Learning Astronomy (and other things) with KM-AstCalc

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Contents

Introduction

Hello

Welcome to KM-AstCalc, the astronomical internet calculator. KM-AstCalc helps you perform scientific calculations with particular attention to astronomical purposes. With KM-AstCalc, you may quickly evaluate some intricate calculus you need for prompt use. When looking for values of some physical or astronomical constants and performing some quick calculations with them, you can count on KM-AstCalc, which you can get directly from its URL on the internet: `http://km-cienctec.rf.gd/astcalc/`, or may have it on your PC after freely downloading from its location in `https://astcalc.sourceforge.io`. KM-AstCalc was fully designed in JavaScript. You may access and modify its code if you feel comfortable with this programming language under the condition it complies with the GNU-GPL, the GNU General Public License. KM-AstCalc is designed to give you numeric results without having to write heavy programs, or when you are developing, you may perform some tests with the aid of KM-AstCalc.

With some training, you can rapidly learn to calculate efficiently and quickly by entering data and applying functions by clicking on the mouse or pressing shortcut keys.

The method of calculations is based on the RPN - Reversed Polish Notation, once introduced by the HP calculators in the 1960s. Even if you are unfamiliar with it, with some exercises presented here, you will rapidly learn to work with it. I am sure that the reader, in the end, will find it very easy and even preferable to the usual notation that modern calculators apply.

The idea that runs through this book is to teach astronomy while instructing in using KM-AstCalc. Newbies in the subject learn, and experts use their knowledge to become aware of the calculator's facilities (and take the opportunity to relive their student days).

So enjoy what KM-AstCalc offers and use it at large whenever you need it.

Chapter 1

Getting Started

Fig. 1.1 shows the screen of KM-AstCalc.

We can divide KM-AstCalc into eight sections:

- The pile of numeric and unit registers (upper-left panel);
- The panel of number precision and notation, and the set of the International System of Units and derived ones (upper-right);
- The panel of the numeric pad and the pile and memory control (middle-left);
- The panel of mathematical functions and the sub-section of astronomic fundamental solvers (middle-right);
- The panels of physical constants (middle-lower left) and astronomical constants (middle-lower right);
- The panel of planets' primary and osculating data (lower-left); and,
- The panel of asteroids and comets orbital elements.

Below everything, on the left, we have links to the sites where I raised the numbers and the constants KM-AstCalc offers to use, and on the right, the manual, user's guide, and sheets to quick access to shortcut keys.

1.1 How to use

You can apply all the operations by clicking on the mouse, but you may use shortcuts on the keyboard to have the same results. The link "Shortcuts"

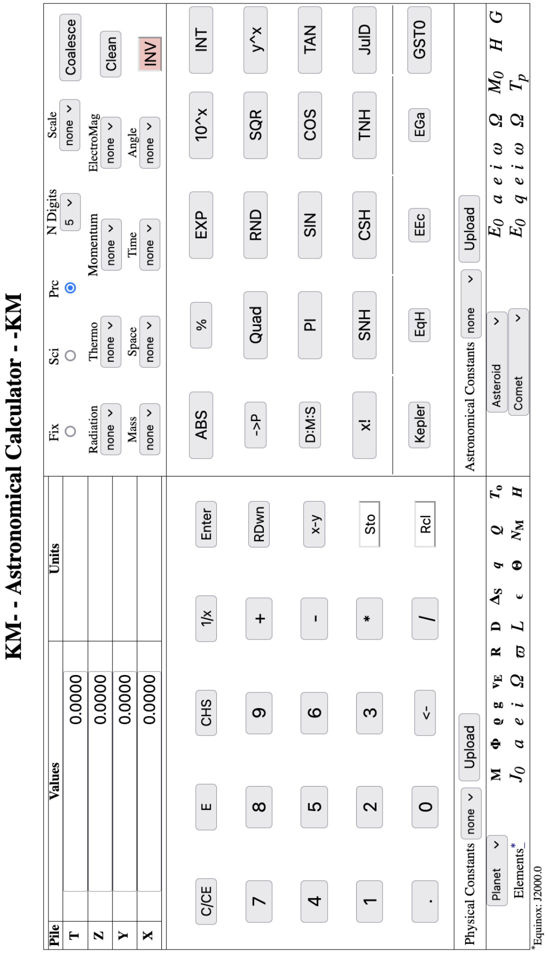


Figure 1.1: The KM-AstCalc screen

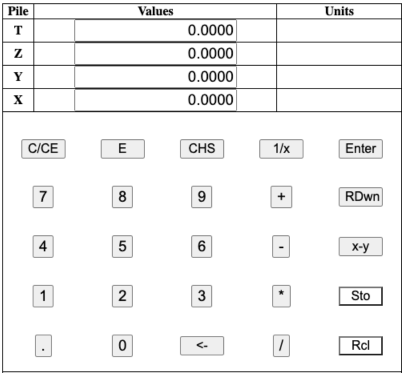


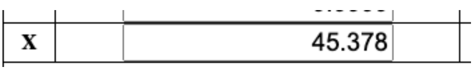
Figure 1.2: Numeric calculation

on the lower right side of the page gives you the sheets of these shortcuts. By pressing the keys directly or by prior pressing the keys "PageDown", "PageUp" or “End”, you can access the commands you can apply with the keyboard. Later on, we will see how to operate it in detail.

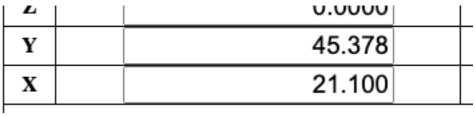
Let’s first observe the numeric registers and the numeric pad. Watch Fig 1.2. We see the numeric registers X, Y, Z, and T on the upper side. On the lower side, a numeric pad and some buttons to control the calculations. Let’s forget the column "Units" for a while. With the buttons on the numeric pad, we may perform all the arithmetic we need for all purposes, not only astronomical.

You click the mouse on a numeric button, and the number appears on the X-register. You keep clicking a digit, dot, or E, and the desired number will be composed on the X-register. To stop composing the number, click on the button Enter. Begin a new number. You see the number you just came to compose going to the Y-register while the new digits appear on the X-register.

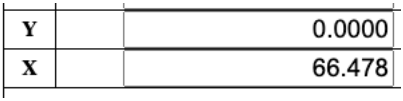
Example: write the number 45.378 by clicking on the numeric buttons in the usual sequence, then click on Enter. You see the register:



Now, enter, in the same way, the number 21.1 and Enter. You see:



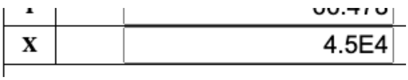
Click on the button +, and you will see the result on the X-register:



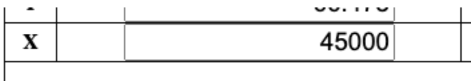
The same procedure may be done with all four arithmetic operators.

With the button CHS, you may change the algebraic signal of the numbers, so by clicking on it, you change the number signal.

A special button, E, helps you to compose big numbers with the 10-exponential notation. After forming the mantissa, click on E to start writing the exponential, namely, the number 4.5×10^4 is written clicking on 4 · 5 E 4. The result is like this:



Clicking on Enter, we have:



Check out that, numerically, it is the same number.

At any time, while composing the number you may fix eventual mistaken by clicking on the key <- you can see at right of the key 0.

The Reversed Polish Notation (RPN)

Hewlett-Packard Co. introduced the so-called Reversed Polish Notation by presenting the HP-2x series of scientific calculators. With the poor means of programming resources at that time, the idea allowed to perform complex calculations without the necessity of the sophistication needed to use the usual sequence of "+", "-", "*", and "/" with the "=" signal. Later, Texas Co. launched on the market its scientific calculator with the resources of the "=" sign. In this case, it needed to add the buttons of opening and closing parentheses, allowing just one additional level of calculation. Even while I became a Texas calculator user, the idea of the RPN remained an elegant solution for these kinds of calculators that I am using to perform very complex calculations without writing partial results on paper. The financial market still uses the RPN, whereas the HP scientific calculators have changed to the usual ones.

The "normal" notation uses to have the sequence:

`[number] [operator] [number] "=",`

while the RPN has the sequence:

`[number] "Enter" [number] [operator].`

With four registers: "X", "Y", "Z," and "T", under the RPN, we can make complex calculations. Example: to make the operation:

$$4.34 + \frac{18 - 7.1}{61 + 3.5} = 4.5090,$$

we do, among other ways,

Operation	Register	
1 8 Enter	**X**	18.000
7 · 1 -	**X**	10.900
6 1 Enter	**X**	61.000
3 · 5 + /	**X**	0.16899
4 · 3 4 +	**X**	4.5090

With some experience, you will know which is the fastest calculation sequence.

Besides the four arithmetic operators, some additional buttons help the calculation. There is the <- button, as seen before, with the rub out function, correcting mistakes you could make. There is the 1/x button, which inverses the value in X-register. The button x-y allows swapping the number in X-register with Y-register. RDwn rolls down the pile, putting the number in X-register into T-register, rolling the original value in T-register into Z-register, and so on. Clicking once on the button C/CE will erase the X-register, and double-clicking on it will reset the entire calculator to its starting state.

The buttons of memory 'Sto' and 'Rcl'

The buttons Sto and Rcl allow you to use memory registers from 0 to 9. You may store the number in the register X in any of the 0-9 records. You click on the button Sto and a 0-9 digit; the number in 'X' will be stored in the memory. When you want this number back, you click on the button Rcl, followed by the same digit you clicked before. The pile will push up, and the number will appear in the X-register.

Exercises

1. Make the calculation in KM-AstCalc:

$$3.4 + \cfrac{1}{6.8 + \cfrac{3}{3.4 + \cfrac{5}{6.8} \cdots}}.$$

   Use the 0-9 memory registers. See to what extent you can go with the series using memory.

2. Idem

$$1.43\left(\frac{28}{2.77 - 1.884} - \frac{0.993 \times 10^3}{(2.12 - 4.3(219 - 12.76)}\right),$$

3. Enter the number:

$$-5.3038 \times 10^{-5},$$

   invert it and multiply it by

$$3.001 \times 10^{-3.}$$

1.2 The Panel of Mathematical Functions

See in Fig. 1.3 the panel of mathematical functions. You can recognize the usual functions in numerical calculations, like absolute value, percentage, exponential functions, the integer part, etc. On the other hand, you should be careful with the trigonometric functions, for they demand angle units, which we will see next.

All the functions operate on registers X, or on X with Y combined. Let's see what each one does when we click on their buttons:

ABS: Always returns the positive value of the number. Ex. -13.4 gives 13.4, and 44.1 gives 44.1;

%: Does the operation: $X * Y/100$ and put the result on register X;

EXP: Executes e^X;

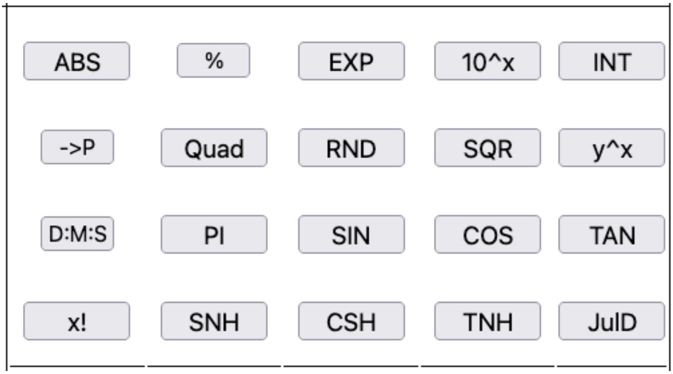


Figure 1.3: Panel of Mathematical Functions

10^x: Executes 10^X;

INT: Find the integer part of the number;

->P: Transforms the values on X and Y from rectangular to polar coordinates. The registers X and Y should be with the same units (see below);

Quad: Reduces the number to the quadrant $[0, 2\pi]$, if needed. The number should be in units of angle (see below);

RND: Generates a random number in the interval 0 to 1 (not included);

SQR: Finds the square of the number;

y^x: Finds y^x;

D:M:S: Converts the number into the format DD:MM:SS.sss. The number should be in degrees or in hours (see below).;

PI: Generates the number π;

SIN, **COS**, **TAN**: Find the trigonometric functions sin, cos and tan respectively. The number should be in units of angle (see below);

Figure 1.4: Mathematical functions with 'INV' button.

x!: Evaluates the factorial of the number. Notice: floating point numbers will not give the Gamma function. The calculation is done as the following :

$$f(x) = (x-1)f(x-1),$$

repeating until $x < 1$;

SNH, **CSH**, **TNH**: Find the hyperbolic sine, cosine and tangent of the number, respectively. The numeric values should be adimensional in this case;

JulD: A special button that gives the Julian Day from the Gregorian date on register-X with the format "dd:mm:yyyy", or else, depending on how the units register is built. We will see it later on.

1.2.1 The Button INV

Just above the panel of mathematical functions, at right, we find the button INV. Clicking on the button INV we switch some important buttons to different functions, some of them being the inverse function of the unswitched buttons. It has a different color (pink) and becomes gray INV, when we click on it. It gets different colors when we hover over it, in both states. The Fig. 1.4 shows the calculator after we click once on INV:

-ABS: The negative of the number's absolute value;

D% : Gives the result of the operation

$$100\frac{X - Y}{Y};$$

LN : Evaluates $\log_e X$;

LOG : Evaluates $\log_{10} X$;

FRAC : Returns the fractionary part of the number in X;

->R : Converts the polar coordinates $X(R), Y(\theta)$ into rectangular coordinates X, Y;

Atan2 : Evaluates the algebraic value of arctan(Y/X). The registers X and Y should be in the same units;

SQRT : Evaluates the square root of X;

x^y : Evaluates X^Y;

D.ddd : Put the number X from the format "DD:MM:SS.sss" into the format "D.ddd";

PI/2 : Evaluates $\pi/2$;

ASIN , **ACOS** , **ATAN** : Evaluates the inverse functions of sine, cosine and tangent, respectively;

ASNH , **ACSH** , **ATNH** : Evaluates the inverse functions of (hyperbolic) sine, cosine and tangent, respectively;

GrgD : Evaluates the Gregorian date, given the Julian day (units in dy, see forward).

1.2.2 The Sub-Panel of Astrometric Functions

Down in the Mathematical Functions Panel, you see five buttons: Kepler, EqH, EEc, EGa, and GST0. They execute functions for different purposes of astrometric and celestial dynamic studies. Later on, we will see them in the appropriate context.

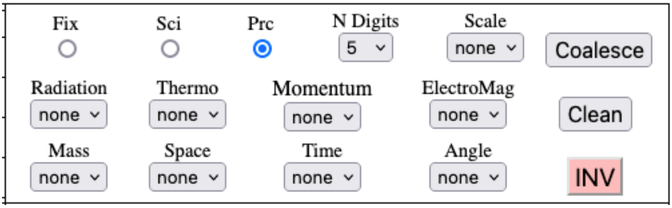


Figure 1.5: The Number Precision and the SI panel

Exercises

1. Calculate
$$2\pi\sqrt{0.911\left(\frac{2}{3.1}-\frac{1}{1.6^{3.2}}\right)}.$$

2. Calculate
$$\frac{5.5!}{5!},$$
and then,
$$\frac{5.4!}{5!}.$$
Do you see some unexpected result? If yes, why?

1.3 The Panel of Number Precision and the SI

The upper-right panel is consecrated to define the numbers' precision format and the units. Look at the Fig. 1.5.

On the first row we see three radio-buttons and a select-combo-panel, this one identified as "N Digits". The remaining dropdown combo e button, we will see later. They define the way the numbers on the numeric pile is shown. Let's see what they do:

Fix: Defines a fixed number of digits in the decimal part of the number;

Sci: The "scientific" notation: the first significant digit, followed by a number of digits and the power of ten needed to define the number;

Prc: Gives the number with its significant digits multiplied by the power of ten necessary to represent the number;

Quantity	Symbol	Basic Unit	Combo's Name
Luminous Intensity	cd	candela	Radiation
Amount of substance	mol	mol	Thermo
Thermodynamic temperature	ºK	kelvin	Thermo
Electric current	A	ampere	ElectroMag
Mass	kg[1]	kilogram	Mass
Length	m	metre	Space
Time	s	second	Time
Angle	rad	radian	Angle
Speed, Force, Energy		-	Momentum

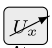
Table 1.1: Basic units and derived ones.

N Digits: May be $0 \ldots 12$. For the notations "Fix" and "Sci", defines the number of significant digits after the period and for "Prc" sets the number of total significant digits.

1.3.1 System of Units

The two rows below the panel of Number Precision and SI are dedicated to units, especially the International System of Units (SI). According to this system's board, the basic units are seven. Aside from them, we find the derived units, which a combination of the basic ones may obtain. The Tab. 1.1 shows the basic units available in KM-AstroCalc. Where applicable, each combo offers multiples of the respective base units. For example, for the unit of the theme Space, you can choose foot, mile, etc.

Following the multiple variety of units, they were assembled according to a physical proxy. For example, the units of speed, pression, energy and so on, were grouped into the theme "Momentum". Nevertheless, the unit "calorie", an energy one, was set under the theme "Thermo".

Once you choose the desired unit or units, you fill-up the column "Units" you see in the numerical pile in the panel at left (see Fig. 1.6). To do it, you click just on the text field next to the register X (unit-X). Hereafter we refer this operation with the icon . Each time you click on unit-X, the chosen unit will be added to it, or its exponent will be increased by one,

[1]To facilitate the adopted basic unit for mass is the gram instead of the kilogram.

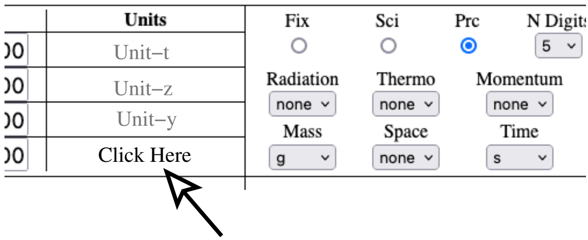


Figure 1.6: Once you have selected the units on the combos panel, click on the 'Unit-x' row of the Units column to load them.

if it is already there. Example: by clicking on the unit meter twice, unit-X will show m^2. To avoid undesired results, all the units are reset to 'none' for each clicking on unit-X. Consider that from the scratch every unit in unit-X has exponent zero. When you add one of them into unit-X, you add one to it. By clicking on the button INV , you add a negative unitary exponent.

1.3.2 The Dropdown Combo Scale

You see, just to the right of the select-combo Digits, another select-combo named Scale. Clicking on it, you will see the scales and their names adopted by the Internation System of Units. You should compose the unit of your choice with one of these scales. Consequently, when you insert the chosen scaled unit into Unit-X, you will see it preceded by the letter of the scale. In Tab 1.2, we find the available scales to be applied, their adopted names, and the keyboard character used to apply the scale on the unit to be uploaded.

1.3.3 In a Nutshell

Let's take this little tutorial:

Problem: A driver takes a trip from his city to another one at 455km in $6^{h}35^{m}$. What is the average speed?

Answer:

1. Compose the number 455, then choose 'k' (kilo) in the combo Scale, 'm' in the combo Space and click on unit-X (U_x) and click on the button Enter . It shows like:

X		455.00	km

Name	Symbol	Base 10	End-key
quecto	q	-30	q
ronto	r	-27	r
yocto	y	-24	y
zepto	z	-21	z
atto	a	-18	a
femto	f	-15	f
pico	p	-12	p
nano	n	-9	n
micro	µ	-6	u
milli	m	-3	m
centi	c	-2	c
deci	d	-1	d
		0	
deca	da	1	D
hecto	h	2	C
kilo	k	3	k
mega	M	6	M
giga	G	9	G
tera	T	12	T
peta	P	15	P
exa	E	18	E
zetta	Z	21	Z
yotta	Y	24	Y
ronna	R	27	R
quetta	Q	30	Q

Table 1.2: Scales available to be applied to the units. Forward, we discuss the functions of the last column.

Now, enter the time with the string: 6 : 35, then, in the combo Time, choose the unit 'hr' and click on unit-X. It will look like:

Y		455.00	km
X		6:35	hr

Before divide, transform 6 : 35hr, into decimal format. Click on INV and then, D.ddd. Now you can divide, by clicking on the button /. The result will be:

Y		0.0000	
X		69.114	hr^{-1}.km

Problem: Enter the density $3.22 g.cm^{-3}$ into KM-AstrCalc.

Answer:

1. Enter the number 3.22 into the numeric register-X. Now, choose the unit 'g' in the combo 'Mass', click on unit-X (U_x). Then, choose 'c' (centi) in the combo Scale, 'm' in the combo Space and click on the button INV and onto unit-X. Repeat the latter procedure twice. Finally click on Enter. You will have:

X		3.2200	$g.cm^{-3}$

2. Other way to do it is entering 3.22g, clicking on Enter and $1cm^3$, by choosing and clicking on unit-X (U_x) three times and making the division.

3. To make $1cm^3$, you may enter 1, choose 'c' in the combo Scale, choose 'm' in the combo Space, click in the text field unit-X (U_x), Enter 3 Enter y^x.

4. Calculate $\sin(35°48'33'')$.

Answer:

3 5 INV :	X	35:	
4 8 INV :	X	35:48:	
3 3	X	35:48:33	
Angle→ deg U_x	X	35:48:33	deg
INV D.ddd	X	35.809	deg
SIN	X	0.58509	

1.3.4 The Button Coalesce

In the upper-right corner of this panel, we find the button Coalesce. Aside from this function applies implicitly to many operations in KM-AstrCalc, you may need it in some conditions. The function Coalesce takes all the units registered in unit-X and put them in terms of the SI-basic units, applying the numerical conversions when applicable. Ex., the length's basic unit is the metre, so if you put the number 8.1km, the Coalesce function will convert it to 8100.0m. Many functions coalesce the set of units in unit-X before performing the asked operation.

Notice: the function Coalesce does not operate on a format having the character ":".

1.3.5 The button (Inv) Convert

You may convert the units of a number by doing the following: Choose the units you want to convert into and click INV Convert.

Ex. convert 1AU into kilometers:

1. 1, choose AU in the combo Space, click on 'unit-X', Enter, choose 'k' in the combo Scale, choose 'm' in the combo Space, INV Convert :

1 Space→ AU U_x	X	1	AU
Scale→ k Space→ m	X	1	AU
INV Convert	X	1.4960e+8	km

This is the value of a AU in km.

1.3.6 Operating with Units

Some mathematical functions work only with the appropriate set of units, for instance, the trigonometric functions sine, cosine and tangent operate only if the number is in angle units. It could be any (of the options in the combo Angle). If the number is not in an angle unit, the calcul will not be done.

The conversion of polar coordinates from rectangular ones demands both coordinates in the same units. The reverse one requires the angular coordinate in the angle unit. Arithmetic operations of summation and subtraction should be with both numbers in the same units. Multiplication and division will operate on the units like they do with numbers. The percent operator [%] works similarly to multiplication or division, but percent variation [D%] acts like summation or subtraction.

Logarithm/exponential functions require adimensional exponents. It includes the hyperbolic functions as well as the factorial [x!] operator.

The special operator [JulD] requires the units to work as template for the numerical input. There should be three fields split with signal ":", giving hints on what each field means. Ex., if the numerical input is in the form "yyyy:mm:dd", the unit-X should be setup as "yr.mth.dy". For the inverse, [GrgD], unit-X should be in days (dy).

Exercise

1. Calculate the number of days between May 15, 2023 and Jan 01, 2001.

Answer:

[5] Time→ [mth] [U_x]	**X**	5	mth
[:] [15] Time→ [dy] [U_x]	**X**	5:15	mth.dy
[:] [2023] Time→ [yr] [U_x]	**X**	5:15:2023	mth.dy.yr
[JulD]	**X**	2460079.50000	dy

you may set the notation to 'Fix'. Now, let's find the Julian day of Jan 01, 2001:

[1] Time→ [dy] [U_x]	**X**	1	dy
[:] [1] Time→ [mth] [U_x]	**X**	1:1	dy.mth
[:] [2001] Time→ [yr] [U_x]	**X**	1:1:2001	dy.mth.yr
[JulD]	**X**	2451910.50000	dy

and we just have to make the difference:

[-]	**X**	8169.00000	dy

1.4 Manipulating the Pile

The following buttons allow you to manipulate the pile:

C/CE: clears both register-X and unit-X with one click and all registers and the unit pile with double click;

Enter: enters the number into register-X. When the number is complete, it pushes the pile up;

INV **Down**: pulls the pile down;

RDwn: Rolls the pile down;

RUp: Rolls the pile up;

x-y: Swaps the register-X and unit-X with register-Y and unit-Y;

INV **LstX**: Retrieves the last register-X and unit-X;

Clean: Special button to clean the unit-X (one-click), or all units (double-click).

Exercises

1. The Sun's absolute magnitude is 4.82, what is it apparent magnitude? Answer: the astrophysical absolute magnitude is the star apparent magnitude when it put at 10pc distance from the Earth.

2. Enter the number 2451545.466 into the register X. Then, alternatively click on the radios "Fix", "Sci" and return to "Prc" and see how this number is shown. Change NDigits and repeat the operation for different values of number of digits, to see how the same number is shown in the register. Thus, you will find out the way to put a number in the format and precision you need.

3. The french aircraft carrier Charles de Gaulle is capable of sailing at 27 knots (Wikipedia). Calculate it in km/h, then in mi/h, and nmi/h (nautical miles).

Chapter 2

The Panel of Constants

The two penultimate lower panels are dedicated to operating with some of the physical and astronomical constants. On the left, a combo contains some main physical constants, and on the right, the astronomical ones. A combo at each one allows you to choose what constant you want. When you hover over the correspondent symbol, you've got a quick explanation. A button at the right of each combo Upload allows you to put the constant value into the register-X with its units, if applicable.

Once you upload the constant, you may convert it to other units since they are appropriate. If the units to be converted are not compatible, the result of the conversion operation will end with the combination of the units that is possible.

2.1 The Physical Constants

The physical constants are shown in the Tab. 2.1 / 2.2. It contains some of the most important constants with current use in physics, namely, the light's speed (c), Plank's constant (h), the vacuum's electric permittivity (ϵ_0), and magnetic permeability (μ), among others.

2.1.1 Forwarding

You might play 'Maxwell' and figure out that the light speed on the vacuum is the inverse of the square root of the product between the electric permittivity

Constant	Symbol	Value	Unit	PgUp-key
alpha particle mass	A_P	6.644657×10^{-27}	kg	A
angstron star	A^*	1.000015×10^{-10}	m	*
atomic mass	m_a	1.660539×10^{-27}	kg	a
Avogadro constant	N_A	$6.022141 \times 10^{+23}$	mol^{-1}	N
Bohr magneton	μ_B	9.274010×10^{-24}	$J\,T^{-1}$	b
Bohr radius	a_0	5.291772×10^{-11}	m	B
Boltzmann constant	k_B	1.380649×10^{-23}	$J\,^oK^{-1}$	K
Impedance of vacuum	Z_0	376.7303	Ω	w
electron radius	r_e	2.817940×10^{-15}	m	!
deuteron mass	D_M	3.343584×10^{-27}	kg	Q
electron mass	m_e	9.109384×10^{-31}	kg	e
electron volt	eV	1.602177×10^{-19}	J	v
elementary charge	e	1.602177×10^{-19}	C	q
Faraday constant	F	96485.33	$C\,mol^{-1}$	F
fine-structure constant	α	0.007297353		f
Hartree energy	E_h	4.359745×10^{-18}	J	T
Josephson constant	K	$4.835978 \times 10^{+14}$	$Hz\,V^{-1}$	l
lattice parameter Si	a	5.431021×10^{-10}	m	t
molar gas constant	R	8.314463	$J\,mol^{-1}\,^oK^{-1}$	R
nuclear magneton	μ_N	5.050784×10^{-27}	$J\,T^{-1}$	(

Table 2.1: Physical Constants

Constant	Symbol	Value	Unit	PgUp-key
Planck constant	h	6.626070×10^{-34}	$J\,Hz^{-1}$	h
proton mass	P	1.672622×10^{-27}	kg	&
Rydberg constant	R_∞	$1.097373 \times 10^{+7}$	m^{-1}	x
speed of light	c	$2.997925 \times 10^{+8}$	$m\,s^{-1}$	c
acceleration of gravity	g	9.806650	$m\,s^{-2}$	g
standard atmosphere	Atm	101325.0	Pa	m
Stefan-Boltzmann constant	Σ	5.670374×10^{-8}	$W\,m^{-2}\,{}^oK^{-4}$	s
electric permittivity	ε	8.854188×10^{-12}	$F\,m^{-1}$	n
magnetic permeability	μ	0.000001256637	$N\,A^{-2}$	U
von Klitzing constant	R_k	25812.81	Ω	V

Table 2.2: Physical Constants

and the magnetic permeability. Let's see. Try the sequence:

Phys.Const.→ ϵ_0 Upload	X	8.8542e-12	F.m^{-1}
Phys.Const.→ μ Upload	X	0.0000012566	N.A^{-2}
*	X	1.1127e-17	N.A^{-2}.F.m^{-1}
Coalesce	X	1.1127e-17	m^{-2}.s^{2}
1/x	X	8.9876e+16	m^{2}.s^{-2}
INV SQRT	X	2.9979e+8	m.s^{-1}
Momentum→ c	X	2.9979e+8	m.s^{-1}
INV Convert	X	1.0000	c

In other words, you made the product of the electric permittivity with the magnetic permeability, then, inverted, and got the square root of the result. Then you converted what was issued (that should be in $m.s^{-1}$ units) to c as a speed unit. The result is just 1c, meaning that the operation issue is the exact value of the light speed.

2.2 The Astronomical Constants

The astronomical constants are shown in the Tab. 2.3 and 2.4.

2.2.1 Forwarding

Notice that there are four types of years in the astronomical constants list. The first one is the Julian Year, 365.25 days. This is the value of the year duration in the Julius Cesar times in Rome. When Julius Cesar adopted the calendar, this was the year's duration value they could measure with the means they had. Later on, when Pope Gregorius II adopted the actual calendar in 1582, the measured duration of the year was 365.2425 days, so this is the Gregorian year. Recently, after the IAU deliberation, the tropical year is known to be of 365.24219 days. This is the sun's average time to cross the vernal equinox successively. Finally, the sidereal year is that of the tropical one, considering the precession of the equinoxes, so it is 365.256363004 days. This latter one is adopted for the orbital periods in the gravitation theory, for it is the only one that considers the earth's movement as seen out of our planet.

We may understand some important issues about celestial mechanics theory, namely, in deducing the Gauss constant (registered as the 'k' symbol to be selected in the combo). According to Gauss, the Earth-Moon system orbits around the Sun, considering simplifying this motion as a two-body prob-

Constant	Symbol	Value	Unit	PgUp-key
Astronomical Unit	AU	$1.495979 \times 10^{+11}$	m	X
parsec	pc	$3.085678 \times 10^{+16}$	m	p
lightyear	ly	$9.460730 \times 10^{+15}$	m	Y
Modified Julian Day	$mdj0$	2400001	dy	j
Julian Year	$JulYr$	365.2500	dy	J
Julian Century	$JulCy$	36525.00	dy	y
Epoch 2000-Jan-1.5TD	$J2000.0$	2451545	dy	0
Besselian Epoch	$B1950.0$	2433282.42350	dy	%
Sidereal year	Syr	365.256363040	dy	/
Tropical year	Tyr	365.24219	dy	[
Gregorian year	Gyr	365.2425		
acceleration of gravity	g	9.806650	$m\,s^{-2}$	g
constant of gravitation	G	6.674280×10^{-11}	$m^3\,kg^{-1}\,s^{-2}$	G
mass of Sun	S	$1.988550 \times 10^{+30}$	kg	S
Gaussian gravitational constant	k	0.000008169351	$m^{1.5}\,kg^{-0.5}\,s^{-1}$	k
Equatorial radius for Earth	R_e	6378137	m	O
Earth ellipticity	e	0.003352820		i
Geocentric gravitational constant	GE	$3.986004 \times 10^{+14}$	$m^3\,s^{-2}$	D
Earth mass / Moon mass	$1/\mu$	81.30056		u

Table 2.3: Astronomical Constants

Constant	Symbol	Value	Unit	PgUp-key
Precession in longitude	ρ	0.02438029	rad	r
Precession term in m	m	3.0750	$^{s}.yr^{-1}$	Z
Precession term in n	n	20.043	$''yr^{-1}$	
Obliquity of ecliptic	ε	0.4089906	rad	o
Sidereal rate	ϵ	0.997269624573		:
Constant of nutation	N	0.000004462823	rad	2
Constant of aberration	κ	0.000009936509	rad	1
Heliocentric gravitational constant	GS	$1.327244 \times 10^{+20}$	$m^3\,s^{-2}$	L
Sun mass / Earth mass	S/E	332946.1		E
Sun mass / Earth+Moon mass	$S/E+M$	328900.6		#
Hubble constant	H_0	70.10000	$km\,s^{-1}\,Mpc^{-1}$	H
Solar luminosity	L_0	3.939×10^{26}	W	W

Table 2.4: Astronomical Constants (cont.)

lem, with a rate of 0.886 degrees per day, or nearly 0.0172 radians per day. This value is deduced by the relation $k : (rad/d) = (GM)_{\odot}^{0.5} AU^{-1.5}$ (`https://en.wikipedia.org/wiki/Gaussian_gravitational_constant`), where G is the Gravitational Constant and $M_{\odot}$ is the mass of the Sun. In the combo of the Astronomical Constant this product has the symbol GS.

Let's check it out: pop out the combo labeled Astronomical Constants, select the value of the Heliocentric Gravitational Constant (GS), and upload it to the Unit-x register:

Astr.Const.→ GS Upload	**X**	1.3272e+20	$m^{-3}sc^{-2}$

now, find the square root of it:

INV SQRT	**X**	1.1521e+10	$m^{-1.5}sc^{-1}$

Upload the constant AU:

Astr.Const.→ AU Upload	**X**	1.4960e+11	m

and raise it to the power 1.5:

1 · 5 y^x	**X**	5.7861e+16	$m^{1.5}$

Now, divide:

/	**X**	1.9911e-7	sc^{-1}

Since the result is supposed to be an angular speed, the unit 'rad' is added to the result that has units of s^{-1} (frequency, or rate).

Now it is to convert to units of day:

Time→ dy	**X**	1.9911e-7	sc^{-1}
INV Convert	**X**	0.017203	dy^{-1}

Exercise:

1. See how G is related to k.

2.3 Natural Units × Terrestrial Units

We might go forward on the constants registered in KM-AstCalc by deducing the so-called Planck units or natural units. In 1899, Max Planck published a paper introducing units that could be considered natural because they don't depend on arbitrary "terrestrial" standards. Among others, we have the Planck length, considered the least distance the actual physics science could describe a phenomenon. It is:

$$l_p = \sqrt{\frac{\hbar G}{c^3}} = \sqrt{\frac{hG}{2\pi c^3}} = 1.616255(18)\times 10^{-35} m.$$

The '18' between parentheses refers to the fluctuation of the value's last two digits for more or less. In the KM-AstCalc we could do:

Phys.Const.→ h Upload	X	6.6261e-34	$\mathrm{J.Hz^{-1}}$
2 / PI /	X	1.0546e-34	$\mathrm{J.Hz^{-1}}$
Phys.Const.→ c Upload	X	2.9979e+8	$\mathrm{m.sc^{-1}}$
3 y^x /	X	3.9139e-60	$\mathrm{m^{-3}.sc^{3}.J.Hz^{-1}}$
Astr.Const.→ G Upload	X	6.6743e-11	$\mathrm{m^{3}.kg^{-1}.sc^{-2}}$
*	X	2.6123e-70	$\mathrm{kg^{-1}.sc.J.Hz^{-1}}$
INV SQRT Coalesce	X	1.6163e-35	m

Exercises

1. Find Planck units for:

   (a) Mass:

   $$m_p = \sqrt{\frac{\hbar c}{G}};$$

   (b) Time:

   $$t_p = \sqrt{\frac{\hbar G}{c^5}};$$

   (c) Temperature:

   $$T_p = \sqrt{\frac{\hbar c^5}{G\, k_B^2}}.$$

2. And some derived ones:

   (a) area:

   $$l_p^2 = \frac{\hbar G}{c^3};$$

   (b) volume:

   $$l_p^3 = \sqrt{\frac{(\hbar G)^3}{c^9}};$$

(c) momentum:

$$m_p c = \sqrt{\frac{\hbar c^3}{G}};$$

(d) energy:

$$E_p = \sqrt{\frac{\hbar c^5}{G}}$$

(e) force:

$$F_p = \frac{c^4}{G};$$

(f) density:

$$\rho_p = \frac{c^5}{\hbar G^2};$$

(g) acceleration:

$$a_p = \sqrt{\frac{c^7}{\hbar G}}.$$

3. Find the Earth's gravity acceleration in natural units.

2.3.1 The Hubble constant

After Hubble introduced the general redshift law for galaxies[1], the proportionality constant of the galaxy's receding velocity with its radial distance has been measured and revised several times. The actual value is registered in KM-AstCalc: $H_0 = 70.100\,\mathrm{kms^{-1}Mpc^{-1}}$. Let's upload this constant to the register-X and coalesce it:

Astr.Const.→ H0 Upload	X	70.100	km.sc^{-1}.Mpc^{-1}
Coalesce	X	2.2718e-18	s^{-1}

You see that, on a shell, the Hubble constant is expressed in units of frequency. If we invert it:

1/x	X	4.4018e+17	s

and convert it into gigayears:

Scale→ G Time→ yr	X	4.4018e+17	s
INV Convert	X	13.948	Gyr

or 13.95 gigayears. It is called the Hubble time and it is supposed to be the age of the universe.

[1] Today, the discovery of Hubble's law is credited to several authors. See `https://en.wikipedia.org/wiki/Hubble%27s_law`

Exercise

1. Calculate the Hubble time in Planck time units.

2. Calculate the Hubble length: $l_H = cH_0^{-1}$.

2.3.2 The Schwartzchild Radius

Karl Schwartzchild stated that, from the General Relativity Theory, there is a radius in a sphere containing a certain amount of mass that the equations lead to a singularity, meaning that every quantity vanishes or diverges to infinity, namely, time stops and the space-time curvature goes to the infinity. From his calculation:

$$R_S = 2\frac{GM}{c^2}$$

it is now called the Schwartzchild Radius, or the Event Horizon Radius. G is the gravitational constant, M is the mass inside the sphere and c is the light speed.

Many "classical" academics say that the force is so that nothing escape, even the light. It is the reason that bodies like these are called "black holes". There is no limit to how much mass is put inside. Theoretically, anything having mass may be a black hole, since the limit of the radius this mass is put inside obey the relation above.

We may use KM-AstCalc to evaluate a black hole containing the mass of the Sun:

Astr.Const.→ G Upload		X	6.6743e-11	$m^3.kg^{-1}.sc^{-2}$
Astr.Const.→ S Upload	*	X	1.3272e+20	$m^3.sc^{-2}$
Phys.Const.→ c Upload		X	2.9979e+8	$m.sc^{-1}$
SQR /		X	1476.7	m
2 *		X	2953.4	m

Thus, if the Sun suddenly collapsed to a sphere of a radius no greater than 3km, it would become a black hole.

Exercises

1. Evaluate the size of a black hole for the Earth's and for the Moon's masses.

2. Idem for the electron.

3. According to `https://www.cfa.harvard.edu/news/mass-milky-way`, the mass of the Milky Way is between 1.2 and 1.9×10^{12} solar-mass. Evaluate a black hole containing the mass of our galaxy. Put it in AU.

4. It is assumed that there is a black hole with a mass of 4×10^6S, or Sagitarius* in the center of the Milky Way. Evaluate the "radius" of this black hole, taking the Event Horizon as base.

5. Suppose that the solar disk is $32'$ diameter. Find the Sun's diameter in length and compare it with the "radius" of the black hole in the galactic center.

   Answer: the hint is that the apparent diameter of the Sun comes from the distance it is from the Earth (1AU)

32 Angle→ min $\cancel{U_x}$	X	32	min
TAN	X	0.0093087	
AU Upload	X	1.4960e+11	m
*	X	1.3926e+9	m
Scale→ G Space→ m	X	1.3926e+9	m
INV Convert	X	1.3926	Gm

The adopted Sun's diameter is[2] 1.3927 Gm.

2.3.3 Imaging Resolution Power of a Telescope

An optical device, once it has an aperture, presents a resolution power. If the aperture is circular, then the resolution power is symmetrical. According to the Rayleigh criterium, this resolution is

$$r''_D = 0.25\frac{\lambda}{\phi},$$

where λ is the radiaton's wavelength in μm, and ϕ is the aperture in meters.

Exercise

1. Find the resolution power of a binocular 4cm aperture.

Answer: It is usual to adopt the central human vision sensibility spectrum wavelength 5500Å to estimate the resolution power of a telescope, so

[2] `https://science.nasa.gov/sun/facts/`

5500 Space→ AA U_x	X	5500	AA
Enter	X	5500.0	AA
Scale→ mu Space→ m	X	5500.0	AA
INV Convert	X	0.55000	mum
4 Scale→ c Space→ m	X	4	cm
Enter	X	4.0000	cm
Space→ m INV Convert	X	0.040000	m
/ Clean	X	13.750	
Angle→ sec U_x	X	13.750	sec
0.25 *	X	3.4375	sec

Taking into account that our eyes are capable of resolving $1'$, we may use an ocular that gives us an approximation factor of $250\times$ without resolution loss.

2.3.4 Telescope Limiting Magnitude

The limiting magnitude of a telescope is evaluated with the formula:

$$m_l = 17.1 + 5 \log \phi,$$

being ϕ the telescope aperture in meters.

Exercise

1. Evaluate the limiting magnitude of a binocular 4cm aperture.

Answer:

4	X	4	
Scale→ c	X	4	
Space→ m U_x	X	4	cm
Coalesce Clean	X	0.040000	
INV LOG	X	-1.3979	
5 *	X	-6.9897	
17.1 +	X	10.110	

Chapter 3

Positioning and Celestial Coordinates

3.1 The Calendar

Astronomy is in charge of counting time and setting up the calendar, which counts days, months, years, etc. Formerly, it was a religious task, but the clerical people used astronomy to do it. Universally, astronomy uses to count the days through the Julian date, counted from Jan 1.5, 4712 bC. This date was set for a number of reasons, one of them being before any known historical record. The Gregorian Calendar was set in 1582 (anglo speaking countries got it in 1642 when it was adopted in England) by Pope Gregory II. For astronomy, the day is counted from noon. It is the reason why the day is set to have a half more.

You may transform Gregorian to Julian date and vice-versa by using the button JulD / INV GrgD in the lower-right side of the functions panel. You input the Gregorian date on the Reg-X composing the numbers separated with colons. The meaning of each number is determined by the sequence of the names in the register unit-X. Ex[1].

X	23:9:2023	dy.mth.yr

Means that the date is in the format "dd/mm/yyyy". Clicking on the button JulD you get the correspondent Julian Day:

[1]The symbol : is reached by pressing the sequence INV ·.

23 : 9 : 2023	X	23:9:2023	
Time→ dy $\cancel{U_x}$	X	23:9:2023	dy
Time→ mth $\cancel{U_x}$	X	23:9:2023	dy.mth
Time→ yr $\cancel{U_x}$	X	23:9:2023	dy.mth.yr
JulD	X	2460210.50000	dy

The inverse is done straightforwardly:

INV GrgD	X	23.00000:9:2023	dy.mth.yr

Notice that the day is written in the format given by what you have set to in the format panel. The issue from the button INV GrgD is always in the format "`dd:mm:yyyy`".

3.2 Solar and Sidereal Time

We know that the Earth turns around the poles' North-South axis (as Galileo Galilei stated, "Et pur si muove") in the rotation motion. This rotation determines the march of the clock, marking the hours and days. We determine the advance of time by looking at the sky. However, there are two ways of assessing the tick-tack of the clock. The first one, the time adopted by the civil (and military) society, is defined by the Sun. The other is determined by the stars. The former is called Solar Time, and the latter is called Sidereal Time. To see the difference, let's look at the Fig. 3.1.

Imagine that, at noon, when the Sun crosses the local meridian, you can see the Sun and a fixed star at precisely the same local meridian. At this moment, you start two clocks, A and B. Let them work until the next day. You stop clock A when you see the star you saw the day before crossing the local meridian again, and stop clock B when you see the Sun doing the same. You notice that the two clocks are not showing the same value. It is because, in the course of a day, the Earth has moved $360/365.25\,\text{deg} = 0\overset{\circ}{.}98563 = 59'8''\!.2546$ in its orbit around the Sun, in the julian scale. If you scale both clocks to their reference of time origin, you would call clock A a sidereal clock and clock B the solar clock. Considering the day-a-day of such a variation, at the end of a year, the sidereal time will gain a whole day compared to the solar time. Thus, to convert the time measured by the Solar clock to the sidereal one, we use the factor $\epsilon = 365.25/366.25$, called the sidereal rate. You may retrieve its value in KM-AstCalc from the Astronomical Constant combo list (ϵ).

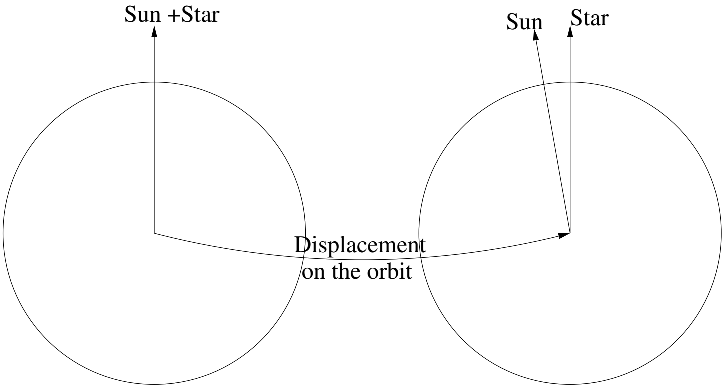


Figure 3.1: Solar Time and Sidereal Time. In a day, because of the movement on the orbit path, the mark of the Sun's position has displaced to the left, overlapping positions already covered in the rotation movement.

Exercise

1. Convert 18^{h} from Sidereal to Solar time.

Answer:

18 Time→ hr U_x	X	18	hr
Astr.Const.→ ϵ Upload	X	0.99727	
*	X	17.951	hr
D:M:S	X	17:57:3.0717	hr

2. Calculate how much time does the sidereal time gain over the Solar one in 24^{h}.

Answer:

24 Time→ hr U_x	X	24	hr
Astr.Const.→ ϵ Upload	X	0.99727	
/	X	24.065	hr
24 Time→ hr U_x	X	24	hr
- D:M:S	X	0:3:56.550	hr

3.2.1 Greenwich Sidereal Time at 0^{h}

The sidereal time origin in Greenwich (GST) is when the vernal equinox (gamma-point) crosses its prime meridian. There is an equation that computes the sidereal time for 0^{h} of the solar time in Greenwich. It is, in degrees[2]

$$\mathrm{ST}_0 = 100.46061837 + 36000.77053608\, S + 0.000387933\, S^2 - \frac{S^3}{38710000.0}, \tag{3.1}$$

where S is the fraction of the Julian Century of the difference of the Julian Day to J2000.0:

$$S = \frac{T - 2451545.0}{36525},$$

being T the Julian Day for the date.

Elsewhere, the sidereal time (LST) is evaluated considering the local longitude. If it is west, we subtract the longitude. Otherwise, we add the longitude to the GST value.

The button[3] GST0 allows us to evaluate the GST at 0^{UT} for a given Julian day in the X-Register.

Exercise:

1. Find GST at 0^{UT} for May, 15, 2023.

Answer:

5 : 15 : 2023	X	5:15:2023	
Time→ mth $\cancel{U_x}$	X	5:15:2023	mth
Time→ dy $\cancel{U_x}$	X	5:15:2023	mth.dy
Time→ yr $\cancel{U_x}$	X	5:15:2023	mth.dy.yr
JulD	X	2.4601e+6	dy
GST0	X	567.50	hour

The result does not account for the quadrant, so it is much greater than 24^{h}. There are two ways to reduce it to the quadrant. The first is clicking the button Quad, which gives the value in radians. You should convert it again to hours. The second way is by dividing it by 24, obtaining the fractional part, and multiplying it again by 24:

[2] Jean Meeus, 2000, *Astronomical Algorithms*, 2nd Edition, Willmann-Bell, Inc.

[3] Notice that this function GST0 gives the result in angle's, not in time's units. The reason is an easier way to deal with spheric trigonometry as we will see later on.

2 4 /	X	23.646	hour
INV FRAC	X	0.64574	hour
2 4 *	X	15.498	hour
D:M:S	X	15:29:52.181	hour

2. Determine the local ST0 of the longitude 43°27′38″W for the GST0 of the last exercise.

Answer:

INV D.ddd	X	15.498	hour
43 : 27 : 38	X	43:27:38	
Angle→ deg U_x	X	43:27:38	deg
D.ddd	X	43.461	deg
Angle→ hour INV Convert	X	2.8974	hour
-	X	12.601	hour
D:M:S	X	12:36:2.1600	hour

3.2.2 Greenwich Solar Time

As we have seen, the origin of solar time is when the Sun crosses the prime meridian. One needs to tie the clocks around the world to the Greenwich's. The globe is then split into 24 time zones, each one having a number of integer hours of difference from Greenwich[4]. The zones at the west of Greenwich are negative, and at the east, positive. We have then time zones that vary from 0h to -12h and from 0h to +12h. The opposite side to the prime meridian in Greenwich marks the line of changing the date since when the Sun crosses the prime meridian in Greenwich, it is noon there, and on the other side is midnight. Each zone is referenced with the name "±N GUT", where N is the number of integer hours (or plus or minus half-hour) that is summed or subtracted from the Greenwich Solar (or Universal) Time.

3.2.3 Greenwich Sidereal Time

Once you know the sidereal time for 0^{h} of solar time in Greenwich, you may know the sidereal time for any solar time. To do it, you must convert the solar time scale to the sidereal's. Use the Sidereal Rate ϵ in Astronomical Constants dropdown combo of KM-AstCalc for the conversion.

[4] India and Afghanistan adopt half-hour time zones.

Exercise:

1. Determine the sidereal time for $13^{\mathrm{h}}23^{\mathrm{m}}48^{\mathrm{s}}$ of solar time.

Answer:

13 : 23 : 4 8	X	13:23:48	
Angle→ hour U_x	X	13:23:48	hour
INV D.ddd	X	13.397	hour
Astr.Const.→ ϵ Upload	X	0.99727	
/ D:M:S	X	13:26:1.2444	hour

Having GST0 and the solar time converted to sidereal time, you may obtain the sidereal time for the date in Greenwich.

Exercise

1. Calculate the sidereal time in Greenwich of May, 15, 2023, at $13^{\mathrm{h}}23^{\mathrm{m}}48^{\mathrm{s}}$.

Answer: You already obtained the GST0 from the last exercise, so use it[5].

INV D.ddd	X	13.434	hour
+	X	25.115	hour

The result exceeds 24^{h} so you subtract it to get the result in the quadrant:

24 Angle→ hour U_x	X	24	hour
-	X	1.1150	hour
D:M:S	X	1:6:54.000	hour

3.2.4 Local Sidereal Time

To calculate the local sidereal time from the local solar time involves the following operations:

1. Take the local solar time and subtract the time zone (positive eastward) to establish the correspondent Greenwich solar time;
2. Convert it to the sidereal scale by dividing it by the sidereal rate;
3. Sum it to the Greenwich sidereal time at 0UT for the date;

[5] Important: KM-AstCalc does not operate the format “D:M:S”, so, before doing anything revert it to the usual format “D.ddd”.

4. Subtract it from the local longitude (positive westward);

5. Correct it to the right quadrant if necessary.

Mathematically, this procedure may be written as:

$$ST = Q\left[(LT - F)/\epsilon + GST0 - \lambda\right].$$

Where $Q[]$ means the quadrant operator and F, the time zone.

Exercise:

1. Determine the local sidereal time of the longitude $43°27'38''$ for the May, 15, 2023 at $21^{\mathrm{h}}30^{\mathrm{m}}$. Time zone: -3^{h}.

Answer:

2023 Time→ yr U_x	X	2023	yr
:5 Time→ mth U_x	X	2023:5	yr.mth
:15 Time→ dy U_x	X	2023:5:15	yr:mth:dy
JulD	X	2.4601e+6	dy
GST0	X	567.50	hour
24 /	X	23.646	hour
INV FRAC	X	0.64574	hour
24 *	X	15.498	hour
43 : 27 : 38	X	43:27:38	
Angle→ deg U_x	X	43:27:38	deg
D.ddd	X	43.461	deg
Angle→ hour INV Convert	X	2.8974	hour
-	X	12.600	hour
21 INV :	X	21:	
30 Angle→ hour U_x	X	21:30	hour
INV D.ddd	X	21.500	hour
3 CHS	X	-3	
Angle→ hour U_x	X	-3	hour

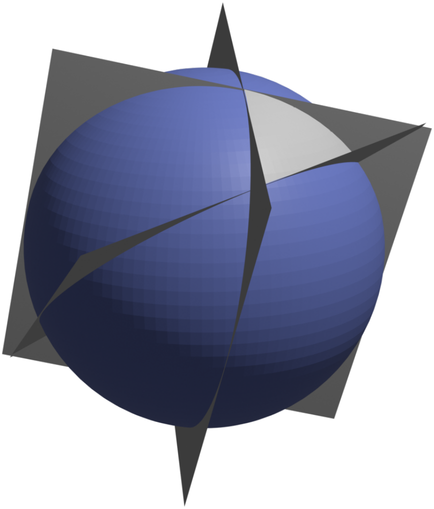


Figure 3.2: Spheric triangle definition: three non-coincident great circles define it.

-	X	24.500	hour
Astr.Const.→ ϵ Upload /	X	24.567	hour
+	X	37.168	hour
24 Angle→ hour U_x -	X	13.168	hour
D:M:S	X	13:10:3.0195	hour

3.3 Spherical Coordinates

Anyone who is working with or understand astronomy is supposed to have minimal notions of spherical trigonometry. Here, KM-AstCalc comes to help you to solve the central problem in this field: equations involving spheric triangles.

3.3.1 Spheric Triangle

Imagine that the sky over you is a huge sphere containing the celestial bodies we can see. A great circle is a line on this sphere defined by the crossing of a plane passing by the sphere's center and its surface. We define a spheric triangle as a triangle drawn by three non-coincident great circles (Fig. 3.2).

We call this sphere the celestial sphere. By convention, we set its radius as unitary, then the distances on this sphere are in angle units. Consequently, the sides of a spheric triangle are done in angle units, too. The three vertices also define three angles. In astronomy, we represent the position of any object of study in terms of positions in the celestial sphere[6]. Eventually, if we know the distance to the object, we adopt the radial distance, making its positioning in terms of polar spheric coordinates.

Usually, we tag the vertices with capital letters, like A, B, and C. The sides get the lowercase of the opposite vertex's letter: side $\underline{a}$ is the opposite of vertex A, and so on.

Solving a spheric triangle problem is to determine its vertices and sides. It is known that if we have three elements of a spheric triangle, we may get the lacking ones. That is to say, if we have two vertices and one side, we may find out the third vertex and the two other sides. If we have one vertex and two sides, we are in measure to find out the other elements. The most frequent problem in the astronomy of position is the reference frame transform, which generally represents solving a spheric triangle problem.

It is possible to generate the following equations that give us the way to solve a spheric triangle. They make three sets of equations whose components can turn in a cyclical pattern since they are of the same type, namely:

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}, \tag{3.2}$$

known as the "sine law".

Similarly, we have three equations of the cosine law:

$$\begin{gathered} \cos a = \cos b \cos c + \sin b \sin c \cos A \\ \vdots \end{gathered} \tag{3.3}$$

We may re-write it by replacing "a" with "b" and, then, with "c" since the vertex changes to be the same capital letter as the lowercase letter on the left side.

A third set is six equations of the type:

$$\begin{gathered} \sin a \cos C = \cos c \sin b - \sin c \cos b \cos A \\ \vdots \end{gathered} \tag{3.4}$$

and so on.

KM-AstCalc can give you the solution for the most common coordinates transforms in astronomy: equatorial system to ecliptic one, local one, galactic one, and vice-versa. We are now going to see what these coordinate systems are.

[6] Even knowing that the Earth is not in the center of the universe.

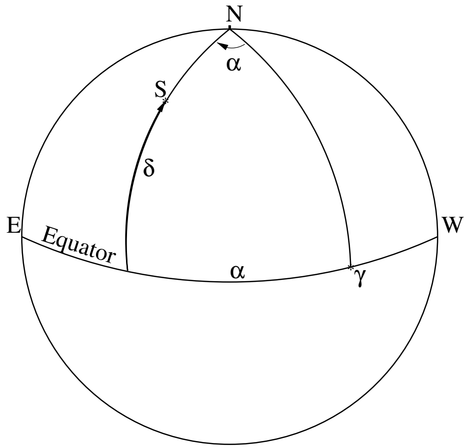


Figure 3.3: Definition of the Equatorial Coordinates

3.4 Equatorial Reference System

Let's look at the Fig. 3.3. We define the Equator, the Great circle, perpendicular to the Earth's rotation axis, that divides the celestial sphere into the northern and southern hemispheres. A celestial body is then positioned with two spherical coordinates: the right ascension and the declination, usually denoted with the Greek letters α and δ, respectively.

The declination is counted from the Equator, measuring positive through the northern hemisphere and negative oppositely. Its absolute values go from $0°$ to $90°$ ($\pi/2\,\mathrm{rad}$) maximum.

The right ascension is measured on the Equator from the spring equinox point, which is usually denoted with the Greek letter γ, increasing to the east. Its value usually is done in hours and varies from 0^{h} to 24^{h}, equivalent to $0 - 360°$ ($2\pi\,\mathrm{rad}$).

Since the point γ is not fixed (see forward in Section 3.7), we have to refer the Equatorial Coordinates to the point of the time the equinox is set. We call this instant Epoch. Modern texts cite the Epoch at J2000.0, which means the Julian day 2451545.0, or the Gregorian Date at noon of the January, first, 2000.

If you are on the Earth's surface, you may be located by your geographical coordinates. They are the longitude and the latitude, usually denoted by the Greek letters λ and ϕ.

The longitude is measured along the equator from the Greenwich (a London suburb) meridian. For political reasons, the longitude increases in both eastward and westward directions, denoted with the letters 'E' or 'W', according to the place east or west of Greenwich.

The latitude is measured from the equator. Again, for political reasons, one denotes the latitude with the letter 'N' or 'S', depending on whether the place is northward or southward from the equator.

In astronomy, we must differentiate between the geocentric and the topographic coordinates. The former ones are taken considering the center of the Earth. The latter ones are taken regarding the geographical position on the Earth's surface. The differences involve the relationship between the sidereal distances and the Earth's radius. They are negligible for stars but somewhat crucial for solar system bodies (the closer, the more critical). The correction for this effect is called parallax.

In astronomy, one traditionally has adopted longitude increasing eastward and latitude northward (following the right ascension and declination standards). Nowadays, we must be careful about what directions we are dealing with.

Exercise

1. Get J2000.0 from the Astronomical Constants combo into register-X and convert it to Gregorian date.

3.5 Ecliptic's Reference System

The coordinates referenced to the ecliptic are celestial longitude (λ) and celestial latitude (β) to mark the differences from the geographic (or geocentric) coordinates. The ecliptic is determined by the mean orbit of the Earth. The equator coordinates and the ecliptic coordinates are such the coordinates in the base's plane have the same origin, the crossing point for the Sun going to the northern hemisphere (equinox point: γ). As a consequence, the (mean) Sun always has null latitude.

The equatorial to the ecliptic coordinates transform is straightforward. Watch Fig. 3.4. The coordinates of the body in S are defined: α, δ as equatorial, and λ, β as ecliptic, respectively. Let 'P' be the North geocentric pole, 'K' be the celestial north pole and 'S' be the position of the body. We have the spheric triangle $\widehat{\mathrm{PKS}}$ with the sides PK having the value ε, the obliquity of the ecliptic, PS, having the value $\frac{\pi}{2} - \delta$ and KS, having the value $\frac{\pi}{2} - \beta$. Drawing the great circle connecting P to γ, we may measure along the equator the right ascension. From the figure, the angle P is $\frac{\pi}{2} + \alpha$. Doing the same for K, along the ecliptic, we have the angle K: $\frac{\pi}{2} - \lambda$.

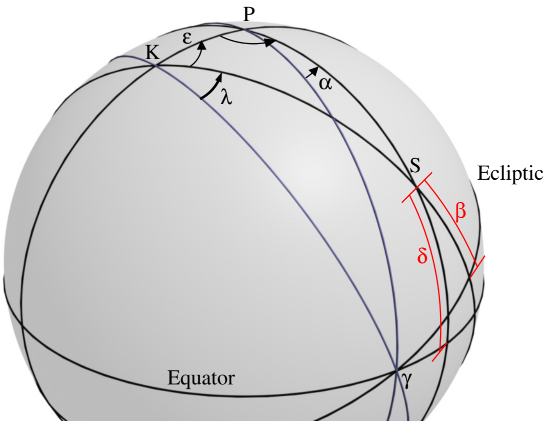


Figure 3.4: Ecliptic / Equatorial transform geometry. Converting Equatorial to Ecliptic or vice-versa is solving the triangle PKS.

We have to solve the triangle problem PKS to transform both, from Equatorial to Ecliptic coordinates and vice-versa. Then, from the geometry of Fig. 3.4 we have:

- Angle in vertex K: $\frac{\pi}{2} - \lambda$;
- Angle in vertex P: $\frac{\pi}{2} + \alpha$;
- Side $\widehat{\mathrm{KS}}$: $\frac{\pi}{2} - \beta$;
- Side $\widehat{\mathrm{PS}}$: $\frac{\pi}{2} - \delta$;
- Side $\widehat{\mathrm{KP}}$: ε.

KM-AstCalc transforms in both directions from equatorial to ecliptic coordinates systems with the buttons EEc and INV EcE . "Base" angles (those that are measured along the baseline, spanning values til 2π) should be in Register-X and "cote" angles, into Register-Y. Ex: to transform from equatorial coordinates, we put the declination into Reg-Y and right ascension into Reg-X, then press on EEc .

Exercise:

1. Convert the star Spica equatorial coordinates $\alpha = 13^{\mathrm{h}}25^{\mathrm{m}}12^{\mathrm{s}}$, $\delta = -11°09'41''$, equinox J2000.0, to ecliptic coordinates λ, β.

Answer:

11 : 9 INV : 41	X	11:9:41	
Angle→ deg U_x	X	11:9:41	deg
INV D.ddd CHS	X	-11.161	deg
1 3 INV :	X	13:	
2 5 INV :	X	13:25:	
12 Angle→ hour U_x	X	13:25:12	hour
INV D.ddd	X	13.42000	hour

We have, then, the data on the registers:

	Y	-11.161	deg
	X	13.420	hour

So, we press on EEc and we have on the pile:

	Y	-2.05628	deg
	X	-156.15721	deg

The value in Reg-X should be done in the range $0° \to 360°$, or $0^h \to 24^h$, so the negative value is out of the quadrant. To fix it, we put it in the quadrant and transform it to hours units:

Quad	X	3.55773	rad
Angle→ hour INV Convert	X	13.58952	hour
D:M:S (λ)	X	13:35:22.26940	hour

To have β, we do[7]

INV D.ddd ↓	X	-2.05628	deg
D:M:S (β)	X	-2:3:22.62136	deg

It is usual to present λ in degrees, too, so you may convert it to.

The reverse calculation can easily be done by clicking on the button INV EcE.

3.6 Local Reference System

The sky constantly moves because of the Earth's rotation and around the sun, so we have to consider the time and the local positioning system when

[7]The first operation INV D.ddd is to avoid undesirable issues on the last value in the format D:M:S.

positioning the sky bodies. There are two local reference frames: the local equatorial and the local horizontal ones.

3.6.1 Local Equatorial Reference Frame

The local equatorial reference frame keeps the declination as a coordinate, but we must convert the right ascension axis to a local reference. Let t_S be the sidereal time for a given instant. We define the local coordinate Hour Angle (H) to be

$$H = t_S - \alpha. \tag{3.5}$$

H increases eastward and is zero when the object is on the local meridian. The hour angle is usually done in hours. But if you need it in degrees you multiply it by the factor 15, or, in KM-AstCalc, convert hour (hour) angle to degree.

If you have a telescope with equatorial mounting, you point it to the object with coordinates hour angle (h) and declination δ.

Exercise

The star Spica has $\alpha = 13^{\mathrm{h}}25^{\mathrm{m}}11^{\mathrm{s}}$ for a given epoch. For a sidereal time $t_S = 10^{\mathrm{h}}35^{\mathrm{m}}31^{\mathrm{s}}$, determine its hour angle H in hours and degrees.

13 : 25 : 11 *	X	13:	13:25:11
Angle→ hour U_x	X	13:25:11	hour
INV D.ddd	X	13.41972	hour
10 : 35 : 31	X	10:35:31	
Angle→ hour U_x	X	10:35:31	hour
INV D.ddd	X	10.58917	hour
x-y -	X	-2.82778	hour
D:M:S	X	-2:49:40.000	hour

* You may use the keyboard. See Appendix.

The object is eastward.

3.6.2 Local Horizontal Reference Frame

Some telescopes are mounted in the horizontal system frame, described here.

Zenith is the point defined by the local vertical crossing upward of the celestial sphere. Nadir is the opposite downward. The local meridian is the great circle containing the zenith and the geographical poles. It divides the local sky into two hemispheres.

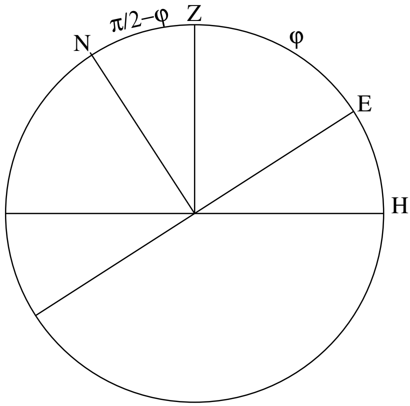


Figure 3.5: Local latitude. The meridian is drawn on the paper plane.

The Fig. 3.5 shows the projection of the local meridian on the plane of the paper. The horizon (H) is the great circle determined by the plane perpendicular to the zenith (Z). The angular distance between the equator (E) and the zenith is the local latitude: φ.

The distance from the zenith is called the zenithal distance (z). Conversely, the distance from the horizon is called altitude (h). Obviously, $z + h = \pi/2\,\mathrm{rad}$.

Looking at the north, we count angles in the horizontal plane as azimuth (A). In the northern hemisphere, the origin is the north pole. The opposite happens in the southern hemisphere.

Imagine the local meridian defined by the paper plane. On the celestial sphere, a body is placed at point S in the eastern hemisphere. In Fig. 3.6, we see the spheric triangle defined by the vertices Z (zenith), N (north pole), and S. The vertices and sides defined are shown in the picture. There is the paralactic angle S which usually has no practical use. The distance from S to the north pole is just the complement of the declination, while its distance to Z, as we have seen, is the zenithal distance.

We may want to transform the local equatorial coordinates to the horizontal ones and vice-versa. To transform from equatorial coordinates, it is supposed that we have H, δ and the local latitude φ. Use the button EqH to do it. You should add the value of φ onto Reg-Z.

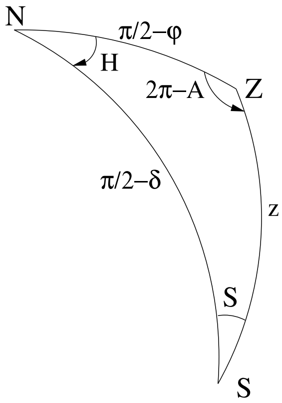


Figure 3.6: Frame transform between equatorial and horizontal reference systems. The local meridian is paralel to the plane of the paper. The object is in the eastern hemisphere.

Exercise

1. Let us be at the Kitt Peak Observatory ($\varphi = 31:58:48\,\text{deg}$), what are the horizontal coordinates of the star Spica ($\delta = -11:09:41\,\text{deg}$), for $t_S = 10:35:31\text{hour}$?

Answer: Let's use the result of the last exercise. We have, in X-register the value of $H = -2^{\rm h}\!.82778$. So,

31 : 58 : 48	**X**	31:58:48	
Angle→ deg U_x	**X**	31:58:48	deg
INV D.ddd	**X**	31.98000	deg
x-y 1 1 CHS† :	**X**	-11:	
09 : 41	**X**	-11:09:41	
Angle→ deg U_x	**X**	-11:09:41	deg
INV D.ddd	**X**	-11.161	deg
x-y	**X**	-2.82778	hour

† You may change the signal, but you must to do it before enter the ':'

Now, we click on the button EqH. The result will be:

	(ϕ)**Z**	31.98000	deg
	(z)**Y**	59.21283	deg
	(A)**X**	129.61768	rad

3.6.3 The Galactic Reference Frame

It may be helpful to put the body of interest into the galactic system of reference, especially for the extragalactic observers interested in knowing if there are galactic clouds that may absorb the radiation. Given the equatorial coordinates in the usual way, we have the galactic coordinates b (g-longitude) and l (g-latitude) by clicking on the button EGa.

Exercise

1. Find out the equatorial coordinates of the center of the Milky Way, the radio source Sagittarius A, the central black hole.

Answer:

0 Angle→ deg $\cancel{U_x}$	**X**	0	deg
Enter Enter INV GaE	**X**	265.61084	deg

The result of the object in the center of the Milky Way $(0, 0)$, when we apply the transform Galactic-to-Equatorial is its equatorial coordinates.

D:M:S	**X**	265:36:39.03851	deg
INV D.ddd	**X**	265.61084	deg
Angle→ hour INV Convert	**X**	17.70739	hour
D:M:S	**X**	17:42:26.60257	hour

and the coordinates on the pile are:

(δ)	**Y**	-28.91679	deg
(α)	**X**	17:42:26.60257	hour

It should be noticed that these coordinates refer to the epoch B1950.0.

3.7 Movements of the Earth

We know that our planet Earth is not fixed in space. On the contrary, it rotates and translates around the Sun (and goes within the overall trajectory in the galaxy). In addition to the orbital path, this one is affected by the

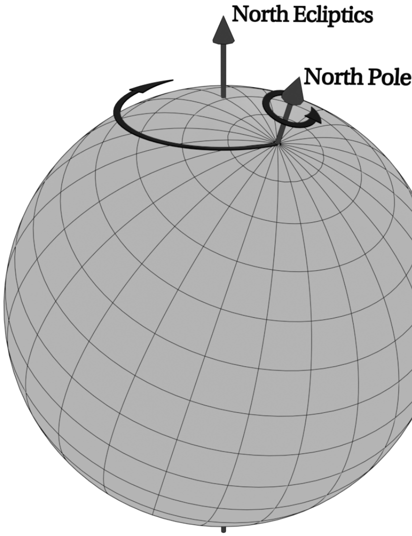


Figure 3.7: Precession: the Earth rotates around the axis of the equator, while this one recedes around the axis of the ecliptics.

gravity of other planets in the solar system, mainly the bigger ones: Jupiter, Saturn, and so on, given into variations of the orbital eccentricity, the length of the year, the orbital inclination, and others.

Its rotation is more complex, too. Together with the pure rotation comes the precession and the nutation motions.

3.7.1 Precession and Nutation

One of the noticeable motions of the Earth is the precession of the equinoxes (Fig. 3.7). It is the motion of the Earth's rotation axis around the ecliptic axis, the axis perpendicular to the mean Earth's orbit. While the Earth rotates eastward, its axis recedes around the north of the ecliptic axis. Consequently, the equinoxes recedes around 1 degree every 72 years, performing an entire cycle in 25,772 years. The Greeks already observed it long ago. From one year to the next, you may lose your celestial object from the ocular field, so it is better to correct its coordinates for this effect.

The Earth's rotation axis makes the northern ecliptics an angle of $23\overset{\circ}{.}433$ In KM-AstCalc, this value is denoted by its usual symbol: ε, and may be found among the Astronomical Constants.

The precession results mainly from the gravitational pull from the Sun and, in smaller proportion, from the Moon, provoking a short-period movement called nutation.

The precession makes the equinox walk through the ecliptic at a $50''\!.288$ yr^{-1} rate (ρ in the Astronomical Constants of KM-AstCalc). Consequently, the right ascension and declination of the sky bodies vary all the time. The adopted values for these coordinates are referred to some dates used as standards. Ultimately we say the equinox for J2000.0, which means January noon of first, 2000. The Julian Day for this date is tagged as J2000.0 in the Astronomical Constants of KM-AstCalc.

To place the coordinates to the desired date (we say to precess to the date), we use a simple formula (in a reasonable approximation) derived from the trigonometric analysis of the movement of the equinox by the ecliptic. It is

$$\begin{array}{rcl} \Delta\alpha & = & (m + n\sin\alpha\tan\delta)\,\Delta T \\ \Delta\delta & = & n\cos\alpha\Delta T \end{array} , \tag{3.6}$$

where ΔT is the time spent in (Julian) years, and

$$\begin{array}{rcl} m & = & \rho\cos\varepsilon \\ n & = & \rho\sin\varepsilon \end{array} .$$

We know, already, that ρ is the variation of the equinox in celestial longitude and ε is the ecliptic obliquity. There is a tiny variation of these numbers over time, but it isn't interesting to those who are not experts in this specific area.

Because the precession motion, the basic unit of time for orbits computation should is not the Julian year (365.25dy), but $20^{\mathrm{m}}24^{\mathrm{h}}\!.5$ longer, i.e. 365.256363004 dy.

Exercise:

1. Calculate the values of m and n in hsec.yr^{-1} from the precession in longitude constant ρ and the ecliptic obliquity ε. Calculate n in arcsec.yr^{-1}, too.

Answer:

Astr.Const.→ ρ Upload	X	0.02438	rad.Cen−1
Astr.Const.→ ε Upload	X	0.40899	rad
COS *	X	0.022369	rad.Cen^{-1}
Angle→ hsec	X	0.022369	rad.Cen^{-1}
Time→ yr INV Convert	X	3.0760	hsec.yr^{-1}
Astr.Const.→ ρ Upload	X	0.024380	rad.Cen^{-1}
Astr.Const.→ ε Upload	X	0.40899	rad
SIN *	X	0.0096956	rad.Cen^{-1}
Angle→ hsec Time→ yr	X	0.0096956	rad.Cen^{-1}
INV Convert	X	1.3332	hsec.yr^{-1}
Angle→ sec Time→ yr	X	1.3332	hsec.yr^{-1}
INV Convert	X	19.985	sec.yr^{-1}

The actual values for m and n are $3^{h}.075$ and $1^{h}.336$, or $20''.043$. These ones are taken considering other influences over the precession.

3.7.2 Aberration

Aberration is the name we give to the variation in the position of the celestial bodies as a consequence of their relative longitudinal speed. There are two types of aberration: stellar and planetary aberrations.

The stellar aberration has three components:

Annual: from the Earth's orbital movement around the Sun;

Diurnal: from the Earth's rotation;

Secular: from the entire solar system's movement.

The planetary aberration affects the solar system bodies' movement and has two terms, too:

Aberration of light: the usual one from relative speed, and

Light-time displacement: from the time the light travels to the virtual observer's geocentric position.

The annual aberration has well-known formulas to correct the position in celestial coordinates:

$$\begin{aligned} \Delta\lambda &= \kappa \frac{-\cos(\odot - \lambda) + e\cos(\varpi - \lambda)}{\cos\beta} \\ \Delta\beta &= -\kappa \sin\beta\,(\sin(\odot - \lambda) - e\sin(\varpi - \lambda)) \end{aligned} , \qquad (3.7)$$

where κ is the constant of aberration, e the Earth's orbital eccentricity, $\odot$ the Sun's longitude and ϖ is the longitude of the perihelion of Earth's orbit. κ and e are given in KM-AstCalc, but don't confuse the latter with "Earth ellipticity", which is the index of the flattening of our planet. The Earth's orbital eccentricity is given in the Section of orbital data of the planets, that we will see later. The value of κ is of the order of $2''$.

Exercise:

1. Calculate the terms of the annual aberration for the star Spica, at May, 10 2023, given $\varpi = 102^\circ\!.94719$ and $e = 0.0167102$. At this date $\odot = 49^\circ 35' 51''$.

Answer: From former exercise, we know, for Spica: $\lambda = 13^h 35^m 21^h\!.6$, $\beta = -2^\circ 3' 21''\!.24$, so

49:35:51	X	49:35:51	
Angle→ deg $\cancel{U_x}$	X	49:35:51	deg
INV D.ddd	X	49.597	deg
13:35:21.6	X	13:35:21.6	
Angle→ hour $\cancel{U_x}$	X	13:35:21.6	hour
INV D.ddd	X	13.589	hour
Angle→ deg	X	13.589	hour
INV Convert Sto 0	X	203.83	deg
- Sto 1	X	-154.24	deg
COS	X	-0.90062	
102.94719 Angle→ deg $\cancel{U_x}$	X	102.94719	deg
Rcl 0 - Sto 0	X	-100.88	deg
COS CHS	X	0.18875	
x-y 0.0167102	X	0.0167102	
Sto 3 *	X	-0.015050	

+		X	0.17370	
-2:3:21.24 Angle→ deg U_x		X	-2:3:21.24	deg
INV D.ddd Sto 2		X	-2.0559	deg
COS /		X	0.17386	
Astr.Const.→ κ Upload		X	0.0000099365	rad
*		X	0.0000017276	rad
Angle→ sec		X	0.0000017276	rad
INV Convert	$(\Delta\lambda)$	X	0.35634	sec
Rcl 0 SIN		X	-0.98202	
Rcl 1 SIN		X	-0.43460	
Rcl 3 *		X	-0.0072622	
-		X	-0.97476	
Rcl 2 SIN		X	-0.035875	
* Astr.Const.→ κ Upload		X	0.0000099365	rad
* CHS		X	-3.4747e-7	rad
Angle→ sec		X	-3.4747e-7	rad
INV Convert	$(\Delta\beta)$	X	-0.071671	sec

These values should be added to the original λ and β, and, then, be converted to equatorial coordinates. Because the denominator $\cos\beta$ in $\Delta\lambda$ correction, it could be significant for high values of latitude.

Chapter 4

Solar System Bodies

4.1 Introduction

KM-AstCalc offers you ways to determine positions of solar system bodies following the Kepler laws for two-body problems, taking the Sun as the central main body. Thus, for a given instant, you can determine positions of planets, asteroids and comets. KM-AstCalc has the necessary data to find out the positions of all the planets and hundreds of asteroids and comets.

Generically, one has to have the orbital parameters defined according to the Kepler's laws and the celestial transforming equations. Look at Fig. 4.1. We treat the data as generically elliptical orbits being referenced to the equinox (point γ) for a standard date. Each class of body will have its own characteristics that we will see when discussing each one. We have:

a: semi-major axis;

e: eccentricity of the orbit $b = a\sqrt{1-e^2}$, with b being the semi-minor axis;

i: inclination of the orbital plane to the ecliptics;

ω: argument of perihelion, the angle coming from the ecliptics crossing line to the direction of the perihelion in the orbital plane;

Ω: longitude of the ascending node, the angle in the ecliptics coming from the equinox direction (γ) to the crossing line with the orbital plane;

t_0: an epoch of reference, ex. time of passage through the perihelion;

γ: direction of the equinox, taken as the reference for the ecliptic (and equatorial) coordinates.

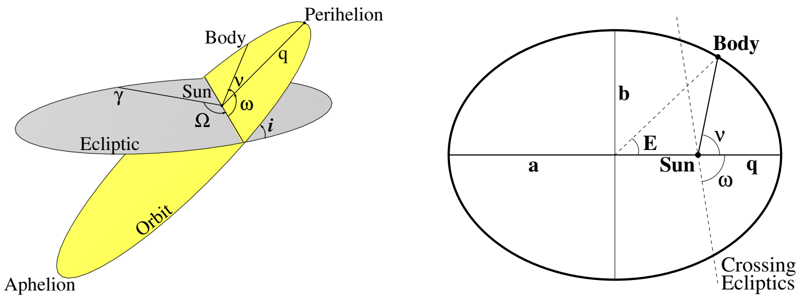


Figure 4.1: Graphical illustration for the orbital parameters for the two-body Kepler problem. Left: a 3-D vision of the orbit crossing the ecliptics, the reference plane, with the Sun in the center; right: the schematic illustration of the Kepler's problem.

A solar system body has a complicate movement in the long term, but it can be described with simple equations if we consider small periods of time. It is the so-called Kepler equations for two-body problems. The Kepler equation for an elliptical orbit is, given an instant t:

$$E - e \sin E = M, \tag{4.1}$$

where e is the eccentricity, M is the mean anomaly. $M = n(t - t_0)$, where n is called mean motion and t_0 is the reference time. E is the eccentric anomaly. In KM-AstCalc, this equation is solved by pressing the button Kepler that you find just at the left of the panel of astronomical functions, below the panel of mathematical functions. The rule is to put the value of the eccentricity in the register 'Y' and the mean anomaly in 'X'. You have to obey the right unit for KM-AstCalc to perform the calculation correctly. Thus, while the eccentricity is unit-less, M should be in units of angle (E is given in angle units).

Exercise:

1. Solve the Kepler equation for $e = 0.5$ and $M = 30^o$.

   Answer:

0.5 Enter	**X**	0.5	
30 deg U_x	**X**	30	deg
Kepler	**X**	0.92201	rad

You may convert the result to degrees. Notice that the register Y maintains the eccentricity value.

Following the script, we should obtain the vector radius and the true anomaly ν to know a body's position in the sky. If we know the eccentric anomaly, then

$$r = a(1 - e\cos E)$$
$$\tan\frac{\nu}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2} \quad . \tag{4.2}$$

Knowing that ν is in the same half circle as E. If $E < \pi$, ν will be, too. The same for $E > \pi$, so applying the formula to find ν in Eq. (4.2), we use this property to eliminate the ambiguity in applying the solution. Usually, we need a value for $\cos\nu$ and the value of $\sin\nu$ to solve this ambiguity.

Exercise:

1. Use the result of last exercise to find the true anomaly ν.

   Answer:

   In the last exercise, the issue of Kepler button operator is the value of E in register X, and e in the register Y, so

Sto 0 2 / TAN	X	0.49670	
x-y Sto 1	X	0.5	
1 +	X	1.5000	
1 Rcl 1 -	X	0.50000	
/ INV SQRT	X	1.7321	
* INV ATAN	X	0.71045	rad
2 * Sto 2	X	1.4209	rad

The true anomaly ν obtained from the above operations is the true angle positioning the body in its orbit.

2. Now, let's find out the radius vector modulus lying the body to the Sun as the center of the reference frame. Suppose the semimajor axis being $a = 3$AU.

Answer: From Eq. (4.2):

Rcl 0 COS	X	0.60422	
Rcl 1 *	X	0.30211	
CHS 1 +	X	0.69789	
3 AU U_x	X	3.0000	AU
* Sto 3	X	2.0937	AU

3. With these data (ν and r), you may find the Cartesian coordinates in the orbital plane centered in the Sun:

$$x_0 = r \cos \nu$$

$$y_0 = r \sin \nu$$

Rcl 2 COS	X	0.14934	
Rcl 3 *	X	0.31266	AU
Rcl 2 SIN	X	0.98879	
Rcl 3 *	X	2.0702	AU

4.1.1 Finding the Mean Motion n for a Body in the Solar System

The mean motion n is obtained from the relation

$$n = \frac{2\pi}{T},$$

where T is the period of the body's motion. The period is related to semi-major axis by the Kepler's third law:

$$\frac{a^3}{T^2} = \frac{\mathrm{G}(\mathrm{M}_\odot + \mathrm{m_b})}{(2\pi)^2} \tag{4.3}$$

for all the bodies in the solar system. If we adopt units in terms of AU for space, $S + m_{♁}$ (solar mass + Earth's mass) for mass and Syr[1] for time, this constant is unitary. Thus,

$$T = \frac{2\pi}{k}\sqrt{\frac{a^3}{M_\odot(1+\mu_b)}},$$

[1]Sidereal year: Earth's orbital period as seen from stars' referential frame. Beyond the 365.25 dy, we must consider the precession movement.

where μ_b is the reduced mass of the considered body ($\mu_b = m_b/M_\odot$).

From Eq. (4.3) we have:

$$n = k.\sqrt{\frac{M_\odot + m_b}{a^3}}, \tag{4.4}$$

where k is the Gauss gravitational constant.

Exercise

1. Find the mean motion for a body of neglectable mass that the semi-major axis is:

$$a = 3\text{AU}.$$

Answer:

Astr.Const.→ [S] [Upload]	X	1.98855e+30	k,g
[3] Space→ [AU] [U_x] [Enter]	X	3.0000	AU
[3] [y^x]	X	27.000	AU^3
[/] [INV] [SQRT]	X	2.7139e+14	$\text{AU}^{-1.5}.\text{kg}^{0.5}$
[k] [*] [Coalesce]	X	3.8316e-8	$\text{s}^{-1}.\text{rad}$
Time→ [dy] [INV] [Convert]	X	0.0033105	rad.dy^{-1}

Thus, the mean motion is $n = 0.0033105\,\text{rad.dy}^{-1}$. If you are curious about the absolute value of the period, simply divide this number by 2π rad, invert the result and convert it to years.

4.2 Heliocentric and Geocentric Positions

When calculating the orbit of solar system bodies, we finish by finding the coordinates for a reference system centered on the Sun. But we are interested in knowing the position centered on the Earth. As a matter of fact, we want them centered on our observation location. We say, then, that we calculate the positions in heliocentric coordinates and want to convert them to geocentric and, then, to topocentric ones. Undoubtedly, the conversion to geocentric from heliocentric coordinates is the most important. Watch the Fig 4.2.

If we have the Sun's reference frame and the Earth's, we must consider the radius vectors $\vec{R}_{SB}$, $\vec{R}_{SE}$, and $\vec{R}_B$ to transform the body's coordinates from the former to the latter. From the figure:

$$\vec{R}_B = \vec{R}_{SB} - \vec{R}_{SE}. \tag{4.5}$$

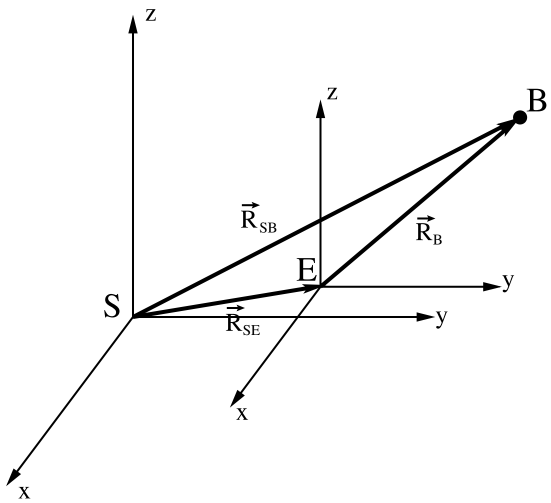


Figure 4.2: Heliocentric to Geocentric coordinates conversion.

It means that, for a given instant, we have to know the body's and the Earth's relative position to the Sun. If the body's motion follows the universal gravitational laws, we must solve their coordinates equations along with the Earth's.

Considering the mean ecliptic we may set that the z component of the Earth is null:

$$\begin{array}{rcl} x_B &=& R_{SB}\cos\theta_B\cos b_B - R_{SE}\cos\theta_E \\ y_B &=& R_{SB}\sin\theta_B\cos b_B - R_{SE}\sin\theta_E \\ z_B &=& R_{SB}\sin b_B \end{array} . \tag{4.6}$$

Once we know $\vec{R}_B = (x_B, y_B, z_B)$, we deduce the ecliptic coordinates:

$$\begin{array}{rcl} R_B &=& \sqrt{x_B^2 + y_B^2 + z_B^2} \\ \lambda_B &=& \arctan\dfrac{y_B}{x_B} \\ \beta_B &=& \arcsin\dfrac{z_B}{R_B} = \arcsin\left(\dfrac{R_{SB}}{R_B}\sin b_B\right) \end{array} . \tag{4.7}$$

If the body is on the ecliptic, the equations simplify. See Fig 4.3. From the cosine law for triangles:

$$R_B^2 = R_{SB}^2 + R_{SE}^2 - 2R_{SB}R_{SE}\cos(\theta_{SB} - \theta_{SE}). \tag{4.8}$$

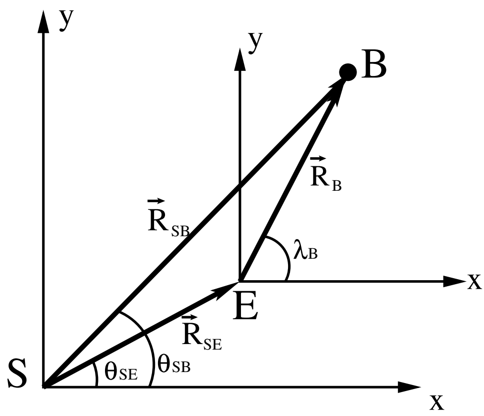


Figure 4.3: Body on the ecliptic.

Comparing with the modulus of the polar form of Eq. (4.5):

$$\tan \lambda_B = \left(\frac{R_{SB} \sin \theta_{SB} - R_{SE} \sin \theta_{SE}}{R_{SB} \cos \theta_{SB} - R_{SE} \cos \theta_{SE}} \right) \quad ; \quad \beta_B = 0 \ . \tag{4.9}$$

The angle λ_B will be the ecliptic Earth's longitude. The angle β_B is the Earth's latitude.

4.2.1 Elongation - Phase Angle

It is helpful to know the elongation, the angle the body does with the sun from the earth-observer point of view. It determines the body's brightness since the elongation defines the illuminated region of the body. The body's apparent magnitude is given by:

$$m = H + 5.0 \log \Delta_S + 5.0 \log R_E + \Theta(\vartheta), \tag{4.10}$$

where $\Delta_S = \left|\vec{R}_{SB}\right|$ and $R_E = \left|\vec{R}_B\right|$ are the distance to the Sun and to the Earth, respectively, ϑ is the elongation, $\Theta()$ is an albedo-dependent function, and H is the absolute magnitude ($\Delta_S = R_E = 1$AU, and $\vartheta = 180°$-Sun at opposition). The function $\Theta()$ is known as phase function and usually is put in empirical terms of a power function summation in ϑ[2].

[2] Authors usually name the phase angle with α, which I avoid here so as not to mix it up with right ascension.

The phase angle may be determined by the simple relation:

$$\cos\vartheta = \frac{\vec{\mathrm{R}}_B \cdot \vec{\mathrm{R}}_{ES}}{R_B R_{ES}}.$$

Considering that the Sun and the Earth share the same plane, which is the ecliptic, a simple calculation gives us:

$$\cos\vartheta = \cos(\lambda_B - \lambda_S)\cos\beta_B. \tag{4.11}$$

The elongation is the "algebraic" version of the phase angle, being positive if it goes 'beyond' the sun on the sky. It suffices to consider the difference $\lambda_B - \lambda_S$.

4.3 Planets, Asteroids and Comets

KM-AstCalc gives us the orbital data for the planets, asteroids, and comets. Their data are distributed through three dropdown combos. The left panel is dedicated to the solar system's planets, and the right panel is to the asteroids and comets.

In the left panel, the planet's area, there are further data, such as mass, diameter, density, etc. They have been obtained from methods other than the orbital (Kepler's laws) properties. These data are marked differently from the orbital ones. The former are in boldface and the latter in italics.

To upload data from these panels to the pile's register X, choose a body (planet, asteroid, or comet) from the concerned dropdown combo and click on the letter relative to the parameter you want to upload. Hovering through the letters, you get a description of the parameters and their units. For example, to upload Jupiter's mass, you choose Jupiter in the select combo of planets, then click with the mouse on the letter **M** , just at the right. The mass of Jupiter will appear in register X of the pile.

Exercise:

1. Compare the mass of Jupiter with the Earth's.

   Answer:

Jupiter $\mapsto$ **M**	**X**	1.8980e+27	kg
Earth $\mapsto$ **M**	**X**	5.9700e+24	kg
/	**X**	317.83	

Conclusion: Jupiter is 317.83 more massive than Earth.

4.4 Explanation of the Planetary Data

Data were referred to the equinox of J2000.0. Below you see the orbital parameters for the planets obtained from `https://nssdc.gsfc.nasa.gov/planetary/factsheet/`.

4.4.1 Planets

$\boxed{J_0}$ **:** Julian date when mean longitude was taken;

$\boxed{a}$ **:** Semi-major axis;

$\boxed{e}$ **:** Eccentricity of the orbit;

$\boxed{i}$ **:** Inclination of the orbital plane to the ecliptic;

$\boxed{\Omega}$ **:** Longitude of the ascending node;

$\boxed{\varpi}$ **:** Longitude of the perihelion, usually $\varpi = \Omega + \omega$, where ω is the argument of the perihelion;

$\boxed{L}$ **:** Mean longitude, usually $L = \Omega + \omega + M_0$, where M_0 is the initial mean anomaly.

We have further data for the planets from[3] `https://nssdc.gsfc.nasa.gov/planetary/factsheet`:

$\boxed{\mathbf{M}}$ **:** Mass in kg;

$\boxed{\mathbf{\Phi}}$ **:** Diameter in km;

$\boxed{\rho}$ **:** Density in kg/cm^3;

$\boxed{\mathbf{g}}$ **:** Gravity in m/s^2;

$\boxed{\mathbf{v_E}}$ **:** Escape velocity in km/s;

$\boxed{\mathbf{R}}$ **:** Rotation period in hr;

$\boxed{\mathbf{D}}$ **:** Length of the day hr;

$\boxed{\mathbf{\Delta_S}}$ **:** Distance from the Sun in km;

[3] Not too accurate. These data are just for illustration.

$\boxed{\boldsymbol{q}}$: Perihelion in km;

$\boxed{\boldsymbol{Q}}$: Aphelion in km;

$\boxed{\boldsymbol{T_0}}$: Orbital period in dy;

$\boxed{\epsilon}$: Obliquity of the orbit in degrees;

$\boxed{\boldsymbol{\Theta}}$: Mean temperature in C;

$\boxed{\boldsymbol{N_M}}$: Number of Moons;

$\boxed{\boldsymbol{H}}$: Visual absolute magnitude.

Some quantities, like gravity ($\mathbf{g}$), escape velocity ($\mathbf{v_E}$), distance, and others, may be derived. The gravity is done by

$$\mathrm{g} = 4G\frac{\mathbf{M}}{\mathbf{\Phi}^2},$$

while the escape velocity is

$$\mathbf{v_E} = \sqrt{\frac{4G\mathrm{M}}{\mathbf{\Phi}}}.$$

Exercises:

1. Calculate the gravity and escape velocity for the Earth from the expression above and compare them with the given values in the planet data panel.

Astr.Const.→ G Upload	X	6.6743e-11	$m^3.kg^{-1}.s^{-2}$
4 *	X	2.6697e-10	$m^3.kg^{-1}.s^{-2}$
Earth→ M * Enter	X	1.5944e+15	$m^3.s^{-2}$
$\mathbf{\Phi}$ SQR / Coalesce	X	10.081	$m.s^{-1}$
g	X	9.8000	$m.s^{-1}$
INV D%	X	-2.7897	
x-y $\mathbf{\Phi}$ / Coalesce	X	1.2678e+8	$m^2.s^{-2}$
INV SQRT	X	11260	$m.s^{-1}$
$\mathbf{v_E}$ Coalesce	X	11200	$s^{-1}.m$
INV D%	X	-0.53051	

There is a difference of $\sim 3\%$ between the calculated and the given values of the gravity of the Earth. The calculated value is obtained, assuming the Earth is a perfect homogenous sphere. The given value is the adopted one and is the result of the average on the measured values from the gravimetry. For the escape velocity ($\sim 0.5\%$), the difference is likely due to measurement fluctuations.

4.4.2 Longitude of the Perihelion

Exceptionally the planets have the value of "Longitude of the Perihelion" (ϖ), which would be the coordinate of the perihelion on the ecliptic instead of its position in the planet's orbit. The reason is that the planets' orbit inclination from the ecliptic is too small. In practice, it is as if the orbit were parallel to the ecliptic. Let's see: the greatest inclination over the planets is Mercury's: 7°. This value will bring a difference in the calculations of about 0.75%. For this reason, we adopt the sum of the Longitude of the Ascending Node (Ω) and the Argument of the Perihelion (ω) and set it to the Longitude of the Perihelion only for the planets, and perform 2^{nd} order corrections.

At the same time, the value $\boxed{J_0}$ is taken as an "osculating date", meaning that the data for each planet is fixed at this date, including the 'mean longitude' $\boxed{L}$, such that, to have the mean motion M for a date J, we do:

$$M = n(J - J_0) + M_0, \tag{4.12}$$

where M_0, the initial mean anomaly is

$$M_0 = L - \varpi.$$

4.4.3 Earth's Heliocentric Coordinates

It will be helpful to know the Earth's Heliocentric coordinates, so let's calculate it for a given date. Let's take the date May, 15 2023 at 21^{h} (GMT). Let's first obtain the Julian Date:

15:5:2023	**X**	15:5:2023	
Time→ dy U_x	**X**	15:5:2023	dy
Time→ mth U_x	**X**	15:5:2023	dy.mth
Time→ yr U_x	**X**	15:5:2023	dy.mth.yr
JulD	**X**	2.4601e+6	dy

To see this value in the "usual" format, we put it into the mode "Fix":

	X	2460079.50000	dy

Because it is at 21 hours:

21	X	21	
Time→ hr $\cancel{U_x}$ Enter	X	21	hr
Time→ dy INV Convert	X	0.87500	dy
+	X	2460080.37500	dy
Sto 0	X	2460080.37500	dy

Let's save the Julian day for the date in registry R_0. Now, let's get the time since the osculating date for the Earth.

Earth → J_0 -	X	8535.37500	dy

According to the Gauss gravitational theory, the mean motion of a body in orbit of the Sun would be (other systems would have $M_\odot$ replaced by the central body mass):

$$n = k.\sqrt{\frac{M_\odot + m_b}{a^3}}.$$

Astr.Const.→ k Upload	X	0.00001	$m^{1.5}.kg^{-0.5}.sc^{-1}.rad$
* Coalesce	X	190.51269	$m^{1.5}.rad.g^{-0.5}$
Astr.Const.→ S Upload	X	1.98855e+30	kg
Earth → M +	X	1.9886e+30	kg
INV SQRT * Coalesce	X	8.4956e+18	$m^{1.5}.rad$
Earth → a 1.5 y^x /	X	8.4956e+18	$AU^{-1.5}.m^{1.5}.rad$
Coalesce	X	146.82656	rad
L ϖ -	X	-2.48284	deg
Coalesce +	X	146.78323	rad
Quad	X	2.26997	rad

Let's upload the orbital eccentricity and swap the terms to put in the position to solve the Kepler's equation[4]:

Earth → e x-y Kepler	X	2.28262	rad

This is the value of the eccentric anomaly (E). Let's save it in the memory R_1 and calculate the true anomaly and the radius vector modulus from Eq. (4.2), then calculate the polar coordinates of the Earth in the ecliptic plane. First, we calculate the modulus of the radius vector, using Eq. (4.4.3), and storing it in memory R_2:

[4] Assuming that the planet Earth is set on the dropdown combo.

Sto 1 COS *	X	-0.01092	
CHS 1 +	X	1.01092	
Earth→ a * Sto 2	X	1.01092	AU

Then, we evaluate the true anomaly:

Rcl 1 2 /	X	1.14131	rad
TAN	X	2.18340	
1 e +	X	1.01671	
1 e -	X	0.98329	
/ INV SQRT	X	1.01685	
* INV ATAN	X	1.14760	rad
2 *	X	2.29520	rad

And, sum it to ϖ to find the heliocentric longitude of the Earth. We put it in arc-degrees for unit compatibility of the terms. We store it in R_1 replacing the eccentric anomaly (supposing we are not using it anymore):

Angle→ deg INV Convert	X	131.50544	deg
ϖ + Sto 1	X	234.45263	deg

This is in good approximation to the Earth's coordinates in the ecliptic, with the Sun centered in the reference frame.

Until now, the position of our memory base is:

$$\begin{array}{llll} \mathrm{R}_0 & \leftarrow & J & \text{Julian day} \\ \mathrm{R}_1 & \leftarrow & \theta_{\text{♁}} & \text{Earth's heliocentric longitude} \\ \mathrm{R}_2 & \leftarrow & R_{\text{♁}} & \text{Earth's radius vector to the Sun} \end{array}$$

The registry R_1 was used as a buffer for the Earth's eccentric anomaly.

Obviously, for the Sun position in the geocentric reference, we sum 180° to the longitude. Eventually we put the result in the quadrant:

180 Angle→ deg U_x +	X	414.45263	deg
360 Angle→ deg U_x -	X	54.45263	deg

The longitude is referred to equinox J2000.0. The apparent value is yet to be obtained after corrections of aberration, parallax, precession and nutation.

Exercise:

1. Calculate the equatorial coordinates of the Sun at 21^{h} (GMT) of the May, 15 2023 for

(a) Equinox J2000.0

(b) Equinox of the date

Answer:

(a) Having the ecliptic longitude of the sun, we know that its latitude is null, so

0 deg U_x	X	0	deg
x-y INV EcE	X	52.089528	deg
Angle→ h INV Convert	X	3.47264	hour
D:M:S $(\alpha_\odot)$	X	3:28:21.48665	hour
INV D.ddd x-y	X	18.87890	deg
D:M:S $(\delta_\odot)$	X	18:52:44.06898	deg

(b) To consider the precession, it is convenient to apply it to the ecliptic coordinates, so

Rcl 0 *J0* -	X	8535.3750	dy
Earth→ ρ Upload	X	0.024380292	rad.Cen^{-1}
* Coalesce	X	0.0056972296	rad
Angle→ deg INV Convert	X	0.32642721	deg
Rcl 1	X	234.45263	deg
180 Angle→ deg U_x	X	180	deg
-	X	54.452626	deg
+	X	54.77905	deg
0 Angle→ deg U_x	X	0	deg
x-y INV EcE	X	52.424889	deg
Angle→ hour INV Convert	X	3.49499	hour
D:M:S $(\alpha_\odot)$	X	3:29:41.973	hour
INV D.ddd x-y	X	18.958530	deg
D:M:S $(\delta_\odot)$	X	18:57:30.706	deg

4.4.4 Planet's Heliocentric and Geocentric Coordinates

Let's regard that, doing the example of the Sec. 4.4.3 above, we have recorded in the memory: $\mathbf{R_0} \leftarrow \mathbf{J}$, the Julian day for the date 2023, May 15 at 21$^{\mathrm{h}}$GMT; $\mathbf{R_1} \leftarrow \theta_{\text{♁}}$, the heliocentric longitude for the Earth in the date; and $\mathbf{R_2} \leftarrow R_{\text{♁}}$, the Earth's radius vector to the Sun. Now, let's do the same calculation for Jupiter.

First of all, let's get the orbital period from the osculating data for Jupiter, using data already raised by KM-AstCalc[5].

Astr.Const.→ [S] [Upload]	X	1.98855e+30	kg
[Jupiter] ↦ [M] [+]	X	1.99044813e+30	kg
[Jupiter] ↦ [a] [3] [y^x] [/]	X	1.4129e+28	$\mathrm{AU}^{-3}.\mathrm{kg}$
[INV] [SQRT]	X	1.1886e+14	$\mathrm{AU}^{-1.5}.\mathrm{kg}^{0.5}$
Astr.Const.→ [k] [Upload]	X	0.0000081694	$\mathrm{m}^{1.5}.\mathrm{kg}^{-0.5}.\mathrm{sc}^{-1}.\mathrm{rad}$
[*] [Coalesce]	X	1.6782e-8	$\mathrm{rad.s}^{-1}$
[Rcl] [0] [Jupiter] ↦ [J_0] [-]	X	8535.37500	dy
[*] [Coalesce] [Quad]	X	6.09295	rad
[Jupiter] ↦ [L]	X	34.404	deg
[Jupiter] ↦ [ϖ] [-]	X	19.65053	deg
[Coalesce] [+] [Quad]	X	0.15273	rad
[Jupiter] ↦ [e] [x-y] [Kepler]	X	0.16046	rad
[Sto] [3] [COS]	X	0.98715	
[*] [CHS] [1] [+]	X	0.95223	
[Jupiter] ↦ [a] [*] [Sto] [4]	X	4.95479	AU
[Rcl] [3] [2] [/] [TAN]	X	0.08040	

and procedure to usual calculations[6]:

[1] [Jupiter] ↦ [e] [+]	X	1.04839	
[1] [Jupiter] ↦ [e] [-] [/]	X	1.10171	
[INV] [SQRT] [*]	X	0.08439	
[INV] [ATAN] [2] [*]	X	0.16839	rad
Angle→ [deg]	X	1	deg
[INV] [Convert]	X	9.64795	deg

Now, we have the value of $\nu_{\text{♃}}$ for Jupiter, the true anomaly measured from the perihelion. We have to add the argument of the perihelion ω to have the angle of Jupiter on the orbit, say ψ, from the crossing point with

[5] https://nssdc.gsfc.nasa.gov/planetary/factsheet/

[6] Until here, the calculations were carried out by toggling modes "Fix" and “Prc” for the number's boxes to fit in the columns.

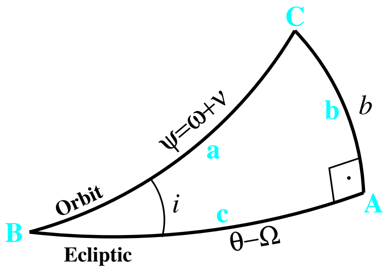


Figure 4.4: Spherical triangle of orbital to ecliptic coordinates conversion.

the ecliptic, the ascending node, that's say, $\psi = \nu + \omega$. We don't have ω directly, but we know that: $\omega = \varpi - \Omega$, so, with Jupiter chosen on the dropdown planet's combo:

Jupiter $\mapsto$ ϖ	+	**X**	24.40180	deg
Jupiter $\mapsto$ Ω	- Sto 9	**X**	-76.15435	deg

This is the value of ψ. Now, we are at the point where we should do the second order calculations on Jupiter's position because its orbital inclination is tiny ($1^\circ\!.85$), still we may take the solutions facility of KM-AstCalc. Watch Fig. 4.4. If we nominate the vertices and sides as the cyan letters in the figure, in lights of Eqs. (3.2), (3.3) and (3.4) we find the solutions:

$$\begin{array}{lcl} \sin b & = & \sin i \sin(\omega + \nu) \\ \cos b \sin(\theta - \Omega) & = & \cos i \sin(\omega + \nu) \\ \cos b \cos(\theta - \Omega) & = & \cos(\omega + \nu) \end{array} . \tag{4.13}$$

We get the solution of these three equations to have:

$$\begin{array}{lcl} b & = & \arcsin\left(\sin i \sin(\omega + \nu)\right) \\ \theta & = & \Omega + \arctan_2\left(\cos i \sin(\omega + \nu), \cos(\omega + \nu)\right) \end{array} , \tag{4.14}$$

the "index 2" in the operator arctan means the button INV Atan2 in KM-AstCalc.

SIN	X	-0.97094392	
Jupiter → i SIN *	X	-0.031355	
INV ASIN	X	-0.031355296	rad
Angle→ deg INV Convert	X	-1.79682	deg
Sto 5 D:M:S (b)	X	-1:47:48.5542	deg
D.ddd Jupiter → i COS	X	0.999478	
Rcl 9 SIN *	X	-0.97044	
Rcl 9 COS	X	0.23931	
INV Atan2	X	-1.32902	rad
Jupiter → Ω Coalesce	X	1.75504	rad
+	X	0.42601	rad
Angle→ deg INV Convert	X	24.40875	deg
Sto 3 D:M:S (θ)	X	24:24:31.49516	deg

Thus, we have the coordinates $R_{\jupiter} = 4.95579\text{AU}$, $\theta_{\jupiter} = 24^\circ 49' 31''\!.5$, and $\beta_{\jupiter} = -1^\circ 47' 48''\!.6$ for Jupiter in the ecliptic with the frame centered in the Sun. To have these coordinates for the geocentric frame, $\rho_{\jupiter}, \lambda_{\jupiter}, \beta_{\jupiter}$, we must transform to the coordinates of the Earth relative to the Sun.

Let's look at the state of the memory registry:

$$\begin{array}{cccl} \mathrm{R}_0 & \leftarrow & J & \text{Julian day} \\ \mathrm{R}_1 & \leftarrow & \theta_{\earth} & \text{Earth's heliocentric longitude} \\ \mathrm{R}_2 & \leftarrow & R_{\earth} & \text{Earth's radius vector to the Sun} \\ \mathrm{R}_3 & \leftarrow & \theta_{\jupiter} & \text{Jupiter's heliocentric longitude} \\ \mathrm{R}_4 & \leftarrow & R_{\jupiter} & \text{Jupiter's radius vector to Sun} \\ \mathrm{R}_5 & \leftarrow & b_{\jupiter} & \text{Jupiter's heliocentric latitude} \end{array} .$$

We use the formulae given in Eqs. (4.8) and (4.9) to find out the geocentric coordinates.

Rcl 3	X	0.42601	rad
Rcl 1 Coalesce -	X	-3.66596	rad
COS	X	-0.86564	
Rcl 2 *	X	-0.87509	AU
Rcl 4 *	X	-4.33590	AU^2
2 * CHS	X	8.67179	AU^2
Rcl 2 SQR +	X	9.69374	AU^2
Rcl 4 SQR +	X	34.24372	AU^2
INV SQRT Sto 6	X	5.85181	AU

We have $\Delta_{♃} = 5.85$AU, Jupiter's distance to the Earth. Now, we go to calculate λ_J. New data into the memory: $R_6 \leftarrow r_{♃}$, Jupiter's radius vector to Earth.

Rcl 4	X	4.95479	AU
Rcl 3 SIN *	X	2.04754	AU
Rcl 2	X	1.01092	AU
Rcl 1 SIN * -	X	2.87005	AU
Rcl 4	X	4.95479	AU
Rcl 3 COS *	X	4.51194	AU
Rcl 2	X	1.01092	AU
Rcl 1 COS * -	X	5.09966	AU
INV Atan2 Sto 7	X	0.51261	rad
Angle→ hour INV Convert	X	1.95803	hour
D:M:S ($\lambda_{♃}$)	X	1:57:28.92145	hour

Thus, $\lambda_{♃} = 1^{\mathrm{h}}57^{\mathrm{m}}28^{\mathrm{h}}.92$, and in memory, $R_7 \leftarrow \lambda_{♃}$, Jupiter's geocentric longitude.

All these coordinates are referred to the Equinox J2000.0. It is time to consider the precession to transfer the equinox to the date.

Rcl 0 Jupiter → *J0* -	X	8535.3750	dy
Astr.Const.→ ρ Upload	X	0.024380	rad.Cen^{-1}
* Coalesce	X	0.0056972	rad
Rcl 7 + Sto 7	X	0.51831	rad

Let's go to the ecliptic latitude evaluation:

Rcl 4	X	4.95479	AU
Rcl 6 /	X	0.84671	
Rcl 5 SIN *	X	-0.02655	
INV ASIN Sto 8	X	-0.02655	rad
deg INV Convert	X	-1.52120	deg
D:M:S ($\beta_{♃}$)	X	-1:31:16.74033	deg

And $\beta_{♃} = -1°31'16''.7$.

The final state of the memory registry is:

R_0	$\leftarrow$	J	Julian day
R_1	$\leftarrow$	$\theta_{♁}$	Earth's heliocentric longitude
R_2	$\leftarrow$	$R_{♁}$	Earth's radius vector from the Sun
R_3	$\leftarrow$	$\theta_{♃}$	Jupiter's heliocentric longitude
R_4	$\leftarrow$	$R_{♃}$	Jupiter's radius vector from Sun
R_5	$\leftarrow$	$b_{♃}$	Jupiter's heliocentric latitude
R_6	$\leftarrow$	$\Delta_{♃}$	Jupiter's radius vector from Earth
R_7	$\leftarrow$	$\lambda_{♃}$	Jupiter's geocentric longitude
R_8	$\leftarrow$	$\beta_{♃}$	Jupiter's geocentric latitude .

Now, we have to apply the equations in Eq. (3.7) for the aberration correction. Remind that, in equations, $\odot$ is the geocentric longitude of the Sun: $\theta_{♁} + \pi$:

Rcl 1	X	234.45263	deg
180 Angle→ deg U_x	X	180	deg
+ Quad	X	0.95038	rad
Rcl 7 - Sto 9	X	0.43207	rad
COS CHS	X	-0.90810	
Jupiter →ϖ Coalesce	X	0.25750	rad
Rcl 7 -	X	-0.26081	rad
COS Jupiter →e *	X	0.04676	
+	X	-0.86135	
Rcl 8 COS /	X	-0.86165	
Astr.Const.→ κ Upload *	X	-17.660	rad

The correction of the aberration from the relative speed of Jupiter to the Earth is, then, $-17''.7$.

Jupiter's elongation will be:

Rcl 7	X	0.51831	rad
Rcl 1	X	234.45	deg
180 Angle→ deg U_x -	X	54.453	deg
Coalesce -	X	-0.43207	rad

The minus signal is important here. It means that Jupiter is 'behind' the Sun (on the sky).

COS	X	0.90810	
Rcl 8	X	-0.026552	rad
COS *	X	0.90778	
INV ACOS	X	0.43283	rad
Angle→ deg INV Convert	X	24.799	deg
Sto 9	X	24.799	deg

The last free unit of memory, R_9, then, stores the phase angle. Because the difference between Jupiter's longitude and Sun's is negative, we also use the elongation negative: $\vartheta_{♃} = -24.^\circ 8$[7].

The Jupiter's equatorial coordinates comes straightforward:

Rcl 8 Rcl 7	X	0.51831	rad
INV EcE	X	28.16635	deg
Angle→ hour INV Convert	X	1.87776	hour
D:M:S ($\alpha_{♃}$)	X	1:52:39.92375	hour
D.ddd x-y	X	9.8237	deg
D:M:S ($\delta_{♃}$)	X	9:56:18.08193	deg

Exercise:

1. Estimate the Jupiter's apparent magnitude for May, 15, 2023 at 21^{h}. From Eq. (4.10) (supposing the memory is yet populated with data of this Section):

Jupiter→ H	X	-9.39500	
Rcl 4	X	4.95479	AU
Rcl 6	X	5.85181	AU
* Clean	X	28.99452	
INV LOG 5 *	X	7.31158	
+	X	-2.08342	

This should be Jupiter's magnitude for phase angle null. The empirical phase function for Jupiter, for phase angle > 12°[8] is:

$$\Theta(\vartheta) = -0.033 - 2.5\log(1 - 1.507x - 0.363x^2 - 0.062x^3 + 2.809x^4 - 1.876x^5),$$

[7] I use ϑ here, instead of the usually adopted by the community β to not overlap the celestial latitude.

[8] ARXIV1808.01973

with

$$x = \frac{\vartheta}{180^\circ},$$

ϑ in arc-degree. Going on:

Rcl 9	X	24.79945	deg
180 Angle→ deg $\cancelto{}{U_x}$ /	X	0.13777	
Sto 9 5 y^x	X	0.00005	
1.876 CHS *	X	-0.00009	
Rcl 9 SQR SQR	X	0.00036	
2.809 * +	X	0.00092	
Rcl 9 3 y^x	X	0.00262	
0.062 * -	X	0.00076	
Rcl 9 1.507 * -	X	-0.20687	
1 +	X	0.79313	
INV LOG	X	-0.10066	
2.5 * -	X	-1.83178	
0.033 -	X	-1.86478	

4.4.5 Planetary Perturbations

We found a way to locate a body orbiting around the Sun by solving the Kepler's equation. We did it by assuming that we are dealing with the two-body problem, that is to say, by isolating the Sun-body system from all the rest of the solar system. What we do by solving Kepler's equation is, then, an approximation. Other bodies, especially giant planets, significantly influence a body's trajectory inside the solar system. They generate changes in the body's movement we call perturbation.

The easier way to face the planetary perturbation problem is to put each orbital element of a body in terms of a power series with the time, usually until the third order. The time is given in a fraction of century.

The orbital parameters we have in KM-AstCalc are, then, the 'zero-order' components.

The first-order components are called secular perturbation and are more important for L, Ω, and ϖ. Second-order components are far less significant[9].

[9] Comprehensible details may be found in Jean Meeus, 2000, *Astronomical Algorithms*, 2nd Edition, Willmann-Bell, Inc.

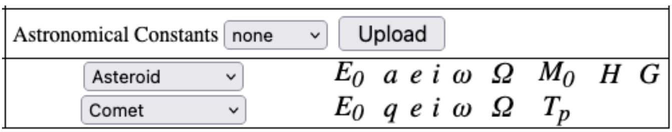
4.5 Asteroids and Comets

Asteroids and comets are classified as small bodies. They are like pebbles going back and forth through outer space. However, their motions provide essential clues about their origin and the solar system's. They are very alike, but we differentiate them from each other essentially by their chemical constitution. Ancient Greeks called comets (long-haired) because of the long tail they used to present. They are formed of "dirty-icy" snow called CHONdrites: Carbone-Hydrogen-Oxygen-Nitrogen varieties of molecules. These are volatile and sublimate when they come to the Sun's neighborhood. Pushed by the Sun's radiation pressure, their dust forms the tail, and solar wind will act on the molecules (yet not destroyed by the UV radiation) to generate the comma. They present beautiful pictures when passing by the Earth's vicinity.

Nevertheless, through the centuries, the comets were considered a bad omen. Even in modern times, at the beginning of the XXth Century, people thought that Halley's comet tail would poison the atmosphere and kill all of us! They were wrong, for our sake.

Asteroids are made of rocks and metals. Minerals, in short. They are not volatile; we notice an asteroid by its quick movement relative to the stars in the background. Possibly some comets may have a core like an asteroid.

The JPL-Caltech Solar System Dynamics Institute[10] has listed few less than four thousand comets and 620 thousand asteroids with their orbital elements. For the KM-AstCalc purposes, I decided to limit the asteroids to those orbiting inside the Jupiter zone and absolute magnitude less than 10, so we could observe it with a binocular, and the comets to the orbital period of a hundred years. This limitation puts available 701 asteroids and 776 comets. We may get them on the lower-right panel, just below the Astronomical Constants one:



The elements are essentially the same, except for the length element: asteroids have the semi-major axis and comets have the perihelion distance; and asteroids have brightness parameters, whereas comets have not. The common features are:

[10] https://ssd.jpl.nasa.gov/sb/elem_tables.html

$\boxed{E_0}$ Epoch of initial values in MDJ0, the Modified Julian Date constant, which means:

$$E_0 = J_0 - \mathrm{mdj}_0,$$

where J_0 is the full Julian Date and mdj_0 is 2400000.5, so to get J_0 it suffices to add the mdj_0 found among the Astronomical Constants dropdown combo just above;

$\boxed{e}$ Eccentricity of the orbit;

$\boxed{i}$ Inclination of the orbit relative to the mean ecliptic in degrees;

$\boxed{\omega}$ Argument of the perihelion in degrees;

$\boxed{\Omega}$ Longitude of the ascending node in degrees.

4.5.1 Asteroids

The exclusive orbital elements for asteroids are:

$\boxed{a}$ Semi-major axis in AU;

$\boxed{M_0}$ Initial mean anomaly, such that, to have the mean anomaly for the date:

$$M = n(J - J_0) + M_0. \tag{4.15}$$

$\boxed{H}$ Absolute magnitude. Just as for the planets, it is the magnitude for $\Delta_A = R_A = 1\mathrm{AU}$, phase angle $= 0°$;

$\boxed{G}$ Magnitude slope. Asteroids rarely are spheric and rotate relatively fast, so they usually have complex ways to evaluate their magnitude. After a long debate within the International Astronomical Union in 1985, one has concluded that[11]

$$m_A = H + 5\log R_A\Delta_A - 2.5\log\left[(1-G)\,\Phi_1 + G\Phi_2\right], \tag{4.16}$$

with

$$\begin{aligned} \Phi_1 &= \exp\left[-3.33\left(\tan\frac{\vartheta}{2}\right)^{0.63}\right] \\ \Phi_2 &= \exp\left[-1.87\left(\tan\frac{\vartheta}{2}\right)^{1.22}\right] \end{aligned}, \tag{4.17}$$

where ϑ is the phase angle. This formula is valid for $0° \leq \vartheta \leq 120°$.

[11] Jean Meeus, *Op. cit.*

Exercise:

1. Calculate the equatorial coordinates for the equinox J2000.0 of the asteroid Ceres at 21H GMT of May 15, 2023 and its magnitude.

Answer: For the equatorial coordinates we need to know where the Earth is. Then, we must to do the procedure done in Sec. 4.4.4 to have in memory:

$$\begin{array}{llll} \mathrm{R}_0 & \leftarrow & J & \text{Julian day} \\ \mathrm{R}_1 & \leftarrow & \theta_\oplus & \text{Earth's heliocentric longitude} \\ \mathrm{R}_2 & \leftarrow & R_\oplus & \text{Earth's radius vector to the Sun} \end{array} .$$

Going forward, we now start to calculate the position for Ceres.

First, let's retrieve the Julian Date stored in R_0 and evaluate the mean anomaly:

Rcl 0	X	2460080.37500	dy
Asteroid Ceres → E_0	X	59800.00000	dy
-	X	2400280.375000	dy
Astr.Const.→ mdj0 Upload	X	2400000.50000	dy
-	X	279.87500	dy

For the Eq. (4.4), we consider the asteroid mass too little to be considered, so:

Astr.Const.→ S Upload	X	1.9885e+30	kg
Ceres → a 3 y^x /	X	9.3905e+28	$\mathrm{AU}^{-3}.\mathrm{kg}$
INV SQRT	X	3.0644e+14	$\mathrm{AU}^{-1.5}.\mathrm{kg}^{0.5}$
Astr.Const.→ k Upload *	X	2.5034e+9	$\mathrm{m}^{1.5}.\mathrm{sc}^{-1}.\mathrm{rad}.\mathrm{AU}^{-1.5}$
* Coalesce	X	1.0462	rad

The mean anomaly is, then, obtained by adding the initial mean anomaly, which is in degrees and should be coalesced before:

Ceres → M_0 Coalesce +	X	6.8813	rad
Quad	X	0.59814	rad

Now, let's prepare the data and apply the Kepler's equation solver:

Ceres → e x-y Kepler	X	0.64544	rad

Having the eccentric anomaly, we are in position to find out the heliocentric radius vector value and the true anomaly.

Sto 3 COS *	X	0.062817	
CHS 1 +	X	0.93718	
Ceres→ a *	X	2.5928	AU
Sto 4 (R_C)	X	2.5928	AU
Rcl 3 2 /	X	0.32272	rad
TAN 1 Ceres→ e +	X	1.07864	
1 Ceres→ e -	X	0.92136	
/ INV SQRT *	X	0.36183	
INV ATAN 2 *	X	0.69435	rad
Angle→ deg INV Convert	X	39.783	deg

This is the true anomaly. We sum the argument of the perihelion ω to be ready to solve the triangle problem to calculate the ecliptic projection of Ceres position in the orbit.

Ceres→ ω + $(\omega + \nu)$	X	113.31	deg

Let's recall the Eqs. (4.13) that give us the way to get the heliocentric ecliptic coordinates. Thus,

Sto 9 SIN	X	0.91834	
Ceres→ i SIN *	X	0.16872	
INV ASIN	X	0.16953	rad
Angle→ deg INV Convert	X	9.7135	deg
Sto 5 (b_C)	X	9.7135	deg
Rcl 9 SIN	X	0.91834	
Ceres→ i COS	X	0.98298	
Rcl 9 COS	X	-0.39579	
INV Atan2	X	1.9840	rad
Ω Coalesce +	X	3.3849	rad
Sto 3 (θ_C)	X	3.3849	rad

Reminding the memory configuration until now:

$$
\begin{array}{llll}
\mathrm{R}_0 & \leftarrow & J & \text{Julian day} \\
\mathrm{R}_1 & \leftarrow & \theta_{\oplus} & \text{Earth's heliocentric longitude} \\
\mathrm{R}_2 & \leftarrow & R_{\oplus} & \text{Earth's radius vector to the Sun} \\
\mathrm{R}_3 & \leftarrow & \theta_C & \text{Ceres' heliocentric longitude} \\
\mathrm{R}_4 & \leftarrow & R_C & \text{Ceres' radius vector to the Sun} \\
\mathrm{R}_5 & \leftarrow & b_C & \text{Ceres' heliocentric latitute}
\end{array}.
$$

Now, let's calculate the three rectangular coordinates of Ceres according to the transformations of the Eqs. (4.6):

Rcl 4	X	2.5928	AU
Rcl 3 COS *	X	-2.5165	AU
Rcl 5 COS *	X	-2.4804	AU
Rcl 2	X	1.0109	AU
Rcl 1 COS *	X	-0.58772	AU
- Sto 7 (x_C)	X	-1.8927	AU
Rcl 4	X	2.5928	AU
Rcl 3 SIN *	X	-0.62467	AU
Rcl 5 COS *	X	-0.61572	AU
Rcl 2	X	1.0109	AU
Rcl 1 SIN *	X	-0.82252	AU
- Sto 8 (y_C)	X	0.20680	AU
Rcl 4 Rcl 5	X	9.7135	deg
SIN * Sto 9 (z_C)	X	0.43747	AU

and we use the Eqs. (4.7) to obtain the ecliptic geocentric coordinates:

Rcl 7 SQR	X	3.5821	AU^2
Rcl 8 SQR +	X	3.6249	AU^2
Rcl 9 SQR +	X	3.8163	AU^2
INV SQRT	X	1.9535	AU
Sto 6 (Δ_C)	X	1.9535	AU
Rcl 8 Rcl 7	X	-1.8927	AU
INV Atan2	X	3.0328	rad
Angle→ deg INV Convert	X	173.76442	deg
Sto 7 (λ_C)	X	173.76442	deg
Rcl 4 Rcl 6 /	X	1.32725	
Rcl 5 SIN *	X	0.22394	
INV ASIN	X	0.22585	rad
Angle→ deg INV Convert	X	12.94038	deg
Sto 8 (β_C)	X	12.94038	deg

The phase angle of Ceres is important to estimate its magnitude, so, using Eq. (4.7):

Rcl 7 Rcl 1 -	X	-60.68820	deg
COS CHS	X	-0.48956	
Rcl 8 COS *	X	-0.47713	
INV ACOS	X	2.06818	rad
Angle→ deg INV Convert	X	118.49805	deg
Sto 9	X	118.49805	deg

Being $\lambda_c - \theta_{\oplus}$ negative and less than 180°, we assure that $\lambda_C - \lambda_{\oplus}$ is positive, then, Ceres is 118°.5 *beyond* the Sun in the sky.

Let's again actualize the memory content:

R_0	$\leftarrow$	J	Julian day
R_1	$\leftarrow$	$\theta_{\oplus}$	Earth's heliocentric longitude
R_2	$\leftarrow$	$R_{\oplus}$	Earth's radius vector to the Sun
R_3	$\leftarrow$	θ_C	Ceres' heliocentric longitude
R_4	$\leftarrow$	R_C	Ceres' radius vector to the Sun
R_5	$\leftarrow$	b_C	Ceres' heliocentric latitute
R_6	$\leftarrow$	Δ_C	Ceres' radius vector to the Earth
R_7	$\leftarrow$	λ_C	Ceres' geocentric ecliptic longitude
R_8	$\leftarrow$	β_C	Ceres' geocentric ecliptic latitude
R_9	$\leftarrow$	ϑ_C	Ceres' elongation

Now, let's estimate the Ceres magnitude with the aid of the Eqs. (4.16) and (4.17). Let's first rewrite the term inside the function log in Eq. (4.16):

$$(1 - G)\Phi_1 + G\Phi_2 \to \Phi_1 - G(\Phi_2 - \Phi_1).$$

So, we evaluate Φ_1 and Φ_2 from Eq. (4.17):

Rcl 9 2 / TAN	X	1.68078	
Enter 0.63 y^x	X	1.38699	
3.33 CHS * EXP	X	0.00987	
x-y	X	1.68078	
1.22 y^x	X	1.88419	
1.87 CHS * EXP	X	0.02950	
x-y Enter ↓ -	X	0.01963	
Ceres→ G *	X	0.00236	
CHS ↑ +	X	0.00751	
INV LOG	X	-2.12436	
2.5 CHS *	X	5.31089	

We have reached the slope contribution of the magnitude. Now, let's evaluate the distances components:

Rcl 6 Rcl 4 *	X	5.06518	AU^2
Clean INV LOG	X	0.70459	
5 *	X	3.52297	
Ceres→ H + +	X	12.16386	

You'll need a 10 cm aperture telescope to see Ceres. A binocular is not sufficient.

Let's calculate the equatorial coordinates for Ceres.

Rcl 8 Rcl 7	X	173.76442	deg
INV EcE	X	179.52269	deg
Angle→ hour INV Convert	X	11.96818	hour
D:M:S (α_C)	X	11:58:5.44551	hour
D.ddd x-y	X	14.33349	deg
D:M:S (δ_C)	X	14:20:0.57422	deg

On May 15, 2023, Ceres seemed to be wandering through the constellation of Leo near the border with Coma Berenices.

4.5.2 Comets

Two specific elements characterize the Comets panel. They are:

- q Perihelion distance to the Sun, instead of the semi-major axis;
- T_p The comet's Julian day of the Perihelion passage, instead of mean anomaly.

No brightness data. The explanation for why there is no such information is that the comets are (very) dirty ice. Imagine the snow in a street full of people, automobiles, debris, and all sort of dust polluting it. It is ugly and dark. Comets are the same. The sunlight does reflect very little, if it doesn't reflect at all.

If comets do bright, it is because in the sun's proproximity, they warm, and the volatile material sublimates and fills up the region with gas that shines under the sun's radiation, in a process we call it fluorescence: the molecules absorb the UV radiation and then re-emit it in visible frequences. They, then, form the comma, and the dust forms the tail.

Knowing the perihelion instant, we have the mean anomaly simply by applying the following equation:

$$M = \frac{2\pi}{P_c}\left(J - T_P\right),$$

where we know the comet's period from the Kepler's law, reminding Eq. (4.4):

$$P_c = \frac{2\pi}{k}\sqrt{\frac{a^3}{M_\odot}},.$$

A comet's mass isn't significant in this calculation. So,

$$M = k\sqrt{\frac{M_\odot}{a^3}}\left(J - T_P\right).$$

The problem is that we have not the semi-major axis. However we may have it from:

$$a = \frac{q}{(1-e)}.$$

Then, we follow the same routine for locating a solar body in the sky.

Exercise

1. Find out the position of the Halley's comet at 21H GMT May 15, 2023.

Answer: For someone involved with this comet's observations in the 1980s as a member of the International Halley Watch, a global consortium for collecting as much information as possible about this mythic comet, I am particularly curious to know where has she been in present times.

Let's assume that the Earth's data for the date are already stored in the KM-AstCalc memory.

Let's find out the period[12]:

Comet 1P/Halley $\rightarrow$ q	X	0.57472	AU
1 1P/Halley $\rightarrow$ e -	X	0.03206	
/ Sto 9 (a)	X	17.92782	AU

We store the value of the semi-major axis (a) into the memory R_9, for we will need it later. So long, we get it to calculate the period:

[12]Put the format mode on "Fix", with five digits.

3 y^x	**X**	5762.1	AU^3
Astr.Const.→ S Upload /	**X**	2.8976e-27	$kg^{-1}.AU^3$
INV SQRT	**X**	5.3830e-14	$kg^{-0.5}.AU^{1.5}$
Astr.Const.→ k Upload /	**X**	6.5892e-9	$m^{1.5}.kg^{-0.5}.sc^{-1}.rad$
Coalesce	**X**	3.8126e+8	$sc.rad^{-1}$
2 PI Angle→ rad U_x *	**X**	2.3955e+9	sc
Time→ yr INV Convert (P_C)	**X**	75.962	yr

Notice the orbital period of Halley's comet: 75.9yr. This is one feature that makes it so "charming". It follows nearly a human generation cycle. Every generation might say it has seen Halley's comet at least once. Halley's comet is the celestial body that confirmed the validity of Newton's theory of celestial mechanics. In 1705, Edmond Halley, the friend that convinced Newton to publish his Principia Mathematica (which stayed in the drawer for twenty years), made the calculations that the comet that appeared in 1682 was the same that came so many times before and would return in 1758, what happened (four years later, it is true, but, then, they realized that the big planets perturbations were significant, too, and that it was one more aspect of the gravitational theory).

Now we go to find the mean anomaly:

Rcl 0 1P/Halley→ T_p -	**X**	13610.22610	dy
x-y / Coalesce	**X**	0.49088	
360 Angle→ deg U_x *	**X**	176.71698	deg

We see that this value almost 180° shows that Halley's comet is near the aphelion. Knowing M, we are ready to apply the Kepler's equation solver, arranging the parameters:

1P/Halley→ e x-y Kepler	**X**	3.11247	rad

Having the eccentric anomaly, we might obtain the radius vector and the true anomaly:

Sto 3 COS *	**X**	-0.96753	
CHS 1 +	**X**	1.96753	
Rcl 9 * Sto 4	**X**	35.27356	AU

Halley's comet is now in the middle way between Neptune and Pluto's orbits. Due to its orbital inclination ($\sim 162°$), it passes close to their orbit planes, which makes this comet's path sensitive to their perturbations.

Rcl 3 2 /	X	1.55624	rad
TAN	X	68.694	
1 1P/Halley $\mapsto$ e +	X	1.96794	
1 1P/Halley $\mapsto$ e - /	X	61.38846	
INV SQRT *	X	538.11444	
INV ATAN 2 *	X	3.13788	rad
Angle$\to$ deg INV Convert	X	179.78705	deg

The true anomaly is added to the argument of perihelion (ω) to have the orbital side of the spheric triangle in Fig. 4.4:

1P/Halley $\mapsto$ ω +	X	292.04484	deg

Now, retaking Eqs. (4.13), we do:

Enter SIN Enter	X	-0.92689	
1P/Halley $\mapsto$ i SIN	X	0.30590	
* INV ASIN	X	-0.28748	rad
Angle$\to$ deg INV Convert	X	-16.47117	deg
Sto 5 (b_C)	X	-16.47117	deg

The comet is on the "southern" of the ecliptic, with the Sun in the center. Let's, now, evaluate the heliocentric ecliptic longitude:

$\downarrow$	X	-0.92689	
1P/Halley $\mapsto$ i COS *	X	0.88246	
x-y COS	X	0.37533	
INV Atan2	X	1.16865	rad
deg INV Convert ($\theta - \Omega$)	X	66.95874	deg

Let's consider the third equation of Eqs. (4.13). We know that $\cos b$ is always positive, so we should examine what happens to $\cos(\omega + \nu)$. We already know that $\omega + \nu = 292^\circ\!.04$, so it is in the fourth quadrant, that is to say, $\cos(\omega+\nu)$ is positive, so is $\cos(\theta-\Omega)$, what puts it in the first or fourth quadrant. The $\tan(\theta - \Omega)$ is positive, so its parameter should be in the first or third quadrant. We conclude that $\theta - \Omega$ is in the first quadrant.

Adding Ω, the longitude of the ascending node, we have got the heliocentric longitude (θ) of Halley's comet:

1P/Halley $\mapsto$ Ω + Sto 3	X	126.07322	deg

Let's remind, as we did in the asteroid Ceres discussion, the state of the memory in KM-AstCalc:

$$
\begin{array}{llll}
\mathrm{R}_0 & \leftarrow & J & \mathrm{Julian\,day} \\
\mathrm{R}_1 & \leftarrow & \theta_{\male} & \mathrm{Earth's\,heliocentric\,longitude} \\
\mathrm{R}_2 & \leftarrow & R_{\male} & \mathrm{Earth's\,radius\,vector\,to\,the\,Sun} \\
\mathrm{R}_3 & \leftarrow & \theta_H & \mathrm{Halley's\,heliocentric\,longitude} \\
\mathrm{R}_4 & \leftarrow & R_H & \mathrm{Halley's\,radius\,vector\,to\,the\,Sun} \\
\mathrm{R}_5 & \leftarrow & b_H & \mathrm{Halley's\,heliocentric\,latitute} \\
\mathrm{R}_9 & \leftarrow & a_H & \mathrm{Halley's\,semi-major\,axis}
\end{array} \quad ,
$$

with further the semi-major axis stored in R_9, since it is not directly accessed in Comet's panel.

To transform the heliocentric coordinates to the geocentric ones, we retrieve the procedure we have done to asteroid Ceres, using the transformation equations Eqs. (4.6):

Rcl 4	X	35.27356	AU
Rcl 3 COS *	X	-20.76973	AU
Rcl 5 COS *	X	-19.91739	AU
Rcl 2	X	1.0109	AU
Rcl 1 COS *	X	-0.58772	AU
- Sto 7 (x_C)	X	-19.32967	AU
Rcl 4	X	35.27356	AU
Rcl 3 SIN *	X	28.51039	AU
Rcl 5 COS *	X	27.34040	AU
Rcl 2	X	1.0109	AU
Rcl 1 SIN *	X	-0.82252	AU
- Sto 8 (y_C)	X	28.16291	AU
Rcl 4 Rcl 5	X	-16.47117	deg
SIN * Sto 9 (z_C)	X	-10.00121	AU

To obtain the ecliptic geocentric coordinates, we use Eq. (4.7) :

Rcl 7 SQR	X	373.63619	AU^2
Rcl 8 SQR +	X	1166.78593	AU^2
Rcl 9 SQR +	X	1166.78593	AU^2
INV SQRT	X	35.59228	AU
Sto 6 (Δ_C)	X	35.59228	AU
Rcl 8 Rcl 7	X	-19.32967	AU
INV Atan2	X	2.17230	rad
Angle→ deg INV Convert	X	124.46383	deg
Sto 7 (λ_C)	X	124.46383	deg
Rcl 4 Rcl 6 /	X	0.99105	
Rcl 5 SIN *	X	-0.28099	
INV ASIN	X	-0.28483	rad
Angle→ deg INV Convert	X	-16.31954	deg
Sto 8 (β_C)	X	-16.31954	deg

Let's update the state of the registers in KM-AstCalc memory:

$$
\begin{array}{lcll}
R_0 & \leftarrow & J & \text{Julian day} \\
R_1 & \leftarrow & \theta_{\oplus} & \text{Earth's heliocentric longitude} \\
R_2 & \leftarrow & R_{\oplus} & \text{Earth's radius vector to the Sun} \\
R_3 & \leftarrow & \theta_H & \text{Halley's heliocentric longitude} \\
R_4 & \leftarrow & R_H & \text{Halley's radius vector to the Sun} \\
R_5 & \leftarrow & b_H & \text{Halley's heliocentric latitute} \\
R_6 & \leftarrow & \Delta_H & \text{Halley's radius vector to the Earth} \\
R_7 & \leftarrow & \lambda_H & \text{Halley's geocentric ecliptic longitude} \\
R_8 & \leftarrow & \beta_H & \text{Halley's geocentric ecliptic latitude} \\
R_9 & \leftarrow & a_H & \text{Halley's semi} - \text{major axis}
\end{array}.
$$

Then, let's find out the equatorial coordinates of Halley's comet.

Rcl 8 Rcl 7	X	124.46383	deg
INV EcE	X	122.95403	deg
Angle→ hour INV Convert	X	8.19694	hour
D:M:S (α_H)	X	8:11:48.96727	hour
D.ddd x-y	X	3.25928	deg
D:M:S (δ_H)	X	3:15:33.40359	deg

There is no standard way to determine a comet's magnitude. It is composed of "dirty ice", like the snow in a big city like Paris. Asteroids, composed of rocks, are far more brilliant. Recent estimations from the JPL give us values of 25.6 for Halley's magnitude, that is to say, far less brilliant than the sky background itself ($21 - 22^{\text{mag}}$).

Finally, we must explain that all the above calculations were made considering the two-body Kepler's problem. It is not what happens in the actual cases. Thus, our results here may differ from those in the professional calculations. Even so, if you compare, you will see that they are not in much disagreement.

4.6 Perihelion and Aphelion

Determining a solar system body's perihelion and aphelion time is relatively easy. It suffices to set

$$\nu = E = M = 2j\pi,$$

where $j = 0, 1, 2, 3, \ldots$, for the perihelion, and

$$E = \nu = M = (2j + 1)\pi,$$

for the aphelion. These are the conditions that E and ν coincide, independent of the eccentricity. For both conditions, $\sin E = 0$ in Kepler's equation.

The way it changes is the particularities of each kind of body.

4.6.1 Planets

Eq. (4.12) says that

$$M = n(J - J_0) + M_0,$$

with

$$M_0 = L - \varpi.$$

The condition is that, for the perihelion, an instant where

$$\frac{2\pi}{P}(J_q - J_0) + M_0 = 2\pi j,\ j = 0, 1....$$

P, being the orbital period. The number j will be determined by how many periods P fits in the difference $J_q - J_0$. Let's find the "first" pass-by through the perihelion after the given osculation date J_0:

$$\frac{2\pi}{P}(J_q - J_0) + M_0 = 0,$$

or

$$J_{q0} = J_0 - (L - \varpi)\,\frac{P}{2\pi}.$$

So, the condition for any perihelion pass-by:

$$\frac{J_q - J_{q0}}{P} = j.$$

Now, let's go to determine the integer number j. Suppose you want to know the next perihelion of a body after today. Let's call it J. Then, you know that the perihelion will be after some N days. Putting it in the condition just above, we have:

$$\frac{J + N - J_{q0}}{P} = j.$$

Or,

$$\frac{J - J_{q0}}{P} + \frac{N}{P} = j.$$

But, clearly $N < P$, so for the above equation to be true the relation N/P should be replaced by the unity. In other words, we say, mathematically

$$j = \texttt{Ceil}\left[\frac{J - J_{q0}}{P}\right].$$

Or, take the integer of the fraction and add the unity.

Once we know j we apply it into

$$J_q = jP + J_{q0}.$$

In the case of aphelion it is not hard to understand that the "first" one after the osculating date is:

$$J_{Q0} = J_0 - (L - \varpi)\frac{P}{2\pi} + \frac{P}{2}.$$

With similar procedure for the rest.

Exercise

1. Find the next perihelion and aphelion for the Earth from May, 15, 2023.

Answer:

Planet Earth→ L ϖ -	X	-2.48284	deg
Astr.Const.→ Syr Upload *	X	-906.87311	dy.deg
360 Angle→ deg $\cancel{U_x}$ /	X	-2.51909	dy
Earth→ $J0$ x-y - (J_{q0})	X	2451547.51909	dy
15:5:2023	X	15:5:2023	
Time→ dy $\cancel{U_x}$	X	15	dy
Time→ mth $\cancel{U_x}$	X	15:5	dy.mth
Time→ yr $\cancel{U_x}$	X	15:5:2023	dy.mth.yr
JulD	X	2460079.50000	dy
x-y -	X	8531.98091	dy
Astr.Const.→ Syr Upload /	X	23.35888	
INT 1 +	X	24.00000	
Astr.Const.→ Syr Upload *	X	8766.15271	dy
+ (J_q)	X	2460313.67180	dy

This is Julian day of the first Earth's perihelion after May 15, 2023. If you curious about the "normal" date, you do:

INV GrgD	X	4.17180:1:2024	dy.mth.yr

Thus, you know that the next Earth's perihelion will be the January, fourth of 2024. We may go forward and figure out at what time it will be:

JulD Enter	X	2460311.15271	dy
INT	X	2460313.00000	dy
0.5 Time→ dy $\cancel{U_x}$ +	X	2460313.50000	dy
-	X	0.17180	dy
Time→ hr INV Convert	X	4.1232	hr
D:M:S	X	4:7:23.52001	hr

The next Earth's perihelion after May, 15, 2023 will be at Jan, 4, 2024 at $4^{\mathrm{h}}7^{\mathrm{m}}23^{\mathrm{s}}$.

It is not difficult to understand that the next aphelion will be at $J_Q = J_q + P/2$.

D.ddd ↓	**X**	2460313.67180	dy
Astr.Const.→ Syr Upload	**X**	365.25636	dy
2 /	**X**	182.62818	dy
+ (J_Q)	**X**	2460496.29998	dy
INV GrgD	**X**	4.79998:7:2024	dy.mth.yr
JulD Enter INT	**X**	2460496.00000	dy
0.5 Time→ dy U_x +	**X**	2460496.50000	dy
-	**X**	-0.20002	dy
Time→ hr INV Convert	**X**	-4.80048	hr
24 Time→ hr U_x +	**X**	19.19952	hr
D:M:S	**X**	19:11:58.27201	hr

The Earth's aphelion will be at $19^{\mathrm{h}}11^{\mathrm{m}}58^{\mathrm{s}}$, the Jul, 4, 2024. But, we are on May, 2023, or, it is supposed an aphelion will happen before that. We subtract $P/2$ from the calculated perihelion, instead of adding it.

4.6.2 Asteroids

The procedure is similar for the asteroids since the initial mean anomaly is directly furnished in this case. Only that the initial date is in Modified Julian Date, so se should add the constant `mjd0` before taking the calculations:

$$J_{q0} = E_0 + \mathtt{mjd0} - 2\pi\frac{M_0}{P}.$$

The calculations remained are the same.

Exercise

1. Find the next perihelion and aphelion of Ceres.

Answer: Before all we should calculate Ceres' orbital period and store it in the memory, say R_1:

Asteroid Ceres→ a	X	2.7666	AU
3 y^x	X	21.17620	AU^3
Astr.Const.→ S Upload	X	1.98855e+30	k,g
/ Coalesce	X	35.65232	$g^{-1}.m^3$
INV SQRT	X	5.97096	$g^{-0.5}.m^{1.5}$
Astr.Const.→ k Upload	X	8.16935e-6	$m^{1.5}.kg^{-0.5}.sc^{-1}.rad$
/ Coalesce	X	23113000.77390	$sc.rad^{-1}$
2 PI *	X	6.28319	
Angle→ rad U_x *	X	145223266.86740	sc
Time→ dy INV Convert	X	1680.82485	dy
Sto 1	X	1680.82485	dy

And go to determine the first perihelion after the epoch E_0:

Ceres→ M_0	X	334.32717	deg
360 Angle→ deg U_x	X	360	deg
/ Rcl 1 *	X	1560.95948	dy
CHS Ceres→ E_0 +	X	58239.04052	dy
Astr.Const.→ mdj0 Upload	X	2400000.50000	dy
+ Sto 2	X	2458239.54052	dy
INV GrgD (J_{q0})	X	1.04052:5:2018	dy.mth.yr

Now, let's figure out how many Ceres' periods fit into J_{q0} and May, 15, 2023. Since the latter Julian day is stored in R_0,

JulD Rcl 0 -	X	-1839.95948	dy
CHS Rcl 1 /	X	1.09468	
INT 1 +	X	2.00000	
Rcl 1 *	X	3361.64970	dy
Rcl 2 +	X	2461601.19021	dy
INV GrgD (J_q)	X	14.69021:7:2027	dy.mth.yr

Given this date, and knowing the period (4.6yr), we know that the aphelion will come before the perihelion, so we subtract $P/2$ from J_q:

D.ddd Rcl 1	X	1680.82485	dy
2 / -	X	2460760.77779	dy
INV GrgD (J_Q)	X	26.27779:3:2025	dy.mth.yr

4.6.3 Comets

Finding comets' perihelion/aphelion time is perhaps the easiest way because we already have the last perihelion time: T_P. We may directly obtain the number of cycling j:

$$j = \texttt{Ceil}\left[\frac{J - T_P}{P_C}\right].$$

And we may pursue the usual calculations.

Exercise

1. Find the next perihelion and aphelion time for the comet P/Halley.

Answer: Let's first find the orbital period

Comet 1P/Halley→ q	X	0.57472	AU
1 1P/Halley→ e -	X	0.032057	
/	X	17.928	AU
3 y^x	X	5762.1	AU^3
Astr.Const.→ S Upload	X	1.9885e+30	kg
/ INV SQRT	X	5.3830e-14	$kg^{-0.5}.AU^{1.5}$
Astr.Const.→ k Upload	X	0.0000081694	$m^{1.5}.kg^{-0.5}.sc^{-1}.rad$
/ Coalesce	X	3.8126e+8	$sc.rad^{-1}$
2 PI *	X	6.2832	
Angle→ rad U_x	X	6.2832	rad
*	X	2.3955e+9	sc
Time→ dy			
INV Convert Sto 9	X	27726.14876	dy

Now, let's put the actual date and find the number of cycling orbits:

15:5:2023	X	15:05:2023	
Time→ dy U_x	X	15:05:2023	dy
Time→ mth U_x	X	15:05:2023	dy.mth
Time→ yr U_x	X	15:05:2023	dy.mth.yr
JulD	X	2460079.50000	dy
T_p	X	2446470.14890	dy

-	X	13609.35110	dy
/	X	0.49085	

We see that obviously, it didn't occur any Halley's perihelion after the last one! Thus, to know the next one we simply add the period to T_p.

Rcl 9	X	27726.14876	dy
T_p +	X	2474196.29766	dy
INV GrgD (J_q)	X	6.79766:1:2062	dy.mth.yr

There will be just one Halley's comet passage in the XXI$^{\text{th}}$ Century, different from the last Century when there were two (1911 and 1986).

To know the aphelion time is also easy:

D.ddd Rcl 9	X	27726.14876	dy
2 / -	X	2432607.07452	dy
INV GrgD (J_Q)	X	23.72328:1:2024	dy.mth.yr

January 2024! How near! Unfortunately we have no device to detect it. Its magnitude will be beyond 26, far weaker than the sky background itself.

4.7 Body's Speed at the Perihelion and Aphelion

The speed of a solar body at perihelion and aphelion is easily found, as there the displacement is perpendicular to the radius of the vector. They are:

$$V_P = \frac{2\pi a}{T}\sqrt{\frac{1+e}{1-e}}, \tag{4.18}$$

at the perihelion, and:

$$V_A = \frac{2\pi a}{T}\sqrt{\frac{1-e}{1+e}}, \tag{4.19}$$

at the aphelion.

Exercise

1. Evaluate the Earth's speed at the perihelion and aphelion.

Answer:

1 Earth→ e +	X	1.0167	
1 e - /	X	1.0340	
INV SQRT Enter	X	1.0169	
2 PI *	X	6.2832	
Earth→ a *	X	6.2832	AU
Astr.Const.→ Syr Upload /	X	0.017202	dy^{-1}.AU
Coalesce Enter	X	29785	sc^{-1}.m
↓ * (Per)	X	30287	sc^{-1}.m
↓ / (Ap)	X	29291	sc^{-1}.m

4.8 The Moon

Orbital elements data for the Moon are unreliable since most are set to zero in KM-AstCalc. The only data available are the physical parameters, like mass, diameter, etc.

The Moon's motion is complex. It is influenced by the tidal forces from the Sun, which are important, so more than the simple two-body model (Earth-Moon) for the orbital motion is needed to take sufficient accuracy.

Exercise

1. Compare the Moon's length of the day (**D**) and its orbital period (T_0). What conclusions do you find?

Moon→ **D**	X	708.70	hr
Moon→ T_0	X	27.300	dy

The units are not the same, so we convert one of them:

Time→ hr			
INV Convert	X	655.20	hr
-	X	53.500	hr

The length of the Moon's day exceeds its orbital period in $53^{\text{h}}\!.5$, so the Moon's visible side points elsewhere after a complete lunation. No surprise, since what happens is a similar condition than the terrestrial and sidereal times difference for the Earth. For the Moon to always point the same face to the Earth, it cannot have the orbital period equals to the length of the day.

Appendix A

Keyboard Shortcuts

KM-AstCalc lets you take shortcuts to upload units (yet scaled or not) and constants directly to the register-X without needing to click with a mouse here and there on the screen. It is handy if you have to repeat the same operations many times. Applying the keyboard shortcuts saves precious minutes if you need to get your results fast.

You use some shortcuts to access the units, constants and scaling factors with no need to use dropdown combos. Lower at right of KM-AstCalc page, you have the link “Shortcuts”. It gives you the list of all the shortcuts you could do to access the units, constants and functions of KM-AstCalc. Every function we have seen until now may be applied by a shortcut at the keyboard.

You find in Appendices A to E the instructions to get these shortcuts and the list of the keys to have them. Five keyboard configurations are available employing "switching", i.e., pressing keys of control. Five switches of key sets allow you to insert the unit or constant you want, and also perform the operations for the available functions. They are:

1. The keyboard “naked”;
2. PgDn switches the keyboard to the units;
3. End switches the keyboard to scaling units;
4. PgUp switches the keyboard to the constants;
5. f switches to the format control.

The special key \ plays the the screen button INV role.

A.1 The keyboard "naked"

You may hit any numeric key on the keyboard to compose the number you want into the register-X. Press the key E to enter the ten-exponential symbol "E". Another special key is : that composes the string of angles and times in hexadecimal and also dates.

Tab. (A.1) shows the lists of commands you may run by pressing the keys on the keyboard directly. For example, you may evaluate the sine of an angle by pressing the key s on the keyboard[1].

Appendices (B) to (C) explain the use of shortcuts on the keyboard with details.

Exercise

1. Evaluate $\sin 35^\circ 28' 56''$

   Answer:

   (a) Using mouse into commands on the screen:

3 5	X	35	
INV : 2 8	X	35:28	
INV : 5 8	X	35:28:56	
deg U_x	X	35:28:56	deg
INV D.ddd	X	35.482	deg
SIN	X	0.58045	

   (b) Using the keyboard

3 5	X	35	
: 2 8	X	35:28	
: 5 8	X	35:28:56	
PgDn o u *	X	35:28:56	deg
\ ;	X	35.482	deg
s	X	0.58045	

   * - You may use u or U to upload the units

[1] Capitals letters mind. Don't forget that trigonometric functions only evaluate if the units are compatible.

<table>
<tr><th>Function</th><th>Key</th></tr>
<tr><td>Sum X+Y</td><td>+</td></tr>
<tr><td>Difference Y-X</td><td>-</td></tr>
<tr><td>Product Y*X</td><td>*</td></tr>
<tr><td>Division Y/X</td><td>/</td></tr>
</table>

<table>
<tr><th>Function</th><th>Key</th></tr>
<tr><td>QUAD | Atan2</td><td>Q</td></tr>
<tr><td>toPolar | toRect</td><td>p</td></tr>
<tr><td>D:M:S | D.dddd</td><td>;</td></tr>
<tr><td>sin | asin</td><td>s</td></tr>
<tr><td>cos | acos</td><td>c</td></tr>
<tr><td>tan | atan</td><td>t</td></tr>
</table>

<table>
<tr><th>Function</th><th>Key</th></tr>
<tr><td>Exp / Ln</td><td>x</td></tr>
<tr><td>10^X | Log</td><td>X</td></tr>
<tr><td>SQR | SQRT</td><td>q</td></tr>
<tr><td>Y^X | X^Y</td><td>^+</td></tr>
<tr><td>SNH | ASNH</td><td>S</td></tr>
<tr><td>CSH | ACSH</td><td>C</td></tr>
<tr><td>TNH | ATNH</td><td>T</td></tr>
<tr><td>π | π/2</td><td>P</td></tr>
<tr><td>x!</td><td>!</td></tr>
</table>

<table>
<tr><th>Function</th><th>Key</th></tr>
<tr><td>CHS</td><td>_</td></tr>
<tr><td>ABS | -ABS</td><td>b</td></tr>
<tr><td>1/X</td><td><</td></tr>
<tr><td>INT | FRAC</td><td>I</td></tr>
<tr><td>X-Y | LstX</td><td>←</td></tr>
<tr><td>RND</td><td>n</td></tr>
<tr><td>% | D%</td><td>%</td></tr>
</table>

<table>
<tr><th>Function</th><th>Key</th></tr>
<tr><td>Inv*</td><td>\</td></tr>
<tr><td>Upload units</td><td>u / U</td></tr>
<tr><td>Coalesce | Convert</td><td>Home</td></tr>
<tr><td>Convert</td><td>k</td></tr>
<tr><td>Clean | CleanAll</td><td>Insert</td></tr>
<tr><td>C | CE</td><td>Delete</td></tr>
<tr><td>Rdown</td><td>↓</td></tr>
<tr><td>Inv Rup</td><td>↑</td></tr>
<tr><td>Enter | Down</td><td>Enter</td></tr>
<tr><td>STO</td><td>m</td></tr>
<tr><td>RCL</td><td>r</td></tr>
</table>

<table>
<tr><th>Function</th><th>Key</th></tr>
<tr><td>1</td><td>1</td></tr>
<tr><td>2</td><td>2</td></tr>
<tr><td>3</td><td>3</td></tr>
<tr><td>4</td><td>4</td></tr>
<tr><td>5</td><td>5</td></tr>
<tr><td>6</td><td>6</td></tr>
<tr><td>7</td><td>7</td></tr>
<tr><td>8</td><td>8</td></tr>
<tr><td>9</td><td>9</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>-</td><td>-</td></tr>
<tr><td>. | :</td><td>.</td></tr>
<tr><td>E</td><td>E</td></tr>
</table>

<table>
<tr><th>Function</th><th>Key</th></tr>
<tr><td>JulD | GregD</td><td>j</td></tr>
<tr><td>Kepler</td><td>K</td></tr>
<tr><td>EqH | HEq</td><td>!</td></tr>
<tr><td>EEc | EcE</td><td>@</td></tr>
<tr><td>EGa | GaE</td><td>#</td></tr>
<tr><td>GST0</td><td>w</td></tr>
</table>

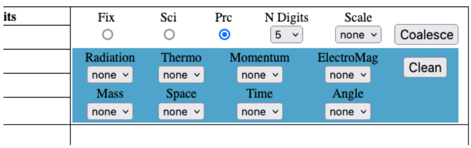
* Functions that accept "Inv" toggle with the signal "\".

Table A.1: Commands on the keyboard

Appendix B

Shortcuts to Units (PageDown sequences)

The keys preceded by the key “PageDown” (PgDn) on the keyboard get the units. When on the page of KM-AstCalc, you press PgDn , you see the panel of units getting a background light blue, indicating that the following key will define the unit that will enter into the register unit-X, like this:



You then choose a letter that corresponds to the unit you want. To get the inverse, press the key \ (function INV) before choosing the unit. The unit will be loaded directly into the register unit-X. After setting the units you want, press u or U on the keyboard, which is equivalent to click on the text field of unit-X with the mouse (U_x).

The lists of the units available on KM-AstCalc are shown in Tables B.1 to B.5.

Radiation			
Unit	**Symbol**	**PgDwn seq**	**Type**
candela	cd	@	Intensity
lumen	lm	W	Luminous Flux
lux	lx	I	Illuminance
becquerel	Bq	k	Radioactivity
gray	Gy	R	Absorbed ionising radiation
slevert	Sv	D	Equivalent absorbed radiation

Thermo			
Unit	**Symbol**	**PgDwn seq**	**Type**
mole	mol	*	Amount of substance
kelvin	oK	K	Temperature
celsius	oC	C	Temperature
farenheit	oF	F	Temperature
calorie	cal	q	Energy,Work,Heat
katal	kat	J	Catalytic Ativity

Mass		
Unit	**Symbol**	**PgDwn seq**
gram	g	g
ton	Tn	T
SunMass	S	S
pound	lb	&
arroba	@	@

Table B.1: Units available on KM-AstCalc: Radiation, Thermo and Mass.

Momentum			
Unit	**Symbol**	**PgDwn seq**	**Type**
kt	knot	.	Speed
mach	M	>	Speed
light speed	c	^+	Speed
gravity	grav	x	Acceleration, Potential field
newton	N	N	Force,Weight
pascal	Pa	4	Pressure,Stress
bar	bar	6	Pressure,Stress
atm	atmosphere	~	Pressure,Stress
joule	J	j	Energy,Work,Heat
erg	erg	7	Energy,Work,Heat
electron volt	eV	v	Energy,Work,Heat
watt	W	t	Power,Radiant Flux
horse-power	hp	2	Power,Radiant Flux
TNT-equiv*	TNT	N	Energy,Work,Heat

*Dynamite-ton equivalent energy

Table B.2: Units available on KM-AstCalc: Momentum & ElectroMag

ElectroMag			
Unit	**Symbol**	**PgDwn seq**	**Type**
ampere	A	A	Electric current
coulomb	C	5	Electric charge
volt	V	V	Electric Potential, Voltage, EMF
farad	Far	d	Capacitance
ohm	Ω	O	Resistance, Impedance, Reactance
siemens	Siem	3	Electrical conductance
weber	Wb	B	Magnetic Flux
tesla	Tes	1	Magnetic Flux Density
henry	H	b	Inductance

Table B.3: Units available on KM-AstCalc: ElectroMag

Space			
Unit	**Symbol**	**PgDwn seq**	**Type**
metre	m	m	Length
inch	in	n	Length
foot	ft	f	Length
yard	yd	‘	Length
mile	mi	8	Length
nautical mile	nmi	w	Length
angstron	AA	-	Length
astronomic unit	AU	U	Length

Space			
Unit	**Symbol**	**PgDwn seq**	**Type**
parsec	pc	p	Length
lightyear	ly	Y	Length
are	are	9	Surface
acre	ac	$	Surface
litre	L	L	Volume
barrel	Bar	‘	Volume
gallon	Gal	G	Volume
pint	Pin	P	Volume

Table B.4: Units avalilable on KM-AstCalc: Space.

Time			
Unit	**Symbol**	**PgDwn seq**	**Type**
second	s	s	Time
minute	mn	u	Time
hour	hr	h	Time
day	dy	a	Time
month	mth	e	Time
year	yr	y	Time
century	Cen	c	Time
hertz	Hz	z	Frequency

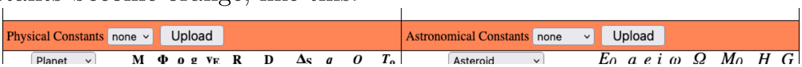
Angle		
Unit	**Symbol**	**PgDwn seq**
radian	rad	r
degree	deg	o
minute	min	’
second	sec	“
grad	grd	0
Hour	hour	H
H-min	hmin	=
H-sec	hsec	+

Table B.5: Units available on KM-AstCalc: Time & Angle.

Appendix C

Shortcuts to the Constants (PageUp sequences)

As well as you get shortcuts to the units in KM-AstCalc, through 'Page-Down' sequences, you may have the constants registered in KM-AstCalc employing 'PageUp' sequences. When you press PgUp on the keyboard, the panels where the dropdown combos of the physical and the astronomical constants become orange, like this:



It is the sign that the next key you press will upload the constant linked to it. Namely, pressing PgUp e will upload the electron mass onto the register X of the KM-AstCalc pile. Otherwise, pressing PgUp U will upload the value of the Astronomical Unit (AU) onto the register X.

The list of the shortcuts to the physical and astronomical constants is shown in the Tabs. C.1 and C.2.

Physical constants		
Name	Symbol	PgUp sequence
alpha particle mass	AP	A
angstron star	A*	*
atomic mass	ma	a
Avogadro constant	NA	N
Bohr magneton	μB	b
Bohr radius	a0	B
Boltzmann constant	kB	K
impedance of vacuum	Z0	w
electron radius	re	!
deuteron mass	DM	Q
electron mass	me	e
electron volt	eV	v
elementary charge	e	q
Faraday constant	F	F
fine-structure constant	α	f
Hartree energy	Eh	T
Josephson constant	K	l
lattice parameter Si	a	t
molar gas constant	R	R
nuclear magneton	μN	(
Planck constant	h	h
proton mass	P	&
Rydberg constant	R∞	x
speed of light	c	c
acceleration of gravity	g	g
standard atmosphere	Atm	m
Stefan-Boltzmann constant	Σ	s
electric permittivity	ϵ	n
magnetic permeability	μ	X
von Klitzing constant	Rk	V

Table C.1: Physical Constants.

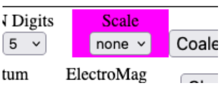
Astronomical constants		
Name	Symbol	PgUp sequence
Astronomical Unit	AU	U
parsec	pc	p
lightyear	ly	Y
Modified Julian Day	mdj0	j
Julian Year	JulYr	J
Julian Century	JulCy	y
Epoch 2000-Jan-1.5TD	J2000.0	0
Besselian Epoch	B1950.0	%
Sidereal Year	Syr	/
Tropical Year	Tyr	[
Gregorian Year	Gyr	]
acceleration of gravity	g	g
constant of gravitation	G	G
mass of Sun	S	S
Gaussian gravitational constant	k	k
Equatorial radius for Earth	Re	O
Earth ellipticity	e	i
Geocentric gravitational constant	GE	D
Earth mass / Moon mass	$1/\mu$	u
Precession in longitude	ρ	r
Precession in RA	m	Z
Precession in DEC	n	z
Obliquity of ecliptic	ε	o
Sidereal rate	ϵ	:
Constant of nutation	N	2
Constant of aberration	$\varkappa$	1
Heliocentric gravitational constant	GS	L
Sun mass / Earth mass	S/E	E
Sun mass / Earth+Moon mass	S/E+M	#
Hubble constant	H0	H
Solar luminosity	L0	W

Table C.2: Astronomical Constants.

Appendix D

Shortcuts to the Scale Factor (“End” sequences)

If you press the key End you get the possibility to add a scale factor to the unit you are going to choose. For example, pressing End followed by k will add the factor “kilo” to the unit chosen next, namely, PgDn m ’m’ as meter. If you press PgDn , then m , you will have the unit “k,m”[1] onto the unit-X register of the pile. When you press End , the sub-panel of the combo Scale becomes magenta, like this:



Now, pressing a key linked to a scale, you add the correspondent scale to the unit you are going to choose. The Tab. 1.2 shows the keys linked to the scale factor in the combo “Scale”.

[1]The notation “k,” is to mark that it is a scaling factor, not a unity symbol.

Appendix E

Shortcuts to Number Format and Notation

Pressing on the key f put KM-AstCalc in the number format definition and notation's mode. The format subpanel, in the upper-right panel of the calculator becomes green, like below

You may now to press the keys F to get the 'Fix' notation, S , to get the 'Sci' notation, or E (as enginnering) to the 'Prc' notation[1]. Additionally, press 0 to 9 , or A , B , C to set the number of digits from zero to twelve, which is the max precision JavaScript might give.

[1]Here, all the letters are capital.