblique shock wave vanishes in the main lane quickly. The formation of the OSW pattern in the merging section is shown in Fig. 3.

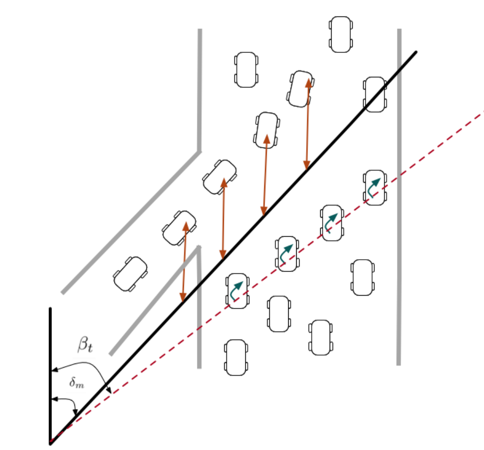

Fig. 3. Oblique Shock Wave(OSW) Pattern in merging section

III. Model Approach

The velocity of vehicles of synchronized flow are approximately the same, but due to the different driving ability, the mean distances between the front car and back car in two synchronized flow are possibly different. Similarly, for sections within a finite area, the maximum of vehicles in this section is also closely to the driving ability of the drivers in this section. So we can generally assume that the congestion of road sections depends largely on the level of driver operation, we propose the parameter of traffic capability k. In our model the parameter of traffic capability is given according to a close analogy with fluid mechanics. In fluid mechanics, specific heat capacity is the physical property of fluid and is used to indicate a fluids ability to absorb heat, which is the essential property of fluid. Essentially, is the parameter to measure the pressure of traffic, which is also the essential property of the traffic flow. To simulate the traffic oblique shock wave in the merging section, we input a stream of merging vehicles which velocity is v_e, the sharp wedge of traffic oblique shock wave is formed while the stream of merging vehicles enter the main lane.

We consider the entering angle β_t when the merging flow is entering the main lane, which β_t is the traffic flow turning angle. The velocity of the main lane flow which formed of vehicles in the main lane is v_{1n}. The wavefront of traffic oblique shockwave is created by the foremost vehicles which close to the merging flow. The wavefront forms the shockwave angle β_t. To ensure the form of shockwave angle conforms to the condition of road intersection, the following constraints are imposed.

$$\arcsin \frac{1}{V} < \beta_t < \frac{\pi}{2} \tag{5}$$

To simulate the traffic oblique shock wave, we apply the oblique shock wave analysis in shock wave theory, and consider the change of OSW before the wavefront and after the wavefront.

$$\frac{\rho_{2f}}{\rho_{1f}} = \frac{\frac{k_t+1}{k_t-1} v_{1f}^2 \sin^2 \beta_t}{\frac{2}{k_t-1} + v_{1f}^2 \sin \beta_t} \tag{6}$$

The density of main lane vehicles close to merging flow is ρ_{1f}. The change of traffic density caused by the influence of the vehicle is different before and after the wavefront. Before the wavefront, the density change from small to large. After the wavefront, the density of traffic flow is the sum of the merging vehicles and the vehicles which avoid the merging flow on the main lane, we define the merging flow is stable and it is the synchronized flow before it meet the main lane vehicles, so we suppose the quantity of merging flow is constant, so the density of total traffic flow after the wavefront change from the small to large.

We consider the sharp wedge which is necessary to form an oblique shock wave and analysis of the air turning angle of the oblique shock wave. The sharp wedge in aerodynamic is a physical entity, while the sharp wedge in traffic flow is not. The sharp wedge in traffic oblique shock wave is a variable model which is the function of the property of merging stream. In our model, the number of vehicles which composed the sharp wedge will decrease over time. Finally, the traffic turning angle turns to zero after the merging flow vanished.

$$\delta_m = \lambda \{t, q_e, \Gamma\} \tag{7}$$

where v_e is the velocity of merging flow, q_e is the quantity of merging flow. Γ is the geometric parameter between the main lane and merging lane.

In the process of changing the speed of main lane vehicles that forms a wavefront in a forced adjustment, we can compute the traffic flow turning angle of our model.

$$\tan \delta_m = \frac{v_{1n}^2 \sin \beta_t - 1}{\left[v_{1n}^2 \left(\frac{k_t+1}{2} - \sin^2 \beta_t\right) + 1\right] \tan \beta_t} \tag{8}$$

where v_{1n} is the velocity of main lane flow before the wavefront, β_t is the angle of the sharp wedge.

The driver's active avoidance of the merging vehicle makes the main lane vehicles speed change after the wavefront, the degree of its change is related to the shock angle β_t, when the shock angle is larger, the deflection angle of the edge vehicle is greater, and the degree of shock wave is larger, the degree of deflection and deceleration of incoming vehicles is larger. To this end, we proposed the relation between the velocity of the vehicles before and after the wavefront.

$$v_{2f}^2 = \frac{v_{1f}^2 + \frac{2}{k_t-1}}{\frac{2k_t}{k_t-1} v_{1f}^2 \sin^2 \beta_t - 1} + \frac{v_{1f}^2 \cos^2 \beta_t}{\frac{k_f-1}{2} v_{1f}^2 \sin \beta_t + 1} \tag{9}$$

where v_{1f} is the velocity of the traffic flow from upstream which is going to meet the wavefront, or the velocity of the incoming traffic flow, v_{2f} is the velocity of the vehicles after the wavefront.

IV. Calibration and Discussion

The study of traffic flow model is usually calibrated by model simulation results, many model simulation results are feasible theoretically, but the applicability of real traffic environment is poor, the model simulation result lacks the data support of real traffic environment. To this end, we select the real world data to verify the traffic oblique shock model. In this paper, a merging section in Tangyan Road, Xi'an is selected as the experimental data acquisition field, which is conform to the characteristics of the merging section we presented in Section II and is suitable for data acquisition.We carry out video acquisition of the vehicles of merging process in this merging section.

According to the analysis of videos, a prototype of wavefront is formed by the front edge of main lane vehicle, and the wavefront gradually dismissed with the vanished of the sharp wedge of merging vehicles, while the wavefront propagate to the upstream constantly with the flow of the main lane stream in the whole process of sharp wedge existence.

When the vehicle in the figure was merging, the sharp wedge was formed. The vehicles in green rectangle are the edge vehicles which form the wavefront of OSW, they formed the OSW by avoiding the merging vehicles for safety. OSW propagated to the upstream of the traffic flow, and vanished while the merging vehicles merged in the main lane.

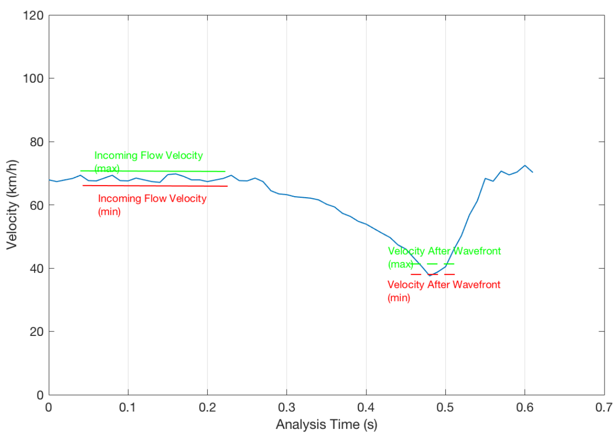

Fig. 4. Estimation based on the real-world data

TABLE I
REAL-WORLD VELOCITY INFORMATION

k_t	v_{1f}	v_{2f}
0.46	65.3	37.4
1.32	54.2	30.2
1.25	68.4	39.2
1.45	57.3	32.4
2.45	68.5	40.7

We obtain the velocities of the edge vehicles that formed the wavefront. The blue line shows the average velocity of the vehicles. According to our calculation of the traffic flow information of the video, we set k_t as 1.25. The fastest and the slowest are used to calculate based on our model, which red line and green line shows their original velocity and the velocity when passing the wavefront. According to the result, the velocity changes of vehicles that passing wavefront are approximate to the real result.

V. CONCLUSION

In this paper, we describe the physical mechanics of the two scenarios that the vehicles are merging. With the similar characteristics, we proposed an analogical model based on the theory of aerodynamic. For traffic flow model, the physical model of aerodynamic is used as an extension. Future research should take into the consideration of the dynamic process to mitigate the congestion in the merging area with large quantity of vehicles. Future research should use a high fidelity microsimulation environment for different demand scenarios and merging facilities.

REFERENCES

[1] Lighthill M J, Whitham J B. On Kinematic Waves: I. Flow movement in Long Rivers. II. A theory of Traffic Flow on Long Crowded Roads[J]. Pharmacology & Therapeutics, 1955, 53(3):275-354.

[2] Richards P I. Shock Waves on the Highway[J]. Operations Research, 1956, 4(1):42-51.

[3] Kerner B S. Experimental Features of Self-Organization in Traffic Flow[J]. Physical Review Letters, 1998, 81(17):3797-3800.

[4] Kerner B S, Rehborn H. Experimental properties of complexity in traffic flow.[J]. Phys.rev.e, 1996, 53(5):R4275-R4278.

[5] Sanwal K K, Petty K, Walrand J, et al. An extended macroscopic model for traffic flow[J]. Transportation Research Part B Methodological, 1996, 30(1):1-9.

[6] Helbing D, Hennecke A, Shvetsov V, et al. MASTER: macroscopic traffic simulation based on a gas-kinetic, non-local traffic model[J]. Transportation Research Part B, 2001, 35(2):183-211.

[7] Ng K M, Reaz M I. A comparative study of the LWR-IM traffic model and shockwave analysis[C] Systems, Process and Control. IEEE, 2016:39-43.

[8] Papageorgiou M. SOME REMARKS ON MACROSCOPIC TRAFFIC FLOW MODELLING[J]. Transportation Research, 1998, 32(5):323-329.

[9] Wolfram S. Cellular automata as models of complexity[J]. Nature, 1984, 311(5985):419-424.

[10] Kai N, Schreckenberg M. Cellular automaton model for freeway traffic. J Phys I (Paris) 2:2221[J]. Journal De Physique I, 1992, 2(12).

[11] Anderson J D. Modern compressible flow : with historical perspective[M]. McGraw-Hill, 1982.

[12] McGrawHill. Fundamentals of Aerodynamics[M]. McGraw-Hill Book Co, 1984.

[13] Lieu H. The Physics of Traffic: Empirical Freeway Pattern Features, Engineering Applications, and Theory[M]. Springer, 2010.

[14] Stephanopoulos S G. An application of shock wave theory to traffic signal control[J]. Transportation Research Part B, 2008, 15(1):35-51.

[15] Lawton D, Rieger J, Steenstrup M. Computational techniques in motion processing[M] Vision, brain, and cooperative computation. MIT Press, 1988:75-83.

[16] Wu X, Liu H X. A shockwave profile model for traffic flow on congested urban arterials[J]. Transportation Research Part B, 2011, 45(10):1768-1786.

[17] Cao W, Shi Z, Feng J H. Traffic Image Classification Method Based on Fractal Dimension[C] IEEE International Conference on Cognitive Informatics. IEEE, 2007:903-907.

[18] Trotsyuk A V, Kudryavtsev A N, Ivanov M S. Mach reflection of shock and detonation waves in steady supersonic chemically reacting flows[M]. 2003.