8. Set Theory - Relations

Relation Between Sets

Recall the Cartesian Product
4. Set Theory - Products and Sums of Sets > Cartesian Product
A **binary** relation from to is **some** set of ordered pairs with and

- In other words, a subset of the Cartesian Product
- i.e.
  We will use to denote relations
  Given finite sets and , there are relations

Examples of Relations

When , instead of saying that is a relation from to , we usually say that is a relation **on**

Examples:

- The full relation , the empty relation
- There is a relation on any set called the *identity* relation
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Digraphs

A *directed graph* (**digraph**) with at most one edge from one vertex to another can be formalised as a binary relation on its set of vertices
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Composition of Relations

if is a relation from to and is a relation from to then is a relation from to defined:

- if there exists such that and
- is sometimes written as
- composition is associative
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Functions are Special Relations

Any function is a special kind of relation from to that satisfies:

- for all there **exists a unique** such that
  - i.e. for all there exists such that
  - if and then
  Given a function , then we want to emphasise that we are talking about the relation, we call it the **graph of**

Composition of function is a *special case* of composition of relations