CE2130: MECHANICS OF SOLIDS AND STRUCTURES

TERM PROJECT →

❖STRESS TRAJECTORIES OF A HIGHWAY BRIDGE WITH TRAPEZOIDAL DEAD LOAD

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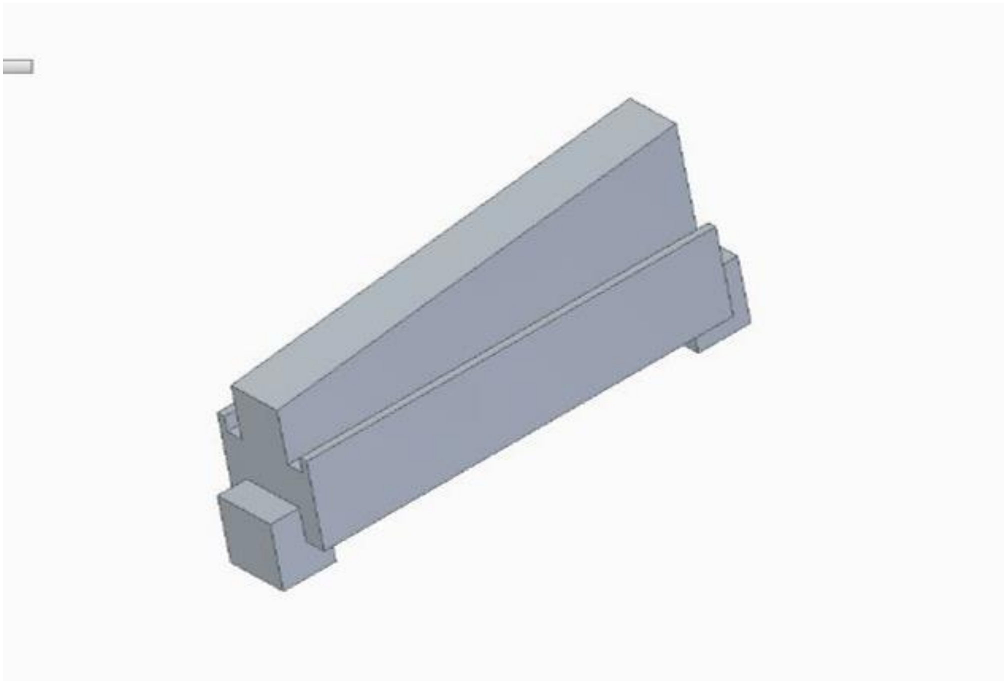
❖ CAD MODEL VIEW OF A SIMPLIFIED HIGHWAY BRIDGE



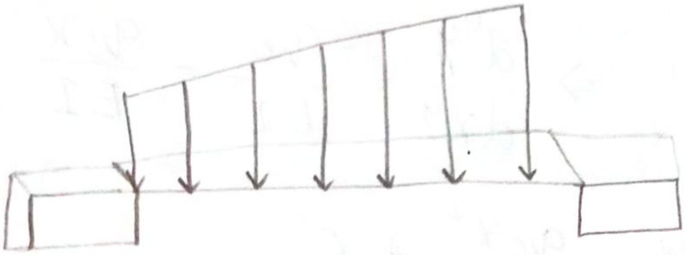
- *The above figure shows a simplified highway bridge structure with trapezoidal loading*

CE2130: Mechanics of Solids and Structures

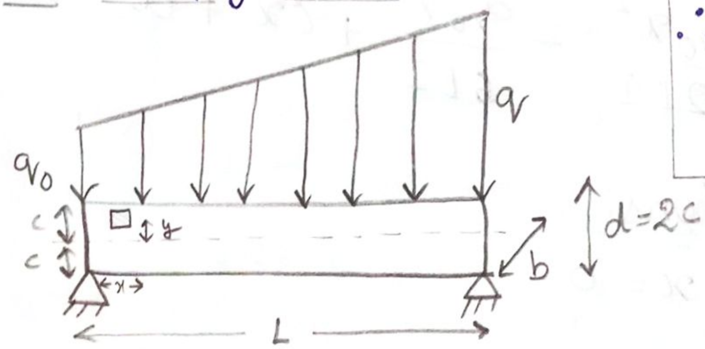
→ Term Project

→ Plotting Stress Trajectories of a highway bridge subjected to Trapezoidal dead load (downwards)

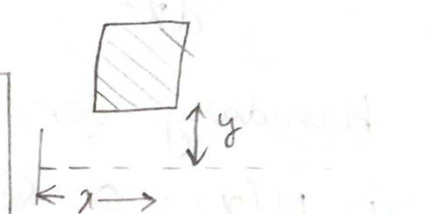
Diagram :



This can be simply drawn as :-



∴ Small element in beam:



→ Assumptions :

Length of bridge : 10c

breadth : b

Depth : d = 2c

→ Rectangular cross-section : $I = \frac{bd^3}{12}$; $A = bd$

↓ Moment of Inertia. ↳ Area of Cross Section

⇒ We can divide Trapezoidal loading into uniformly distributed loading and triangular loading.

for udl : $q_1(x) = q_0$ (∴ Acting downwards)

for linearly varying load : $q_2(x) = (q - q_0)x$ (Assume : $q = 2q_0$)

$q_2(x) = q_0 x$ (Acting downwards)

$\therefore$ by combining both :-

~~Tap~~ Trapezoidal loading : $\boxed{q(x) = -q_0 - q_0 x}$

Finding Bending Moment and Shear force :-

$\Rightarrow$ we know :-

$$\frac{d^4y}{dx^4} = \frac{q(x)}{EI} \Rightarrow \frac{d^4y}{dx^4} = \frac{-q_0}{EI} - \frac{q_0 x}{EI}$$

$$V(x) = \frac{d^3y}{dx^3} = \frac{-q_0 x}{EI} - \frac{q_0 x^2}{2EI} + c$$

$$M(x) = \frac{d^2y}{dx^2} = \frac{-q_0 x^2}{2EI} - \frac{q_0 x^3}{6EI} + cx + d$$

Boundary conditions:-

(i) $M(x) = 0$ @ $x = 0$

$\therefore \boxed{D = 0}$

(ii) $M(x) = 0$ @ $x = L$

$$0 = -\frac{q_0 L^2}{2EI} - \frac{q_0 L^3}{6EI} + cL + 0$$

$$\therefore \boxed{C = \frac{q_0 L}{2EI} + \frac{q_0 L^2}{6EI}}$$

so,

$$V(x) = +\frac{q_0}{EI}\left[-\frac{x^2}{2} - x + \frac{L}{2} + \frac{L^2}{6}\right]$$

$$\therefore \boxed{V(x) = -\frac{q_0}{EI}\left[x + \frac{x^2}{2} - \frac{L}{2} - \frac{L^2}{6}\right]}$$

$$\therefore \boxed{M(x) = -\frac{q_0}{EI}\left[\frac{x^2}{2} + \frac{x^3}{6} - \frac{Lx}{2} - \frac{L^2 x}{6}\right]}$$

As it is a Retangular cross-section:

$$\tau_{xy} = \frac{V(x)Q}{Ib} \Rightarrow \frac{V(x)\ (c+y)\ \frac{b}{2}\ (c-y)}{\frac{bd^3}{12}\cdot b} \Rightarrow \frac{V(x)\cdot(c^2-y^2)\cdot 6}{bd^3}$$

$$\tau_{xy} = \frac{V(x)}{A}\ \cancel{c^2}\left(1-\frac{y^2}{c^2}\right)\cdot \cancel{6}^3 \times \frac{1}{{}_2\cancel{4c^2}} \Rightarrow \boxed{\tau_{xy} = \frac{3V(x)}{2A}\left[1-\frac{y^2}{c^2}\right]}$$

And:- $\sigma_{xx} = \frac{M(x)y}{I} \Rightarrow \frac{M(x)y\cdot 12}{bd^3}$

$$\sigma_{yy} = 0$$

Stress Tensor:- $\begin{bmatrix} \frac{M(x)y\cdot 12}{bd^3} & \frac{3V(x)}{2A}\left(1-\frac{y^2}{c^2}\right) \\ \frac{3V(x)}{2A}\left(1-\frac{y^2}{c^2}\right) & 0 \end{bmatrix}$

$\Rightarrow$ Principal Stress State can be ~~so~~ achieved by rotating the axis by θ_p:

$$\tan(2\theta_p) = \frac{2\tau_{xy}}{\sigma_{xx}-\sigma_{yy}} \Rightarrow \frac{\frac{3V(x)}{A}\left[1-\frac{y^2}{c^2}\right]\cdot bd^3}{M(x)y\cdot 12}$$

$$\tan(2\theta_p) = \frac{V(x)\left[1-\frac{y^2}{c^2}\right]\cdot\frac{1}{\cancel{bd}}\times \cancel{bd}^{\cancel{3}\,2}\times\cancel{3}}{M(x)\cdot y\cdot \cancel{12}\,4}$$

$$\Rightarrow \frac{V(x)\left[1-\frac{y^2}{c^2}\right]\cancel{4}c^2}{M(x)\cdot y\cdot \cancel{4}}$$

$$\tan(2\theta_p) = \frac{-\frac{q\cancel{0}}{\cancel{E}I}\left[x + \frac{x^2}{2} - \frac{L}{2} - \frac{L^2}{6}\right]\cdot\left[1 - \frac{y^2}{c^2}\right]}{-\frac{q\cancel{0}}{\cancel{E}I}\left[\frac{x}{2} + \frac{x^2}{6} - \frac{L}{2} - \frac{L^2}{6}\right]\cdot\frac{xy}{c^2}}$$

$$\tan(2\theta_p) = \frac{\left[x - \frac{L}{2}\right]\left[1 - \frac{y^2}{C^2}\right]}{\left[x - \frac{L}{2} + \frac{x^2}{6} - \frac{L^2}{6}\right]\left[\frac{xy}{C^2}\right]} + \frac{\left[\frac{x^2}{2} - \frac{L^2}{6}\right]\left[1 - \frac{y^2}{C^2}\right]}{\left[x - \frac{L}{2} + \frac{x^2}{6} - \frac{L^2}{6}\right]\left[\frac{xy}{C^2}\right]}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\left\{\frac{\left[x - \frac{L}{2}\right]\left[1 - \frac{y^2}{C^2}\right]}{\left[x - \frac{L}{2} + \frac{x^2}{6} - \frac{L^2}{6}\right]\left[\frac{xy}{C^2}\right]} + \frac{\left[\frac{x^2}{2} - \frac{L^2}{6}\right]\left[1 - \frac{y^2}{C^2}\right]}{\left[x - \frac{L}{2} + \frac{x^2}{6} - \frac{L^2}{6}\right]\left[\frac{xy}{C^2}\right]}\right\}$$

AS, Length $(L) = 10c \Rightarrow 10m$ $\{$Assume : $c = 1m\}$

$c = d/2$ $\{\because d =$ depth of beam$\}$

Taking Portions at :- (x)

$\Rightarrow x = 0, 1c, 2c, 3c, 4c, 5c, 6c, 7c, 8c, 9c, 10c$

And Subsituting 'y' as :-

$\Rightarrow y = 1, 0.8, 0.6, 0.4, 0.2, 0, -0.2, -0.4, -0.6, -0.8, -1$

❖ FINDING (ϴp) BY USING MS EXCEL

→PRINCIPAL STRESS STATE CAN BE ACHIEVED BY ROTAING THE AXIS BY AN ANGLE OF ϴp.

DATA

x	y	ϴp
0	1	45
0	0.8	45
0	0.6	45
0	0.4	45
0	0.2	45
0	0	45
0	-0.2	-45
0	-0.4	-45
0	-0.6	-45
0	-0.8	-45
0	-1	-45

x	y	ϴp
1	1	0
1	0.8	11.68092
1	0.6	22.83523
1	0.4	31.79954
1	0.2	38.86426
1	0	45
1	-0.2	-38.8643
1	-0.4	-31.7995
1	-0.6	-22.8352
1	-0.8	-11.6809
1	-1	0

x	y	ϴp
2	1	0
2	0.8	5.618264
2	0.6	12.60775
2	0.4	21.41434
2	0.2	32.36039
2	0	45
2	-0.8	-5.61826
2	-0.6	-12.6077
2	-0.4	-21.4143
2	-0.2	-32.3604
2	-1	0

x	y	ϴp
3	1	0
3	0.8	3.245969
3	0.6	7.547511
3	0.4	13.98411
3	0.2	25.25361
3	0	45
3	-0.8	-3.24597
3	-0.6	-7.54751
3	-0.4	-13.9841
3	-0.2	-25.2536
3	-1	0

x	y	Θp
4	1	0
4	0.8	1.829392
4	0.6	4.309417
4	0.4	8.307612
4	0.2	17.14699
4	0	45
4	-0.8	-1.82939
4	-0.6	-4.30942
4	-0.4	-8.30761
4	-0.2	-17.147
4	-1	0

x	y	Θp
5	1	0
5	0.8	0.71576
5	0.6	1.694987
5	0.4	3.325874
5	0.2	7.462705
5	0	45
5	-0.8	-0.71576
5	-0.6	-1.69499
5	-0.4	-3.32587
5	-0.2	-7.4627
5	-1	0

x	y	Θp
6	1	0
6	0.8	-0.39561
6	0.6	-0.93746
6	0.4	-1.84374
6	0.2	-4.18994
6	0	-45
6	-0.2	4.189938
6	-0.4	1.84374
6	-0.6	0.937464
6	-0.8	0.395609
6	-1	0

x	y	Θp
7	1	0
7	0.8	-1.80782
7	0.6	-4.25922
7	0.4	-8.21472
7	0.2	-16.9888
7	0	-45
7	-0.8	1.807822
7	-0.6	4.259221
7	-0.4	8.214723
7	-0.2	16.98877
7	-1	0

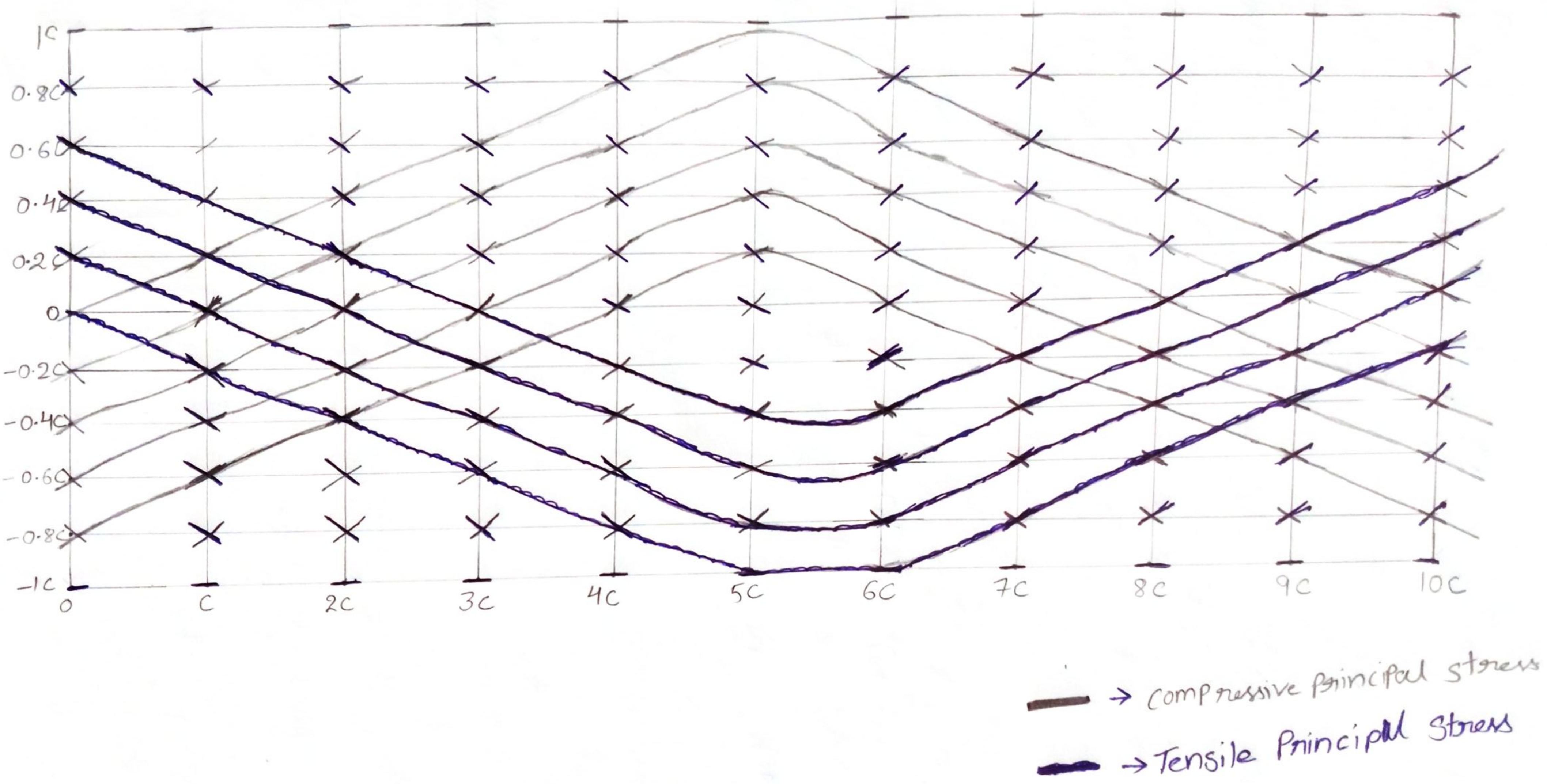
x	y	ϴp
8	1	0
8	0.8	-4.18862
8	0.6	-9.62089
8	0.4	-17.2473
8	0.2	-28.7528
8	0	-45
8	-0.8	4.188621
8	-0.6	9.620891
8	-0.4	17.24732
8	-0.2	28.75283
8	-1	0

x	y	ϴp
9	1	0
9	0.8	-10.3878
9	0.6	-20.9798
9	0.4	-30.2636
9	0.2	-38.0473
9	0	-45
9	-0.8	10.38783
9	-0.6	20.97983
9	-0.4	30.26361
9	-0.2	38.04735
9	-1	0

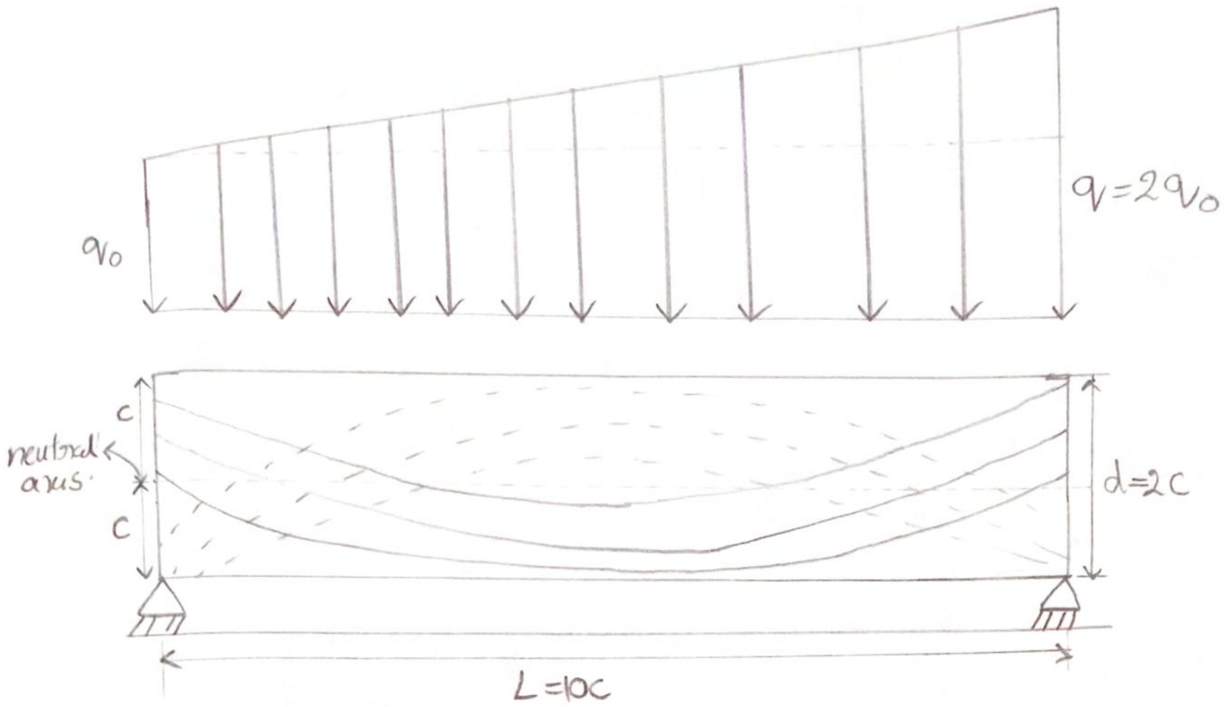
x	y	ϴp
10	1	-45
10	0.8	-45
10	0.6	-45
10	0.4	-45
10	0.2	-45
10	0	-45
10	-0.8	45
10	-0.6	45
10	-0.4	45
10	-0.2	45
10	-1	45

- Hence, by using these values (data) we can plot stress trajectories.

∴ Plotting Stress Trajectories of Trapezoidal Load on highway bridge:



→ Clear Representation of Stress Trajectories:



→ Stress Trajectories :- A system of orthogonal curves which display the variation of Principal Stresses throughout the beam

→ Solid Lines - Tensile Principal Stresses

→ Dashed Lines - compressive Principal Stresses

→ These lines intersect one another at right angles.