Phase Transitions in Financial Markets: An Entropy-Based Approach to Regime Detection

Group 8: Andreas Adamantos, Pranav Bali, Tarreau Boone, Lorenzo Mainetti

Why financial markets?

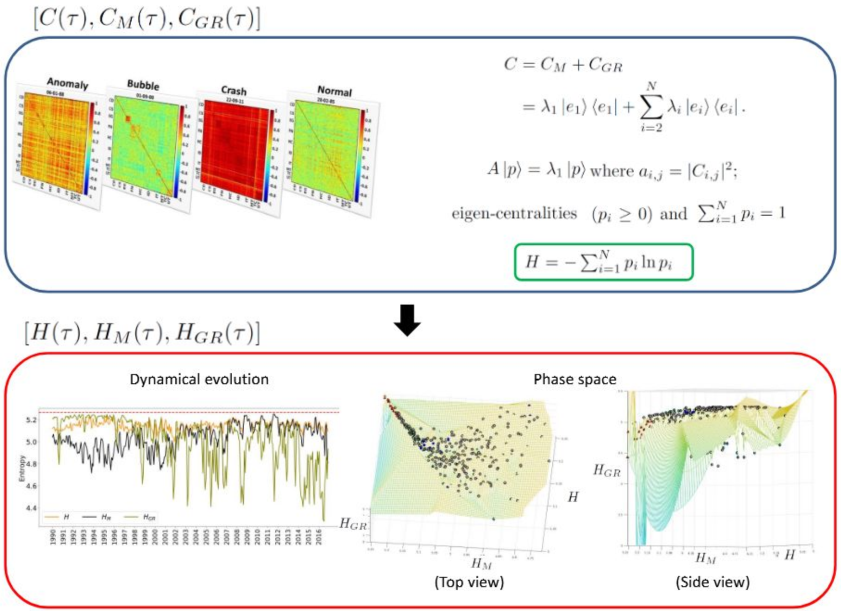
- Financial markets are complex systems where hundreds of assets interact simultaneously, and their collective behavior shifts dramatically during crises

- Asset correlations can determine the behavior of the overall system, and changes in those correlations can act as indicators of a phase transition.

- We want to characterize the dynamics of the market without relying on arbitrary thresholds or fitted parameters

Phase separation and scaling in correlation structures of financial markets

- Provides a methodology that would help understanding and foreseeing tipping points in complex systems.
- extract information about the disorder in the market using the eigen-entropy measure
- Shows how different market events undergo phase separation



Kukreti, V., Pharasi, H.K., Gupta, P., & Kumar, S. (2020). A perspective on correlation-based financial networks and entropy measures. *Frontiers in Physics*, 8, 323.

Research Questions

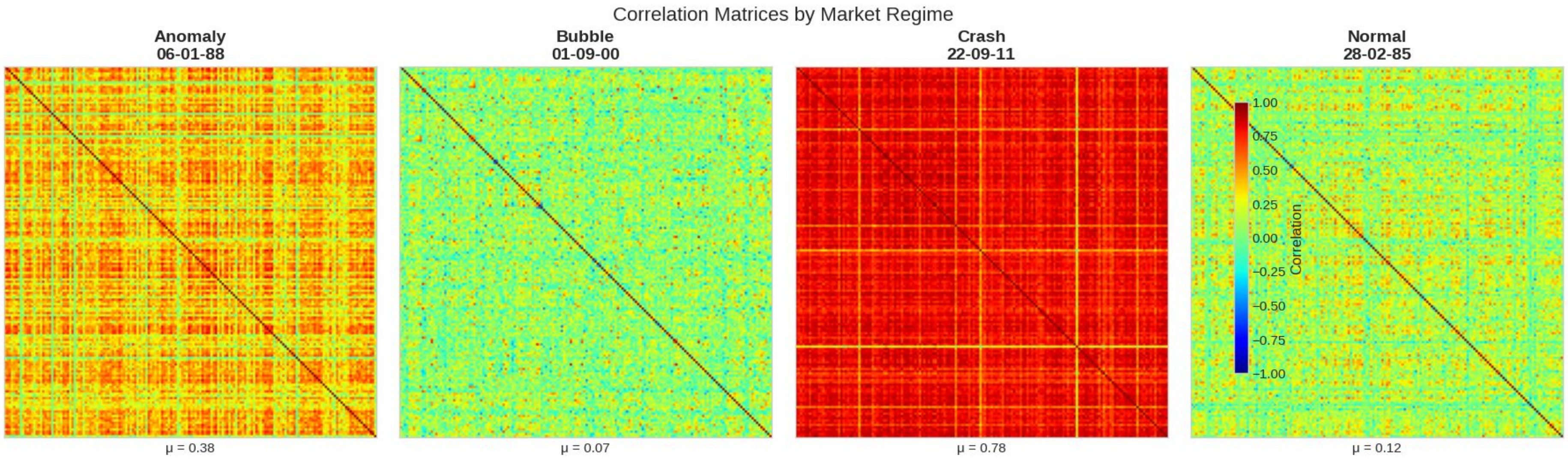
- Can entropy measures derived from correlation matrix eigenvectors reliably distinguish between normal trading periods and market stress events?
- How do the dominant market mode and sector-specific modes behave differently during crashes versus calm periods?
- Does the market's trajectory in a low-dimensional phase space defined by correlation, entropy, and eigenvalue exhibit predictable patterns before major transitions?

Data preparation

- Top 200 stocks of S&P 500, from 1985 to 2025.
- Spans different periods of calm and crisis
- Log returns computed to compare relative changes.
- Back filling and forward filling used for missing data

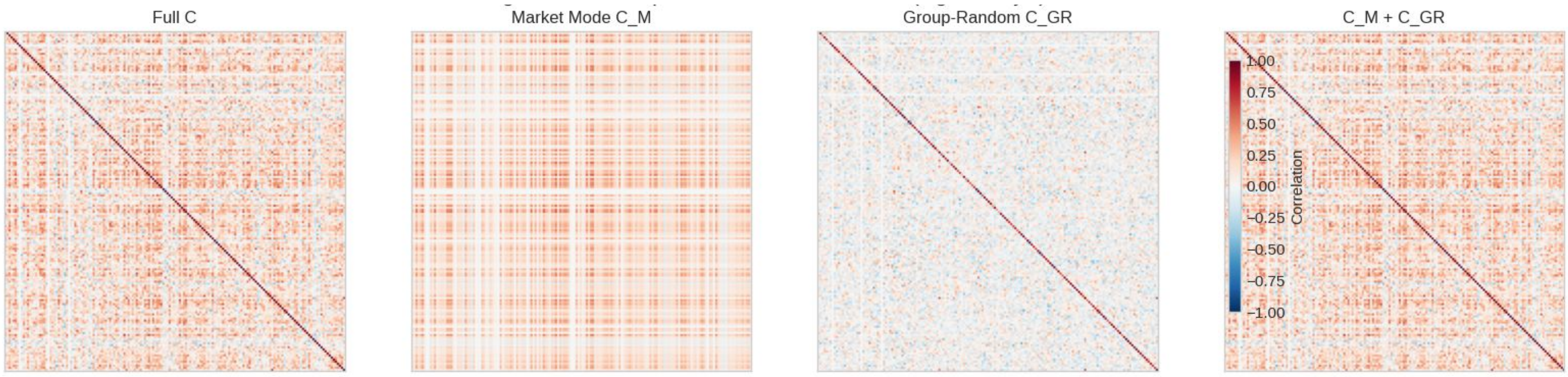
Rolling window

- Rolling window of returns used to capture changing market structure
- Each window is one correlation matrix.
- We used a window of 40 days, and shifted it by 20 days each time



Correlation modes

- Correlation matrix C decomposed into market modes (CM) and composite group plus random modes (CGR)
- CM captures movement of entire system
- CGR captures movements of select sectors (Groups), as well as noise (Random)



From Correlation Matrix to Entropy

1. Transform correlation matrix

$$A_{ij} = |C_{ij}|^2$$

Creates adjacency matrix with positive weights

2. Compute Principal Eigen-Vector

$$A \cdot v = \lambda \cdot v$$

Find eigenvector v for largest eigenvalue of A

$v = [v_1, v_2, ..., v_N]$ where v_i = centrality of stock i

3. Normalize to probability

$$p_i = |v_i| \,/\, \Sigma|v_j|$$

Now $\Sigma p_i = 1$

4. Compute Shannon entropy

$$H = -\Sigma\, p_i \cdot \log(p_i)$$

Measures disorder in centrality distribution

Why eigenvector centrality?

v_i is high if stock i is strongly connected to other stocks that are also highly connected. It captures recursive importance in the network.

Interpretation

Crash: all stocks equally central → p_i uniform → HIGH H

Normal: varied centrality → non-uniform p_i → LOWER H

Reference Entropies and Phase Classification

Correlation Matrix Decomposition

$C = C\_M + C\_GR$

$C\_M = \lambda_1 \cdot e_1 \cdot e_1^T$

$C\_GR = \Sigma_{i=2}^{N} \lambda_i \cdot e_i \cdot e_i^T$

Three Entropies

Full Correlation

$H = \text{entropy}(C)$

Market Mode (rank-1)

$H\_M = \text{entropy}(C\_M)$

Group-Random Mode

$H\_GR = \text{entropy}(C\_GR)$

Phase Space Axes

|H - H_M| = distance from market mode

|H - H_GR| = distance from group-random

Phase Classification

Crash

Low |H-H_M|

High |H-H_GR|

Type-1

High |H-H_M|

High |H-H_GR|

Anomaly

Low |H-H_M|

Low |H-H_GR|

Type-2

High |H-H_M|

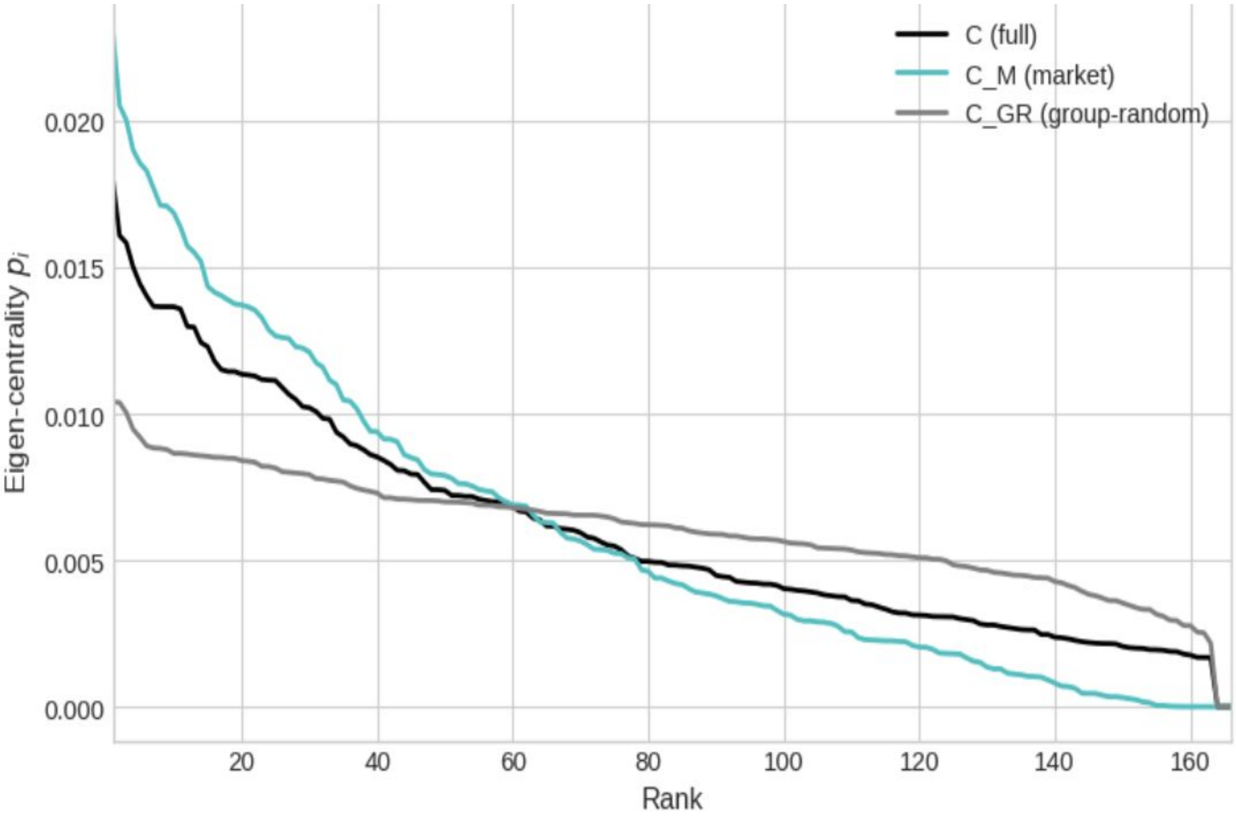
Low |H-H_GR|

Crash: Market dominated by single eigenmode, so H approaches H_M.

Ranked Eigen-Centrality Distribution

1) The market mode is constructed by retaining only the largest eigenvalue–eigenvector pair of the correlation matrix in its spectral decomposition.

2) The largest eigenvalue represents the strongest common factor, keeping only this mode captures market-wide synchronization.



Evolution of Market Indicators

Major Market Crash Events (1987-2020)

19 October 1987

Black Monday

27 October 1997

Asian Financial Crisis

10 March 2000

Dot-com Crash

11 September 2001

September 11 Attacks

15 September 2008

Lehman Brothers Collapse

6 May 2010

Flash Crash

8 August 2011

European Debt Crisis

24 August 2015

China Black Monday

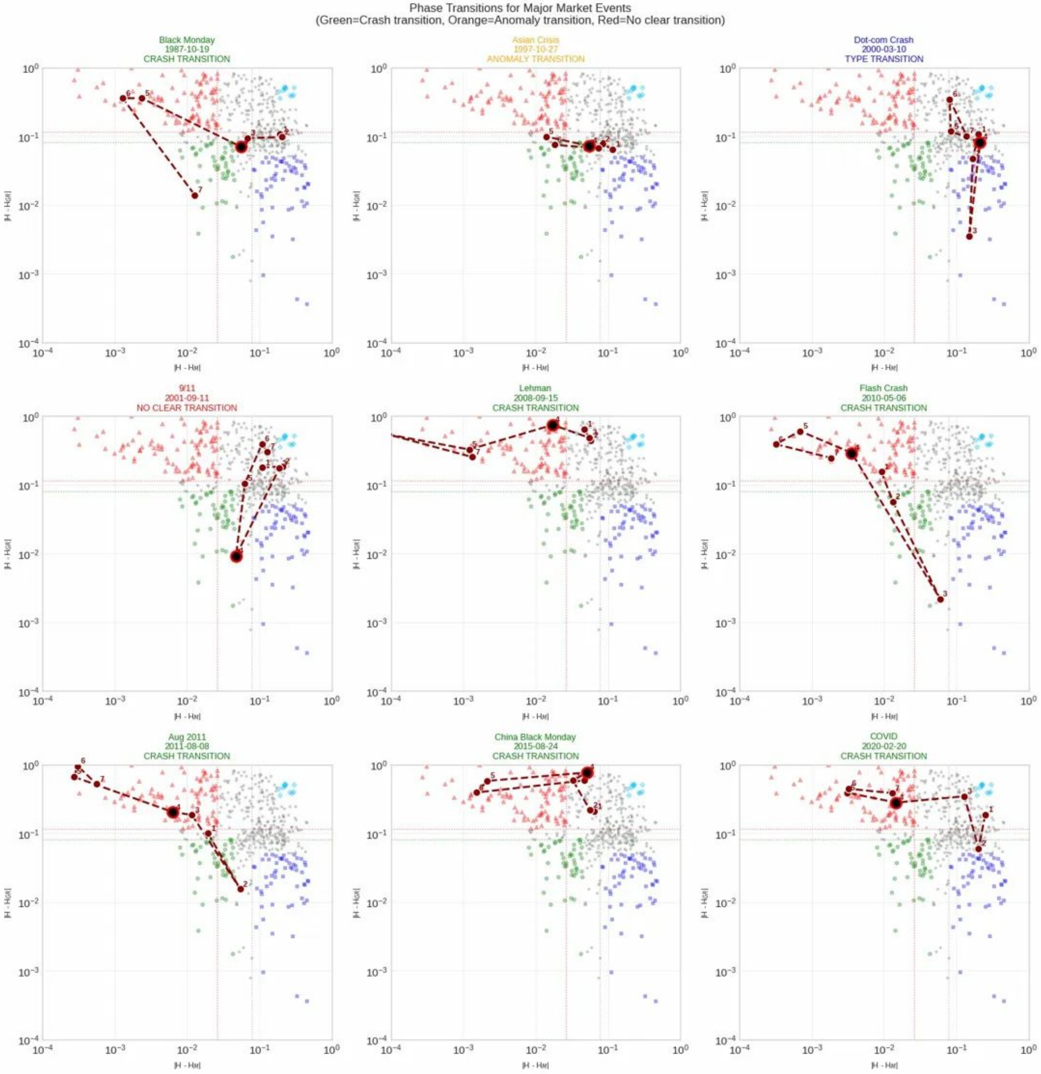
20 February 2020

COVID-19 Market Crash

> These events represent major market disruptions where correlations spiked and the market entered a crash regime in phase space.

Phase Space Transition Plots

- 9 Historical Crash Events
- Clear Phase Transition seen on 6

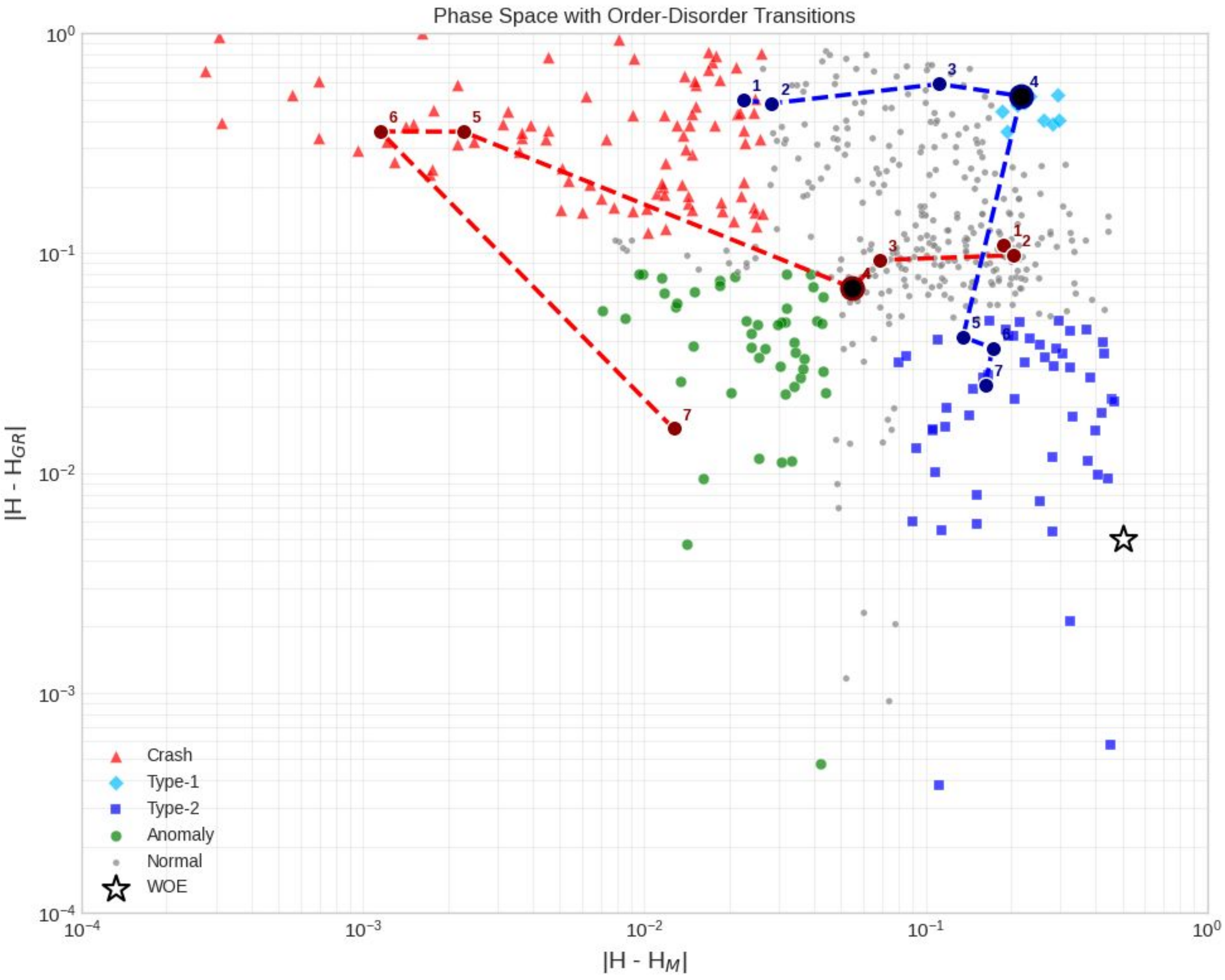


Clearest Phase Transition seen for

Black Monday

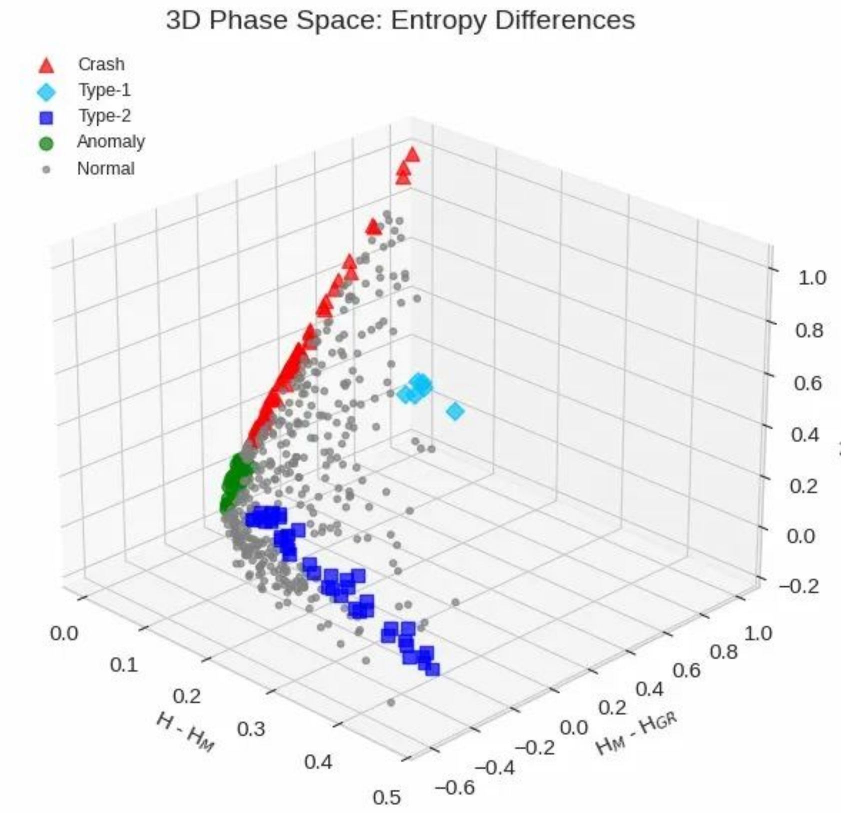
&

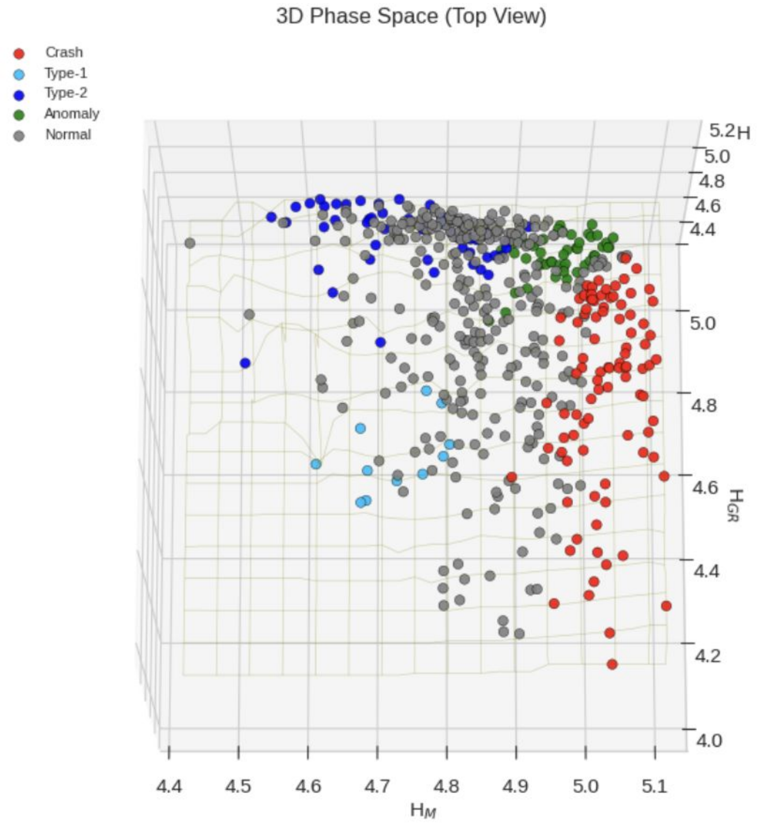
Dot-com Bubble



(* Few Anomalies exist)

3D Phase Space plots for reference





Scaling relation

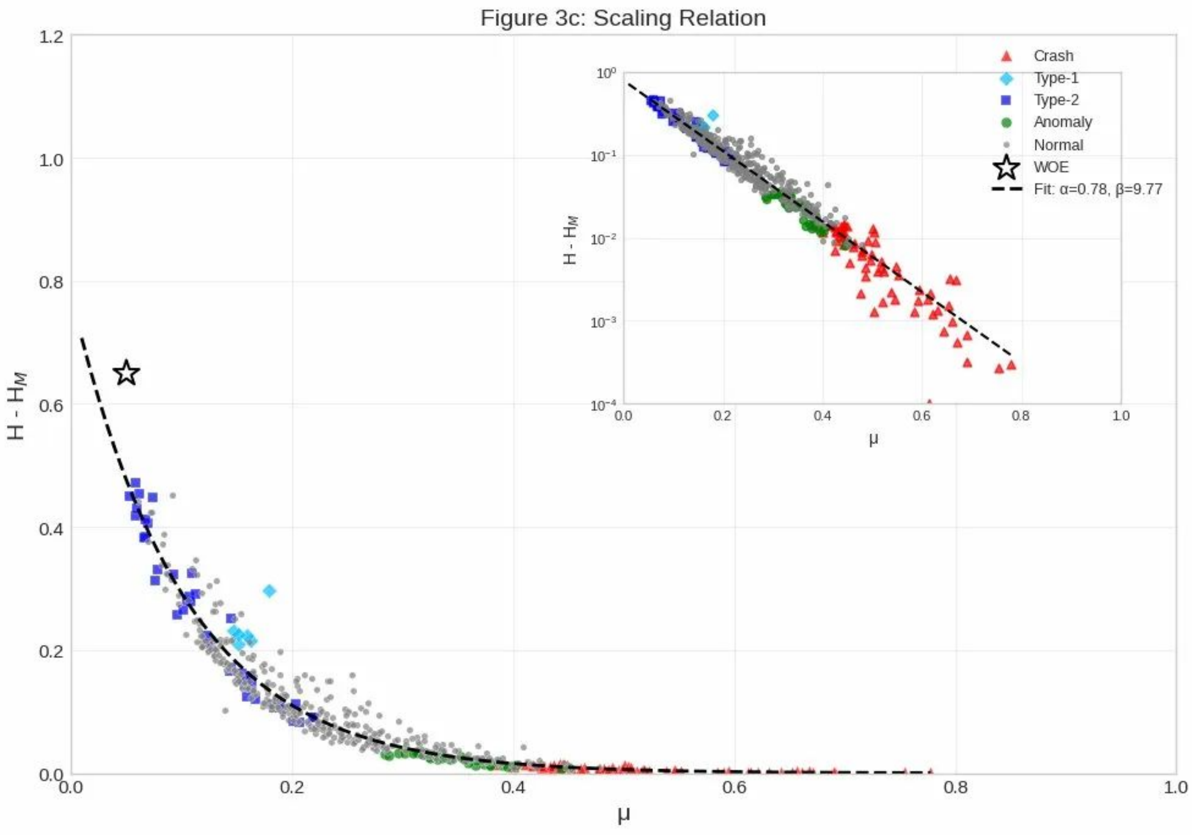
Exponential relation between

H - H_M vs μ

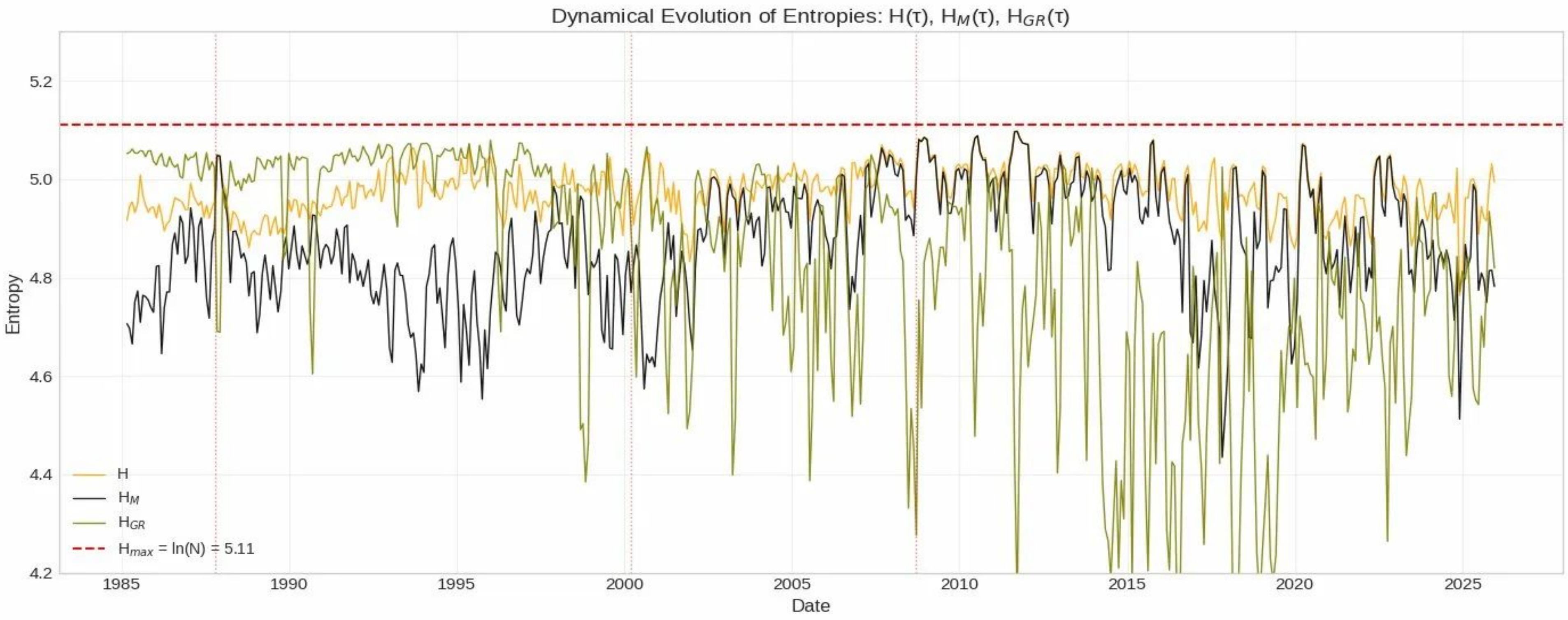
Crash events are located
at the bottom at higher μ
(shows strong correlation in crashes)

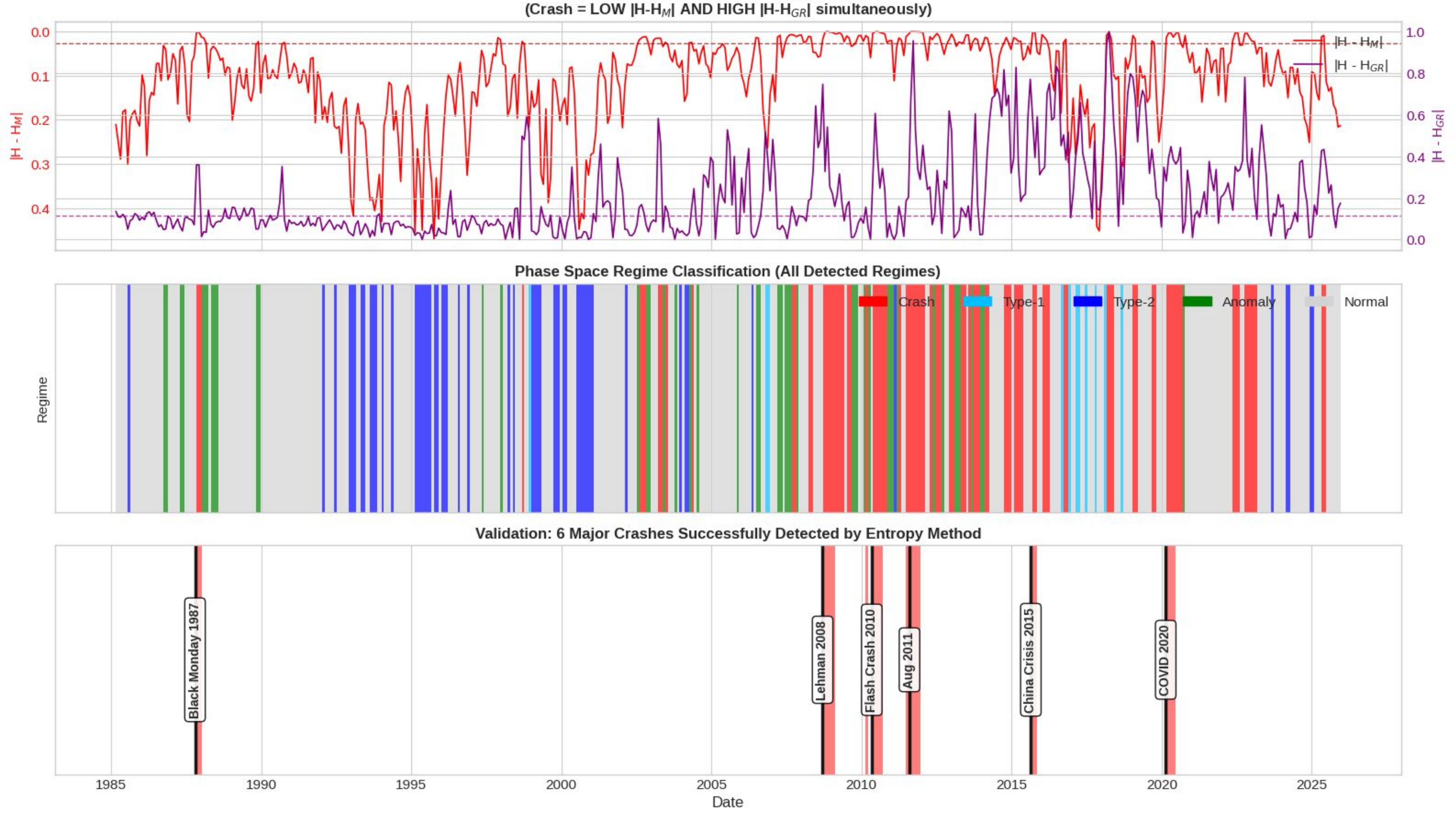
Universal organizing principle

(Inset is log-scaled)



Evolution of Entropies



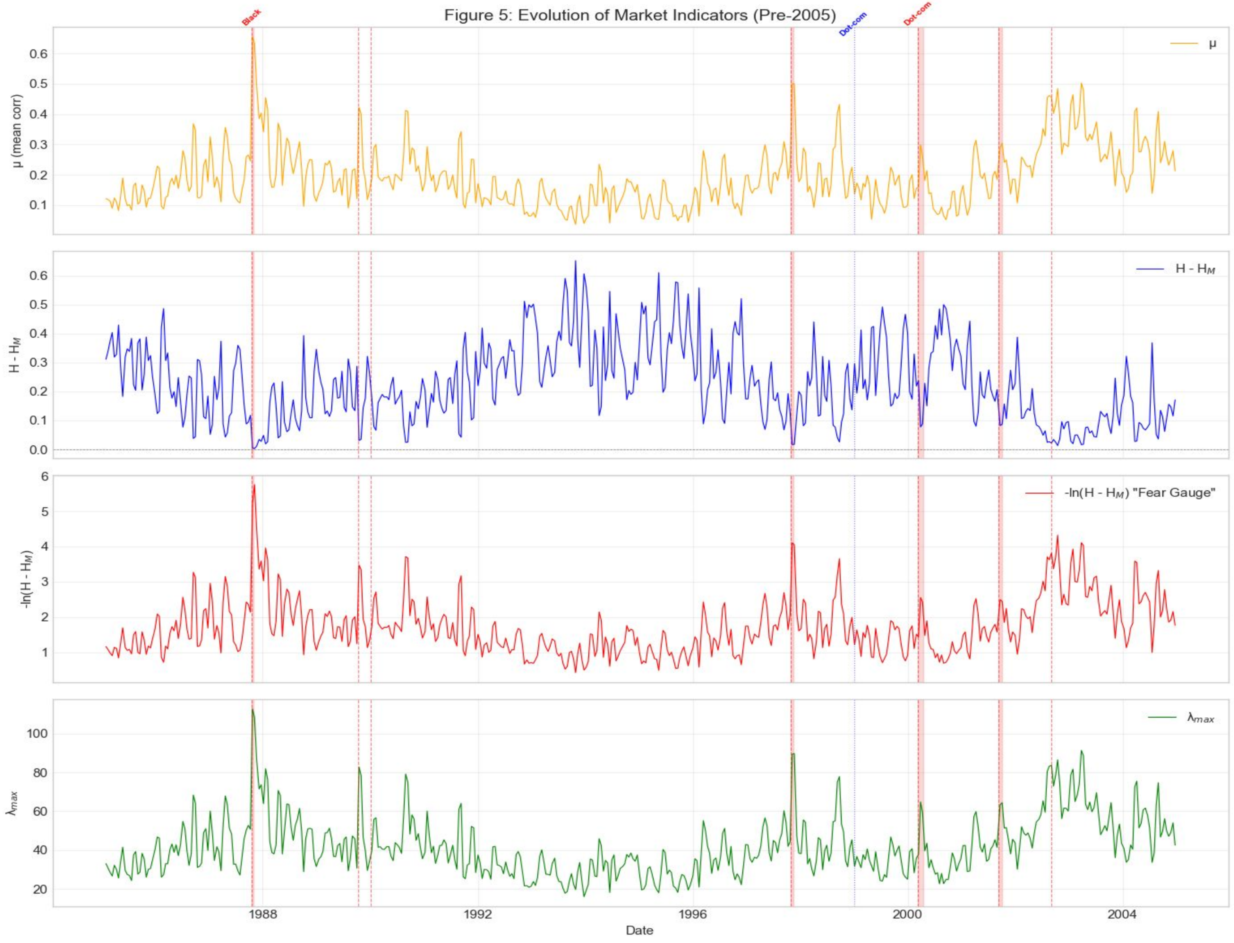

(Crash = LOW |H-H_M| AND HIGH |H-H_GR| simultaneously)
|H - H_M|
|H - H_GR|
Phase Space Regime Classification (All Detected Regimes)
Regime
Crash
Type-1
Type-2
Anomaly
Normal
Validation: 6 Major Crashes Successfully Detected by Entropy Method
Black Monday 1987
Lehman 2008
Flash Crash 2010
Aug 2011
China Crisis 2015
COVID 2020
Date

Can We See a Crash Coming?

- Does the market's trajectory in a low-dimensional phase space defined by correlation, entropy , and eigenvalue exhibit predictable patterns before major transitions?
- Is there a stable, recurring pre-crash region that we can detect?
- Hypothesis: Major market crashes are preceded by a systematic approach toward a common pre-crash region in a low-dimensional phase space defined by correlation, entropy, and eigenvalue structure.

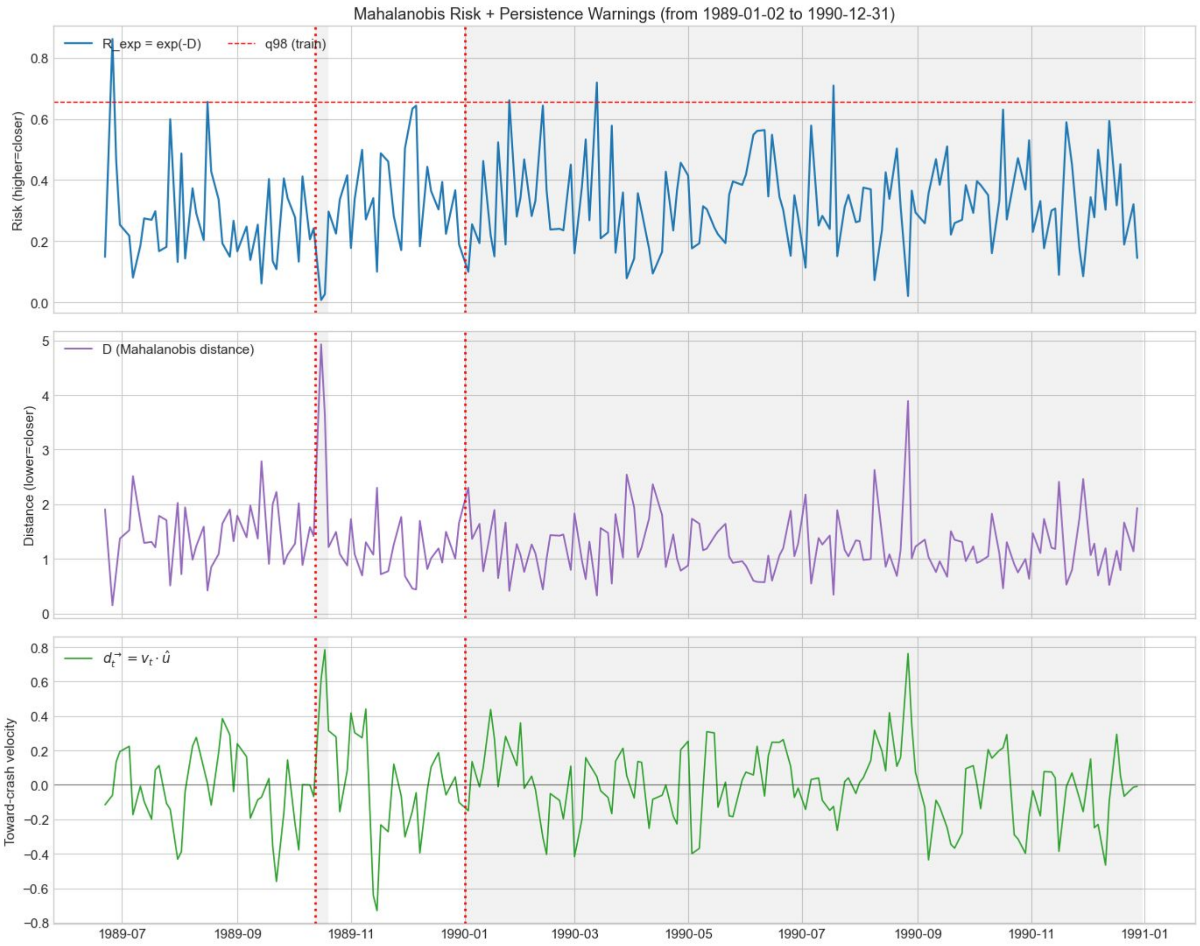
From Correlation to Phase Space

- Z-scored |H−H_M|, μ, λ_max
- Pre-Crash Centroid
- Mahalanobis Distance

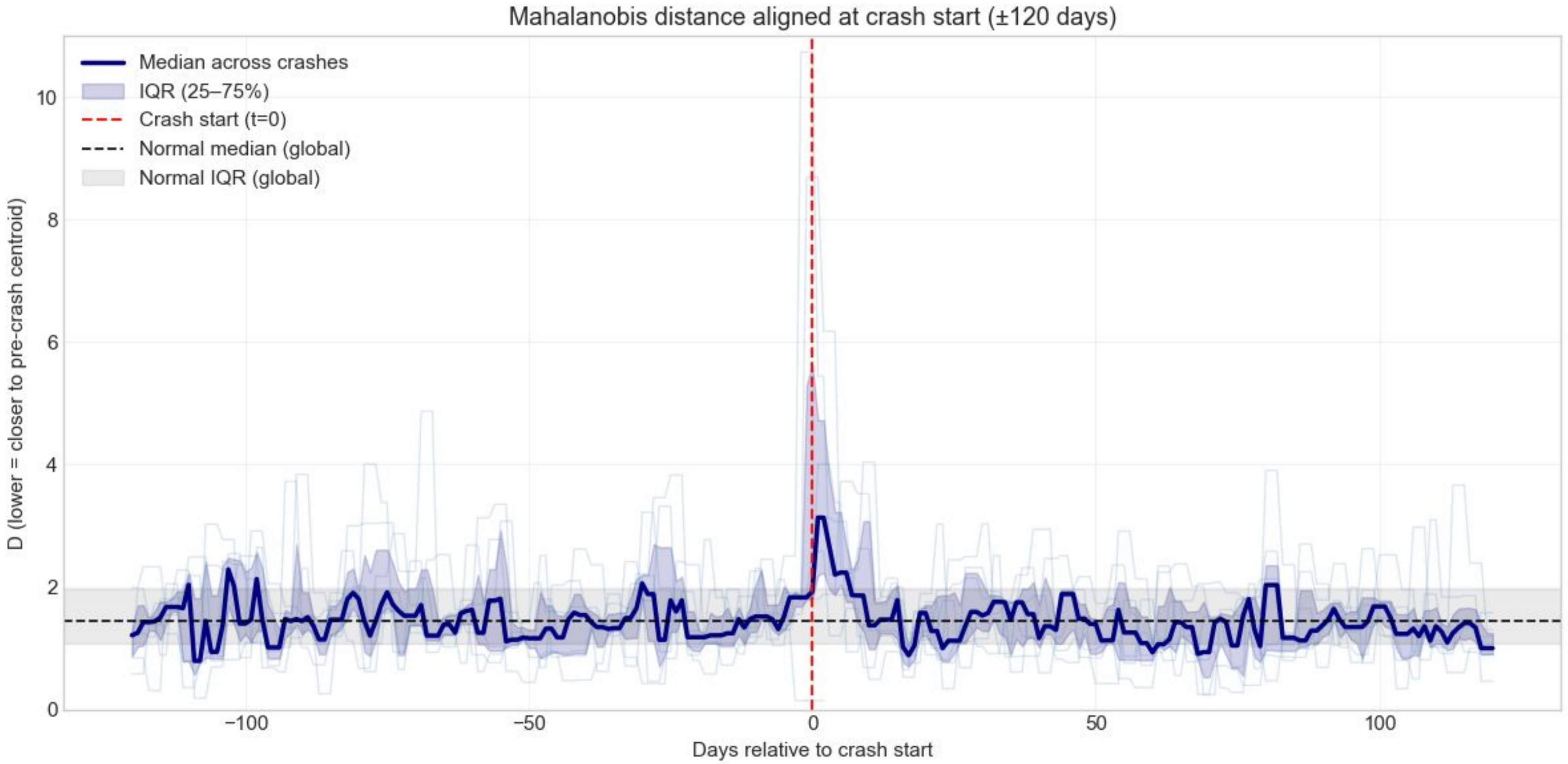


Pipe Line

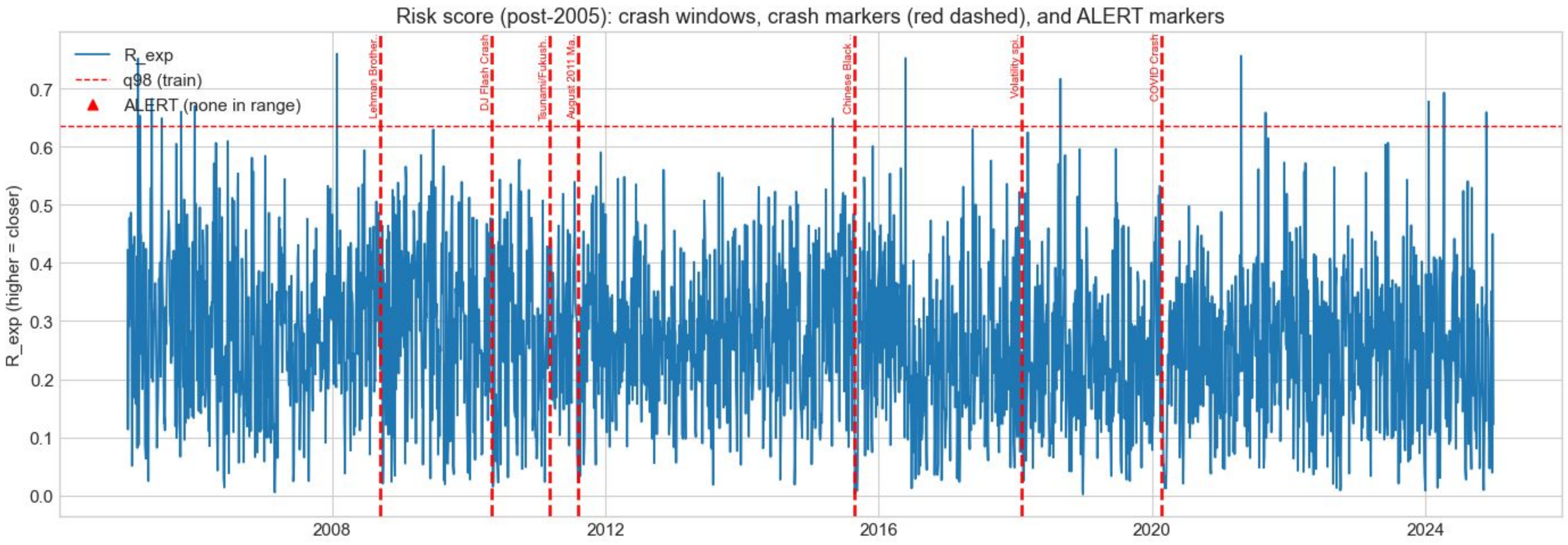
- Risk-Score = Exp(-D)
- 2 days above Threshold
- Moving toward Pre-Crash Region



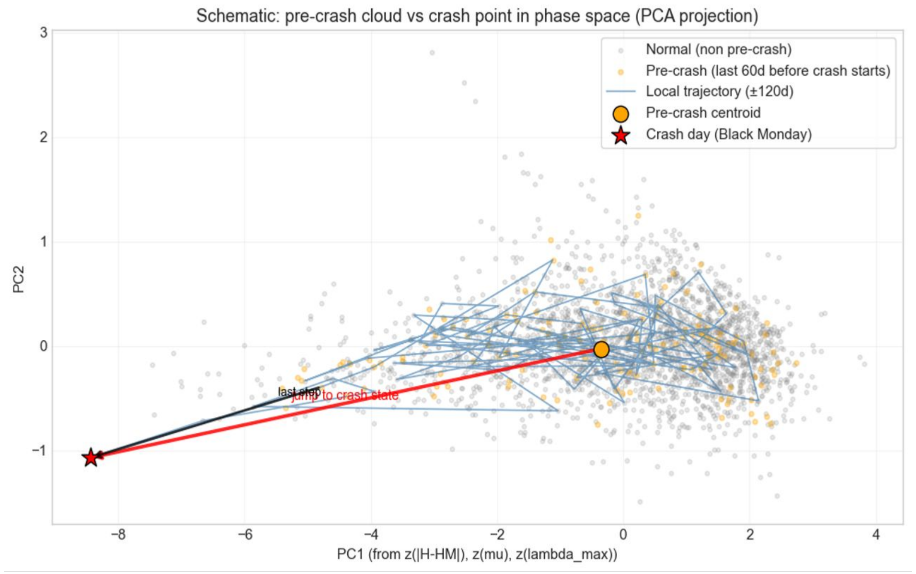
No Systematic Approach to Crashes

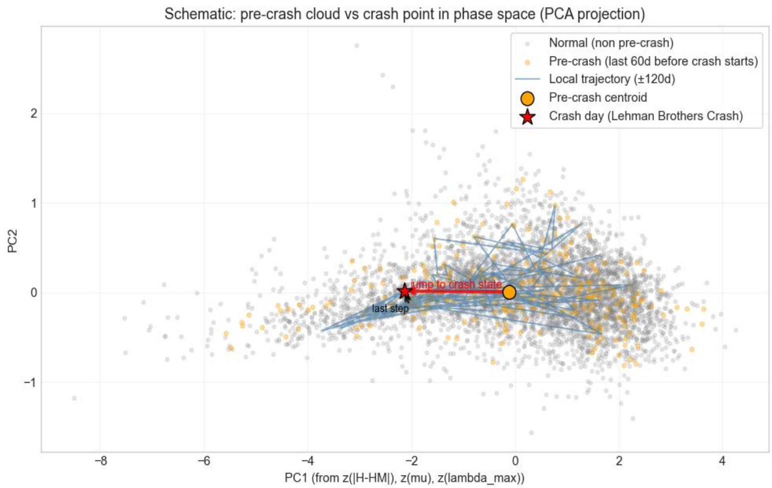


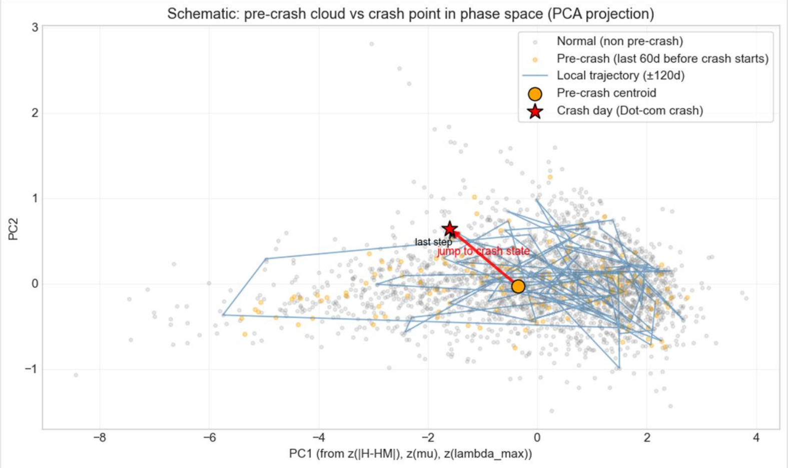
No Predictive Results



Trajectories are Heterogenous







Conclusion

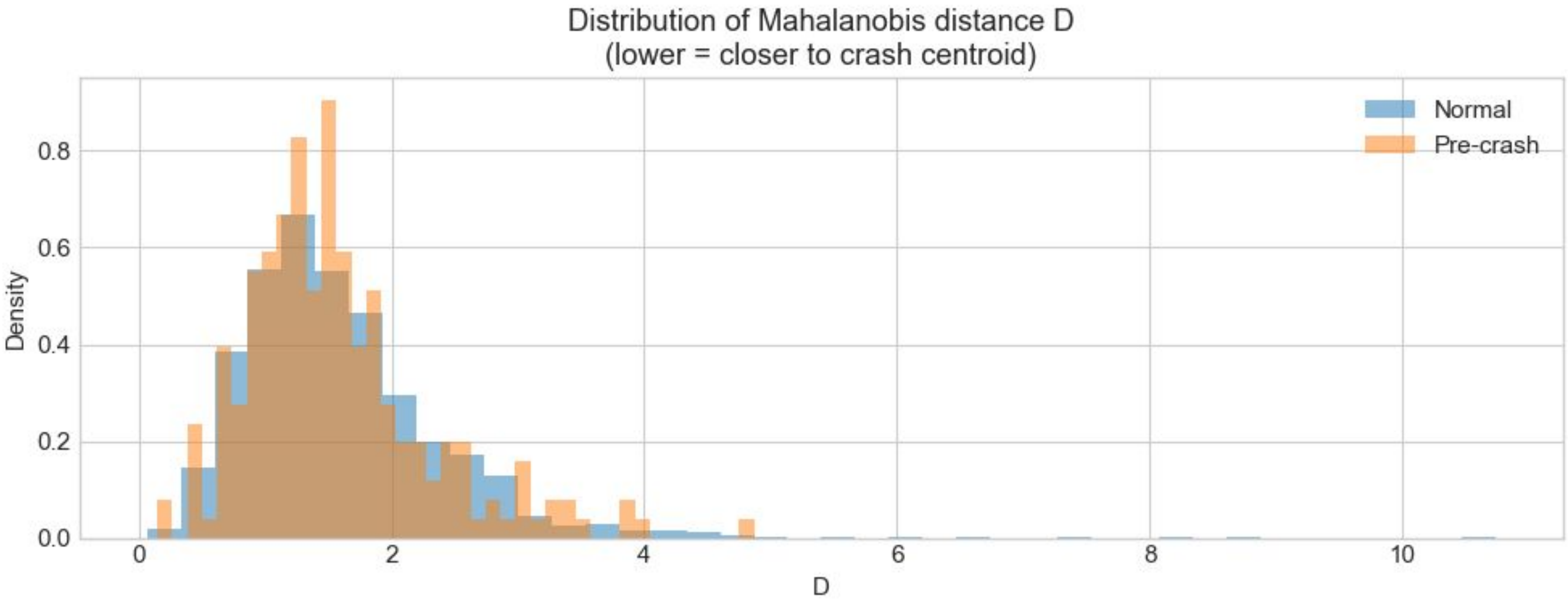
- Market behavior changes clearly between calm periods and crises.
- During crises, the market acts as a single dominant system.
- Correlation-based measures capture these changes very clearly.
- Entropy summarizes market organization in a compact way.
- Calm periods correspond to more distributed correlations and higher entropy.
- Crisis periods are marked by a strong reduction of entropy and loss of diversification.

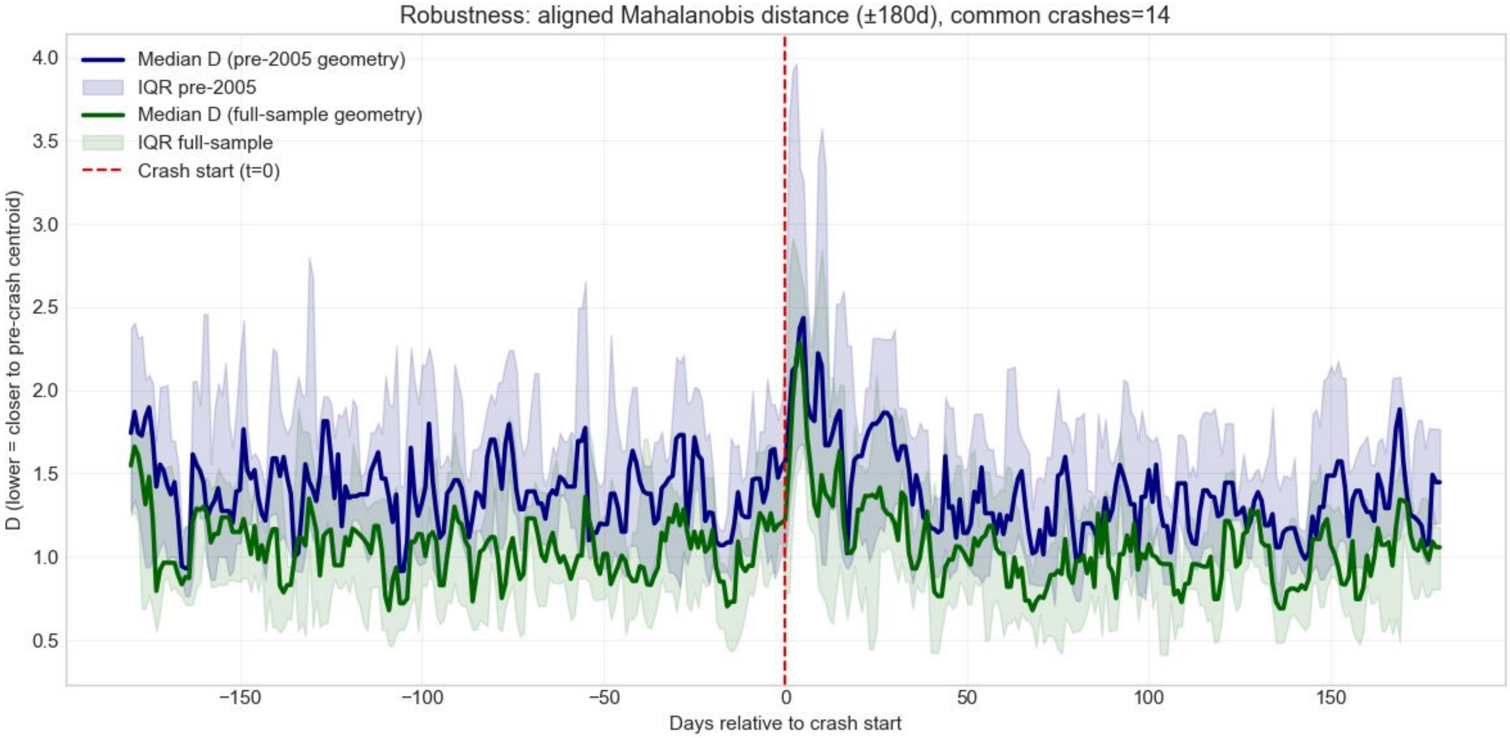
Conclusion

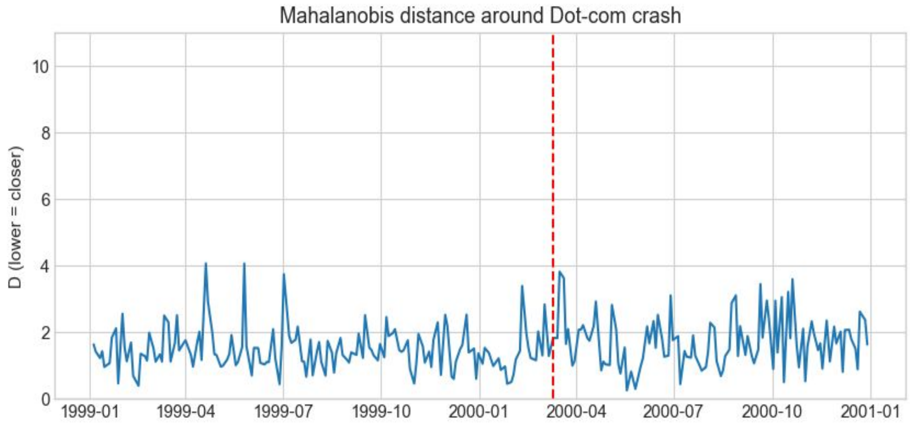
- Known crashes and bubbles tend to occupy different regions in phase space.
- Results reproduce the main qualitative findings of the paper.
- Confirmed a stable relationship between correlation and entropy.
- Crisis transitions are visible but not identical across events.
- Method describes market stress well but does not reliably predict crashes.
- Framework is best suited for monitoring and interpretation.

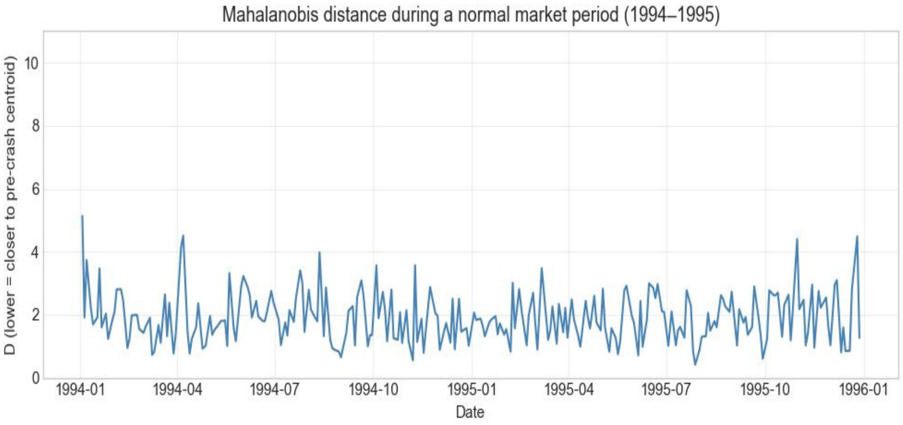
Questions?

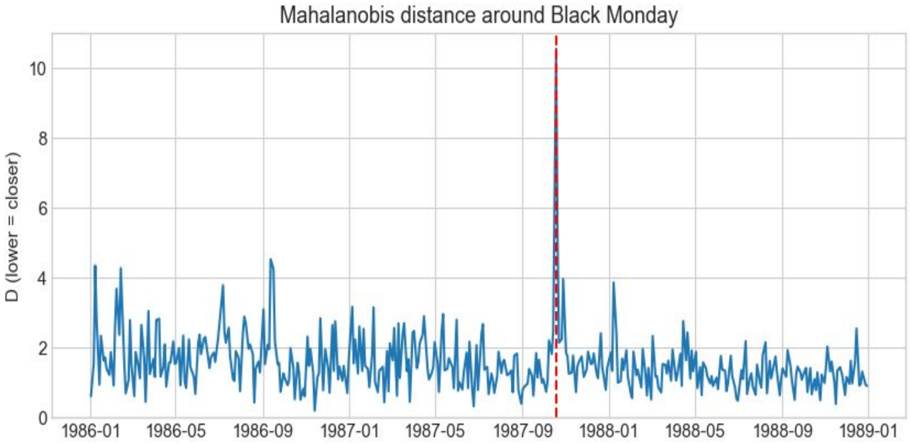
Appendix

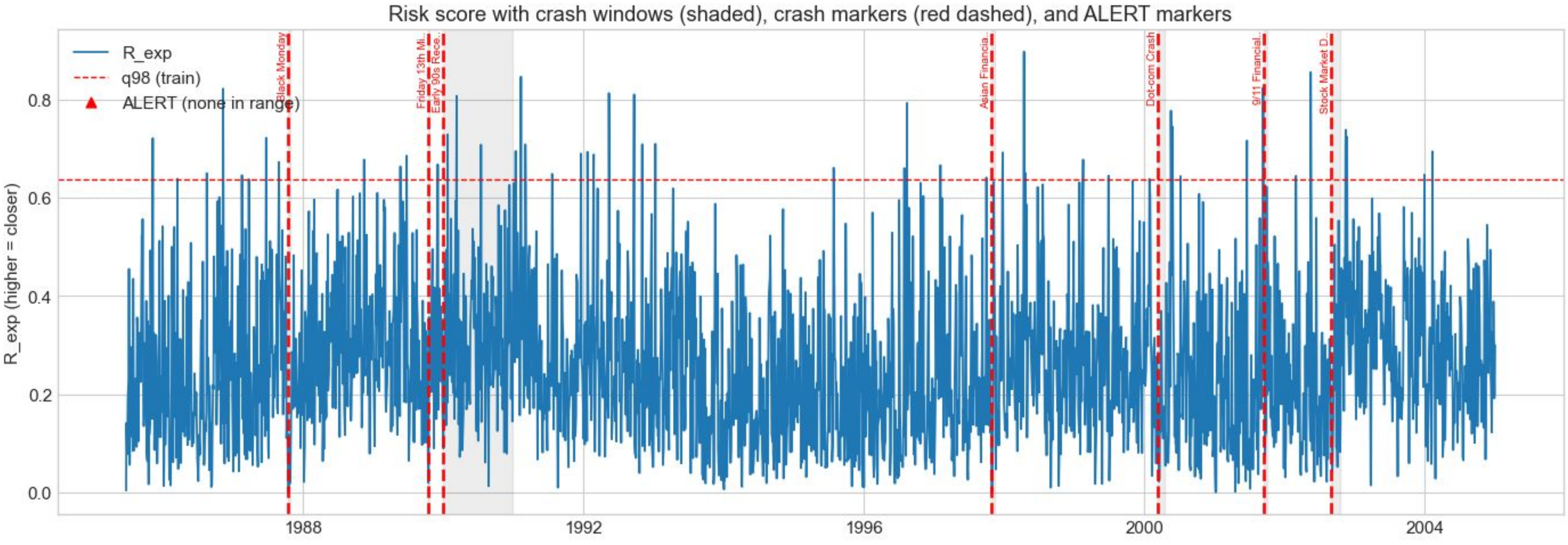

Distribution of Mahalanobis distance D
(lower = closer to crash centroid)
Normal
Pre-crash
Density
0.0
0.2
0.4
0.6
0.8
0
2
4
6
8
10
D


Robustness: aligned Mahalanobis distance (±180d), common crashes=14
Median D (pre-2005 geometry)
IQR pre-2005
Median D (full-sample geometry)
IQR full-sample
Crash start (t=0)
D (lower = closer to pre-crash centroid)
Days relative to crash start
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
−150
−100
−50
0
50
100
150


Mahalanobis distance around Dot-com crash
D (lower = closer)
0
2
4
6
8
10
1999-01
1999-04
1999-07
1999-10
2000-01
2000-04
2000-07
2000-10
2001-01


Mahalanobis distance during a normal market period (1994–1995)
D (lower = closer to pre-crash centroid)
0
2
4
6
8
10
1994-01
1994-04
1994-07
1994-10
1995-01
1995-04
1995-07
1995-10
1996-01
Date


Mahalanobis distance around Black Monday
D (lower = closer)
0
2
4
6
8
10
1986-01
1986-05
1986-09
1987-01
1987-05
1987-09
1988-01
1988-05
1988-09
1989-01


Risk score with crash windows (shaded), crash markers (red dashed), and ALERT markers
R_exp
q98 (train)
ALERT (none in range)
Black Monday
Friday 13th Mi...
Early 90s Rece...
Asian Financia...
Dot-com Crash
9/11 Financial...
Stock Market D...
R_exp (higher = closer)
0.0
0.2
0.4
0.6
0.8
1988
1992
1996
2000
2004