

① 倒数关系:

$$\tan x = \frac{\sin x}{\cos x} = \frac{\sec x}{\csc x}$$

$$\sec x = \frac{\tan x}{\sin x} = \frac{\csc x}{\cot x}$$

$$\csc x = \frac{\sec x}{\tan x} = \frac{\cot x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x} = \frac{\csc x}{\sec x}$$

$$\cos x = \frac{\sin x}{\tan x} = \frac{\cot x}{\csc x}$$

$$\sin x = \frac{\tan x}{\sec x} = \frac{\cos x}{\cot x}$$

② 积化和差:

$$\sin x \cdot \csc x = 1$$

$$\cos x \cdot \sec x = 1$$

$$\tan x \cdot \cot x = 1$$

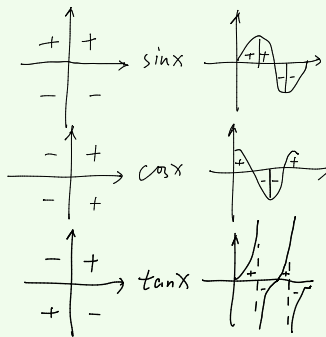
③ 平方关系:

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

$$\begin{aligned} \sin(\alpha \pm n\frac{\pi}{2}) \\ \cos(\alpha \pm n\frac{\pi}{2}) \\ \tan(\alpha \pm n\frac{\pi}{2}) \end{aligned} \quad \begin{cases} \text{奇变偶不变} \\ \text{正负看符号} \\ \text{以当成锐角看} \end{cases}$$



$$\begin{aligned} \sin(\alpha + \frac{\pi}{2}) &= \cos \alpha & \sin(\alpha + \pi) &= -\sin \alpha \\ \cos(\alpha + \frac{\pi}{2}) &= -\sin \alpha & \cos(\alpha + \pi) &= -\cos \alpha \\ \tan(\alpha + \frac{\pi}{2}) &= -\cot \alpha & \tan(\alpha + \pi) &= \tan \alpha \end{aligned}$$

$$\begin{aligned} &\beta (\cos \beta, \sin \beta) \\ &A (\cos \alpha, \sin \alpha) \\ &\vec{OA} \cdot \vec{OB} = \cos(\alpha - \beta) \end{aligned}$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

① 积化和差:

$$\sin \alpha \sin \beta = \frac{\sin(\alpha + \beta) - \sin(\alpha - \beta)}{2}$$

$$\cos \alpha \sin \beta = \frac{\sin(\alpha + \beta) + \sin(\alpha - \beta)}{2}$$

$$\cos \alpha \cos \beta = \frac{\cos(\alpha + \beta) + \cos(\alpha - \beta)}{2}$$

$$\sin \alpha \cos \beta = \frac{\sin(\alpha + \beta) + \cos(\alpha - \beta)}{2}$$

② 和差化积: ①或② $\begin{cases} \alpha = \frac{\alpha + \beta}{2} + \frac{\alpha - \beta}{2} \\ \beta = \frac{\alpha + \beta}{2} - \frac{\alpha - \beta}{2} \end{cases}$

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

③ 倍角:

$$\sin 2\theta = 2 \sin \theta \cos \theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

④ 平方差: 由①②

$$\sin(\alpha + \beta) \sin(\alpha - \beta) = \sin^2 \alpha - \sin^2 \beta$$

$$\cos(\alpha + \beta) \cos(\alpha - \beta) = \cos^2 \alpha - \cos^2 \beta$$

$$-\sin(\alpha + \beta) \sin(\alpha - \beta) = \cos^2 \alpha - \cos^2 \beta$$

⑤ 降幂:

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$\sin \theta \cos \theta = \frac{\sin 2\theta}{2}$$

⑥ 半角:

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \frac{\sin \theta}{1 + \cos \theta} = \frac{1 - \cos \theta}{\sin \theta}$$