

10. Set Theory - Partial Orders

Orders

We all know that $<$, \leq , and

The order symbol is actually a relation:

In layman terms, this means that \leq is defined as:

- for elements x and y in S , there exists (z) an element in S such that

Partial Orders

A **partial order** on a set S is a relation that is Reflexive, Transitive and Anti-symmetric. It allows for a structured way to compare elements.

Example 1

A relation \leq is a partial order as it is:

- Reflexive - $x \leq x$
- Transitive - $x \leq y$ and $y \leq z$ imply $x \leq z$
- Anti-symmetric - if $x \leq y$ and $y \leq x$ then $x = y$

Example 2

Another example of a partial order is the **subset relation** (\subseteq) on the powerset of a set

For \subseteq on

- Reflexive - Every subset
- Transitive - if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$
- Anti-symmetric - If $A \subseteq B$ and $B \subseteq A$ then $A = B$