Fourier Transforms

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1 Introduction

This report discusses Fourier transforms and applies it to musical signals with and without noise. It also gives a brief description of the techniques associated with Fourier transforms.

2 Fourier Tansform

A Fourier transform (FT) is a mathematical transform that decomposes functions depending on space or time into functions depending on spatial frequency or temporal frequency.
An example application would be decomposing the waveform of a musical chord into terms of the intensity of its constituent pitches.
The Fourier transform of a function is a complex-valued function representing the complex sinusoids that comprise the original function. For each frequency, the magnitude (absolute value) of the complex value represents the amplitude of a constituent complex sinusoid with that frequency, and the argument of the complex value represents that complex sinusoid's phase offset.
Given a time signal, g(t), the forward Fourier transform can help evaluate the signal in the frequency domain, $\hat{g}(f)$. The transformation is given by:

$$\hat{g}(f) = \sum_{-\infty}^{\infty} g(t)exp(-2\pi ift)dt \tag{1}$$

Discrete Fourier transform (DFT): If the ordered pairs representing the original input function are equally spaced in their input variable (for example, equal time steps), then the Fourier transform is known as a discrete Fourier transform.
Fast Fourier transform (FFT): An algorithm that computes the discrete Fourier transform (DFT) of a sequence, or its inverse (IDFT).
Figure 1 shows the amplitude of each frequency f. The peaks on the graph represent the key frequencies. Reading the graph, the key frequencies are as follows:

- $\pm$259.74 Hz.
- $\pm$329.67 Hz.
- $\pm$439.56 Hz.
- $\pm$779.22 Hz.
- $\pm$789.21 Hz.

The magnitudes of the other frequencies are negligible.

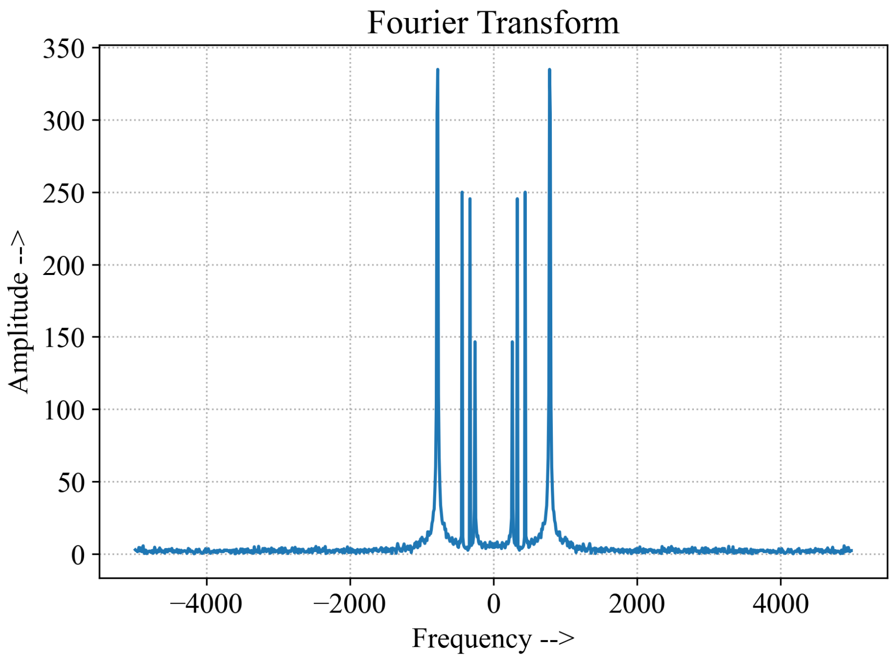

Figure 1: Amplitude VS Frequency diagram.

2.1 Nyquist's sampling theorem:

The Nyquist Sampling Theorem states that a bandlimited continuous-time signal can be sampled and perfectly reconstructed from its samples if the waveform is sampled over twice as fast as it's highest frequency component.
If the sample rate is too low, it will not accurately express the original signal and will be distorted, or show aliasing effects, when reproduced.
If the sample rate is too high, it will needlessly take up extra storage and processing resources.
The Nyquist theorem helps to find the perfect frequency where all the necessary information is recorded but nothing extra.

2.2 Musical notes:

The following table gives the corresponding musical notes for a particular frequency.
We know the key frequencies present in our signal. Using those, we can identify the corresponding musical notes.

- ±259.74 Hz: Note C, Octave 4.
- ±329.67 Hz: Note E, Octave 4.
- ±439.56 Hz: Note A, Octave 4.
- ±779.22 Hz: Note G, Octave 5.

- ±789.21 Hz: Note G, Octave 5.

NOTE	OCTAVE 0	OCTAVE 1	OCTAVE 2	OCTAVE 3	OCTAVE 4	OCTAVE 5	OCTAVE 6	OCTAVE 7	OCTAVE 8
C	16.35 Hz	32.70 Hz	65.41 Hz	130.81 Hz	A piano middle C 261.63 Hz	523.25 Hz	1046.50 Hz	2093.00 Hz	A piano's highest note 4186.01 Hz
C#/D♭	17.32 Hz	34.65 Hz	69.30 Hz	138.59 Hz	277.18 Hz	554.37 Hz	1108.73 Hz	2217.46 Hz	4434.92 Hz
D	18.35 Hz	36.71 Hz	73.42 Hz	146.83 Hz	293.66 Hz	587.33 Hz	1174.66 Hz	2349.32 Hz	4698.63 Hz
D#/E♭	19.45 Hz	38.89 Hz	77.78 Hz	155.56 Hz	311.13 Hz	622.25 Hz	1244.51 Hz	2489.02 Hz	4978.03 Hz
E	20.60 Hz	A bass's lowest note 41.20 Hz	A guitar's lowest note 82.41 Hz	164.81 Hz	329.63 Hz	659.25 Hz	1318.51 Hz	2637.02 Hz	5274.04 Hz
F	21.83 Hz	43.65 Hz	87.31 Hz	174.61 Hz	349.23 Hz	698.46 Hz	1396.91 Hz	2793.83 Hz	5587.65 Hz
F#/G♭	23.12 Hz	46.25 Hz	92.50 Hz	185.00 Hz	369.99 Hz	739.99 Hz	1479.98 Hz	2959.96 Hz	5919.91 Hz
G	24.50 Hz	49.00 Hz	98.00 Hz	A violin's lowest note 196.00 Hz	392.00 Hz	783.99 Hz	1567.98 Hz	3135.96 Hz	6271.93 Hz
G#/A♭	25.96 Hz	51.91 Hz	103.83 Hz	207.65 Hz	415.30 Hz	830.61 Hz	1661.22 Hz	3322.44 Hz	6644.88 Hz
A	A piano's lowest note 27.50 Hz	55.00 Hz	110.00 Hz	220.00 Hz	440.00 Hz	Music Note To Frequency Chart-01	[illegible]	3520.00 Hz	7040.00 Hz
A#/B♭	29.14 Hz	58.27 Hz	116.54 Hz	233.08 Hz	466.16 Hz	932.33 Hz	1864.66 Hz	3729.31 Hz	7458.62 Hz
B	A 5 string bass's lowest note 30.87 Hz	61.74 Hz	123.47 Hz	246.94 Hz	493.88 Hz	987.77 Hz	1975.53 Hz	3951.07 Hz	7902.13 Hz

Figure 2: Music note to frequency chart.

3 Signal with noise

Considering the signal with noise and performing an FFT on it, the following frequency-amplitude diagram is obtained. We know that the amplitude of noise is always less than 0.01. Therefore, the frequencies with the highest amplitude greater than 0.01 are the key frequencies in the true signal. These are as follows:

- ±229.77 Hz.
- ±319.68 Hz
- ±309.69 Hz

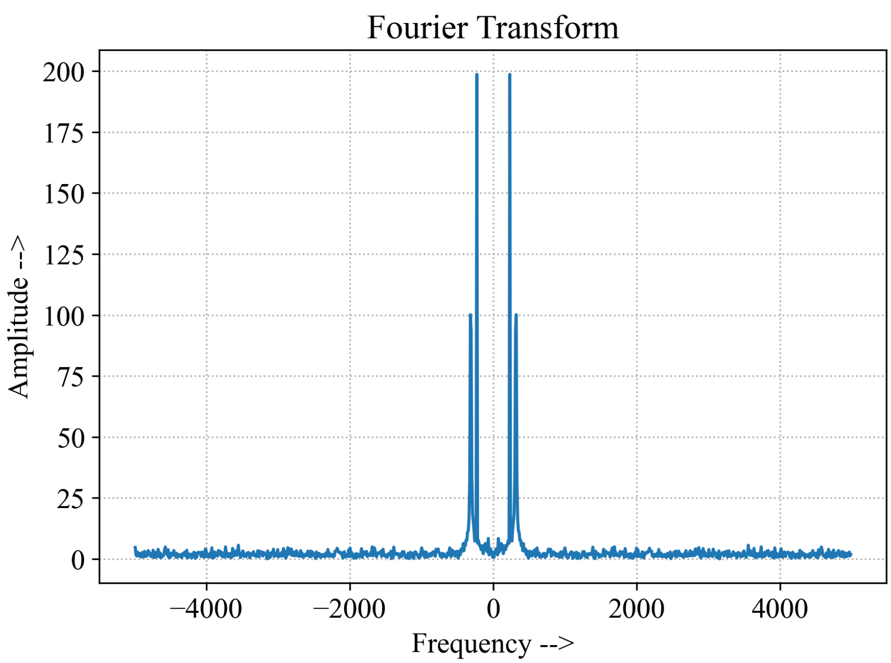

Figure 3: Amplitude VS Frequency diagram.

3.1 Time signal plots:

In this subsection, we see the effect on the time-signal plots when we remove the true frequencies from the signal and plot the time signal overlaid with the original signal. It can be observed that the noise wave without the true frequencies is rapidly varying and irregular in behaviour. The true signal with the noise resembles a distorted sinusoidal wave.

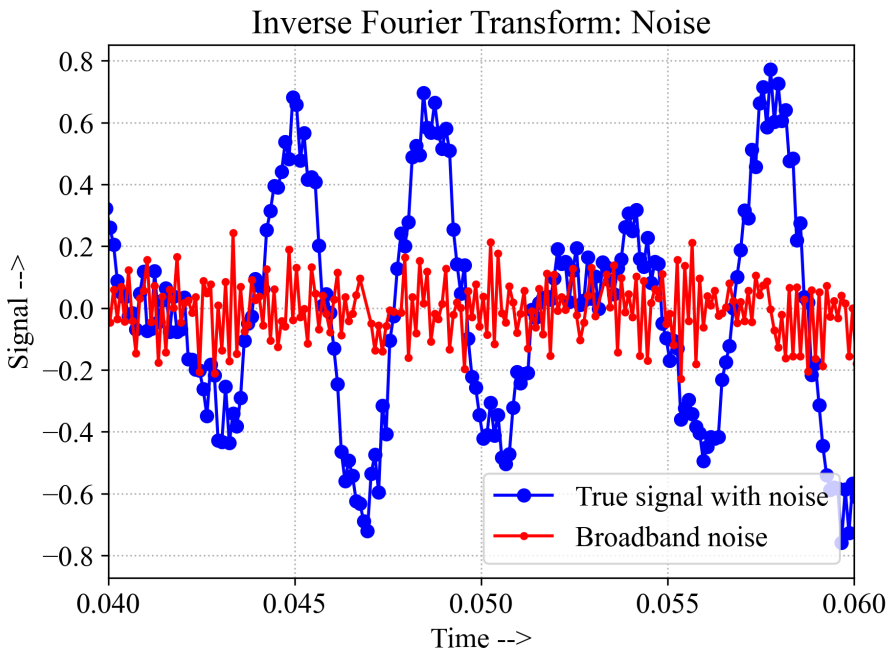

Figure 4: Time-signal plots.

3.2 5 point median average:

A 5 point median average involves taking the median of 5 values surrounding a particular value and plotting that at a particular time.
This process, known as median filtering, significantly reduces noise as the noise signals are ignored while taking the median.
The median filter is a non-linear digital filtering technique, often used to remove noise from a signal. Such noise reduction is a typical pre-processing step to improve the results of later processing.
We can observe from the plot that the noise amplitudes are much lower.
In this graph, the amplitude of the true signal is slightly changed. While the original signal showed maximum amplitudes 198.62, 100.15 and 93.5, this signal has maximum amplitudes 194.90, 102.98 and 84.80.

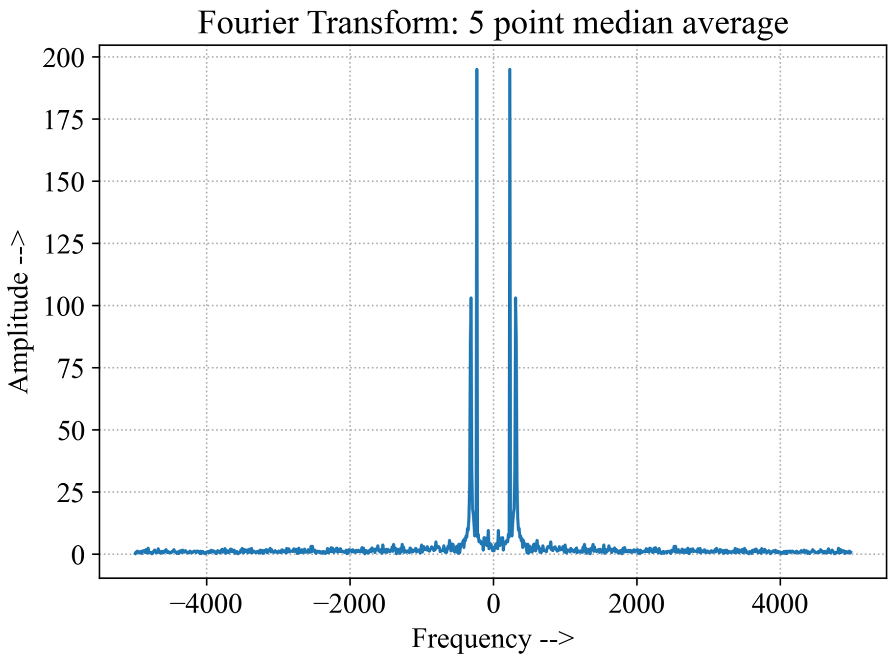

Figure 5: Amplitude VS Frequency diagram.

4 Conclusion

In conclusion, Fourier transforms are extensively used in fields of signals and systems. They are highly efficient algorithms that operate with little loss of information and are extremely advantageous.