Sequencing forklifts operations for enhancing storage assignment and order picking in drive-through racking

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1 Introduction

In many areas of the world, rise of land costs near urban areas have made warehouse buildings increasingly expensive. Particularly for products that which have to be refrigerated or frozen, it is important to have compact storage systems, given that can lead to less lower energy consumption and they provide a solution to storage density problems by storing products closer together without providing a direct access to each single item. Such benefit is possible due to the compaction of multi-deep racks, which results in warehouse layouts with less aisles.

Compact storage systems, in combination with multiple technologies, has led to the advent of completely automated warehouses capable to meet requirements where typically has to retrieval small orders from a large assortment under great time pressure, having to flexibly adjust order fulfillment processes to varying workloads. This case is especially true in the business-to-consumer segment (B2C) of e-commerce, where the deliveries are next-day or even same-day (Yaman et al., 2012). This puts increasing stress on warehouse operations and leads to highly time critical order fulfillment processes. The tight delivery schedules can be managed with intelligent planning approaches in compact storage systems to ensure acceptably fast retrieval times and could even be an alternative for online retailers that offers premium delivery services within the next few hours directly in the city centers (Boysen et al., 2019).

According to the authors on Azadeh et al. (2019), in Western Europe alone, about 40 fully automated warehouses are in operation, and many are under development. In addition to these fully automated warehouses, many partially robotized warehouses have been built. According to Buck Consultants International (2017), in the Netherlands alone, 63 large new warehouses were constructed in the period 2012–2016 using robot technologies.

To boost capacity and productivity in the warehouse, there are different levels of automation in deep-lane systems. The choice of handling equipment will depend on the number of entry and exit movements, the number of SKUs and the quantity of pallets per SKU or batch.

Shuttle-based systems with transfer cars and lift. Generally, two high-density racking units are installed, one on each side of the working aisle. The lifts carry out the vertical movements moving unit loads across tiers, the shuttles carry out the horizontal movements within the storage lanes moving underneath the unit loads and the orthogonal to the storage lanes movements can be performed by a transfer car (to move shuttles between the lanes). The lift can be a continuous or a discrete elevator. The main difference between these two lift types is the number of unit loads that can be handled simultaneously: a continuous elevator is similar to a conveyor and can move multiple unit loads simultaneously, whereas a discrete elevator allows only one unit load to be transferred simultaneously.

Shuttle-based systems with stacker crane. In this system, the stacker crane handles the rack's horizontal and vertical movement, in conjunction with an orthogonal transport mechanism (pallet shuttle) that handles depth movements. The stacker crane moves the pallet shuttle within a lane to store or retrieve the loads, while waits at the front until the pallet shuttle returns.

shuttle-based systems with forklifts. Also, semi-automated systems exist in which shuttles are moved between lanes by manually operated forklifts. Such systems are more flexible than fully automated shuttle-based systems, as multiple forklifts may work in parallel in the same space. In such a forklift-based system, a forklift fetches a shuttle and drives to a particular lane where a load needs to be retrieved. At the retrieval lane, the forklift drops off the shuttle, which drives autonomously on a rail underneath the loads. It lifts the load in the storage lane and brings it to the front end of the storage lane. There, the forklift can pick up the load with the shuttle, or the load without the shuttle, so that the shuttle is available to retrieve another load in that lane. If a system has fewer shuttles than storage lanes, the forklift moves the shuttles between the lanes.

Fully automated shuttle-based systems are similar to semi-automated shuttle-based systems with forklifts. The main difference between these systems is that a forklift can transfer any of the three units: loaded shuttles, only loads, or empty shuttles. However, fully automated shuttle-based systems cannot pick up a load without a shuttle.

In recent years, the application of autonomous shuttles, operating together with forklifts or transfer cars, has become more popular due to dropping prices of the shuttles, the flexibility offered, and a large choice of suppliers offering such systems. Nonetheless, this investment is still large for many owners, so traditional systems continue to be the best alternative for small distribution centers. Some current estimates suggest that up to 80% of warehouses perform their order picking operations manually (Grosse et al., 2017).

Traditional pallet racking. It is comprised of multiple racks with a series of lanes accessed by forklifts to deposit or retrieve pallets. It's the simplest high-density storage system and requires the lowest investment, hence it's commonly found in smaller warehouses where the daily picked volume is lower. In this system, typically the operations are carried out from a single working aisle to minimize the space of the chamber (drive-in racking) , accordingly, the forklift enters the rack storage lane with the pallet raised above the level on which it will be placed. The pallet is placed in the deepest free location, beginning at floor level and moving upwards until reaching the highest tier, while in the unloading process follows the reverse order of loading. The highest pallet is removed first, finishing with the pallet at floor level. Thus, stock is managed following the LIFO (last in, first out) principle.

The principal disadvantages comparing to automated systems is that more manoeuvring time is required for storage and retrieval operations. Furthermore, as commonly a dedicated storage policy is used (each storage lane is reserved for a single SKU), drive-in pallet racking is not recommended for warehouses that manage a broad array of products. However, use a shared storage policy its possible, but increases the risk of generating reshuffles in order picking, lowering the throughput capacity. Reshuffling is defined as the process of removing the unit loads blocking the exit of a unit load that needs to be retrieved (Buckow et al., 2023).

To improve this situation, the called drive-through racking (layaout with two aisles) allows operate in the front and from the back to gain access to the pallets and mitigate reshuffles. The forklifts, commonly loaded through the front aisle and unloaded through the rear aisle. Thus, the loading/unloading sequence is done in line with the FIFO method (first in, first out). This configuration is typically employed when the system is used as a buffer or interim warehouse to effectively regulate flows between two work areas (for example, between production and dispatch or between different manufacturing stages).

To fully benefit from such systems, it is important to schedule the forklifts to store or retrieve the loads. The throughput capacity depends not only on the number of forklifts but also on how they storage and retrievals sequences are planning. Proper deployment may save investment by lead to a reduction in makespan (total completion time). It is important to note that a reduction in makespan ensures an overall good performance of the system. When a truck arrives at the company for picking up products, fast retrieval of customer orders reduces the truck's waiting time. This means that the truck is more likely to be loaded within its expected arrival and departure time-frame. Furthermore, a possible reduction in each truck's waiting time makes possible to serve more trucks during a shift, thereby increasing the overall performance and utilisation of the system.

On the other hand, in deep-freeze warehouses context, it's also a good idea minimize the makespan, given that decreases the total time people have to work in cold circumstances and the open door times, allowing increase worker productivity, control the temperature, reduces the risk of loss of quality of the foodstuff and save energy costs.

This situation shows that an efficient operation in the storage and warehousing processes is crucial for a cost effective supply chain operation and also for an adequate procurement of quality standards. Therefore, developing tools for optimising storage and the retrieval operations appears as a fundamental task for improving the supply chain competitiveness. Such tools are particularly relevant for the case of DTPR systems, where the human factor plays an important role in their performance.

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EXPLICACIÓN DE PROBLEMA

We study this important problem in the multi-deep shuttle system. To do this, we assume a dedicated storage policy in which every lane stores only a single product. This policy is particularly used in production and distribution warehouses storing bulk products, such as fresh produce, general ambient and frozen food products, vaccines, and packaging materials. As the shuttles are battery-powered, they need charging. We assume this is done offline, outside regular working hours. We also assume a single storage level in our base model, but extending the model to multiple levels is straightforward. We focus on retrievals rather than storage requests because these job types are typically split in deep-lane systems. Storage requests result from receiving a production batch or inbound truckload, and the loads must be stored in a few deep lanes. Retrieval requests come with time pressure and require high responsiveness, so optimal shuttle scheduling is important (Boysen, De Koster, Weidinger, 2019; Zou, Xu, De Koster et al., 2016).

Given a set of retrieval requests and a number of shuttles, we aim to schedule forklifts (or transfer cars) and shuttles such that the makespan is minimized. Specifically, the following research questions are addressed:

• In what ways can we determine the optimal choices for (1) the sequence in which the shuttles are released in the lanes of retrieval requests by the forklift (or transfer car), (2) the sequence in which the retrieval requests are processed, and (3) the decision whether a shuttle is dropped off at the I/O point or kept in the lane for another request? • What improvements can be achieved by this paper's proposed method compared to straightforward heuristics used in practice and in the literature.

During the storage process, it is crucial to ensure there are no empty spaces between stored pallets to maximize space utilization. In the retrieval process, the operator aims to minimize the number of reorganizations. This means a pallet needs to be reorganized when it meets three conditions: (i) it's in the same row, (ii) at an equal or lower level, and (iii) at a greater depth compared to the pallet to be retrieved. Access to a pallet is hindered not only by pallets in front of the loading unit that needs to be retrieved but also by pallets stored at lower levels in the same row. This occurs because the lifting crane always needs to enter through the first level and raise the fork to gain access to higher levels.

The problem of finding an optimal storage and retrieval sequence that minimizes the number of reorganizations is referred to as the optimal drive-through loading management problem (DTPR). At each retrieval stage, there are multiple cranes that remove the pallets that need to be retrieved from the chamber. Each of these fork cranes is assigned to a row and a side from which they need to retrieve the pallets. A crane can be assigned to multiple sides, ensuring there is no possibility of collision between cranes when one retrieves a pallet from one side and another crane retrieves a pallet from the opposite side. The objective is minimize the maximum time that the fork cranes use in a retrieval process. This is important because normally a truck is waiting to the process end. Other important objective is the minimization of (...), because is most cost.

The scheduling problems with the forklift and the transfer car are denoted as the Forklift-Shuttle Scheduling (FSS) problem and the Transfer car-Shuttle-Scheduling (TSS) problem. Note that the case of scheduling transfer cars and shuttles is a special case of FSS since when using a transfer car, the shuttle must always be dropped off at the I/O point, as the transfer car cannot leave the shuttle in the lane and move only the load. Therefore, this paper studies scheduling retrievals with a focus on the FSS problem with the forklift, where the shuttle should be either kept in the lane or returned to the I/O point.

ESTUDIOS SIMILARES Y DIFERENCIAS

Surprisingly, this scheduling problem in multi-deep systems has received little academic attention, though the literature on scheduling decisions in single-deep automated systems is abundant . We are aware of only one paper by Dong, Jin, Wang, Kelle (2021) studying a scheduling problem in a multi-deep crane-based AS/RS. They assume that the crane picks up only the load from the shuttle and leaves the shuttle in the lane. This operation differs from our system, since the forklift can pick up only the load or both the load and the shuttle in our paper.

CONTRIBUCIÓN

The contribution of this research is twofold. First, we formulate a novel mathematical model that can be used to obtain the optimal shuttle transfer and load retrieval sequence and the resulting makespan for a set of retrieval requests We show that the problem is NP-hard, and to solve it, we propose an efficient twostage heuristic to produce near-optimal solutions, even for large instances. The model can also be used to decide on the optimal shuttle fleet size, minimizing the total cost. Second, we provide managerial insights into the use of these systems based on realcase data. We show that our two-stage heuristic can decrease the makespan substantially compared with straightforward heuristics used in practice and in the literature.

2 Mathematical model

Next, the mathematical model to solve the problem is presented. In the Table (...) the set and parameters definitions is presented.

Table 1: definition of the sets

set	definition
I	set of pallets
B	set of rows
T	set of tiers
P	set of depths
S	set of storage and retrieval stages
I^D	set of pallet must to be retrieval in a stage of the set S
K	set of forklift
$\vec{\Psi}_{tp}$	set of locations (τ,ρ) that would be blocked by front entry when placing a pallet at location (t,p), where $t \in T, p \in P$
$\overleftarrow{\Psi}_{tp}$	set of locations (τ,ρ) that would be blocked by back entry when placing a pallet at location (t,p), where $t \in T, p \in P$
$\vec{R}_{tp}$	set of locations (τ,ρ) that would be reshuffled their pallets to retrieval a pallet of the location (t,p) by the front entry, where $t \in T, p \in P$
$\overleftarrow{R}_{tp}$	set of locations (τ,ρ) that would be reshuffled their pallets to retrieval a pallet of the location (t,p) by the back entry, where $t \in T, p \in P$
Φ_s	set of pallet stored in the stage s, where $s \in S$
Ω_s	set of pallet retrieval in the stage s, where $s \in S$

Table 2: definition of the parameters

parameter	definition
A_i	stage at which pallet i is storage, where $i \in I$
D_i	stage at which pallet i is retrieval, where $i \in I$
X^i_{btp}	location (b,t,p) of the pallet i at the first stage, where $i \in \Phi_0, b \in B, t \in T, p \in P$

$$\min \sum_{b \in B} \sum_{p \in P} \sum_{s \in S} \sum_{i \in \Phi_s \cap \Omega_s} \left(\vec{y}^{is}_{bp} + \overleftarrow{y}^{is}_{bp} \right) \tag{1}$$

$$\min \sum_{s \in S} F_s \tag{2}$$

subject to

$$\sum_{b \in B} \sum_{t \in T} \sum_{p \in P} x^{is}_{btp} = 1 \qquad , i \in I, s \in A_i..D_i \tag{3}$$

$$\sum_{i \in \Phi_s} x^{is}_{btp} \leq 1 \qquad , s \in S, b \in B, t \in T, p \in P \tag{4}$$

$$x^{i0}_{btp} = X^i_{btp} \qquad , i \in \Phi_0, b \in B, t \in T, p \in P \tag{5}$$

Table 3: definition of the decision variables

variable	definition
$x_{btp}^{is} \in \{0,1\}$	where $x_{btp}^{is} = 1$ if the pallet i in the stage s is storage in the location (b,t,p), $x_{btp}^{is} = 0$ otherwise, where $i \in I, s \in A_i..D_i, b \in B, t \in T, p \in P$
$\vec{y}_{bp}^{is} \in \{0,1\}$	where $\vec{y}_{bp}^{is} = 1$ if the pallet i is reshuffled when in the stage s a pallet is retrieval by the row b and depth p for the front entry, $\vec{y}_{bp}^{is} = 0$ otherwise, where $s \in S, i \in \Phi_s \cap \Omega_s, b \in B, p \in P$
$\overleftarrow{y}_{bp}^{is} \in \{0,1\}$	where $\overleftarrow{y}_{bp}^{is} = 1$ if the pallet i is reshuffled when in the stage s a pallet is retrieval by the row b and depth p for the back entry, $\overleftarrow{y}_{bp}^{is} = 0$ otherwise, where $s \in S, i \in \Phi_s \cap \Omega_s, b \in B, p \in P$
$\vec{\varphi}_{bp}^{jks} \in \{0,1\}$	where $\vec{\varphi}_{bp}^{jks} = 1$ if the pallet k is reshuffled when the pallet j is reshuffled in the stage s by the row b and depth p for the front entry, $\vec{\varphi}_{bp}^{jks} = 0$ otherwise, where $s \in S, j \in \Phi_s \cap \Omega_s, k \in \Phi_s \cap \Omega_s, b \in B, p \in P$
$\overleftarrow{\varphi}_{bp}^{jks} \in \{0,1\}$	where $\overleftarrow{\varphi}_{bp}^{jks} = 1$ if the pallet k is reshuffled when the pallet j is reshuffled in the stage s by the row b and depth p for the back entry, $\overleftarrow{\varphi}_{bp}^{jks} = 0$ otherwise, where $s \in S, j \in \Phi_s \cap \Omega_s, k \in \Phi_s \cap \Omega_s, b \in B, p \in P$
$\vec{w}_{kb}^{s} \in \{0,1\}$	where $\vec{w}_{kb}^{s} = 1$ if the forklift k reshuffled or retrieval pallet in the stage s by the front entry of the row b, $\vec{w}_{kb}^{s} = 0$ otherwise, where $k \in K, s \in S, b \in B$
$\overleftarrow{w}_{kb}^{s} \in \{0,1\}$	where $\overleftarrow{w}_{kb}^{s} = 1$ if the forklift k reshuffles or retrievals pallet in the stage s by the back entry of the row b, $\overleftarrow{w}_{kb}^{s} = 0$ otherwise, where $k \in K, s \in S, b \in B$
$\vec{z}_{bp}^{js} \in \{0,1\}$	where $\vec{z}_{bp}^{js} = 1$ if the pallet j reshuffled in the stage s is placed in the row b and depth p for the front entry, $\vec{z}_{bp}^{js} = 0$ otherwise, where $s \in S, j \in \Phi_s \cap \Omega_s, b \in B, p \in P$
$\overleftarrow{z}_{bp}^{js} \in \{0,1\}$	where $\overleftarrow{z}_{bp}^{js} = 1$ if the pallet j reshuffled in the stage s is placed in the row b and depth p for the back entry, $\overleftarrow{z}_{bp}^{js} = 0$ otherwise, where $s \in S, j \in \Phi_s \cap \Omega_s, b \in B, p \in P$
$\vec{U}_{kb}^{s} \in \mathbb{Z}^{+}$	number of pallet reshuffled or retrieved in the stage s by the forklift k for the front entry of the row b, where $k \in K, b \in B, s \in S$
$\overleftarrow{U}_{kb}^{s} \in \mathbb{Z}^{+}$	number of pallet reshuffled or retrieved in the stage s by the forklift k for the back entry of the row b, where $k \in K, b \in B, s \in S$
$F^{s} \in \mathbb{Z}^{+}$	maximum number of pallet reshuffled or retrieved for a forklift in the stage s, where $s \in S$
$\vec{G}_{is} \in \{0,1\}$	where $\vec{G}_{is} = 1$ if the pallet i is placed by the front entry when is reshuffled in the stage s, $\vec{G}_{is} = 0$ otherwise, where $s \in S, i \in \Phi_s \cap \Omega_s$
$\vec{C}_{bp}^{i} \in \{0,1\}$	where $\vec{C}_{bp}^{i} = 1$ if the pallet i is retrieval from the front entry of the row b and depth p, $\vec{C}_{bp}^{i} = 0$ otherwise, where $b \in B, p \in P, i \in I^{D}$
$\overleftarrow{C}_{bp}^{i} \in \{0,1\}$	where $\overleftarrow{C}_{bp}^{i} = 1$ if the pallet i is retrieval from the back entry of the row b and depth p, $\overleftarrow{C}_{bp}^{i} = 0$ otherwise, where $b \in B, p \in P, i \in I^{D}$

$$\sum_{i\in\Phi_s} x^{is}_{b\nu\varrho} \leq 0 + \left(1 - \sum_{i\in\Phi_s} x^{is}_{btp}\right) + \sum_{i\in\Phi_s} x^{is}_{b\tau\rho} \qquad , s\in S, b\in B, t\in T, p\in P, (\tau,\rho)\in \overrightarrow{\Psi}_{tp}, (\nu,\varrho)\in \overleftarrow{R}_{\tau\rho} \tag{6}$$

$$\sum_{i\in\Phi_s} x^{is}_{b\nu\varrho} \leq 0 + \left(1 - \sum_{i\in\Phi_s} x^{is}_{btp}\right) + \sum_{i\in\Phi_s} x^{is}_{b\tau\rho} \qquad , s\in S, b\in B, t\in T, p\in P, (\tau,\rho)\in \overleftarrow{\Psi}_{tp}, (\nu,\varrho)\in \overrightarrow{R}_{\tau\rho} \tag{7}$$

$$\overrightarrow{C}^i_{bp} + \overleftarrow{C}^i_{bp} \geq \sum_{t\in T} x^{iD_i}_{btp} \qquad , b\in B, p\in P, i\in I^D \tag{8}$$

$$\overrightarrow{y}^{js}_{b\rho} - \left(x^{is}_{btp} + x^{js}_{b\tau\rho}\right) \geq -2 + \overrightarrow{C}^i_{bp} \qquad , s\in S, b\in B, t\in T, p\in P, (\tau,\rho)\in \overrightarrow{R}_{tp}, i\in\Omega_s, j\in\Phi_s\cap\Omega_s \tag{9}$$

$$\overleftarrow{y}^{js}_{b\rho} - \left(x^{is}_{btp} + x^{js}_{b\tau\rho}\right) \geq -2 + \overleftarrow{C}^i_{bp} \qquad , s\in S, b\in B, t\in T, p\in P, (\tau,\rho)\in \overleftarrow{R}_{tp}, i\in\Omega_s, j\in\Phi_s\cap\Omega_s \tag{10}$$

$$\overrightarrow{\varphi}^{jks}_{b\rho} - \left(x^{js}_{btp} + x^{ks}_{b\tau\rho}\right) \geq -2 + \overrightarrow{y}^{js}_{bp} \qquad , s\in S, b\in B, t\in T, p\in P, (\tau,\rho)\in \overrightarrow{R}_{tp}, j\in\Phi_s\cap\Omega_s, k\in\Phi_s\cap\Omega_s : j\neq k \tag{11}$$

$$\overleftarrow{\varphi}^{jks}_{b\rho} - \left(x^{js}_{btp} + x^{ks}_{b\tau\rho}\right) \geq -2 + \overleftarrow{y}^{js}_{bp} \qquad , s\in S, b\in B, t\in T, p\in P, (\tau,\rho)\in \overleftarrow{R}_{tp}, j\in\Phi_s\cap\Omega_s, k\in\Phi_s\cap\Omega_s : j\neq k \tag{12}$$

$$\overrightarrow{\varphi}^{jks}_{bp} \leq \overrightarrow{y}^{ks}_{bp} \qquad , s\in S, b\in B, t\in T, j\in\Phi_s\cap\Omega_s, k\in\Phi_s\cap\Omega_s : j\neq k \tag{13}$$

$$\overleftarrow{\varphi}^{jks}_{bp} \leq \overleftarrow{y}^{ks}_{bp} \qquad , s\in S, b\in B, t\in T, j\in\Phi_s\cap\Omega_s, k\in\Phi_s\cap\Omega_s : j\neq k \tag{14}$$

$$\begin{aligned} x^{ks}_{b\tau\rho} \leq{} & \left(1 - x^{is+1}_{btp}\right) + \sum_{\beta\in B}\sum_{\varrho\in P}\left(\overrightarrow{y}^{ks}_{\beta\varrho} + \overleftarrow{y}^{ks}_{\beta\varrho}\right) + \\ & \left(1 - \sum_{\beta\in B}\sum_{\varrho\in P}\left(\overrightarrow{y}^{is}_{\beta\varrho} + \overleftarrow{y}^{is}_{\beta\varrho}\right)\right) + \overrightarrow{G}_{is} \end{aligned} \qquad , s\in S, i\in\Phi_s\cap\Omega_s, b\in B, t\in T, p\in P, k\in\Phi_s\cap\Omega_s, (\tau,\rho)\in \overrightarrow{R}_{tp} \tag{15}$$

$$\begin{aligned} x^{ks}_{b\tau\rho} \leq{} & \left(1 - x^{is+1}_{btp}\right) + \sum_{\beta\in B}\sum_{\varrho\in P}\left(\overrightarrow{y}^{ks}_{\beta\varrho} + \overleftarrow{y}^{ks}_{\beta\varrho}\right) + \\ & \left(1 - \sum_{\beta\in B}\sum_{\varrho\in P}\left(\overrightarrow{y}^{is}_{\beta\varrho} + \overleftarrow{y}^{is}_{\beta\varrho}\right)\right) + \left(1 - \overrightarrow{G}_{is}\right) \end{aligned} \qquad , s\in S, i\in\Phi_s\cap\Omega_s, b\in B, t\in T, p\in P, k\in\Phi_s\cap\Omega_s, (\tau,\rho)\in \overleftarrow{R}_{tp} \tag{16}$$

$$\overrightarrow{z}^{js}_{bp} \geq \sum_{\beta\in B}\sum_{\rho\in P}\overrightarrow{y}^{js}_{\beta\rho} + \sum_{\beta\in B}\sum_{\rho\in P}\overleftarrow{y}^{js}_{\beta\rho} + \sum_{t\in T} x^{js+1}_{btp} + \overrightarrow{G}_{js} - 2 \qquad , s\in S\setminus\{|S|\}, j\in\Phi_s\cap\Omega_s, b\in B, p\in P \tag{17}$$

$$\begin{aligned} \overleftarrow{z}^{js}_{bp} \geq{} & \sum_{\beta\in B}\sum_{\rho\in P}\overrightarrow{y}^{js}_{\beta\rho} + \sum_{\beta\in B}\sum_{\rho\in P}\overleftarrow{y}^{js}_{\beta\rho} + \\ & \sum_{t\in T} x^{js+1}_{btp} + \left(1 - \overrightarrow{G}_{js}\right) - 2 \end{aligned} \qquad , s\in S\setminus\{|S|\}, j\in\Phi_s\cap\Omega_s, b\in B, p\in P \tag{18}$$

$$\sum_{k\in K} \overrightarrow{w}^s_{kb} = 1 \qquad , s\in S, b\in B \tag{19}$$

$$\sum_{k\in K} \overleftarrow{w}^s_{kb} = 1 \qquad , s\in S, b\in B \tag{20}$$

$$\begin{aligned} \overrightarrow{U}^s_{kb} \geq{} & \sum_{j\in\Phi_s\cap\Omega_s}\sum_{p\in P}\overrightarrow{z}^{js}_{bp} + \sum_{i\in\Phi_s\cap\Omega_s}\sum_{p\in P}\overrightarrow{y}^{is}_{bp} + \\ & \sum_{i\in\Omega_s}\sum_{p\in P}\overrightarrow{C}^i_{bp} - 2|\Phi_s|\left(1 - \overrightarrow{w}^s_{kb}\right) \end{aligned} \qquad , k\in K, b\in B, s\in S \tag{21}$$

$$\begin{aligned} \overleftarrow{U}^s_{kb} \geq{} & \sum_{j\in\Phi_s\cap\Omega_s}\sum_{p\in P}\overleftarrow{z}^{js}_{bp} + \sum_{i\in\Phi_s\cap\Omega_s}\sum_{p\in P}\overleftarrow{y}^{is}_{bp} + \\ & \sum_{i\in\Omega_s}\sum_{p\in P}\overleftarrow{C}^i_{bp} - 2|\Phi_s|\left(1 - \overleftarrow{w}^s_{kb}\right) \end{aligned} \qquad , k\in K, b\in B, s\in S \tag{22}$$

$$F_s \geq \sum_{b\in B}\left(\overrightarrow{U}^s_{kb} + \overleftarrow{U}^s_{kb}\right) \qquad , k\in K, s\in S \tag{23}$$

$$\sum_{\rho\in P}\rho\overrightarrow{y}^{is}_{b\rho} \leq \sum_{\rho\in P}\rho\overleftarrow{y}^{js}_{b\rho} + 1 + |P|\left(2 - \sum_{\rho\in P}\left(\overrightarrow{y}^{is}_{b\rho} + \overleftarrow{y}^{js}_{b\rho}\right)\right) \qquad , b\in B, s\in S, i\in\Phi_s\cap\Omega_s, j\in\Phi_s\cap\Omega_s \tag{24}$$

$$\sum_{\rho\in P}\rho\overrightarrow{y}^{is}_{b\rho} \leq \sum_{\rho\in P}\rho\overleftarrow{C}^{j}_{b\rho} + 1 + |P|\left(2 - \sum_{\rho\in P}\left(\overrightarrow{y}^{is}_{b\rho} + \overleftarrow{C}^{j}_{b\rho}\right)\right) \qquad , b\in B, s\in S, i\in\Phi_s\cap\Omega_s, j\in\Omega_s \tag{25}$$

$$\sum_{\rho\in P}\rho\overrightarrow{C}^{j}_{b\rho} \leq \sum_{\rho\in P}\rho\overleftarrow{y}^{is}_{b\rho} + 1 + |P|\left(2 - \sum_{\rho\in P}\left(\overleftarrow{y}^{is}_{b\rho} + \overrightarrow{C}^{j}_{b\rho}\right)\right) \qquad , b\in B, s\in S, i\in\Phi_s\cap\Omega_s, j\in\Omega_s \tag{26}$$

$$\sum_{\rho\in P}\rho\overrightarrow{C}^{j}_{b\rho} \leq \sum_{\rho\in P}\rho\overleftarrow{C}^{i}_{b\rho} + 1 + |P|\left(2 - \sum_{\rho\in P}\left(\overleftarrow{C}^{i}_{b\rho} + \overrightarrow{C}^{j}_{b\rho}\right)\right) \qquad , b\in B, i\in\Omega_s, j\in\Omega_s \tag{27}$$

$$\sum_{\rho\in P}\rho\overrightarrow{z}^{is}_{b\rho} \leq \sum_{\rho\in P}\rho\overleftarrow{z}^{js}_{b\rho} + 1 + |P|\left(2 - \sum_{\rho\in P}\left(\overleftarrow{z}^{js}_{b\rho} + \overrightarrow{z}^{is}_{b\rho}\right)\right) \qquad , b\in B, i\in\Phi_s\cap\Omega_s, j\in\Phi_s\cap\Omega_s \tag{28}$$

$$x^{is}_{btp} - x^{is+1}_{btp} \geq \sum_{\beta\in B}\sum_{\rho\in P} - \left(\overrightarrow{y}^{\beta\rho}_{is} + \overleftarrow{y}^{\beta\rho}_{is}\right) \qquad , i\in I, s\in A_i..D_i - 1, b\in B, t\in T, p\in P \tag{29}$$

$$x^{is}_{btp} - x^{is+1}_{btp} \leq \sum_{\beta\in B}\sum_{\rho\in P}\left(\overrightarrow{y}^{\beta\rho}_{is} + \overleftarrow{y}^{\beta\rho}_{is}\right) \qquad , i\in I, s\in A_i..D_i - 1, b\in B, t\in T, p\in P \tag{30}$$

$$x^{is}_{btp} \in \{0,1\} \qquad , i\in I, s\in A_i..D_i, b\in B, t\in T, p\in P \tag{31}$$

$$\overrightarrow{y}^{is}_{bp}, \overleftarrow{y}^{is}_{bp} \in \{0,1\} \qquad , s\in S, i\in\Phi_s\cap\Omega_s, b\in B, p\in P \tag{32}$$

$$\overrightarrow{\varphi}^{jks}_{bp}, \overleftarrow{\varphi}^{jks}_{bp} \in \{0,1\} \qquad , s\in S, j\in\Phi_s\cap\Omega_s, k\in\Phi_s\cap\Omega_s, b\in B, p\in P \tag{33}$$

$$\overrightarrow{w}^{s}_{kb}, \overleftarrow{w}^{s}_{kb} \in \{0,1\} \qquad , k\in K, s\in S, b\in B \tag{34}$$

$$\overrightarrow{z}^{js}_{bp}, \overleftarrow{z}^{js}_{bp} \in \{0,1\} \qquad , s\in S, j\in\Phi_s\cap\Omega_s, b\in B, p\in P \tag{35}$$

$$\overrightarrow{c}^{i}_{bp}, \overleftarrow{c}^{i}_{bp} \in \{0,1\} \qquad , i\in I^D, p\in P, b\in B \tag{36}$$

$$\overrightarrow{G}_{is} \in \{0,1\} \qquad , s\in S, i\in\Phi_s\cap\Omega_s \tag{37}$$

$$\overrightarrow{U}^{s}_{kb}, \overleftarrow{U}^{s}_{kb} \in Z^+ \qquad , s\in S, b\in B, k\in K \tag{38}$$

$$F_s \in Z^+ \qquad , s\in S \tag{39}$$

The objective function (1) minimize the number of the reshuffled carried out by the forklifts. The objective function (2) minimize the maximum number of movement carried out by a forklift. The constraint (3) guaranteed that when a pallet is storage in a location of the chamber. On the other hand the constraint (4) guaranteed that each location of the chamber storage no more than one pallet. The constraint (5) assure that the in the stage zero, the pallets are located in the initial location. The constraints (6-7) establish that all empty locations of the storage are accessible by the front or back entry. The constraint (8) assure that the pallet must be retrieved by the front or back entry. The constraints (9 - 10) obtain the pallets that must to be reshuffled for the front and back entry to retrieve the pallets. The constraints (11 - 14) obtain the pallet that must to be reshuffled when other pallets are reshuffled. The constraints (15 - 16) assure to storage the pallets reshuffled no other reshuffled have to be carried out. The constraints (17-18) obtain the row and depth for the pallet reshuffled is storage. The constraints (19-20) assure that each row for the front and back entry have a forklift assign. The constraints (21-22) calculate the number of movement carried out for every forklift in each row. On the other hand, the constraint (23) calculate the maximum number of movement carried out for a forklift. The constraints (24-28) assure that the forklift do not collide in a row. The constraints (29-30) assure that a pallet do not change its location if it is not reshuffled. Constraints (31-39) are about to nature of the decision variables.

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