MATHEMATICS

SECTION A

February 8, 2024

1 Vector:

1. For any two vectors $\vec{a}$ and $\vec{b}$, prove that

$$\left(\vec{a} \times \vec{b}\right)^2 = \vec{a}^2 \ \vec{b}^2 - \left(\vec{a} \cdot \vec{b}\right)^2$$

2. Find the equation of planes passing through the intersection of the planes $\vec{r} \cdot \left(2\hat{i} + 6\hat{j}\right) + 12 = 0$ and $\vec{r} \cdot \left(3\hat{i} - \hat{j} + 4\hat{k}\right) = 0$ and are at a unit distance from origin.

2 Probability:

3. If $P(A) = 0.6$, $P(B) = 0.5$ and $P(B \mid A) = 0.4$, find $P(A \cup B)$ and $P(A \mid B)$.

3 Matrix:

4. Using the properties of determinants, prove the following :

$$\begin{vmatrix} a^2 & bc & ac + c^2 \\ a^2 & b^2 & ac \\ ab & b^2 + bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

4 Differential equation:

5. If $y = 2\sqrt{\sec\left(e^{2x}\right)}$, then find $\dfrac{dy}{dx}$.

6. Find the particular solution of the differential equation: $\left(1 + e^{2x}\right)dy + \left(1 + y^2\right)e^x dx = 0$, given that $y(0) = 1$.

7. If $X^p y^q = (x + y)^{p+q}$, prove that $\dfrac{dy}{dx} = \frac{y}{x}$ and $\dfrac{d^2y}{dx^2} = 0$.

5 Integration:

8. Evaluate:

$$\int_{-1}^{2} \left|x^3 - x\right| dx$$

6 Linear Form:

9. Using integration, find the area of the following region:

$$\left\{(x, y) : x^2 + y^2 \leq 16a^2 and y^2 \leq 6ax\right\}$$

7 Algebra:

10. Let an operation $*$ on the set of natural numbers N be defined by $a * b = a^b$. Find
    (a) whether $*$ is a binary or not, and
    (b) if it is a binary, then is it commutative or not.