

Virtual Memory Questions

1. A certain computer provides its users with a virtual-memory space of 2^{32} bytes. The computer has 2^{18} bytes of physical memory. The virtual memory is implemented by paging, and the page size is 4096 bytes. A user process generates the virtual address 11123456 (this is in hexadecimal (base 16)). Explain how the system establishes the corresponding physical location. Distinguish between software and hardware operations. Feel free to use a diagram (simple ASCII is fine) if you wish, but that's not required.

Answer: The virtual address 11123456 corresponds to 0001 0001 0001 0010 0011 0100 0101 0110 in binary form. Because the page size is 4096 or 2^{12} bytes, the page table size is then $2^{32}/2^{12} = 2^{20}$ bytes. The first 12 bits of the virtual address, 0100 0101 0110, then correspond to the displacement into the page. The next 20 bits of the virtual address, 0001 0001 0001 0010 0011, then correspond to the displacement into the page table.

The system establishes the physical location by utilizing the TLB. All dynamic address translation operations are hardware operations. For example, the page table displacement is looked up in the TLB (hardware operation) whereas software operations handle faulty pages and reading the resulting page.

2. Assume we have a demand-paged memory. The page table is held in registers. It takes 8 milliseconds to service a page fault if an empty page is available or if the replaced page is not modified, but 20 milliseconds if the replaced page was modified. Memory access time is 100 nanoseconds.

Assume that the page to be replaced is modified 70% of the time. What is the maximum acceptable page-fault rate for an effective access time of no more than 200 nanoseconds? Show your work.

Answer: If we let p be the page fault rate, then $(1-p)$ is the rate at which a memory access costs 100 nanoseconds. The probability that a memory access will cost 20 nanoseconds will then be 70% ($0.7 * p$) and the probability that a page fault costs 8 milliseconds will be ($0.3 * p$). 1 millisecond = 1000000 nanoseconds so we derive the following equation.

$$(1 - p) * 100 + 0.7 * p * 20000000 + 0.3 * p * 8000000 = 200$$

Solving for p we get

$$(100 - 100p) + 14000000p + 2400000p = 200$$

$$-100p + 14000000p + 2400000p = 100$$

$$(-100 + 14000000 + 2400000)p = 100$$

$$p = 100 / (-100 + 14000000 + 2400000)$$

$$= 100 / (16399900)$$

$$p = 0.0000061$$

$$\text{Maximum acceptable page fault rate} = p * 100 = 0.0000061 * 100 = 0.00061\%$$