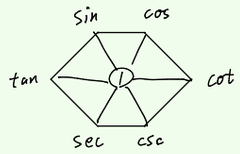


① 商关系.

$$\tan x = \frac{\sin x}{\cos x} = \frac{\sec x}{\csc x}$$

$$\sec x = \frac{\tan x}{\sin x} = \frac{\csc x}{\cot x}$$

$$\csc x = \frac{\sec x}{\tan x} = \frac{\cot x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x} = \frac{\csc x}{\sec x}$$

$$\cos x = \frac{\sin x}{\tan x} = \frac{\cot x}{\csc x}$$

$$\sin x = \frac{\tan x}{\sec x} = \frac{\cos x}{\cot x}$$

② 倒数关系.

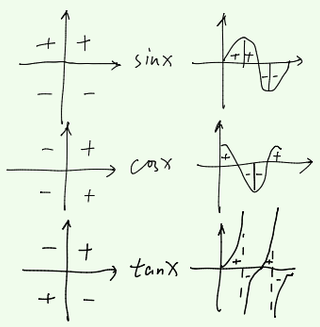
$$\sin x \cdot \csc x = 1$$
$$\cos x \cdot \sec x = 1$$
$$\tan x \cdot \cot x = 1$$

③ 平方和关系.

$$\sin^2 x + \cos^2 x = 1$$
$$\tan^2 x + 1 = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$

$\sin(\alpha \pm n\frac{\pi}{2})$
$\cos(\alpha \pm n\frac{\pi}{2})$
$\tan(\alpha \pm n\frac{\pi}{2})$

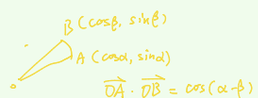
奇变偶不变 正负看相限. α当成锐角看



$\sin(\alpha + \frac{\pi}{2}) = \cos\alpha$ $\sin(\alpha + \pi) = -\sin\alpha$

$\cos(\alpha + \frac{\pi}{2}) = -\sin\alpha$ $\cos(\alpha + \pi) = -\cos\alpha$

$\tan(\alpha + \frac{\pi}{2}) = -\cot\alpha$ $\tan(\alpha + \pi) = +\tan\alpha$



$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta.$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

(下) $\cos(\alpha+\beta-\frac{\pi}{2})$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

① 积化和差:

$$\sin\alpha\sin\beta = \frac{\sin(\alpha+\beta) + \sin(\alpha-\beta)}{2}$$

$$\cos\alpha\sin\beta = \frac{\sin(\alpha+\beta) - \sin(\alpha-\beta)}{2}$$

$$\cos\alpha\cos\beta = \frac{\cos(\alpha+\beta) + \cos(\alpha-\beta)}{2}$$

$$\sin\alpha\sin\beta = \frac{-\cos(\alpha+\beta) + \cos(\alpha-\beta)}{2}$$

② 和差化积: ①式代入 $\begin{cases} \alpha = \frac{\alpha+\beta}{2} + \frac{\alpha-\beta}{2} \\ \beta = \frac{\alpha+\beta}{2} - \frac{\alpha-\beta}{2} \end{cases}$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

③ 倍角:

$$\sin 2\theta = 2\sin\theta\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

万能公式

④ 平方差: 由①②

$$\sin(\alpha+\beta)\sin(\alpha-\beta) = \sin^2\alpha - \sin^2\beta$$
$$\cos(\alpha+\beta)\cos(\alpha-\beta) = \cos^2\alpha - \sin^2\beta$$
$$-\sin(\alpha+\beta)\sin(\alpha-\beta) = \cos^2\alpha - \cos^2\beta$$

⑥ 降幂:

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\sin^2\theta\cos^2\theta = \frac{1-\cos 4\theta}{8}$$

⑤ 半角:

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$$