Release Time Scheduling and Hub Location for Next-Day Delivery

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Inspired by a real-life problem faced by one of the largest ground-based cargo companies of Turkey, the current study introduces a new facet to the hub location literature. The release time scheduling and hub location problem aims to select a specified number of hubs from a fixed set of demand centers, to allocate each demand center to a hub, and to decide on the release times of trucks from each demand center in such a way that the total amount of cargo guaranteed to be delivered to every potential destination by the next day is not below a threshold and the total routing cost is minimized. The paper introduces integer programming models to solve this problem in the special cases when the cargo uniformly arrives to each demand center during the day and the more realistic pattern of when the cargo arrivals exhibit a piecewise linear form. Several classes of valid inequalities are proposed to strengthen the formulations. Extensions with multiple service levels and discrete sets for release times are also discussed. Computational results show the computational viability of the models under realistic scenarios as well as the validity of the proposed problems in answering several interesting questions from the cargo sector's perspective.

Subject classifications: hub location; cargo delivery; time definite delivery; release times; valid inequalities.
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1. Introduction

Recently, with the growth in the Internet usage, there has been an increase in the business-to-consumer form of e-commerce. An increasing number and variety of firms are aiming to gain a share of this growing buyer's market. A prominent determinant of business performance is the role of logistics in assuring the timely flow of the materials. Most of the firms exploiting such e-commerce businesses outsource their distribution functions to third-party logistics companies. Consequently, national and international cargo companies have become the major players of this delivery market. Ultimately, gaining the competitive edge through improved service quality is critical for the cargo companies.

This study was initiated after we were approached by the local office of a national cargo delivery company, which currently operates one of the most widespread and effective ground based distribution networks in Turkey. The company wanted a systematic way of solving their truck departure synchronization problem so as to maximize the total cargo reaching its destination by the next day. This company provides delivery services all over Turkey. As is typical in the sector, the cargo does not travel directly from the originating branch office to the consignee branch office. Alternatively, there are operation centers that consolidate cargo from different origins and disseminate the cargo according to its destinations. Our company utilizes 22 operation centers, and every branch office is assigned to one such center. The typical journey of a cargo parcel starts

at the branch office where it is dropped. There it receives a label identifying the consignee branch office as well as the consignee's operation center. Upon arrival at the origin's operation center, the parcel gets consolidated with other cargo parcels destined toward the consignee's operation center. A typical route for cargo consists of three segments: from the branch office to the operation center serving this branch office, then to the consignee's operation center, and finally to the consignee branch office. Note that some of these segments might not exist, depending on the origin and/or destination branch offices being operation centers or origin and destination branch offices both being served by the same operation center.

In the existing terminology of location theory, the operating characteristics of our company fall under the "hubbing" course of action, with operation centers taking the role of hubs and the branch offices representing the demand centers or nonhub nodes.

Delivery time is a major distinguishing factor in the competition of the cargo companies—the primary reason for the current study. A common characterization of "service quality" on both the customer and the service provider side is the time elapsed between a drop-off at a branch office and the arrival at the consignee, with a high value placed on small delivery times. In light of this, the cargo companies strive to provide delivery time guarantees to their customers, and the most commonly practiced service level within Turkish companies is the next-day delivery. Due to

the existing highway network structure and the geographical disparity of the population centers in Turkey, it is not possible to serve each origin-destination pair within 24 hours when utilizing solely ground-based transportation. Yet, with appropriately selected departure times from originating branch offices, it might still be possible to fulfill the next-day delivery guarantee for the majority of the cargo. We highlight here that a cargo package is considered to be "delivered by the next day" if it arrives at its consignee before 18:00 on the day following the day of its drop-off at the branch office.

Short of guaranteeing next-day delivery to all branch offices, our company seeks to maximize the amount of cargo delivered by the next day. In the existing hub location models of the literature, it is customarily assumed that all the cargo of each demand center is ready to depart at the fixed ready time, usually 18:00 for cargo delivery companies. After 18:00, trucks full of the day's cargo depart toward their hubs. However, as a result of this treatment of truck release times as fixed parameters, cargo arriving at a branch office at 8:00 might not reach its destination by the next day. In this study, we propose to relax the assumption of fixed truck release times and to synchronize the truck departures so that every cargo package that arrives at its branch office before the scheduled release time receives the next-day delivery service guarantee.

The primary objective of our cargo company was to design a network such that the amount of cargo that is delivered by the next day is maximized. This objective is defined solely based on service level considerations and ignores the routing costs. However, it might be unrealistic and uncommon to design a hub network with the absence of cost considerations. Moreover, it might be interesting for cargo companies to analyze the trade-off between the cost and the delivery performance.

Against this background, we define the "*release time scheduling and hub location for next day delivery problem* (*RSHL*)" as the problem of finding a design and a release time schedule with minimum routing cost and a guarantee that the total amount of cargo delivered by the next day is not below a certain threshold. In addition to locating hubs and allocating demand centers to hubs, we set release times for trucks so that any cargo package arriving at a demand center before its release time is delivered to its destination, wherever it might be, by 18:00 on the next day. We assume that the hub network is complete. We extend our study to multiple service levels and provide a model to find a hub network design and a release time schedule of minimum total routing cost that ensures that specific amounts of cargo are delivered by certain time bounds.

In summary, our contribution is to provide tools that aid the decision maker in designing networks with a desired service quality at minimum cost by addressing strategic, tactical, and operational-level decisions simultaneously. Although the initiating application area is that of a ground-based cargo carrier, the findings of this research are readily applicable to other modes of transport utilizing the hubbing course of action.

The paper is organized as follows. In §2, we review the hub location literature related to our problem. Section 3 introduces integer programming formulations for uniform and piecewise linear cargo arrival patterns as well as discrete release times and multiple service levels. Several classes of valid inequalities to strengthen the formulations are derived in §4. In §5, we report the results of our experiments. Finally, concluding remarks are given in §6.

2. Literature Review

Hub location problems involve the location decisions of the hubs and the allocation decisions of the demand centers to these hubs. Many variants of the problem with different objectives and constraints are studied in the literature. In terms of allocation decisions, the problem has two variants: single allocation and multiple allocation. In single allocation, all incoming and outgoing flow of each demand center is distributed via a single hub; whereas in multiple allocation, each demand center can receive and send its flow via multiple hubs. The application that we consider in this paper fits into the single allocation category; hence, we focus on single allocation papers in the rest of this section.

Campbell (1994) classifies the hub location problems with respect to their objectives into four classes: minimization of total transportation cost (*p*-hub median), minimization of total transportation cost plus fixed cost of locating and operating hubs (hub location with fixed costs), minimization of the maximum transportation cost (*p*-hub center), and minimization of the number of hubs while serving each origin-destination pair within a predetermined threshold value (hub covering problem).

The hub location problems also have uncapacitated and capacitated variants. In uncapacitated problems, each hub has sufficient capacity to serve all demand centers. In capacitated problems, upper limits are imposed on the total amount of incoming traffic at a hub or on the amount of traffic transiting through a hub (see Ernst and Krishnamoorthy 1999, Labbé et al. 2005, and Costa et al. 2008 for different definitions of capacity).

The hub location problem is first posed by O'Kelly (1986). This problem is a single allocation *p*-hub median problem according to Campbell's classification. O'Kelly (1987) presents a quadratic integer program, which later is linearized in different ways (see, e.g., Campbell 1996, Ernst and Krishnamoorthy 1996, O'Kelly et al. 1996, and Skorin-Kapov et al. 1996).

O'Kelly (1992) introduces the hub location problem with fixed costs, which is first linearized by Campbell (1994). Hamacher et al. (2004), Labbé and Yaman (2004), Labbé et al. (2005), and Marin et al. (2006) study the facial properties of the polyhedra associated with the problem and valid inequalities. For recent heuristics, we refer the reader to Ernst and Krishnamoorthy (1999), Topcuoglu

et al. (2005), Canovas et al. (2007), Cunha and Silva (2007), Chen (2007), and Silva and Cunha (2009).

Kara and Tansel (2000) and Ernst et al. (2009) give integer programming formulations for the *p*-hub center problem. Pamuk and Sepil (2001) and Meyer et al. (2009) propose heuristic algorithms, and Baumgartner (2003) derives valid inequalities and gives a branch-and-cut algorithm. For the hub covering problem, Kara and Tansel (2003) and Wagner (2008) provide integer programming formulations and computational studies. The interested reader is referred to Campbell et al. (2002), Alumur and Kara (2008), and Kara and Taner (2011) for surveys.

Success in solving the hub location problems arising from a particular application area depends highly on the inclusion of the application specific aspects. The current work expands the literature along this line of research. Marsten and Muller (1980) and Kuby and Gray (1993) analyze the networks of Flying Tiger Line and Federal Express for fixed hub locations, respectively. Hall (1996) studies the routing decisions for pickup and delivery for overnight carriers on an already established network. Lin (2001) considers the problem of selecting the segments over a network with fixed hub locations.

Kara and Tansel (2001) analyze a hub location problem arising in the cargo delivery sector and identify certain characteristics specific to this application area. One of these is that the departing vehicles from hubs should wait for all the incoming cargo. This property has been incorporated in the hub location models by Kara and Tansel (2001). The authors define the resulting problem as the "latest arrival hub location problem." They first define the minisum, minimax, and covering versions of the problem and then focus on the minimax version. Later, Tan and Kara (2007) focus on the distribution network design problem of a typical cargo delivery company operating in Turkey; this study is a variant of the latest arrival hub covering problem.

Recently, in the freight transportation literature there are studies on "time-definite" delivery, where the service time between each origin-destination pair needs to be within a service level guarantee. Lin et al. (2003), Lin and Chen (2004), and Chen et al. (2008) focus on routing decisions over a given network. Lin and Chen (2004) allow two modes of transport and determine the fleet size of each mode. Chen et al. (2008) design tree structures over the already established network while minimizing the violations of the service levels. Campbell (2009) and Sim et al. (2009) consider the hub network design decisions together with the time-definite transportation. Campbell (2009) minimizes the total transportation cost while ensuring the time-definite transportation by considering feasible assignments only. Sim et al. (2009) consider the *p*-hub center problem and allow stochasticity on the travel times. Other examples are Yaman et al. (2007), where stopovers are incorporated; Yaman (2009), where a hierarchical hub network is designed; and Alumur et al. (2009) and Alumur and Kara (2009), where incomplete hub networks are allowed.

In this study, we consider a hub location problem with single allocation and no capacity restrictions. The *release time scheduling and hub location for next-day delivery problem* belongs to the class of problems with time-definite deliveries.

3. Notation and Formulation

In this section, we provide a mathematical formulation for the RSHL. Given a fixed set of demand centers, our task is to choose *p* centers as hubs, allocate each demand center to a hub, and decide on the release times of trucks from each demand center. We assume that all trucks at a given demand center leave at the same time, i.e., there is a unique release time for each demand center. Our aim is to minimize the total routing cost subject to the constraint that the amount of cargo delivered by time β is at least ν units. By performing a parametric analysis on ν, the decision maker can observe the trade-off between the routing cost and the cargo delivered within the service level β. After the general model is provided, several variations are presented in order to guide the decision maker in the cost-service trade-off analysis.

3.1. Minimum Cost Release Time Scheduling and Hub Location Model

Let N denote the set of demand centers and $H \subseteq N$ denote the set of candidate hub locations. We denote with ω_{im} the amount of cargo demand from node $i \in N$ to node $m \in N$. We assume full cross-traffic, i.e., there is cargo demand from any demand center to every other demand center. Among the nodes of set H, p nodes are to be chosen as hub nodes. We denote by τ_{ij} the time to travel from node $i \in N$ to node $j \in N$. We assume that travel times satisfy the triangle inequality (but they are not necessarily symmetric). The time to travel between two hub nodes is discounted by a factor of $0 \leqslant \alpha \leqslant 1$ due to the use of larger and faster vehicles. All nodes stop receiving cargo packages from customers at closing time γ. By time β, the cargo packages that leave the origin nodes at their release times should arrive at their destinations. Let $f_i(t)$ be the amount of cargo demand that accumulates at node $i \in N$ up to time $t \in [0, \gamma]$.

Let d_{ij} denote the cost of routing a unit traffic from node $i \in N$ to node $j \in N$ and $0 \leqslant \alpha^c \leqslant 1$ denote the reduction factor in unit routing cost for the traffic travelling between two hubs. The discount due to hubbing might be different in cost and time parameters. We assume that the routing costs satisfy the triangle inequality.

We define the following variables. Let x_{ij} be 1 if node $i \in N$ is assigned to hub $j \in H$ and 0 otherwise. A node is assigned to itself if and only if it is a hub. Additionally, we define r_i to be the release time of node $i \in N$, $\hat{D}_j$ to be the departure time from node $j \in H$ to other hubs and D_j to be the departure time from node $j \in H$ to destinations. We use the classical three index flow variables to model the traffic

flow in the hub network (see Ernst and Krishnamoorthy 1996); g_{jl}^i is the flow that originates at node $i \in N$ and that travels from hub $j \in H$ to hub $l \in H \setminus \{j\}$. Now, the minimum cost RSHL can be modeled as follows:

$$\min \sum_{i \in N} \sum_{j \in H} \left(d_{ij} \sum_{m \in N} \omega_{im} + d_{ji} \sum_{m \in N} \omega_{mi} \right) x_{ij}$$

$$+ \sum_{i \in N} \sum_{j \in H} \sum_{l \in H \setminus \{j\}} \alpha^c d_{jl} g_{jl}^i, \tag{1}$$

$$\text{s.t.} \quad \sum_{j \in H} x_{ij} = 1 \quad \forall i \in N, \tag{2}$$

$$x_{ij} \leqslant x_{jj} \quad \forall i \in N, \ j \in H \setminus \{i\}, \tag{3}$$

$$\sum_{j \in H} x_{jj} = p, \tag{4}$$

$$\hat{D}_j \geqslant (r_i + \tau_{ij}) x_{ij} \quad \forall i \in N, \ j \in H, \tag{5}$$

$$D_j \geqslant (\hat{D}_i + \alpha \tau_{ij}) x_{ii} \quad \forall i \in H, \ j \in H, \tag{6}$$

$$(D_j + \tau_{ji}) x_{ij} \leqslant \beta \quad \forall i \in N, \ j \in H, \tag{7}$$

$$r_i \leqslant \gamma \quad \forall i \in N, \tag{8}$$

$$\sum_{i \in N} f_i(r_i) \geqslant \nu, \tag{9}$$

$$\sum_{l \in H \setminus \{j\}} g_{jl}^i - \sum_{l \in H \setminus \{j\}} g_{lj}^i = \sum_{m \in N} \omega_{im}(x_{ij} - x_{mj})$$

$$\forall i \in N, \ j \in H, \tag{10}$$

$$g_{jl}^i \geqslant 0 \quad \forall i \in N, \ j \in H, \ l \in H \setminus \{j\}, \tag{11}$$

$$x_{ij} \in \{0, 1\} \quad \forall i \in N, \ j \in H, \tag{12}$$

$$r_i \geqslant 0 \quad \forall i \in N, \tag{13}$$

$$D_j, \hat{D}_j \geqslant 0 \quad \forall j \in H. \tag{14}$$

We refer to this model as *min cost RSHL*. Constraints (2)–(4) and (12) ensure that p nodes in H are hub nodes and that each node in N is assigned to exactly one hub node. Constraints (5) imply that to depart from a hub node toward other hub nodes, a vehicle needs to wait for all the trucks arriving from the nodes that are assigned to this hub. To depart from a hub node toward the destination nodes, a vehicle needs to wait for all the trucks coming from other hub nodes, and this is ensured with Constraints (6). Constraints (7) imply that the arrival time to any destination should happen by time β. Due to Constraints (8), the trucks should be released no later than time γ. Constraint (9) ensures that at least ν units of cargo are delivered to the destination by time β. Constraints (10) are flow balance constraints. The objective function (1) is equal to the total routing cost in the network where hub-to-hub traffic is routed at a discounted rate of α^c.

We linearize Constraints (5)–(7) by replacing them with the following set of constraints:

$$\hat{D}_j \geqslant r_i + \tau_{ij} x_{ij} - \gamma(1 - x_{ij}) \quad \forall i \in N, \ j \in H, \tag{15}$$

$$D_j \geqslant \hat{D}_i + \alpha \tau_{ij} x_{ii} \quad \forall i \in H, \ j \in H, \tag{16}$$

$$D_j + \tau_{ji} x_{ij} \leqslant \beta \quad \forall i \in N, \ j \in H. \tag{17}$$

Remark that due to Constraints (16), D_j can be positive even if j is not a hub node. But if j is not a hub node, then it is assigned to a hub node $m \in H \setminus \{j\}$. Let $i \in H$. Constraint (16) for i and j is $D_j \geqslant \hat{D}_i + \alpha \tau_{ij} x_{ii}$, and for i and m it is $D_m \geqslant \hat{D}_i + \alpha \tau_{im} x_{ii}$. We know that $D_m + \tau_{mj} \leqslant \beta$ due to Constraints (17). Hence $\hat{D}_i + \alpha \tau_{im} x_{ii} + \tau_{mj} \leqslant \beta$. As the travel times satisfy the triangle inequality, $x_{ii} \in \{0, 1\}$, and $0 \leqslant \alpha \leqslant 1$, we have $\beta \geqslant \hat{D}_i + \alpha \tau_{im} x_{ii} + \tau_{mj} \geqslant \hat{D}_i + \alpha \tau_{im} x_{ii} + \alpha \tau_{mj} x_{ii} \geqslant \hat{D}_i + \alpha \tau_{ij} x_{ii}$. This shows that even though D_j might be positive when j is not a hub, Constraints (15)–(17) do not eliminate any feasible vectors x and r.

Note that if we set β to a very large value and let $\nu = 0$, the above problem reduces to the well-known p-hub median problem.

3.2. Maximum Flow Release Time Scheduling and Hub Location Model

To find the range of possible values for parameter ν in Constraint (9), we model the problem of computing the maximum amount of cargo that is delivered by the next day as

$$\max \sum_{i \in N} f_i(r_i),$$

$$\text{s.t.} \quad (2)–(4), \ (8), \ (12)–(17).$$

We call this model *max flow RSHL*. Note that this model in essence answers the problem of our cargo company, i.e., to find the hub locations, allocations, and release times that will deliver the maximum amount of cargo by the next day.

3.3. Uniform and Piecewise Linear Arrival Patterns

The models presented above are for an arbitrary function representing the cargo arrivals to demand centers. There is not much restriction on such an arrival function except that, logically, it should be a nondecreasing function of time. If the arrivals are uniform over the time period $[0, \gamma]$, then for $t \in [0, \gamma]$ and $i \in N$, $f_i(t) = w_i(t/\gamma)$, where w_i is the total amount of cargo demand that arrives at node i during the period $[0, \gamma]$.

Assuming that the cargo will arrive at a constant rate throughout the day (working hours: 8:00 through 18:00) does not fully represent the cargo flow structure that is taking place in reality. The information we accumulated from the national cargo delivery company in Turkey revealed the following flow structure: Throughout the day, the cargo arrives at an increasing rate with the majority of the arrivals taking place during the last couple of hours. It is reasonable to assume three time intervals for which the cargo arrival rates differ considerably. During the morning hours (8:00 untill 12:00) about 10% of total daily cargo arrives uniformly. The following interval corresponds to the next four hours of the day, where it is assumed that 20% of a day's cargo arrives uniformly. Finally, 70% of the daily cargo finds its way to the corresponding center within the last two

Figure 1. Piecewise linear structure of the cargo arrival.
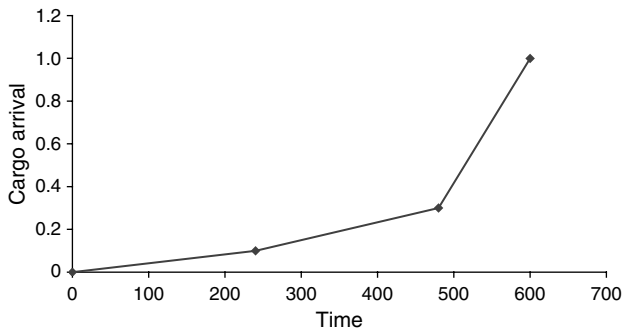
hours, i.e., 16:00 until 18:00. The resulting arrival model is depicted in Figure 1, where the time axis ranging from 0 through 600 minutes corresponds to the working hours 8:00 through 18:00. To model the three linear pieces of our arrival function, we adopted the linearization scheme of Sherali (2001). We add the following constraints to our models:

$$y_{1i} + y_{2i} + y_{3i} = 1 \quad \forall i \in N, \tag{18}$$

$$z_{1i}^L + z_{1i}^R = y_{1i} \quad \forall i \in N, \tag{19}$$

$$z_{2i}^L + z_{2i}^R = y_{2i} \quad \forall i \in N, \tag{20}$$

$$z_{3i}^L + z_{3i}^R = y_{3i} \quad \forall i \in N, \tag{21}$$

$$r_i = 240(z_{1i}^R + z_{2i}^L) + 480(z_{2i}^R + z_{3i}^L) + 600z_{3i}^R \quad \forall i \in N, \tag{22}$$

$$y_{1i}, y_{2i}, y_{3i} \in \{0, 1\} \quad \forall i \in N, \tag{23}$$

$$z_{1i}^L, z_{1i}^R, z_{2i}^L, z_{2i}^R, z_{3i}^L, z_{3i}^R \geqslant 0 \quad \forall i \in N, \tag{24}$$

and substitute $\sum_{i \in N} f_i(r_i) = \sum_{i \in N}(0.1w_i(z_{1i}^R + z_{2i}^L) + 0.3w_i(z_{2i}^R + z_{3i}^L) + w_i z_{3i}^R)$. Here, y_{ki} is 1 if r_i falls into the kth interval and 0 otherwise for $k = 1, 2, 3$. Constraints (18) and (23) ensure that one interval is picked for $i \in N$. Constraints (19)–(21) say that the sum of z_{ki}^L and z_{ki}^R is 0 if r_i is not in interval k, and this sum is equal to 1 if r_i is in interval k, for $k = 1, 2, 3$. Then, through Constraints (22) and (24), r_i is written as a convex combination of the end points of the interval into which it falls.

3.4. Discrete Release Times

In our previous models, we assume that the cargo arrival patterns are independent of the release times. In reality, the arrivals might be affected by the release times if the carrier announces its release times and guarantees next-day delivery for arrivals before the release time. In such a case, the amount of cargo that arrives at a demand center by a given time t depends on the time t and the release time of the demand center. Furthermore, our previous models assume the truck departure times to be continuous. In some applications, it might not be possible to depart trucks from each demand center at all times. To answer these concerns, we propose another model where we discretize the time period $[0, \gamma]$. Let R_i denote the set of possible release times

for demand center $i \in N$. We assume that R_i is a finite set for all $i \in N$. Let $\phi_i(t)$ be the amount of cargo that arrives at the demand center $i \in N$ by time $t \in R_i$ if the release time of this demand center is t. We define the binary variable a_{it} to be 1 if the release time of demand center $i \in N$ is equal to $t \in R_i$ and 0 otherwise. Then the following system can be used in the formulations presented above:

$$\sum_{t \in R_i} a_{it} = 1 \quad \forall i \in N, \tag{25}$$

$$r_i = \sum_{t \in R_i} t a_{it} \quad \forall i \in N, \tag{26}$$

$$f_i(r_i) = \sum_{t \in R_i} \phi_i(t) a_{it} \quad \forall i \in N, \tag{27}$$

$$a_{it} \in \{0, 1\} \quad \forall i \in N, t \in R_i. \tag{28}$$

Here, Constraints (25) and (28) ensure that one release time is picked for each demand center. Constraints (26) and (27) compute the release time and the amount of cargo that arrives at a demand center by the release time given the choice of release time for that center.

3.5. Multiple Service Levels

Up to now, our emphasis has been on a single service level. In a more general application setting, one might wish to refine the next day service level so as to distinguish between say overnight deliveries by the morning or in the afternoon. Next, we incorporate multiple service levels into our minimum routing cost release time scheduling problem.

Suppose we have K possible service levels, each defined by an upper bound on the delivery time. Let β^k be the upper bound for service level $k = 1, \ldots, K$ with $\beta^1 < \beta^2 < \cdots < \beta^K$. We would like to find the location of hubs, the allocations of nonhub nodes to hub nodes, and the release times in order to minimize the total routing cost subject to the constraint that at least ν^k units of cargo arrives at its destination by time β^k for $k = 1, \ldots, K$.

We define the variable u_i^k to be 1 if node $i \in N$ receives its incoming cargo in service level $k = 1, \ldots, K$. We also define h_{ij}^k to be the amount of cargo sent from node $i \in N$ to node $j \in N$ that is delivered in service level $k = 1, \ldots, K$. Let $\delta_i(t)$ be the percentage of cargo demand that accumulates at node $i \in N$ up to time $t \in [0, \gamma]$. Our multiple service level RSHL model is as follows.

$$\min \sum_{i \in N} \sum_{j \in H} \left(d_{ij} \sum_{m \in N} \omega_{im} + d_{ji} \sum_{m \in N} \omega_{mi} \right) x_{ij}$$
$$+ \sum_{i \in N} \sum_{j \in H} \sum_{l \in H \setminus \{j\}} \alpha^c d_{jl} g_{jl}^i, \tag{29}$$

s.t. (2)–(4), (8), (12)–(16)

$$D_j + \tau_{ji} x_{ij} - (\beta^K - \beta^1)(1 - x_{ij}) \leqslant \sum_{k=1}^{K} \beta^k u_i^k$$
$$\forall i \in N, j \in H, \tag{30}$$

$$\sum_{k=1}^{K} u_i^k = 1 \quad \forall i \in N, \tag{31}$$

$$\sum_{k=1}^{K} h_{ij}^k \leqslant \omega_{ij}\delta_i(r_i) \quad \forall i \in N, \, j \in N, \tag{32}$$

$$h_{ij}^k \leqslant \omega_{ij} u_j^k \quad \forall i \in N, \, j \in N, \, k = 1, \ldots, K, \tag{33}$$

$$\sum_{l=1}^{k} \sum_{i \in N} \sum_{j \in N} h_{ij}^l \geqslant \nu^k \quad \forall k = 1, \ldots, K, \tag{34}$$

$$u_i^k \in \{0, 1\} \quad \forall i \in N, \, k = 1, \ldots, K, \tag{35}$$

$$h_{ij}^k \geqslant 0 \quad \forall i \in N, \, j \in N, \, k = 1, \ldots, K. \tag{36}$$

Here, Constraints (30), (31), and (35) ensure that each demand center receives its demand by the time corresponding to the service level it is assigned to. Constraints (32) and (33) make sure that the cargo demand from node i to node j delivered in service level k is $\omega_{ij}\delta_i(r_i)$ if j receives its incoming cargo until time β^k. Finally, the total amount of cargo arriving at its destination by time β^k is bounded from below by a fixed parameter ν^k for $k = 1, \ldots, K$ via Constraints (34).

4. Valid Inequalities

Let F be the set of vectors $(x, r, D, \hat{D}, g)$ that satisfy Constraints (2)–(4), (8)–(17). In this section, we derive several families of valid inequalities for the set F. We note here that these inequalities are valid for any demand arrival pattern and can be easily adapted to the multiple service level model.

For $i \in N$ and $j \in N \setminus \{i\}$, we define $\lambda_{ij}^1 = \min_{m \in H \setminus \{j\}} (\alpha\tau_{im} + \tau_{mj})$, $\lambda_{ij}^2 = \min_{m \in H \setminus \{i\}} (\tau_{im} + \alpha\tau_{mj})$, and $\lambda_{ij}^3 = \min_{m \in H \setminus \{i,j\}} \min_{l \in H \setminus \{i,j\}} (\tau_{im} + \alpha\tau_{ml} + \tau_{lj})$. If node i is a hub and node j is not a hub, then it takes at least λ_{ij}^1 units of time to travel from node i to node j. Similarly, if node i is not a hub and node j is a hub, then λ_{ij}^2 is a lower bound on the time to travel from i to j. Finally, λ_{ij}^3 is a lower bound on the travel time from node i to node j if none of these nodes is a hub.

PROPOSITION 1. *For $i \in H$ and $j \in H \setminus \{i\}$, inequalities*

$$\beta \geqslant r_i + \alpha\tau_{ij} + (\lambda_{ij}^1 - \alpha\tau_{ij})(1 - x_{jj})$$
$$\quad + \min\{\lambda_{ij}^3 - \lambda_{ij}^1, \lambda_{ij}^2 - \alpha\tau_{ij}\}(1 - x_{ii}), \tag{37}$$

$$\beta \geqslant r_i + \lambda_{ij}^3 + (\lambda_{ij}^2 - \lambda_{ij}^3)x_{jj}$$
$$\quad + \min\{\lambda_{ij}^1 - \lambda_{ij}^3, \alpha\tau_{ij} - \lambda_{ij}^2\}x_{ii} \tag{38}$$

are valid for F.

PROOF. We prove the validity of inequality (37). The validity of the inequality (38) can be proved in a similar way. If both nodes i and j are hubs then inequality (37) simplifies to $\beta \geqslant r_i + \alpha\tau_{ij}$ and is valid because it takes $\alpha\tau_{ij}$ units to travel from i to j. If node i is a hub and node j is not

a hub, then the inequality simplifies to $\beta \geqslant r_i + \lambda_{ij}^1$ and is valid due to the definition of λ_{ij}^1. If node i is not a hub and node j is a hub, then we know that $\beta \geqslant r_i + \lambda_{ij}^2$. Because $r_i + \lambda_{ij}^2 \geqslant r_i + \alpha\tau_{ij} + \min\{\lambda_{ij}^3 - \lambda_{ij}^1, \lambda_{ij}^2 - \alpha\tau_{ij}\}$, inequality (37) is satisfied. If neither node i nor node j is a hub, then $\beta \geqslant r_i + \lambda_{ij}^3 \geqslant r_i + \lambda_{ij}^1 + \min\{\lambda_{ij}^3 - \lambda_{ij}^1, \lambda_{ij}^2 - \alpha\tau_{ij}\}$. Hence inequality (37) is valid. $\square$

Inequalities similar to (37) using the same idea as here are used by Yaman et al. (2007) to solve the latest arrival hub location problem with stopovers.

Inequality (37) can be obtained by fixing x_{ii} and x_{jj} to one and then lifting the valid inequality $\beta \geqslant r_i + \alpha\tau_{ij}$ first with x_{jj} and then with x_{ii}. We can obtain inequality (38) by fixing x_{ii} and x_{jj} to zero and lifting the valid inequality $\beta \geqslant r_i + \lambda_{ij}^3$ first with x_{jj} and then with x_{ii}.

If we fix x_{ii} and x_{jj} to one and lift the valid inequality $\beta \geqslant r_i + \alpha\tau_{ij}$ first with x_{ii} and then x_{jj}, we obtain

$$\beta \geqslant r_i + \alpha\tau_{ij} + \min\{\lambda_{ij}^3 - \lambda_{ij}^2, \lambda_{ij}^1 - \alpha\tau_{ij}\}(1 - x_{jj})$$
$$\quad + (\lambda_{ij}^2 - \alpha\tau_{ij})(1 - x_{ii}). \tag{39}$$

Similarly, if we fix x_{ii} and x_{jj} to zero and lift the valid inequality $\beta \geqslant r_i + \lambda_{ij}^3$ first with x_{ii} and then with x_{jj}, we obtain

$$\beta \geqslant r_i + \lambda_{ij}^3 + (\lambda_{ij}^1 - \lambda_{ij}^3)x_{ii}$$
$$\quad + \min\{\lambda_{ij}^2 - \lambda_{ij}^3, \alpha\tau_{ij} - \lambda_{ij}^1\}x_{jj}. \tag{40}$$

If $\lambda_{ij}^3 + \alpha\tau_{ij} \geqslant \lambda_{ij}^2 + \lambda_{ij}^1$, then inequality (39) reads

$$\beta \geqslant r_i + \alpha\tau_{ij} + (\lambda_{ij}^1 - \alpha\tau_{ij})(1 - x_{jj}) + (\lambda_{ij}^2 - \alpha\tau_{ij})(1 - x_{ii})$$

and is the same as inequality (37). Inequality (40) reads

$$\beta \geqslant r_i + \lambda_{ij}^3 + (\lambda_{ij}^1 - \lambda_{ij}^3)x_{ii} + (\lambda_{ij}^2 - \lambda_{ij}^3)x_{jj}$$

and is the same as inequality (38). If $\lambda_{ij}^3 + \alpha\tau_{ij} < \lambda_{ij}^2 + \lambda_{ij}^1$, then inequality (39) is the same as (38), and inequality (40) is the same as inequality (37).

Inequalities (37) and (38) only use the information of whether a node becomes a hub node or not in order to derive lower bounds on travel times. If a node is not a hub node, there is no information about which hub node it is assigned to. In the next proposition, we present valid inequalities that use the information of the assignment of one endpoint of a traffic demand to derive lower bounds on travel times.

PROPOSITION 2. *Inequalities*

$$\beta \geqslant r_i + \sum_{h \in H \setminus \{i\}} \left(\tau_{ih} + \min_{m \in H \setminus \{i\}} (\alpha\tau_{hm} + \tau_{mj}) \right) x_{ih}$$
$$\quad + \min_{m \in H} (\alpha\tau_{im} + \tau_{mj}) x_{ii} \quad \forall i \in H, \, j \in N \setminus \{i\}, \tag{41}$$

$$\beta \geqslant r_i + \sum_{h \in H} \left(\tau_{ih} + \min_{m \in H} (\alpha\tau_{hm} + \tau_{mj}) \right) x_{ih}$$
$$\quad \forall i \in N \setminus H, \, j \in N \setminus \{i\}, \tag{42}$$

$$\beta \geqslant r_i + \sum_{h \in H \setminus \{j\}} \left(\min_{m \in H \setminus \{j\}} (\tau_{im} + \alpha \tau_{mh}) + \tau_{hj} \right) x_{jh}$$
$$+ \min_{m \in H} (\tau_{im} + \alpha \tau_{mj}) x_{jj} \quad \forall i \in N, \, j \in H \setminus \{i\}, \quad (43)$$

$$\beta \geqslant r_i + \sum_{h \in H} \left(\min_{m \in H} (\tau_{im} + \alpha \tau_{mh}) + \tau_{hj} \right) x_{jh}$$
$$\forall i \in N, \, j \in N \setminus (H \cup \{i\}) \quad (44)$$

are valid for F.

PROOF. We prove the validity of inequality (41). The validity of the other inequalities can be proved in a similar way. Let $i \in H$ and $j \in N \setminus \{i\}$. If node i is a hub, i.e., $x_{ii} = 1$, then $\min_{m \in H} (\alpha \tau_{im} + \tau_{mj})$ is a lower bound on the travel time from node i to node j. Hence $\beta \geqslant r_i + \min_{m \in H} (\alpha \tau_{im} + \tau_{mj})$. If node i is not a hub, then it is assigned to a hub, say h, and the travel time from node h to node j is at least $\min_{m \in H \setminus \{i\}} (\alpha \tau_{hm} + \tau_{mj})$. Hence $\beta \geqslant r_i + \tau_{ih} + \min_{m \in H \setminus \{i\}} (\alpha \tau_{hm} + \tau_{mj})$. □

Finally, we present valid inequalities that are based on lower bounds on travel times.

PROPOSITION 3. *For $i \in N$, $j \in N \setminus \{i\}$ and $l \in H \setminus \{i, j\}$, the inequality*

$$\beta \geqslant r_i + \sum_{h \in H} (\tau_{ih} + \alpha \tau_{hl}) x_{ih} + \tau_{lj} x_{jl} \quad (45)$$

is valid for F.

PROOF. Let $i \in N$, $j \in N \setminus \{i\}$, and $l \in H \setminus \{i, j\}$. Suppose node i assigned to hub node h and node j is assigned to node l. Then the travel time between node i and node j is $(\tau_{ih} + \alpha \tau_{hl}) + \tau_{lj}$, so inequality (45) is satisfied. If node j is not assigned to node l, then inequality (45) reduces to $\beta \geqslant r_i + \tau_{ih} + \alpha \tau_{hl}$, which is satisfied because $\tau_{ih} + \alpha \tau_{hl}$ is a lower bound on the travel time from node i to node l. □

5. Computational Results

In this section, we report our computational results with data from Turkey and the standard CAB data (O'Kelly 1987) with 25 nodes. Because our space is limited, we provide some of our results in an online companion. An electronic companion to this paper is available as part of the online version that at http://dx.doi.org/10.1287/opre.1120.1065.

All instances are solved using GAMS 22.5 and CPLEX 11.0.0 on an AMD Opteron 252 processor (2.6 GHz) with 2 GB of RAM operating under the system CentOS (Linux version 2.6.9-42.0.3.ELsmp). Default settings of the solver are used.

5.1. Data Sets

In the Turkey network, we aggregated the branch offices into 81 cities (see also Tan and Kara 2007 and Yaman 2009). Hubs can be chosen from a subset of 22 cities that are currently serving as hubs for our company. A map

with the 81 cities and the 22 candidate hub locations is available in Appendix A2 in the online companion. The company's estimations of time and cost savings in transportation among hub nodes due to the utilization of specialized trucks were 10% and 20%, respectively. Hence, in all our test instances with the Turkey network, parameter α is fixed to 0.9 and α^c is fixed to 0.8. The next-day delivery guarantee limit β is fixed at 2,040 minutes, which accounts for the time elapsed between opening hours at 8:00 and the next day's closing time at 18:00 hours. Because the typical working hours in the cargo sector are 8:00 through 18:00 (namely, 10 hours), the latest release time of the trucks γ is taken as 600 minutes. In the CAB data, we let $\alpha^c = 0.8$ and $\alpha = 0.8$. We scale the travel times by dividing the distances with 1.5 so that $\beta = 2,040$ is achievable for this data set as well. We take $\gamma = 600$ and use the original CAB data for the routing costs. A map for this data set is also available in Appendix A2.

5.2. Valid Inequalities

In Appendix A1 in the online companion, we present the results of our experiment with the valid inequalities. We solve the max flow RSHL with and without the valid inequalities with a time limit of one hour. We could not find optimal solutions for the instances with Turkey data and uniform arrival model in one hour of computation time without valid inequalities. With the inclusion of valid inequalities, the same instances were solved to optimality in less than a minute. We observed that using inequalities (41)–(44) resulted in considerable savings in the computation times. Inequalities (37) and (38) were also helpful for some of the instances. Adding inequalities (45) decreased the duality gaps and the number of nodes in the branch-and-cut trees but did not reduce the cpu times for the uniform arrival model. The results are mixed for the piecewise linear arrival model; all inequalities were useful in reducing the computation times for some instances.

We also solved the min cost RSHL model with our inequalities. As expected, the min cost model turned out to be computationally more demanding, sometimes requiring an hour to find an optimal solution.

5.3. The Trade-Off Between Service Guarantees and Routing Costs

In this section we shall attempt to answer the following key questions for our instances:
- What is the cost of providing a given level of service, and what is the corresponding network design?
- How much does the decision maker have to pay in order to improve the service level?

To this end we perform the following experiment. For each instance, we first solve the max flow RSHL using the model of §3.2 to compute the maximum amount of flow ν^* that can be delivered to its destination by the next day. Then we solve the min cost RSHL to compute the minimum routing cost to deliver the maximum amount ν^* of flow by

the next day. We also solve the classical single allocation p-hub median problem and refer to its optimal value as c^*. For the CAB instances, the longest delivery times in the optimal solutions of the p-hub median problem are longer than $\beta = 2{,}040$, which means that even if all trucks leave their origins at time zero, the latest delivery will be later than time 2,040. This implies that the optimal solution of the p-hub median problem is not feasible in our min cost model even if we set the flow bound $\nu = 0$. To find the flow delivered by the next day by the p-hub median network design, we fix the hub locations and allocations as given in the median solution, relax the nonnegativity of the release times, and maximize the amount of flow that originates at nodes with nonnegative release times and that is delivered to its destination by the next day. We refer to this as the "median flow."

5.3.1. Results for the CAB Data. For the CAB data, in Table 1, we report the optimal values and the locations of hubs in the optimal solutions for our min cost RSHL with $\nu = \nu^*$ and the p-hub median problem for both arrival patterns. For each problem, we report the percentage of the total flow delivered by the deadline in the column "% flow," and the location of hubs in the corresponding optimal solution in the column "hubs." For the min cost RSHL, the column "% increase in cost" gives the percentage increase in the total cost compared to c^*, the optimal value of the p-hub median problem.

For the uniform arrival, with two hubs, around 74% of the whole flow can be delivered by the next day over a network that is 14.2% more expensive than the p-hub median network. When the number of hubs is increased to five, almost 95% of the flow can be delivered within the time bound, but the percentage increase in the routing cost is around 22%. For the piecewise cargo arrival pattern, the percentage flow delivered by the next day ranges from around 52% to 82%, with 16% to 30% additional costs.

Table 1 reveals that for both arrival models, the amount of cargo that can be delivered by the next day with a p-hub median network is quite small when compared to the max flow values; for instance, 6.2% instead of 74.4% with two hubs, and 36.3% instead of 94.6% with five hubs in the uniform arrival model.

The geographical dispersions of the hubs in the optimal solutions of our min cost RSHL model are very different than those of the p-hub median solutions. This result is not surprising because the differences in the corresponding routing costs are also quite large. Also note that the same hubs are used for the uniform and piecewise arrival patterns for large p.

This preliminary analysis shows that for the CAB data, ignoring service quality can result in poor performance; whereas, imposing the requirement that the maximum amount of cargo receives the next-day delivery guarantee can be quite costly. To see the trade-off, we computed the minimum routing costs obtained when we imposed the constraint that at least a given percentage (named as % max flow bound) of the max flow value ν^* is delivered by the next day. For each percentage value, we report the percentage increase in the routing cost compared to c^* and the locations of hubs in the corresponding optimal solution in Table 2. We also plot the percentage increases in the routing costs against the percentages of max flow delivered by the next day for the uniform and piecewise linear arrival functions in Figures 2 and 3, respectively.

Here, we observe that requiring delivery times to be no more than βï¿½ brings an additional cost of 1.92% over c^* even if we do not impose any lower bound on the amount of cargo that is delivered by the next day ($\nu = 0$). With an allowance of 5% increase in the routing cost, we can deliver around 90% and 75% of ν^* next day for the uniform and piecewise linear arrival models, respectively. The increase in the cost is significantly higher for larger percentages of flow requirements.

Next, we investigate the steep increases in the routing cost in the plots of Figures 2 and 3. For the uniform demand arrival pattern, up to 81% of the max flow can be delivered by the next day using the same structured network with 1, 4, 8, 12, and 18 as hubs. There is a steep increase in the routing cost when we increase the percentage flow bound to 92% of the max flow. This is the first time that Seattle appears in the set of optimal hub locations. Seattle remains in the hub set in the optimal solutions with flow bounds greater than or equal to 92%. An interesting behavior is that of Pittsburgh. It becomes a hub for 90% flow bound, then is replaced with Cleveland in the 91% solution, becomes a hub again in 92%, remains as a hub until 99%, and is replaced by Philadelphia at 100%. The steepest increase in the cost occurs when we increase the flow bound from

Table 1. Max flow and min cost results for the CAB data.

Arrival	p	Min cost RSHL with $\nu = \nu^*$			p-hub median	
		% flow	% increase in cost	Hubs	% flow	Hubs
Uniform	2	74.4	14.2	8, 21	6.2	12, 20
	3	81.8	10.9	8, 13, 20	18.5	2, 4, 12
	4	89.4	30.8	19, 21, 22, 23	28.2	1, 4, 12, 18
	5	94.6	21.9	13, 18, 19, 22, 23	36.3	1, 4, 7, 12, 18
Piecewise	2	52.4	15.9	11, 25	1.6	12, 20
	3	58.4	22.0	8, 11, 25	5.7	2, 4, 12
	4	71.7	30.8	19, 21, 22, 23	10.1	1, 4, 12, 18
	5	82.1	21.9	13, 18, 19, 22, 23	18.9	1, 4, 7, 12, 18

Table 2. The effect of service guarantees on the routing costs for the CAB data with $p = 5$.

% max flow bound	Uniform arrival		Piecewise arrival	
	% increase in cost	Hub locations	% increase in cost	Hub locations
0	1.92	1, 4, 8, 12, 18	1.92	1, 4, 8, 12, 18
60	1.92	1, 4, 8, 12, 18	1.92	1, 4, 8, 12, 18
65	1.92	1, 4, 8, 12, 18	2.15	1, 4, 7, 8, 18
70	1.92	1, 4, 8, 12, 18	3.24	1, 2, 4, 7, 8
75	1.92	1, 4, 8, 12, 18	4.96	4, 8, 12, 13, 17
80	1.92	1, 4, 8, 12, 18	9.40	2, 4, 19, 22, 23
85	2.15	1, 4, 7, 8, 18	10.39	12, 20, 21, 22, 23
90	5.19	8, 17, 20, 21, 24	10.71	12, 20, 21, 22, 23
95	13.14	19, 20, 21, 22, 23	13.12	12, 20, 21, 22, 23
100	21.86	13, 18, 19, 22, 23	21.86	13, 18, 19, 22, 23

99% to 100% even though there is a single hub exchange (Pittsburgh and Philadelphia).

Similar observations also hold for the piecewise arrival pattern. As can be seen from Figure 3, the two largest cost increases are for the last 2% of the flow. Similar to the uniform arrival case, Seattle and San Francisco are not among the selected hubs for smaller flow bounds. Seattle becomes a hub at 78% and San Francisco at 79%, and they remain hubs thereafter.

For both arrival patterns, the effect of the last percent of the flow on the increase in cost is significant.

In summary, we can say that the p-hub median solutions perform poorly in terms of service guarantees for the CAB data, and a considerable amount of flow can be delivered by the next day at small additional cost by designing the network and scheduling the release times accordingly.

5.3.2. Results for the Turkey Data. We report the results for the Turkey data with uniform arrival model and $p \in \{5, 10, 15, 20, 22\}$ in Table 3. We observe that with five hubs, the maximum amount of flow that we can deliver by the next day is about 75%, and this brings a cost increase of around 16% over the median cost value c^*. For 20 and 22 hubs, 79.2% of total flow can be delivered by the next day with around 4% more cost than the min cost value. For the Turkey data, the p-hub median network can deliver a good amount of flow by the next day. This result is in contrast with the one for the CAB data.

As a second experiment, we set the number of hubs p to 22 and compute the min cost RSHL optimal values for different percentage flow bounds. The results show that we can deliver up to 98% of the max flow value at a routing cost equal to c^*. To deliver 99% of the max flow value by the next day, we need to pay an additional cost of 0.2% and 0.3%; however, the max flow value is reached at an additional cost of around 4% and 2.3% for the uniform and piecewise linear arrival models, respectively.

In this trade-off analysis, we obtained different results for the CAB and Turkey data. The p-hub median solutions performed poorly in terms of next-day delivery for the CAB data, whereas their performances for the Turkey data were not as poor. This might be because the CAB data set might be driven by the relative scarcity of nodes and the relative remoteness of some cities. Still, for both data sets it was possible to improve the next day delivery quantities by scheduling release times appropriately with not much additional cost.

5.4. Discrete Release Times

Here, we consider the situation where the release times are restricted to belong to a discrete set. We experiment with four potential release times, i.e., $R_i = \{0, 240, 480, 600\}$. When we force the release times to take on one of the above mentioned values, the percentage of the total amount of cargo receiving the next day delivery in the CAB data
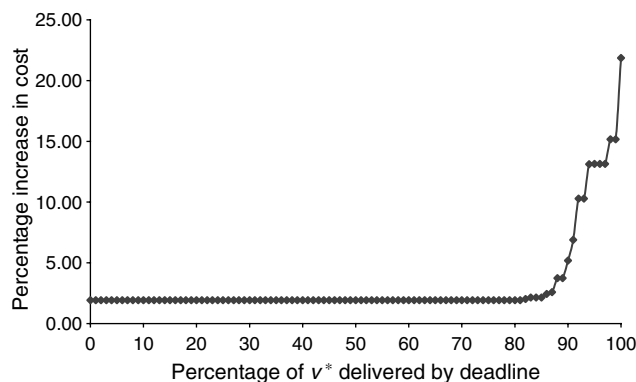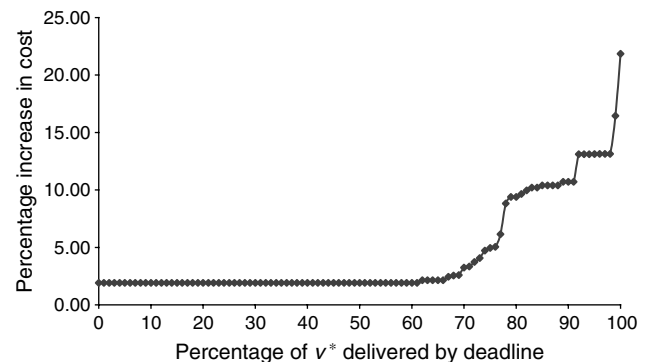
Figure 2. Trade-off chart for $p = 5$, uniform arrivals.

Figure 3. Trade-off chart for $p = 5$, piecewise arrivals.

Table 3. Max flow and min cost results for the Turkey data with uniform arrival.

	Min cost RSHL with $\nu = \nu^*$		p-hub median
p	% flow	% increase in cost	% flow
5	74.9	15.9	59.8
10	78.1	20.0	72.4
15	78.9	19.8	76.6
20	79.2	4.1	77.6
22	79.2	4.0	77.6

set for $p = 3$ decreased from 58.4% to 49.6% and in the Turkey data for $p = 22$ from 58.7% to 51.0%.

In our second experiment, to account for the effect of the announcement of release times on the arrival pattern, we have incremented the arrival rates for the first two intervals of the piecewise arrival pattern by 5%. In other words, the new arrival rates are estimated as 15%, 25%, and 60% for the three intervals. Even in this conservative estimation of the influence of the announcement of release times, the percentages moved up to 52.5% and 54.2% for the CAB and Turkey networks, respectively. Considering the managerial advantages of common release times, the above solutions, although with worse max flow values, could be nice alternatives for the decision makers.

5.5. Multiple Service Levels

Requiring every cargo package received by the release time to reach its destination, wherever it might be, by the next day is a very compelling requirement. To guarantee the next-day service between extreme origin destination pairs, the maximum flow model might force early release times. The overall service might be improved by eliminating this compelling requirement from some destination centers. To make this analysis, we use two service level requirements, namely, $\beta^1 = 2{,}040$ (34 hours) and $\beta^2 = 3{,}120$ (52 hours), which correspond to next-day delivery and second-day delivery by noon, respectively. Table 4 depicts these results for the CAB data under uniform arrival.

By an adaptation of our multiple service level RSHL model, we find the maximum amount of cargo reaching its destination by time β^1 under the constraint that every cargo demand that is received by the release time reaches its destination the latest by time β^2. The percentage of cargo receiving the strict delivery service (delivered by time β^1) is listed under the % flow column in Table 4. Using these flow values as minimum requirements in the

Table 4. Results for the CAB data with uniform arrival and multiple service levels.

p	% flow	Hub locations	Excluded centers	% increase in cost
2	84.0	12, 21	19, 22, 23	15.1
3	87.5	13, 17,19	22, 23	19.3
4	92.2	13, 17, 19, 22	23	24.4
5	95.7	11, 12, 17, 22, 24	23	32.1

multiple service level RSHL model, we find the minimum cost network design that delivers this amount of flow by the next day. The hub locations and the demand centers that do not receive the next-day service (excluded centers) are also provided in Table 4. Finally, the last column in this table corresponds to the % increase in cost of this design over the minimum cost c^*.

A comparison of Tables 1 and 4 reveals quite intuitive results. In all the instances, it is possible to exclude Seattle (23) and improve the amount of flow delivered by the next day. When two hubs are to be chosen, this improvement can be as large as 10% at an additional cost of less than 1%. The excluded cities are the three west coast cities that originate the least amount of flow. To improve the flow, Los Angeles (12)—the third-largest flow-originating city, is replacing a more central city, Denver (8), in the list of hub locations and thus resulting in an additional cost. When three hubs are located, excluding only Seattle and San Francisco improves the flow by 6% at an additional cost of 9%. The two hub locations Denver and Pittsburgh (20) are replaced with New York (17) and Phoenix (19) to accommodate more of the large flows originating from LA and NY. The results for $p = 4$ are appealing. In this design, not only is the flow increased by 3% but also the cost is decreased by 6%, and all this is realized by excluding only Seattle's service. Finally, the additional improvement of 1% in the flow for $p = 5$ costs about 10% more.

5.6. Release Times

We conclude our discussions with an analysis of the solutions for the CAB and Turkey data to see the relationship between the release times and the geographic dispersion of the demand nodes. In our maps, we use the solutions that deliver the maximum amount of flow by the next day at minimum cost. The nodes with release times in the intervals [8:00–12:00), [12:00–16:00), and [16:00–18:00) are denoted with circles, squares, and triangles, respectively. The pentagons correspond to nodes with release times equal to 18:00. The hub nodes are circled. We report only the results for the uniform arrival model because the results for the piecewise linear arrival model were similar.

In Figure 4, we give the map for the CAB data with $p = 4$. The maps for $p = 2, 3, 5$ are available in

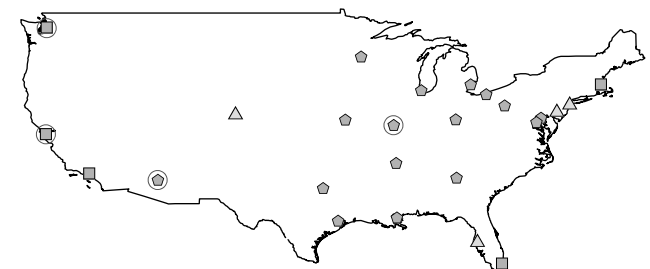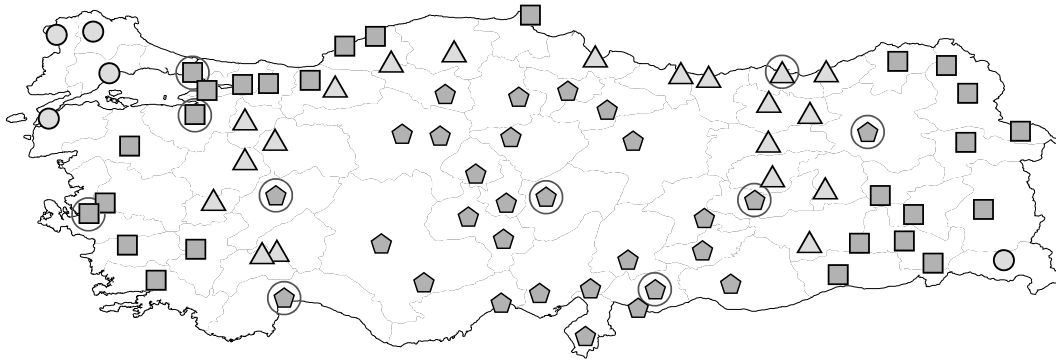**Figure 4.** The solution for the uniform arrival model and the CAB data with four hubs.

Figure 5. The solution for the uniform arrival model and the Turkey data with 10 hubs.

Appendix A2 in the online companion. Here, we observe that the release times of nodes that are located centrally or in the east are mostly later than the release times of those located on the west coast. This is mainly a result of the skewed pattern of the demand data and the geographic locations of nodes. The release times depend both on the location and the magnitude of flow. Also, the release times tend to increase with increasing p values as the number of links with reduced travel times increases.

In Figure 5, we depict the solution for the Turkey data with 10 hubs. The maps of the solutions with 5, 15, and 20 hubs are available in Appendix A2 in the online companion. In these maps, we observe that the demand centers located in the middle parts of Turkey tend to have the latest possible release times corresponding to the closing hours of 18:00. Moving a little bit toward east, west, and north directions from these central nodes, we locate our triangles corresponding to the evening time release times. The squares that are dispersed farther east and west in Turkey correspond to the afternoon category between noon and 16:00. Finally, in the southeast and northwest regions of Turkey lie the circles corresponding to cities with earliest (morning) release times.

For both data sets, we observe that there are concentric circular geographical bands for release times. Given a hub network, let l_{ij} be the length of the path from node i to node j. Then we know that $r_i + l_{ij} \leqslant \beta$ in any feasible solution. Hence, for any demand point i, its longest trip influences the geographic band it belongs to. In the Turkey data, nodes with the latest release times are located in the central part of the country, whereas in the CAB data, these nodes are skewed toward the part of the country that generates more demand. Such a difference is not surprising because the demand nodes are spread rather uniformly in the Turkey data and those in the CAB data are located mostly in the eastern part of the United States.

6. Conclusion

The current study introduces new hub location problems that have emerged from a cargo delivery company. In these problems, we relax the assumption that trucks leave their demand centers for their allocated hubs at the same

hour, typically the closing hour of the centers. We propose models that decide on the locations of hubs, the allocations of demand centers to hubs, and the release times of trucks simultaneously to improve service quality at minimum cost. We present several families of valid inequalities that enable us to solve the Turkey instances with 81 nodes in reasonable times and perform a trade-off analysis.

Our results for the CAB data and the Turkey data show that the network designs with only cost or service level considerations behave poorly for the left-out objective and that significant improvements are possible if we consider both objectives. Our methodology provides the decision makers with a tool to observe the trade-off between the service quality and the cost.

The current study has limitations resulting from the complete hub network, single allocation, and flows between all pairs assumptions. The role of time zones in the network designs and the effect of travel times on the profit are ignored. All these limitations were present in the motivating cargo application. They are also common in the existing literature. A few exceptions are Hall (1989), incorporating the role of time zones in the designs; Yaman et al. (2007), allowing stopovers; and Alumur et al. (2009), designing incomplete hub networks. Removal of some or all of these assumptions will render more realistic and more challenging future research directions.

Electronic Companion

An electronic companion to this paper is available as part of the online version at http://dx.doi.org/10.1287/opre.1120.1065.

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