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NSLS Design Project

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Introduction

The North Suburban Library System (NSLS) is a state-funded library system that delivers interlibrary loan items (books, video cassettes, etc.) to its member libraries in the suburbs north of Chicago. Although NSLS counts with 769 member libraries, the **49 public libraries** it serves account for 89% of the total demand and are the focus of attention for NSLS management.

The system is undergoing a budget crisis due to stagnant state funding, while at the same time experiencing a 35.8% increase in volume in the last year. NSLS would like to provide the greatest possible visit frequency to its members and hence is creating “hub” libraries: the vans will visit only a subset of the 49 public libraries, which will act as hubs; the other libraries will be responsible for collecting material from the hubs and delivering material to the hubs. There are 2 vans with overtime constraints and the model needs to be dynamic catering to the demand expansion for NSLS.

The second part of the consultation involves solving the routing sequence using the vehicle routing constraints. The demand of each hub and the concerned libraries need to be considered to make the model robust and drive with “no overtime” constraints. The van will depart from the depot and visit the hubs (which also have the demand of the libraries they will be serving) and then return within the driver shift length to the depot. The stopping time is a function of the demand served to each library.

Objective

Our goal for this project is the minimize the travel time between hubs and libraries given the demand ie. minimizing the aggregate weighted distance. For example, if a library is far away and has a small demand, both the distance and the demand will be weighted into our model, making it a lower weight in our outcome. This decision comes from our inclination to serve the community fairly given distance and demand variation.

In the first part, the clear objective was to find the set of libraries that become hubs. Since these libraries are selected as hubs they will act as depots for the smaller libraries whose demands need to be fulfilled by this hub. The cumulative demand (of the hub and the libraries it serves in that region) shall be considered to find the optimum route, again within the driver time length. The objective is to find the time taken by each driver to travel from the hub library to the next library in sequence to fulfill the weighted demand and then again return to the hub library, which will be studied and must be below 6 hours.

Modeling Assumptions

To model the hubs and libraries, the **P-median model** is used because this will give us the weighted distance for the libraries and hubs they are being supplied from. The *weighted distance* method takes into account the distance between libraries and hubs and also the demand of the libraries given the size of their community. Given the 49 available libraries, a model will be run to determine which libraries are most effective hub locations. Our goal is to Maximize Service of the libraries. It is assumed that the distance between the depot to each hub is 15 minutes, as opposed to taking into account the given distance data of distance between libraries. The budgetary constraint is taken into account in terms of a maximum labor quantity of 12 hours.

The Vehicle route needs to be discovered to cover the demands of the hub and the libraries the hub is serving. To formulate, the assumption is that there are only 2 vans and 2 drivers. Each driver can work up to 6 hours and no overtime would be considered. The assumption that no driver would work overtime means that the overall travel time for each van's route, including stops at hubs, must not exceed the driver's shift length. For the formulation, the vehicle capacity is not considered to be a constraint. It is assumed that the demand for each library remains constant and deterministic throughout the planning horizon. Assume that each hub library has set time periods for delivery and pickups. This simplifies the routing problem by limiting the time slots available for each stop, assuring resource efficiency and adherence to schedule requirements. Considering that the libraries would travel to hubs to pick up the books the vehicle is delivering to hubs. It is assumed that all the hubs are visited only once. This ensures that each hub's demand is met on time and there is no issue or delay in delivering the books.

Formulation Part 1

Parameters:

The parameters of our model include the set of libraries, the set of potential hub locations, the demand of libraries, and the time between libraries and hubs.

I: set of libraries

J: set of potential hub locations

Di: demand of library i

Tij : time between libraries and hub

Decision variables:

The decision variables of our model are **whether a hub is opened at certain libraries** and **which hubs are assigned to which libraries**.

Yj: opening a hub at library j $\forall$ j $\in$ J

1= if hub j is opened

0= otherwise

Xij: assignment variable (if demand is filled) $\forall$ i $\in$ I, $\forall$ j $\in$ J

1= if a hub (j) is assigned to library i

0= otherwise

Objective Function:

Based on our parameters and decision variables, our objective function is to minimize the weighted distance between libraries and hubs. This model represents the sum from [i to I] and [j to J] of (whether a hub is assigned to a library)*(time between hubs and libraries)*(the demand of library i). Therefore, our objective function is accounting for the time between libraries and hubs along with the demand of the libraries.

$$\text{Minimize: } \sum_{i=1}^{49} \sum_{j=1}^{49} Xij \times Tij \times Di$$

Constraints:

- $\sum_{j}^{J} 15 \times Yj + 2 \times 15 + \sum_{i}^{I} \frac{Di}{10} \leq 2 \times 6(\times 60)(minutes)$ (Each driver works only for 6 hours and there are 2 drivers)

- $\sum Xij = 1$ $\forall$ i $\in$ I , $\forall$ j $\in$ J (All Libraries should be assigned to one hub)
- Xij <= Yj $\forall$ i $\in$ I, $\forall$ j $\in$ J (Libraries can be assigned to hub, if the hub is opened)

- $X_{ij} \in \{0,1\}$ $\forall\, i \in I, \forall j \in J$ (If library is assigned to hub)
- $Y_j \in \{0,1\}$ $\forall\, j \in J$ (If the hub is opened or not)

Formulation Part 2

Parameters:
N: Number of hubs(1,2,3…16) (customer nodes).

N0: Depot and Hubs (0,1,2…16)

v: Number of vehicles.

w{i}: Load of customer node i.

c{i,j}: Distance associated with traveling from node i to node j.

Decision variables:

The decision variables of our model are the load of the vehicle on the route up to the node i and whether node j is visited after node i.

y{i}: Variable to track the load of the vehicle on the route up to customer node i.

x{i,j}: Binary variable indicating whether customer node j is visited after customer node i

Objective function:
The objective is to minimize the total tour cost, which is the sum of distances traveled:

$$Minimize: \sum_{I \epsilon N0} \sum_{J \epsilon N0} Cij \times Xij$$

Constraints:

$$\sum_{i \epsilon N0,\, i \neq j} Xij = 1$$ $\forall$ j $\in$ N (Each node (excluding the depot) has exactly one incoming link)

$$\sum_{j \epsilon N0, j \neq i} Xij = 1$$ $\forall$ i $\in$ N (Each node (excluding the depot) has exactly one outgoing link)

$\sum_{j \epsilon N} X0j \leq V$ (The number of vehicles departing from the depot is limited by the total number of Vehicles)

$C0i + Wi/10 \leq Yi \quad \forall$ i ∈ N (Ensures that the load of each vehicle when leaving the depot is sufficient to cover the initial distance)

$C0i + Yi \leq 360 \quad \forall$ i ∈ N (Sets a time limit for each vehicle's route.)

$Yi + (C0i + Wi/10) \times Xij \leq Yj + 360 \times (1 - Xij)$ ∀ i ∈ N, ∀ j ∈ N ,i≠j

Solution Methods Part 1

Model on AMPL:

```
reset;

set I:= 1..49; /*librarys to visit*/
set J:=1..49; /* potential hub locations*/

param D{I}; /*demands for libraries in set I*/
param t{I, J} default 999; /*time between libraries and hubs*/

var Y{J} binary; /*opening a hub at library j*/
var X{I,J} binary; /*if library i is being assigned to hub j*/

minimize weighteddistance: sum{i in I, j in J}X[i,j]*t[i,j]*D[i];

subject to Time: 15*sum{j in J}Y[j] + 30 + sum {i in I} D[i]/10 <= 720;
subject to Open{i in I}: sum {j in J}X[i,j] = 1; /*if hub j is assigned to library i*/
subject to Nonneg{i in I, j in J}: X[i,j] <= Y[j]; /*hubs are less than number of libraries availbable*/


data NSLS2.dat;
option solver cplex;
solve;


param demand{J};
for {j in J} {let demand[j]:= sum{i in I} D[i]*X[i,j];}


display Y, X, weighteddistance, demand;
```

Data file attached to model:

```
param D:=
1 180
2 24
3 76
4 83
5 84
6 118
7 206
8 36
9 225
10 118
11 196
12 44
13 51
14 27
15 16
16 150
17 56
18 92
19 40
20 59
21 50
22 6
23 138
24 105
25 9
26 78
27 233
```

```
param t:=

        0     1    2    3    4    5    6    7    8     9    10   11   12   13
        Depot      Alg  Ant  Arl  Bar  Car  Cook      Cry  Dee  Des  Dun  Ela
0   Depot   0      29   29   4    15   23   13   30   7    5    24   13   9
1   Alg 29  0      28   25   15   8    28   5    34   32   7    18   25   46
2   Ant 29  28     0    28   22   20   17   23   28   34   29   20   34   41
3   Arl 4   25     28   0    11   19   14   26   11   7    19   10   6    21
4   Bar 15  15     22   11   0    8    15   15   19   19   10   4    14   32
5   Car 23  8      20   19   8    0    20   6    27   27   8    10   22   40
6   Cook    13     28   17   14   15   20   0    26   11   18   25   11   20
7   Cry 30  5      23   26   15   6    26   0    33   33   10   17   27   46
8   Dee 7   34     28   11   19   27   11   33   0    9    29   17   16   14
9   Des 5   32     34   7    19   27   18   33   9    0    26   17   9    14
10  Dun 24  7      29   19   10   8    25   10   29   26   0    14   19   40
11  Ela 13  18     20   10   4    10   11   17   17   17   14   0    14   30
12  Elk 9   25     34   6    14   22   20   27   16   9    19   14   0    23
13  Eva 17  46     41   21   32   40   25   46   14   14   40   30   23   0
14  FXL 26  20     8    24   16   13   16   15   27   31   21   15   29   41
15  FXR 22  9      21   18   7    2    19   8    26   25   7    9    20   39
16  Fre 14  24     15   14   11   15   4    21   14   19   21   8    19   28
17  Gai 26  9      33   21   14   13   29   14   32   28   5    17   19   42
18  GLC 11  40     33   16   26   33   17   40   7    11   35   23   20   8
19  GLN 8   37     35   12   23   31   18   38   7    6    32   21   15   9
20  Gra 19  25     10   18   14   16   7    21   18   23   23   11   24   32
21  HPK 10  37     29   14   23   30   13   37   4    12   33   20   19   12
```

Solution Methods Part 2

Model on AMPL:

```
reset;
param n; /*number of hubs*/
param v; /*number of vehicles*/

set N:=1..n; /*set of customer nodes*/
set N0:=0..n;/*set of customer nodes plus depot*/
set V:=1..v;

param w{N}; /* load of customer node i */
param c{N0,N0}; /* Distance associated with going from i to j */
param Number_of_Vehicles := v;

var y{N}; /* Variable to keep track of load of vehicle on route up to customer i */
var x{N0,N0} binary; /*1 if customer j is visited after i */

minimize cost: sum{i in N0, j in N0} c[i,j]*x[i,j]; /*objective is to minimize total tour cost */

subject to incoming {j in N}: sum{i in N0: i != j} x[i,j] =1; /*Each node has one link coming into it*/
subject to outgoing {i in N}: sum{j in N0: j != i} x[i,j] = 1; /*Each node has one link going out of it */
subject to maximum_number_of_vehicles: sum{j in N} x[0,j] <= Number_of_Vehicles; /*Each node has one link going out of it */
subject to initial_load {i in N}: y[i] >= c[0,i]+0.1 *w[i];
subject to vehicletimelimit {i in N}: y[i] + c[i,0] <= 360;
subj to increasingload {i in N, j in N: i != j}: y[i] + (w[j]*0.1 + c[i,j]) * x[i,j] <= y[j] + 360 * (1-x[i,j]);

data NSLS_VRP.dat;
option solver cplex;
solve;
display y, x, cost, _ampl_time;


param next{N} default 0; /*to indicate what customer follows each customer i in the route */
for {i in N, j in N} { if x[i,j] ==1 then {let next[i]:=j;}}

param a; /* Counter that will represent the vehicle whose route we are forming*/
param number{V}; /* counter for number of customers visited by vehicle k */
param current{V}; /* dummy parameter keeping track of the current customer visited by vehicle k as we build the route*/
param route{V,N} default 0; /* To keep track of the route each vehicle follows*/
param travel_time {V}; /*travel time of each stop*/

let a:=1;
for {i in N} {
    if x[0,i]==1 then{ /* New vehicle a starts route */
        let travel_time[a]:= c[0,i];
        let number[a]:=1; /* we set the counter of visits to 1 */
        let route[a,1]:= i; /* We assign that first cutomer i as first stop, stop 1*/
        let current[a]:=i; /* Now we move the current vehicle postion to customer i*/
        repeat while (next[current[a]]!=0) { /* for all possible follow up stops of the vehicle (at most n stops)*/
            let number[a]:= number[a]+1;
            let route[a, number[a]]:= next[current[a]];
            let travel_time[a]:= travel_time[a]+c[current[a],next[current[a]]];
            let current[a]:=next[current[a]];

        let travel_time[a]:= travel_time[a]+c[current[a],0];                    }
        if a<v then let a:=a+1;
    }
}

display route, travel_time;
```

Data file associated with model file:

```
param n:= 16; /*number of hubs*/
param v:= 2; /*number of vehicles*/

param w:=
1   180
2   433
3   370
4   225
5   190
6   340
7   138
8   99
9   257
10  647
11  223
12  344
13  244
14  191
15  456
16  106;


param c:
    0   1   2   3   4   5   6   7   8   9   10  11  12  13  14  15  16:=
0   0   29  13  30  5   24  13  34  12  24  10  6   2   22  12  15  25
1   29  0   28  5   32  7   18  5   35  26  38  35  28  21  20  44  40
2   13  28  0   26  18  25  11  33  8   11  21  14  13  12  20  22  14
3   30  5   26  0   33  10  17  8   34  22  39  35  28  17  22  44  37
4   5   32  18  33  0   26  17  37  15  28  6   7   5   27  13  13  29
5   24  7   25  10  26  0   14  11  31  26  32  30  22  21  14  38  38
6   13  18  11  17  17  14  0   22  18  15  23  18  12  11  12  27  25
7   34  5   33  8   37  11  22  0   40  30  43  40  33  25  25  49  44
8   12  35  8   34  15  31  18  40  0   19  16  8   13  20  23  15  14
9   24  26  11  22  28  26  15  30  19  0   32  25  23  5   27  34  15
10  10  38  21  39  6   32  23  43  16  32  0   8   11  31  19  7   29
11  6   35  14  35  7   30  18  40  8   25  8   0   8   25  19  9   22
12  2   28  13  28  5   22  12  33  13  23  11  8   0   21  11  16  26
13  22  21  12  17  27  21  11  25  20  5   31  25  21  0   23  34  20
14  12  20  20  22  13  14  12  25  23  27  19  19  11  23  0   26  34
15  15  44  22  44  13  38  27  49  15  34  7   9   16  34  26  0   28
16  25  40  14  37  29  38  25  44  14  15  29  22  26  20  34  28  0 ;
```

Analysis and Conclusion Part 1

Results:

```
Y [*] :=
 1 1     6 1    11 1    16 0    21 0    26 1    31 0    36 0    41 1    46 0
 2 0     7 1    12 0    17 0    22 0    27 1    32 0    37 0    42 0    47 1
 3 0     8 0    13 0    18 0    23 1    28 0    33 1    38 1    43 0    48 0
 4 0     9 1    14 0    19 0    24 0    29 0    34 0    39 0    44 0    49 1
 5 0    10 1    15 0    20 0    25 0    30 0    35 1    40 1    45 0
;
```

Based on our results, (libraries with 1 in front of them) a total of 16 hubs at the following locations will be opened: **1, 6, 7, 9, 10, 11, 23, 26, 27, 33, 35, 38, 40, 41, 47, 49**. The output for X indicates which hubs supply to which libraries. Our output indicates that hub 6, for example, supplies to libraries 16, 43, and 44. Adequate service to all libraries is being provided because since the model runs, it is certain that the demands of all libraries are covered. Relocating a hub is a big deal, so our team would not recommend considering hub relocations due to the large efforts of implementing this new infrastructure.

Analysis and Conclusion Part 2

```
route  :=
1 1      6
1 2      5
1 3      1
1 4      7
1 5      3
1 6     13
1 7      9
1 8     16
1 9      2
1 10     8
2 1     12
2 2     14
2 3      4
2 4     10
2 5     15
2 6     11
;

travel_time
1  319
2   96
```

```
y [*] :=
 1  105
 2  330.1
 3  168.8
 4  102
 5   80
 6   47
 7  123.8
 8  348
 9  247.2
10  172.7
11  354
12   36.4
13  210.2
14   66.5
15  322.7
16  272.8
;

x [*,*]
:    0   1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16
0    0   0   0   0   0   0   1   0   0   0   0   0   1   0   0   0   0
1    0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0
2    0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0
3    0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0
4    0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0
5    0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
6    0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0
7    0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0
8    1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
9    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1
10   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0
11   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
12   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0
13   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0
14   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0
15   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0
16   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0
```

From part 1, the demand for the hubs was changed because now the hubs are also responsible for the demand for the libraries they serve. Part 1 now gives the proper demand for 16 hubs.
Then the VRP model was re-run with the fixed 16 hub demands. This now outputs the following route for each of the drivers:

Driver 1: 6 -> 5 -> 1 -> 7 -> 3 -> 13 -> 9 -> 16 -> 2 -> 8
Driver 2: 12 -> 14 -> 4 -> 10 -> 15 -> 11

Currently, vehicle 1 is utilizing 319 minutes out of 360 minutes, and vehicle 2 is utilizing only 96 minutes out of 360 minutes. This leaves a total of 305 minutes still available for time to deliver books from the depot to the hubs. For example, if the demand of all hubs were to increase by 10%, the drivers would still be within their capacity of 360 minutes of driving time. This model therefore, allows the customer to easily assess the route depending on the demand fluctuation. Overall, there is enough buffer room within the model with the constraint of the 12 hour driving time, so the model can handle a fluctuating demand for books.

Possible Modifications Part 1

In order to ensure adaptability and responsiveness to future changes, our system allows for flexible adjustments to data inputs such as demand and distances between libraries. For instance, the addition of a new library involves updating the corresponding demand and distance parameters in the data file. Similarly, modifications to existing libraries demand values can be easily accommodated within the same data file structure.

To enhance service provision for communities with greater needs, recommendations for model adjustments can be made. For instance, if a particular library experiences a surge in demand, signifying a heightened urgency for swift service, the model can designate it as a hub. Consequently, as demand patterns evolve, the selection of hub libraries may be revised to ensure optimal coverage for the broader community. A constraint can be added telling the model to locate a hub to account for marginalized communities where higher service is desired.

In conclusion, considering the constraint of limited resources, such as only two drivers available for six-hour shifts, the utilization of the P-Median model enables us to efficiently allocate hub resources. This approach ensures equitable service distribution, thereby serving the community fairly.

Possible Modifications Part 2

The observations suggest that the vehicles travel 96 minutes and 319 minutes out of the 360 minutes per driver to cover the respective routes with the vehicle capacity. One modification could be changing the cumulative capacity of the hubs and the libraries. It is observed that the distribution of driver hours to cover each route was 96 and 319 minutes with a lot of buffer time, therefore additional demands could be assigned in case any in the buffer time.

This will lead to a reallocation of the hub and libraries to support the new demands. This could also help us fulfill flexible demands if any.

Conclusion

Minuteman Consulting will stay in touch with the customer for any questions, concerns or changes to the proposed solution. It should be taken into account that this solution approach allows NSLS to re-run as needed when their demand volumes and other parameters change without our support. Thank you for coming to Minuteman Consulting for our services.