

Master Theorem:

If $f(n) \in \Theta(n^d)$ or $f(n) = c \cdot n^d$ where $d \geq 0$
in recurrence $T(n) = aT(n/b) + f(n)$ then

$$T(n) \in \begin{cases} \Theta(n^d) & \text{if } a < b^d \\ \Theta(n^d \log n) & \text{if } a = b^d \\ \Theta(n^{\log_b a}) & \text{if } a > b^d \end{cases}$$

$$1) T(n) = 8T(n/2) + 1000n^2$$

$$\text{ult } T(n) = aT(n/b) + f(n)$$

$$a=8 \quad b=2 \quad f(n) = cn^d \\ = 1000n^2 \\ c=1000, d=2$$

Since

$$a > b^d$$

$$\text{i.e. } 8 > 2^3$$

master theorem case ③

$$T(n) = \Theta(n^{\log_b a})$$

$$= \Theta(n^{\log_2 8})$$

$$\boxed{T(n) = \Theta(n^3)}$$

$$2) T(n) = 2T(n/2) + n^2$$

$$a=2 \quad b=2 \quad c=1 \quad d=2$$

Since

$$a < b^d$$

$$2 < 2^2$$

case ①

$$T(n) = \Theta(n^d)$$

$$\boxed{T(n) = \Theta(n^2)}$$

$$3) T(n) = 2T(n/2) + 10n$$

$$a=2 \quad b=2 \quad c=10 \quad d=1$$

Since

$$a = b^d$$

$$2 = 2^1$$

case ②

i.e

$$T(n) = \Theta(n^d \log n)$$

$$\boxed{T(n) = \Theta(n \log n)}$$