

Load Flow Analysis Using Gauss-Seidel and Newton Raphson Algorithm

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Gauss-Seidel Derivation

The Gauss-Seidel method is based on rearranging the nodal current equation.

Starting from Kirchhoff's Current Law (KCL), the current injection at bus i is:

$$I_i = \sum_{j=1}^N Y_{ij} V_j$$

Since $S_i = P_i + jQ_i = V_i I_i^*$, we can write $I_i = \frac{P_i - jQ_i}{V_i^*}$. Substituting this into KCL equation:

$$\frac{P_i - jQ_i}{V_i^*} = Y_{ii} V_i + \sum_{j \neq i} Y_{ij} V_j$$

Solving for V_i , we obtain the iterative formula used in our code:

$$V_i^{(k+1)} = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{(V_i^{(k)})^*} - \sum_{j < i} Y_{ij} V_j^{(k+1)} - \sum_{j > i} Y_{ij} V_j^{(k)} \right]$$

Newton-Raphson Algorithm

The Newton-Raphson method uses the first two terms of the Taylor series expansion to linearize the non-linear power equations.

We define the relationship between power mismatch and state variable updates as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix}$$

The diagonal element H_{ii} (part of J_1) is derived by differentiating P_i with respect to θ_i . Using the "Q-substitution trick" to simplify the summation:

$$H_{ii} = \frac{\partial P_i}{\partial \theta_i} = -Q_i - V_i^2 B_{ii}$$

Similarly, for L_{ii} (part of J_4), we differentiate Q_i with respect to V_i :

$$L_{ii} = \frac{\partial Q_i}{\partial V_i} = \frac{Q_i}{V_i} - V_i B_{ii}$$

Admittance Table

From	To	Resistance (R)	Reactance (X)	Total Shunt (B)	Shunt per end (B/2)
1	2	0.02	0.06	0.06	0.03
2	3	0.06	0.18	0.04	0.02
1	4	0.08	0.24	0.05	0.025
3	5	0.04	0.12	0.03	0.015
4	5	0.01	0.03	0.02	0.01

```

%% GAUSS-SEIDEL POWER FLOW SOLVER
% Features: Sequential Updates, PV-PQ Switching, Slack Power Calculation
clear; clc;

% ----- 1. SYSTEM PARAMETERS (PU) -----
N = 5; M = 3;
P = [0 1.2 1.0 -1.5 -1.2];
Q = [0 0 0 -0.8 -0.5];
Qmin = [-1000 -1000 -1.5 -1000 -1000];
Qmax = [1000 1000 1.5 1000 1000];
V_mag = [1.05 1.03 1.02 1.0 1.0];
V = V_mag .* exp(1j * [0 0 0 0 0]);
error_tol = 1e-9;
maxIteration = 1000;
bus_status = repmat("Normal", 1, N); % Track PV-PQ switching

% ----- 2. Y-BUS CONSTRUCTION -----
% Using your specific line data
y12 = 1 / (0.02 + 1j*0.06); y14 = 1 / (0.08 + 1j*0.24);
y23 = 1 / (0.06 + 1j*0.18); y35 = 1 / (0.04 + 1j*0.12);
y45 = 1 / (0.01 + 1j*0.03);
y120 = 1j*0.06; y140 = 1j*0.05; y230 = 1j*0.04; y350 = 1j*0.03; y450 = 1j*0.02;

Y = zeros(N,N);
Y(1,1) = y12 + y14 + (y120 + y140)/2;
Y(2,2) = y12 + y23 + (y120 + y230)/2;
Y(3,3) = y23 + y35 + (y230 + y350)/2;
Y(4,4) = y14 + y45 + (y140 + y450)/2;
Y(5,5) = y45 + y35 + (y450 + y350)/2;
Y(1,2)=-y12; Y(2,1)=Y(1,2); Y(1,4)=-y14; Y(4,1)=Y(1,4);
Y(2,3)=-y23; Y(3,2)=Y(2,3); Y(3,5)=-y35; Y(5,3)=Y(3,5);
Y(4,5)=-y45; Y(5,4)=Y(4,5);

% ----- 3. GAUSS-SEIDEL ITERATION -----
fprintf('--- Commencing Gauss-Seidel Iterations ---\n');

```

```

--- Commencing Gauss-Seidel Iterations ---

```

```

for iter = 1:maxIteration
    Vold = V;
    for i = 2:N
        % Step A: Reactive Power Calculation for PV Buses
        if i <= M
            sumYV = Y(i,:) * V.';
            Qi = -imag(conj(V(i)) * sumYV);

            % Check Q Limits
            if Qi > Qmax(i)
                Qi = Qmax(i); isPQ = true; bus_status(i) = "Q-Max Hit";
            elseif Qi < Qmin(i)
                Qi = Qmin(i); isPQ = true; bus_status(i) = "Q-Min Hit";
            else
                isPQ = false; bus_status(i) = "PV-Active";
            end
            Q(i) = Qi;
        else
            isPQ = true; bus_status(i) = "PQ-Load";
        end

        % Step B: Update Voltage
        % V_new = (1/Yii) * [ (P-jQ)/V* - sum(Yij*Vj) ]
        sumYV_excl = (Y(i,:) * V.') - (Y(i,i) * V(i));
        V_new = (1/Y(i,i)) * ((P(i) - 1j*Q(i))/conj(V(i)) - sumYV_excl);

        % Step C: Apply Constraints
        if i <= M && ~isPQ
            V(i) = V_mag(i) * (V_new / abs(V_new)); % Angle only
        else
            V(i) = V_new; % Magnitude and Angle
        end
    end

    if max(abs(V - Vold)) < error_tol, break; end
end

% ----- 4. SLACK BUS POWER & RESULTS -----
S1 = V(1) * conj(Y(1,:) * V.');
P(1) = real(S1); Q(1) = imag(S1);
bus_status(1) = "Slack";

fprintf('Convergence reached in %d iterations.\n', iter);

Convergence reached in 357 iterations.

% ----- FORMATTED OUTPUT -----
fprintf('\n===== FINAL POWER FLOW RESULTS (GAUSS-SEIDEL)
=====');

```

===== FINAL POWER FLOW RESULTS (GAUSS-SEIDEL) =====

```
Bus = (1:N)';
Voltage_Mag = abs(V)';
Angle_Deg = (angle(V)*180/pi)';
Net_P = P';
Net_Q = Q';
Status = bus_status';

Results = table(Bus, Status, Voltage_Mag, Angle_Deg, Net_P, Net_Q);
disp(Results);
```

Bus	Status	Voltage_Mag	Angle_Deg	Net_P	Net_Q
1	"Slack"	1.05	0	1.0271	1.6152
2	"PV-Active"	1.03	1.7532	1.2	-0.41156
3	"Q-Max Hit"	0.97352	-6.1714	1	1.5
4	"PQ-Load"	0.70999	-19.17	-1.5	-0.8
5	"PQ-Load"	0.72901	-18.442	-1.2	-0.5

```
fprintf('===== \n');
```

=====

Newton-Raphson Algorithm

The Newton-Raphson method uses the first two terms of the Taylor series expansion to linearize the non-linear power equations.

We define the relationship between power mismatch and state variable updates as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix}$$

```
%% NEWTON-RAPHSON POWER FLOW SOLVER
% Includes: PV-PQ Switching, Recovery Logic, and Dynamic Jacobian
clear; clc;

% --- 1. SYSTEM DATA ---
N = 5;
% Bus Type: 0=Slack, 1=PV, 2=PQ, 3=PQ_Limited
bus_type = [0; 1; 1; 2; 2];
V = [1.05; 1.03; 1.02; 1.0; 1.0];
theta = zeros(N, 1);
V_setpoint = V;

% Specified Net Power (Gen - Load)
P_spec = [0; 1.2; 1.0; -1.5; -1.2];
Q_spec = [0; 0; 0; -0.8; -0.5];
```

```

% Reactive Power Limits
Qmax = [1000; 1000; 1.5; 1000; 1000];
Qmin = [-1000; -1000; -1.5; -1000; -1000];

% Line Data: [From, To, R, X, B_total_shunt]
line_data = [1 2 0.02 0.06 0.06;
             2 3 0.06 0.18 0.04;
             1 4 0.08 0.24 0.05;
             3 5 0.04 0.12 0.03;
             4 5 0.01 0.03 0.02];

% --- 2. BUILD YBUS ---
Ybus = build_ybus(line_data, N);

Ybus = 5x5 complex
    6.2500 -18.6950i  -5.0000 +15.0000i   0.0000 + 0.0000i  -1.2500 + 3.7500i  ...
   -5.0000 +15.0000i   6.6667 -19.9500i  -1.6667 + 5.0000i   0.0000 + 0.0000i
    0.0000 + 0.0000i  -1.6667 + 5.0000i   4.1667 -12.4650i   0.0000 + 0.0000i
   -1.2500 + 3.7500i   0.0000 + 0.0000i   0.0000 + 0.0000i  11.2500 -33.7150i
    0.0000 + 0.0000i   0.0000 + 0.0000i  -2.5000 + 7.5000i -10.0000 +30.0000i

```

```

% --- 3. ITERATION LOOP ---
tol = 1e-6;
max_iter = 20;

fprintf('--- Commencing Newton-Raphson Iterations ---\n');

```

```

--- Commencing Newton-Raphson Iterations ---

```

```

fprintf('Iter | Max Mismatch\n');

```

```

Iter | Max Mismatch

```

```

fprintf('-----\n');

```

```

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```

```

for iter = 1:max_iter
    [P_calc, Q_calc] = calculate_PQ(V, theta, Ybus);

    % Smart Limit & Recovery Logic
    for i = 1:N
        if bus_type(i) == 1
            if Q_calc(i) > Qmax(i)
                bus_type(i) = 3; Q_spec(i) = Qmax(i);
                fprintf(' (!) Iter %d: Bus %d hit Qmax. Switched to PQ.\n', iter,
i);
            elseif Q_calc(i) < Qmin(i)
                bus_type(i) = 3; Q_spec(i) = Qmin(i);
                fprintf(' (!) Iter %d: Bus %d hit Qmin. Switched to PQ.\n', iter,
i);
            end
        end
    end
end

```

```

        end
    elseif bus_type(i) == 3
        if (Q_spec(i) == Qmax(i) && V(i) > V_setpoint(i)) || ...
            (Q_spec(i) == Qmin(i) && V(i) < V_setpoint(i))
            bus_type(i) = 1; V(i) = V_setpoint(i);
            fprintf('  (+) Iter %d: Bus %d recovered to PV mode.\n', iter, i);
        end
    end
end

th_unk = find(bus_type ~= 0);
v_unk = find(bus_type == 2 | bus_type == 3);

dP = P_spec(th_unk) - P_calc(th_unk);
dQ = Q_spec(v_unk) - Q_calc(v_unk);
Mismatch = [dP; dQ];

max_mis = max(abs(Mismatch));
fprintf(' %2d | %10.3e\n', iter, max_mis);

if max_mis < tol
    fprintf('-----\n');
    fprintf('Convergence reached in %d iterations.\n', iter);
    break;
end

J = build_jacobian(V, theta, Ybus, th_unk, v_unk, P_calc, Q_calc);
dX = J \ Mismatch;

theta(th_unk) = theta(th_unk) + dX(1:length(th_unk));
V(v_unk) = V(v_unk) + dX(length(th_unk)+1:end);
end

```

```

1 | 1.438e+00
2 | 2.883e-01
(!) Iter 3: Bus 3 hit Qmax. Switched to PQ.
3 | 1.696e-01
4 | 1.213e-02
5 | 8.181e-04
6 | 4.268e-06
7 | 1.170e-10
-----
Convergence reached in 7 iterations.

```

% --- 4. FORMATTED FINAL OUTPUT ---

```

fprintf('\n===== FINAL POWER FLOW RESULTS (NEWTON-RAPHSON)
===== \n');

```

```

===== FINAL POWER FLOW RESULTS (NEWTON-RAPHSON) =====

```

```

Bus = (1:N)';
Voltage_Mag = V;

```

```

Angle_Deg = theta * 180/pi;
Type_Code = bus_type;

% Map type codes to readable strings
Type_Names = cell(N,1);
for k = 1:N
    if bus_type(k) == 0, Type_Names{k} = 'Slack';
    elseif bus_type(k) == 1, Type_Names{k} = 'PV';
    elseif bus_type(k) == 2, Type_Names{k} = 'PQ';
    elseif bus_type(k) == 3, Type_Names{k} = 'PQ-Limited';
    end
end

Net_P = P_calc;
Net_Q = Q_calc;
Results = table(Bus, Type_Names, Voltage_Mag, Angle_Deg, Net_P, Net_Q);
disp(Results);

```

Bus	Type_Names	Voltage_Mag	Angle_Deg	Net_P	Net_Q
1	{ 'Slack' }	1.05	0	1.0271	1.6152
2	{ 'PV' }	1.03	1.7532	1.2	-0.41156
3	{ 'PQ-Limited' }	0.97352	-6.1714	1	1.5
4	{ 'PQ' }	0.70999	-19.17	-1.5	-0.8
5	{ 'PQ' }	0.72901	-18.442	-1.2	-0.5

```

fprintf('=====
\n');

```

```

% --- SUPPORTING FUNCTIONS ---

```

```

function Y = build_ybus(data, n)
    Y = zeros(n,n);
    for k = 1:size(data,1)
        f = data(k,1); t = data(k,2);
        z = data(k,3) + 1j*data(k,4);
        y = 1/z; b = 1j*data(k,5)/2;
        Y(f,f) = Y(f,f) + y + b;
        Y(t,t) = Y(t,t) + y + b;
        Y(f,t) = -y; Y(t,f) = -y;
    end
end

function [P, Q] = calculate_PQ(V, th, Y)
    n = length(V); P = zeros(n,1); Q = zeros(n,1);
    for i = 1:n
        for j = 1:n
            d_ij = th(i) - th(j);

```

```

        P(i) = P(i) + V(i)*V(j)*(real(Y(i,j))*cos(d_ij) +
        imag(Y(i,j))*sin(d_ij));
        Q(i) = Q(i) + V(i)*V(j)*(real(Y(i,j))*sin(d_ij) -
        imag(Y(i,j))*cos(d_ij));
    end
end
end

function J = build_jacobian(V, th, Y, th_idx, v_idx, P, Q)
    n = length(V); G = real(Y); B = imag(Y);
    H = zeros(n,n); N = zeros(n,n); M = zeros(n,n); L = zeros(n,n);
    for i = 1:n
        for j = 1:n
            d_ij = th(i) - th(j);
            if i == j
                H(i,i) = -Q(i) - (V(i)^2 * B(i,i));
                N(i,i) = (P(i) / V(i)) + (G(i,i) * V(i));
                M(i,i) = P(i) - (V(i)^2 * G(i,i));
                L(i,i) = (Q(i) / V(i)) - (B(i,i) * V(i));
            else
                H(i,j) = V(i) * V(j) * (G(i,j)*sin(d_ij) - B(i,j)*cos(d_ij));
                N(i,j) = V(i) * (G(i,j)*cos(d_ij) + B(i,j)*sin(d_ij));
                M(i,j) = -V(i) * V(j) * (G(i,j)*cos(d_ij) + B(i,j)*sin(d_ij));
                L(i,j) = V(i) * (G(i,j)*sin(d_ij) - B(i,j)*cos(d_ij));
            end
        end
    end
    J = [H(th_idx, th_idx), N(th_idx, v_idx);
        M(v_idx, th_idx), L(v_idx, v_idx)];
end

```

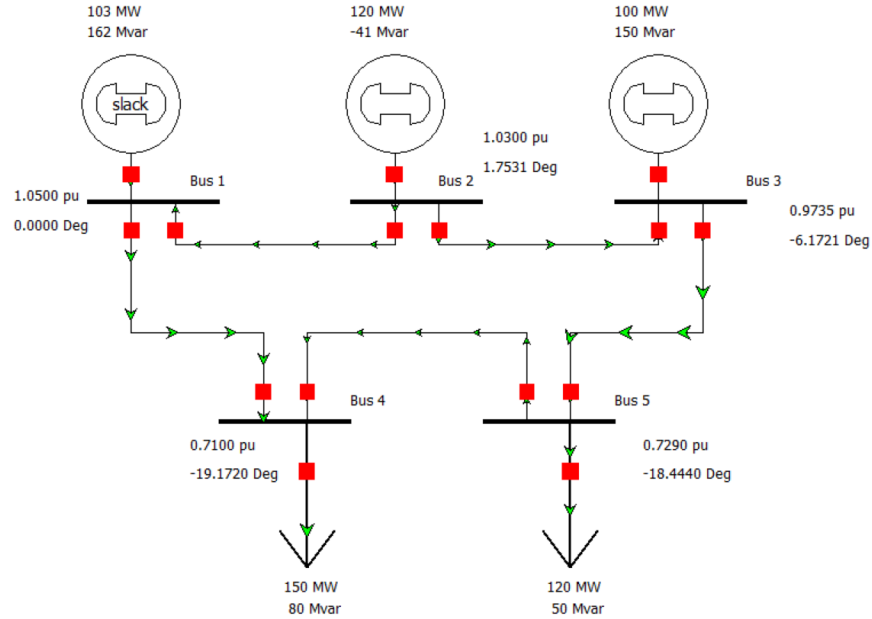
Conclusion

1. Simulation Summary

The results from the PowerWorld simulation are presented below. These values represent the steady-state solution of the network under the specified loading and generation conditions.

Bus	Voltage (pu)	Angle (deg)	P Gen (pu)	Q Gen (pu)	Status
1	1.050	0.00	1.027	1.615	Slack
2	1.030	1.75	1.200	-0.412	PV
3	0.974	-6.17	1.000	1.500	PQ-Limited
4	0.710	-19.17	0.000	0.000	PQ (Load)
5	0.729	-18.44	0.000	0.000	PQ (Load)

5 Bus Load Flow Using Gauss-Seidel and Newton Raphson Algorithm



2. Comparison and Conclusion

Comparing the PowerWorld data to the MATLAB output shows an exact match in the voltage profile and power flow results. Specifically, both environments confirm that **Bus 3 hits its reactive power limit** ($Q_{\max} = 1.5$ pu), causing the voltage to drop below the initial 1.02 pu setpoint to approximately 0.974 pu. This validates the switching logic implemented in the MATLAB code.