Sequencing forklift trucks for enhancing storage assignment and order picking in drive-through pallet racking

Javier Gómez-Lagos David Revillot-Narváez

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1 Introduction

2 Problem descriptions and mathematical model

The compact storage system is made up of a set of shelves forming inner loading aisles, with support lanes for pallets. Storage and retrieval processes are carried out manually using lifting cranes, which need to enter the lanes from the front or the back. This method is known as the "drive-through" storage management system (DTPR), illustrated in Figure 1. This system is commonly used for First In, First Out (FIFO) strategies.

In this situation, pallet retrieval is not a simple process, as it often requires reorganization. Reorganization is defined as the process of moving loading units that block the exit of a loading unit that needs to be retrieved. This system is widely used in refrigeration and freezing chambers in the food industry, where optimal use of controlled temperature storage space is necessary.

During the storage process, it is crucial to ensure there are no empty spaces between stored pallets to maximize space utilization. In the retrieval process, the operator aims to minimize the number of reorganizations. This means a pallet needs to be reorganized when it meets three conditions: (i) it's in the same row, (ii) at an equal or lower level, and (iii) at a greater depth compared to the pallet to be retrieved. Access to a pallet is hindered not only by pallets in front of the loading unit that needs to be retrieved but also by pallets stored at lower levels in the same row. This occurs because the lifting crane always needs to enter through the first level and raise the fork to gain access to higher levels.

The problem of finding an optimal storage and retrieval sequence that minimizes the number of reorganizations is referred to as the optimal drive-through loading management problem (DTPR). At each retrieval stage, there are multiple cranes that remove the pallets that need to be retrieved from the chamber. Each of these fork cranes is assigned to a row and a side from which they need to retrieve the pallets. A crane can be assigned to multiple sides, ensuring there is no possibility of collision between cranes when one retrieves a pallet from one side and another crane retrieves a pallet from the opposite side. The objective is minimize the maximum time that the fork cranes use in a retrieval process. This is important because normally a truck is waiting to the process end. Other important objective is the minimization of (...), because is most cost.

Next, the mathematical model to solve this problem is presented. In the Table (...) the set and parameters definitions is presented.

$$\min \sum_{b\in B}\sum_{s\in S}\sum_{j\in\Phi_s\cap\Omega_s}\left(\overrightarrow{y}^{is}_{bp}+\overleftarrow{y}^{is}_{bp}\right) \tag{1}$$

$$\min \sum_{s\in S} F_s \tag{2}$$

subject to

$$\sum_{b\in B}\sum_{t\in T}\sum_{p\in P} x^{is}_{btp} = 1 \qquad , i\in I, s\in A_i..D_i \tag{3}$$

Table 1: definition of the sets

set	definition
I	set of pallets
B	set of rows
T	set of tiers
P	set of depths
S	set of storage and retrieval stages
I^D	set of pallet must to be retrieval in a stage of the set S
K	set of forklift
$\vec{\Psi}_{tp}$	set of locations (τ, ρ) that would be blocked by front entry when placing a pallet at location (t, p), where $t \in T, p \in P$
$\overleftarrow{\Psi}_{tp}$	set of locations (τ, ρ) that would be blocked by back entry when placing a pallet at location (t, p), where $t \in T, p \in P$
$\vec{R}_{tp}$	set of locations (τ, ρ) that would be reshuffled their pallets to retrieval a pallet of the location (t, p) by the front entry, where $t \in T, p \in P$
$\overleftarrow{R}_{tp}$	set of locations (τ, ρ) that would be reshuffled their pallets to retrieval a pallet of the location (t, p) by the back entry, where $t \in T, p \in P$
Φ_s	set of pallet stored in the stage s, where $s \in S$
Ω_s	set of pallet retrieval in the stage s, where $s \in S$

Table 2: definition of the parameters

parameter	definition
A_i	stage at which pallet i is storage, where $i \in I$
D_i	stage at which pallet i is retrieval, where $i \in I$
X^i_{btp}	location (b, t, p) of the pallet i at the first stage, where $i \in \Phi_0, b \in B, t \in T, p \in P$

$$\sum_{i \in \Phi_s} x^{is}_{btp} \leq 1 \qquad , s \in S, b \in B, t \in T, p \in P \tag{4}$$

$$x^{i0}_{btp} = X^i_{btp} \qquad , i \in \Phi_0, b \in B, t \in T, p \in P \tag{5}$$

$$x^{\iota s}_{b\nu\varrho} \leq 0 + \left(1 - x^{is}_{btp}\right) + x^{is}_{b\tau\rho} \qquad , i \in I, s \in A_i..D_i, \iota \in \Phi_s, b \in B, t \in T, p \in P, (\tau, \rho) \in \vec{\Psi}_{tp}, (\nu, \varrho) \in \overleftarrow{R}_{\tau\rho} \tag{6}$$

$$x^{\iota s}_{b\nu\varrho} \leq 0 + \left(1 - x^{is}_{btp}\right) + x^{is}_{b\tau\rho} \qquad , i \in I, s \in A_i..D_i, \iota \in \Phi_s, b \in B, t \in T, p \in P, (\tau, \rho) \in \overleftarrow{\Psi}_{tp}, (\nu, \varrho) \in \vec{R}_{\tau\rho} \tag{7}$$

$$\vec{C}^i_{bp} + \overleftarrow{C}^i_{bp} \geq \sum_{t \in T} x^{iD_i}_{btp} \qquad , b \in B, t \in T, i \in I^D \tag{8}$$

$$\vec{y}^{js}_{b\rho} - \left(x^{is}_{btp} + x^{js}_{b\tau\rho}\right) \geq -2 + \vec{C}^i_{bp} \qquad , s \in S, b \in B, t \in T, p \in P, (\tau, \rho) \in \vec{R}_{tp}, i \in \Omega_s, j \in \Phi_s \cap \Omega_s \tag{9}$$

$$\overleftarrow{y}^{js}_{b\rho} - \left(x^{is}_{btp} + x^{js}_{b\tau\rho}\right) \geq -1 - \overleftarrow{C}^i_{bp} \qquad , s \in S, b \in B, t \in T, p \in P, (\tau, \rho) \in \overleftarrow{R}_{tp}, i \in \Omega_s, j \in \Phi_s \cap \Omega_s \tag{10}$$

$$\vec{\varphi}^{jks}_{b\rho} - \left(x^{ks}_{btp} + x^{js}_{b\tau\rho}\right) \geq -2 + \vec{y}^{bp}_{js} \qquad , s \in S, b \in B, t \in T, p \in P, (\tau, \rho) \in \vec{R}_{tp}, j \in \Phi_s \cap \Omega_s, k \in \Phi_s \cap \Omega_s : j \neq k \tag{11}$$

$$\overleftarrow{\varphi}^{jks}_{b\rho} - \left(x^{ks}_{btp} + x^{js}_{b\tau\rho}\right) \geq -2 + \overleftarrow{y}^{bp}_{js} \qquad , s \in S, b \in B, t \in T, p \in P, (\tau, \rho) \in \overleftarrow{R}_{tp}, j \in \Phi_s \cap \Omega_s, k \in \Phi_s \cap \Omega_s : j \neq k \tag{12}$$

$$\vec{\varphi}^{jks}_{bp} \leq \vec{y}^{ks}_{p} \qquad , s \in S, b \in B, t \in T, j \in \Phi_s \cap \Omega_s, k \in \Phi_s \cap \Omega_s : j \neq k \tag{13}$$

$$\overleftarrow{\varphi}^{jks}_{bp} \leq \overleftarrow{y}^{ks}_{p} \qquad , s \in S, b \in B, t \in T, j \in \Phi_s \cap \Omega_s, k \in \Phi_s \cap \Omega_s : j \neq k \tag{14}$$

Table 3: definition of the decision variables

variable	definition
$x_{btp}^{is} \in \{0,1\}$	where $x_{btp}^{is} = 1$ if the pallet i in the stage s is storage in the location (b,t,p), $x_{btp}^{is} = 0$ otherwise, where $i \in I, s \in A_i..D_i, b \in B, t \in T, p \in P$
$\vec{y}_{bp}^{is} \in \{0,1\}$	where $\vec{y}_{bp}^{is} = 1$ if the pallet i is reshuffled when in the stage s a pallet is retrieval by the row b and depth p for the front entry, $\vec{y}_{bp}^{is} = 0$ otherwise, where $s \in S, i \in \Phi_s \cap \Omega_s, b \in B, p \in P$
$\overleftarrow{y}_{bp}^{is} \in \{0,1\}$	where $\overleftarrow{y}_{bp}^{is} = 1$ if the pallet i is reshuffled when in the stage s a pallet is retrieval by the row b and depth p for the back entry, $\overleftarrow{y}_{bp}^{is} = 0$ otherwise, where $s \in S, i \in \Phi_s \cap \Omega_s, b \in B, p \in P$
$\vec{\varphi}_{bp}^{jks} \in \{0,1\}$	where $\vec{\varphi}_{bp}^{jks} = 1$ if the pallet k is reshuffled when the pallet j is reshuffled in the stage s by the row b and depth p for the front entry, $\vec{\varphi}_{bp}^{jks} = 0$ otherwise, where $s \in S, j \in \Phi_s \cap \Omega_s, k \in \Phi_s \cap \Omega_s, b \in B, p \in P$
$\overleftarrow{\varphi}_{bp}^{jks} \in \{0,1\}$	where $\overleftarrow{\varphi}_{bp}^{jks} = 1$ if the pallet k is reshuffled when the pallet j is reshuffled in the stage s by the row b and depth p for the back entry, $\overleftarrow{\varphi}_{bp}^{jks} = 0$ otherwise, where $s \in S, j \in \Phi_s \cap \Omega_s, k \in \Phi_s \cap \Omega_s, b \in B, p \in P$
$\vec{w}_{kb}^{s} \in \{0,1\}$	where $\vec{w}_{kb}^{s} = 1$ if the forklift k reshuffled or retrieval pallet in the stage s by the front entry of the row b, $\vec{w}_{kb}^{s} = 0$ otherwise, where $k \in K, s \in S, b \in B$
$\overleftarrow{w}_{kb}^{s} \in \{0,1\}$	where $\overleftarrow{w}_{kb}^{s} = 1$ if the forklift k reshuffles or retrievals pallet in the stage s by the back entry of the row b, $\overleftarrow{w}_{kb}^{s} = 0$ otherwise, where $k \in K, s \in S, b \in B$
$\vec{z}_{bp}^{js} \in \{0,1\}$	where $\vec{z}_{bp}^{js} = 1$ if the pallet j reshuffled in the stage s is placed in the row b and depth p for the front entry, $\vec{z}_{bp}^{js} = 0$ otherwise, where $s \in S, j \in \Phi_s \cap \Omega_s, b \in B, p \in P$
$\overleftarrow{z}_{bp}^{js} \in \{0,1\}$	where $\overleftarrow{z}_{bp}^{js} = 1$ if the pallet j reshuffled in the stage s is placed in the row b and depth p for the back entry, $\overleftarrow{z}_{bp}^{js} = 0$ otherwise, where $s \in S, j \in \Phi_s \cap \Omega_s, b \in B, p \in P$
$\vec{U}_{kb}^{s} \in \mathbb{Z}^+$	number of pallet reshuffled or retrieved in the stage s by the forklift k for the front entry of the row b, where $k \in K, b \in B, s \in S$
$\overleftarrow{U}_{kb}^{s} \in \mathbb{Z}^+$	number of pallet reshuffled or retrieved in the stage s by the forklift k for the back entry of the row b, where $k \in K, b \in B, s \in S$
$F^s \in \mathbb{Z}^+$	maximum number of pallet reshuffled or retrieved for a forklift in the stage s, where $s \in S$
$\vec{G}^{is} \in \{0,1\}$	where $\vec{G}^{is} = 1$ if the pallet i is placed by the front entry when is reshuffled in the stage s, $\vec{G}^{is} = 0$ otherwise, where $s \in S, i \in \Phi_s \cap \Omega_s$
$\vec{C}_{bp}^{i} \in \{0,1\}$	where $\vec{C}_{bp}^{i} = 1$ if the pallet i is retrieval from the front entry of the row b and depth p, $\vec{C}_{bp}^{i} = 0$ otherwise, where $b \in B, p \in P, i \in I^D$
$\overleftarrow{C}_{bp}^{i} \in \{0,1\}$	where $\overleftarrow{C}_{bp}^{i} = 1$ if the pallet i is retrieval from the back entry of the row b and depth p, $\overleftarrow{C}_{bp}^{i} = 0$ otherwise, where $b \in B, p \in P, i \in I^D$

$$\begin{aligned}&x^{ks}_{b\tau\rho} \leq \left(1 - x^{is+1}_{btp}\right) + \sum_{\beta\in B}\sum_{\varrho\in P}\left(\vec{y}^{ks}_{\beta\varrho} + \overleftarrow{y}^{ks}_{\beta\varrho}\right) + \\ &\left(1 - \sum_{\beta\in B}\sum_{\varrho\in P}\left(\vec{y}^{is}_{\beta\varrho} + \overleftarrow{y}^{is}_{\beta\varrho}\right)\right) + \vec{G}_{is}\end{aligned} \qquad \begin{aligned}&, s\in S, i\in\Phi_s\cap\Omega_s, b\in B, t\in T, p\in P,\\ &k\in\Phi_s\cap\Omega_s, (\tau,\rho)\in\vec{R}_{tp}\end{aligned} \tag{15}$$

$$\begin{aligned}&x^{ks}_{b\tau\rho} \leq \left(1 - x^{is+1}_{btp}\right) + \sum_{\beta\in B}\sum_{\varrho\in P}\left(\vec{y}^{ks}_{\beta\varrho} + \overleftarrow{y}^{ks}_{\beta\varrho}\right) + \\ &\left(1 - \sum_{\beta\in B}\sum_{\varrho\in P}\left(\vec{y}^{is}_{\beta\varrho} + \overleftarrow{y}^{is}_{\beta\varrho}\right)\right) + \left(1 - \vec{G}_{is}\right)\end{aligned} \qquad \begin{aligned}&, s\in S, i\in\Phi_s\cap\Omega_s, b\in B, t\in T, p\in P,\\ &k\in\Phi_s\cap\Omega_s, (\tau,\rho)\in\overleftarrow{R}_{tp}\end{aligned} \tag{16}$$

$$\vec{z}^{js}_{bp} \geq \sum_{\beta\in B}\sum_{\rho\in P}\vec{y}^{js}_{\beta\rho} + \sum_{\beta\in B}\sum_{\rho\in P}\overleftarrow{y}^{js}_{\beta\rho} + \sum_{t\in T}x^{is}_{btp+1} + \vec{G}_{js} - 2 \qquad , s\in S\setminus\{|S|\}, j\in\Phi_s\cap\Omega_s, b\in B, p\in P \tag{17}$$

$$\begin{aligned}&\overleftarrow{z}^{js}_{bp} \geq \sum_{\beta\in B}\sum_{\rho\in P}\vec{y}^{js}_{\beta\rho} + \sum_{\beta\in B}\sum_{\rho\in P}\overleftarrow{y}^{js}_{\beta\rho} + \\ &\sum_{t\in T}x^{is}_{btp+1} + \left(1 - \vec{G}_{js}\right) - 2\end{aligned} \qquad , s\in S\setminus\{|S|\}, j\in\Phi_s\cap\Omega_s, b\in B, p\in P \tag{18}$$

$$\sum_{k\in K}\vec{w}^{s}_{kb} = 1 \qquad , s\in S, b\in B \tag{19}$$

$$\sum_{k\in K}\overleftarrow{w}^{s}_{kb} = 1 \qquad , s\in S, b\in B \tag{20}$$

$$\begin{aligned}&\vec{U}^{s}_{kb} \geq \sum_{j\in\Phi_s\cap\Omega_s}\sum_{p\in P}\vec{z}^{js}_{bp} + \sum_{i\in\Phi_s\cap\Omega_s}\sum_{p\in P}\vec{y}^{is}_{bp} + \\ &\sum_{i\in\Omega_s}\sum_{p\in P}\vec{C}^{i}_{bp} - 2|\Phi_s|\left(1 - \vec{w}^{s}_{kb}\right)\end{aligned} \qquad , k\in K, b\in B, s\in S \tag{21}$$

$$\begin{aligned}&\overleftarrow{U}^{s}_{kb} \geq \sum_{j\in\Phi_s\cap\Omega_s}\sum_{p\in P}\overleftarrow{z}^{js}_{bp} + \sum_{i\in\Phi_s\cap\Omega_s}\sum_{p\in P}\overleftarrow{y}^{is}_{bp} + \\ &\sum_{i\in\Omega_s}\sum_{p\in P}\overleftarrow{C}^{i}_{bp} - 2|\Phi_s|\left(1 - \overleftarrow{w}^{s}_{kb}\right)\end{aligned} \qquad , k\in K, b\in B, s\in S \tag{22}$$

$$F_s \geq \sum_{b\in B}\left(\vec{U}^{s}_{kb} + \overleftarrow{U}^{s}_{kb}\right) \qquad , k\in K, s\in S \tag{23}$$

$$\sum_{\rho\in P}\rho\vec{y}^{is}_{b\rho} \leq \sum_{\rho\in P}\rho\overleftarrow{y}^{js}_{b\rho} + 1 + |P|\left(2 - \sum_{\rho\in P}\left(\vec{y}^{is}_{b\rho} + \overleftarrow{y}^{js}_{b\rho}\right)\right) \qquad , b\in B, s\in S, i\in\Phi_s\cap\Omega_s, j\in\Phi_s\cap\Omega_s \tag{24}$$

$$\sum_{\rho\in P}\rho\vec{y}^{is}_{b\rho} \leq \sum_{\rho\in P}\rho\overleftarrow{C}^{j}_{b\rho} + 1 + |P|\left(2 - \sum_{\rho\in P}\left(\vec{y}^{is}_{b\rho} + \overleftarrow{C}^{j}_{b\rho}\right)\right) \qquad , b\in B, s\in S, i\in\Phi_s\cap\Omega_s, j\in\Omega_s \tag{25}$$

$$\sum_{\rho\in P}\rho\vec{C}^{j}_{b\rho} \leq \sum_{\rho\in P}\rho\overleftarrow{y}^{is}_{b\rho} + 1 + |P|\left(2 - \sum_{\rho\in P}\left(\overleftarrow{y}^{is}_{b\rho} + \vec{C}^{j}_{b\rho}\right)\right) \qquad , b\in B, s\in S, i\in\Phi_s\cap\Omega_s, j\in\Omega_s \tag{26}$$

$$\sum_{\rho\in P}\rho\vec{C}^{j}_{b\rho} \leq \sum_{\rho\in P}\rho\overleftarrow{C}^{i}_{b\rho} + 1 + |P|\left(2 - \sum_{\rho\in P}\left(\overleftarrow{C}^{i}_{b\rho} + \vec{C}^{j}_{b\rho}\right)\right) \qquad , b\in B, i\in\Omega_s, j\in\Omega_s \tag{27}$$

$$\sum_{\rho\in P}\rho\vec{z}^{is}_{b\rho} \leq \sum_{\rho\in P}\rho\overleftarrow{z}^{js}_{b\rho} + 1 + |P|\left(2 - \sum_{\rho\in P}\left(\overleftarrow{z}^{js}_{b\rho} + \vec{z}^{is}_{b\rho}\right)\right) \qquad , b\in B, i\in\Phi_s\cap\Omega_s, j\in\Phi_s\cap\Omega_s \tag{28}$$

$$x^{is}_{btp} - x^{is+1}_{btp} \geq \sum_{\beta\in B}\sum_{\rho\in P} - \left(\vec{y}^{\beta\rho}_{is} + \overleftarrow{y}^{\beta\rho}_{is}\right) \qquad , i\in I, s\in A_j..D_j, b\in B, t\in T, p\in P \tag{29}$$

$$x^{is}_{btp} - x^{is+1}_{btp} \leq \sum_{\beta\in B}\sum_{\rho\in P}\left(\vec{y}^{\beta\rho}_{is} + \overleftarrow{y}^{\beta\rho}_{is}\right) \qquad , i\in I, s\in A_j..D_j, b\in B, t\in T, p\in P \tag{30}$$

$$x^{is}_{btp} \in \{0,1\} \qquad , i\in I, s\in A_i..D_i, b\in B, t\in T, p\in P \tag{31}$$

$$\vec{y}^{is}_{bp}, \overleftarrow{y}^{is}_{bp} \in \{0,1\} \qquad , s \in S, i \in \Phi_s \cap \Omega_s, b \in B, p \in P \tag{32}$$

$$\vec{\varphi}^{jks}_{bp}, \overleftarrow{\varphi}^{jks}_{bp} \in \{0,1\} \qquad , s \in S, j \in \Phi_s \cap \Omega_s, k \in \Phi_s \cap \Omega_s, b \in B, p \in P \tag{33}$$

$$\vec{w}^{s}_{kb}, \overleftarrow{w}^{s}_{kb} \in \{0,1\} \qquad , k \in K, s \in S, b \in B \tag{34}$$

$$\vec{z}^{js}_{bp}, \overleftarrow{z}^{js}_{bp} \in \{0,1\} \qquad , s \in S, j \in \Phi_s \cap \Omega_s, b \in B, p \in P \tag{35}$$

$$\vec{c}^{i}_{bp}, \overleftarrow{c}^{i}_{bp} \in \{0,1\} \qquad , i \in I^D, p \in P, b \in B \tag{36}$$

$$\vec{G}_{is} \in \{0,1\} \qquad , s \in S, i \in \Phi_s \cap \Omega_s \tag{37}$$

$$\vec{U}^{s}_{kb}, \overleftarrow{U}^{s}_{kb} \in Z^+ \qquad , s \in S, b \in B, k \in K \tag{38}$$

$$F_s \in Z^+ \qquad , s \in S \tag{39}$$

The objective function (1) minimize the number of the reshuffled carried out by the forklifts. The objective function (2) minimize the maximum number of movement carried out by a forklift. The constraint (3) guaranteed that when a pallet is storage in a location of the chamber. On the other hand the constraint (4) guaranteed that each location of the chamber storage no more than one pallet. The constraint (5) assure that the in the stage zero, the pallets are located in the initial location. The constraints (6-7) establish that all empty locations of the storage are accessible by the front or back entry. The constraint (8) assure that the pallet must be retrieved by the front or back entry. The constraints (9 - 10) obtain the pallets that must to be reshuffled for the front and back entry to retrieve the pallets. The constraints (11 - 14) obtain the pallet that must to be reshuffled when other pallets are reshuffled. The constraints (15 - 16) assure to storage the pallets reshuffled no other reshuffled have to be carried out. The constraints (17-18) obtain the row and depth for the pallet reshuffled is storage. The constraints (19-20) assure that each row for the front and back entry have a forklift assign. The constraints (21-22) calculate the number of movement carried out for every forklift in each row. On the other hand, the constraint (23) calculate the maximum number of movement carried out for a forklift. The constraints (24-28) assure that the forklift do not collide in a row. The constraints (29-30) assure that a pallet do not change its location if it is not reshuffled. Constraints (31-39) are about to nature of the decision variables.