Preparation for Quantitative Finance Interviews

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January 10, 2024

Contents

Part I

Mathematics

Chapter 1

Mathematical Analysis

1 Functions of a real variable

Reference texts: [JP10], [Rud76], [HLP88].

We start by reviewing the real number system. First recall that the Peano axioms define the set of all *natural numbers* $\mathbb{N}$. From this, we can construct the set of integers $\mathbb{Z}$, and then the rationals $\mathbb{Q}$.

An *order* on a set S is a relation, denoted by $<$, with the following two properties: incomparability (only one of the statements $x < y, x = y, y < x$ where $x, y \in S$ is true) and transitivity ($x < y, y < z \implies x < z$ for any $x, y, z \in S$). An ordered set S is said to have the *least-upper-bound property* if the following is true: if a non empty subset $E \subset S$ is bounded above, then $\sup E$ exists in S.

A *field* is a set F with two operations, called addition and multiplication, which satisfy the so-called "field axioms", which includes axioms for addition, axioms for multiplication, and the distributive law: $x(y+z) = zy + xz$ holds for all $x, y, z \in F$.

Theorem 1.1. *There exists an ordered field $\mathbb{R}$ which has the least-upper-bound property. The members of $\mathbb{R}$ are called* real numbers*. Moreover, $\mathbb{R}$ contains $\mathbb{Q}$ as a subfield. $\mathbb{Q}$ is* dense *in $\mathbb{R}$: between any two real numbers there is a rational one.*

Note that Theorem 1.1 is proved by the Dedekind cut method. Now we can define the *extended real number system* $\bar{\mathbb{R}}$ by including the real field $\mathbb{R}$ and two symbols, $+\infty$ and $-\infty$. We preserve the original order in $\mathbb{R}$, and define $-\infty < x < +\infty$ for every $x \in \mathbb{R}$. The extended real number system does not form a field.

For each positive integer k, let $\mathbb{R}^k$ be the set of all ordered k-tuples $\mathbf{x} = (x_1, x_2, \ldots, x_k)$ where $x_1, x_2, \ldots, x_k$ are real numbers, called the coordinates of $\mathbf{x}$. Elements of $\mathbb{R}^k$ where $k \geq 2$ are called *vectors*. For $\mathbf{x}, \mathbf{y} \in \mathbb{R}^k$, the two operations

$$\mathbf{x} + \mathbf{y} = (x_1 + y_1, \ldots, x_k + y_k), \qquad a\mathbf{x} = (ax_1, ax_2, \ldots, ax_k)$$

define the addition and (scalar) multiplication of vectors. These two operations satisfy the commutative, associative, and distributive laws and thus make $\mathbb{R}^k$ into a *vector space over the real field*.

The *inner product* of $\mathbf{x}, \mathbf{y} \in \mathbb{R}^k$ is

$$\mathbf{x} \cdot \mathbf{y} := \sum_{i=1}^{k} x_i y_i,$$

and the *norm* of $\mathbf{x}$ is given by

$$|\mathbf{x}| := (\mathbf{x} \cdot \mathbf{x})^{1/2}$$

Metric spaces

In order to discuss concepts like limits and convergence generally, we define a very general class of spaces (a *space* is a set of all objects of a certain type with rich structures) which includes such standard spaces as the real numbers, complex numbers, vectors, etc.

A set X, whose elements we shall call points, is said to be a *metric space* if with any two points p and q of X there is associated a real number $d(p, q)$, called the *distance* from p to q, such that

(a) $d(p, q) > 0$ if $p \neq q$, $d(p, p) = 0$;

(b) $d(p, q) = d(q, p)$;

(c) $d(p, q) \leq d(p, r) + d(r, q)$ for any $r \in X$

Any function with these three properties is called a *distance function*, or a *metric*.

If $\mathbf{x} \in \mathbb{R}^k$ and $r > 0$, the *open* (or *closed*) *ball* B with center at $\mathbf{x}$ and radius r is defined to be the set of all $\mathbf{y} \in \mathbb{R}^k$ such that $|\mathbf{y} - \mathbf{x}| < r$ (or $|\mathbf{y} - \mathbf{x}| \leq r$).

Definition. Let X be a metric space. All points and sets mentioned below are understood to be elements and subsets of X.

(a) A neighborhood of p is a set $N_r(p)$ consisting of all q such that $d(p, q) < r$, for some $r > 0$. The number r is called the radius of $N_r(p)$.

(b) A point p is a limit point of the set E if every neighborhood of p contains a point $q \neq p$ such that $q \in E$.

(c) If $p \in E$ and p is not a limit point of E, then p is called an *isolated* point of E.

(d) E is closed if every limit point of E is a point of E.

(e) A point p is an interior point of E if there is a neighborhood N of p such that $N \subset E$.

Chapter 2

Differential Equations

4 Solvable first-order ODEs

1. Separable:

$$x' = f(x)g(t) \qquad \Longrightarrow \qquad \int \frac{1}{f(x)}\,\mathrm{d}x = \int g(t)\,\mathrm{d}t + C$$

Exercise 4.1. *Find $f(x)$ such that*

$$f'(x) = f(x)(1 - f(x))$$

Solution. Let $y = f(x)$. Then the equation is separable.

2. Homogeneous of degree k for some arbitrary value a:

$$f(at, ax) = a^k f(t, x)$$

   For example, take $k = 1, a = 1/t$:

$$x' = f(x/t)$$

   Let $y = x/t$, we get

$$y' \cdot t + y = f(y)$$

   which reduces to case 1, a separable equation.

3. More generally, consider

$$x' = f\left(\frac{ax + bt + c}{\alpha x + \beta t + \gamma}\right)$$

   where $a, b, c, \alpha, \beta, \gamma$ are constants.

a) If $c = \gamma = 0$, rewrite it as
$$x' = f\left(\frac{ax/t + b}{\alpha x/t + \beta}\right)$$
which reduces to case 2.

b) If $c, \gamma \neq 0$, but $a/\alpha = b/\beta = 1/k$, we let $y = ax + bt$, then (noting that a is constant so $a' = 0$)
$$y' = a'x + ax' + b = af\left(\frac{y + c}{ky + \gamma}\right) + b$$
which reduces to case 1, a separable equation.

c) If $c, \gamma \neq 0$ and $a/\alpha \neq b/\beta$, we solve the system
$$\begin{cases} ax + bt + c & = 0 \\ \alpha x + \beta t + \gamma & = 0 \end{cases}$$
which must have a solution, say x_0, t_0. Take $y = x - x_0, s = t - t_0$, and we have
$$\frac{\mathrm{d}y}{\mathrm{d}s} = \frac{\mathrm{d}x}{\mathrm{d}t} = f\left(\frac{ax + bt + c - 0}{\alpha x + \beta t + \gamma - 0}\right) = f\left(\frac{ay + bs}{\alpha y + \beta s}\right)$$
which reduces to case 3(a).

4. Exact:
$$M(x,y)\,\mathrm{d}x + N(x,y)\,\mathrm{d}y = 0$$
is exact if there exists a function $g(x,y)$ such that
$$\mathrm{d}g = M(x,y)\,\mathrm{d}x + N(x,y)\,\mathrm{d}y, \qquad M(x,y) = \frac{\partial g}{\partial x}, N(x,y) = \frac{\partial g}{\partial y}$$
This happens if and only if
$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$
Then $g(x,y) = C$ where C is some constant is the solution to this equation.

5. Integrating factors: if $M(x,y)\,\mathrm{d}x + N(x,y)\,\mathrm{d}y$ is not exact but $I(x,y)\,(M\,\mathrm{d}x + N\,\mathrm{d}y)$ is, I is called an *integrating factor*.

   a) If
$$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) =: g(x)$$
is a function of x alone, then
$$I(x,y) = \exp\left(\int g(x)\,\mathrm{d}x\right)$$

b) If

$$\frac{1}{M}\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)=:h(y)$$

is a function of y alone, then

$$I(x,y)=\exp\left(-\int h(y)\,\mathrm{d}y\right)$$

Then it is reduced to case 4.

6. Linear nonhomogeneous equation:

$$x'(t)=k(t)x+a(t)$$

We multiply each side by

$$K(t):=\exp\left\{-\int_{t_0}^{t}k(s)\,\mathrm{d}s\right\}$$

Then

$$K(t)\left(x'-kx\right)=(Kx)' \qquad \Longrightarrow \qquad \int \mathrm{d}(Kx)=\int K(t)a(t)\,\mathrm{d}t+C$$

which yields the result

$$x(t)=\frac{1}{K(t)}\left(\int_{t_0}^{t}K(s)a(s)\,\mathrm{d}s+x(t_0)\right)$$

7. Some nonlinear first-order ODEs:

a) Bernoulli:

$$x'(t)=F_tx+g(t)x^n, \qquad n\neq 0,1$$

We let $y=x^{1-n}$ and have

$$\frac{\mathrm{d}y}{\mathrm{d}t}=(1-n)x^{-n}x'=(1-n)x^{-n}\left(f(t)x+g(t)x^n\right)=(1-n)\left(fy+g\right)$$

b) Ricatti:

$$x'(t)=f(t)x+g(t)x^2+h(t)$$

Suppose we already has a particular solution $p(t)$ so that $p'=fp+gp^2+h$. Subtract it from the original equation and let $y=x-p$:

$$\begin{aligned} x'-p'&=f(x-p)+g(x+p)(x-p)\\ y'&=fy+g(y+2p)y=\left(f(t)+g(t)2p(t)\right)y(t)+g(t)y^2(t)\end{aligned}$$

which reduces to a Bernoulli equation.

5 Second-order linear ODEs

We start by considering a_0, a_1 real constants for

$$y''(x) + a_1 y'(x) + a_0 y(x) = 0$$

Solve the the *characteristic function*

$$\lambda^2 + a_1\lambda + a_0 = 0$$

and get two solutions λ_1, λ_2.

- Case 1: if $\lambda_1, \lambda \in \mathbb{R}, \lambda_1 \neq \lambda_2$:

$$y = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$$

- Case 2: if $\lambda_{1,2} = a \pm bi, b \neq 0$ (noting that roots must be conjugate pairs):

$$y = d_1 e^{(a+bi)x} + d_2 e^{(a-bi)x} = e^{ax} \cdot (c_1 \cos(bx) + c_2 \sin(bx))$$

- Case 3: if $\lambda_1 = \lambda_2 = \lambda$:

$$y = (c_1 + c_2 x)e^{\lambda x}$$

Exercise 5.1. *Solve $4y'' - 4y' + y = 0$ with initial conditions $y(0) = 1, y'(0) = 0$.*

Solution. Consider $4\lambda^2 - 4\lambda + 1 = 0$ whose solutions are $\lambda_{1,2} = 1/2$. It is a case 3 senario, and we let

$$y = (c_1 + c_2 x)e^{x/2}$$

Plugging the initial conditions, we get $c_1 = 1, c_2 = -1/2$.

Now, suppose $a_1 = p(x), a_0 = q(x)$, i.e. we consider

$$y''(x) + p(x)y'(x) + q(x)y(x) = 0$$

- Case 1: Suppose p, q are analytic, then we can let $y(x) = \sum_{n=0}^{\infty} A_k x^k$ and apply the *power series method.*

- Case 2: A *Cauchy-Euler* equation has the form

$$t^2 y'' + aty' + by = 0 \qquad \Longleftrightarrow \qquad y'' + \frac{ay'}{t} + \frac{by}{t^2} = 0$$

  We assume $y = t^m$ and plugging in we get $m^2 + (a-1)m + b = 0$. Suppose this equation has two solutions $m_{1,2}$.

– if $m_1 \neq m_2 \in \mathbb{R}$, $y = c_1 t^{m_1} + c_2 t^{m_2}$.
– if $m_1 \neq m_2 \in \mathbb{C}$, they are conjugates, so we let $m_{1,2} = \alpha \pm \beta i$ and have

$$y = c_1 \cdot t^\alpha \cos(\beta \log t) + c_2 \cdot t^\alpha \sin(\beta \log t)$$

– if $m_1 = m_2 = m$, they can only be reals, and we have

$$y = c_1 \cdot t^m \log t + c_2 \cdot t^m$$

Indeed, if it is an ODE with higher order, and m becomes a root with multiplicity k, then the basis solutions are:

$$t^m, t^m \log t, t^m (\log t)^2, t^m (\log t)^3, \ldots, t^m (\log t)^{k-1}$$

- Case 3: the *method of Frobenius*: the differential equation

$$w'' + p(z)w' + q(z)w = 0$$

has an *isolated singular point* at the origin if the coefficients p and q are analytic and single-valued in a disk $|z| < R$ except at $z = 0$ (analytic in the punctured disk). The origin is a *regular singular point* if p has a pole of order at most one and q a pole of order at most two there. In other words, if the origin is a regular singular point then $p(z) = z^{-1}P(z)$ and $q(z) = z^{-2}Q(z)$ where P and Q are analytic and single-valued in the full disk, including the origin. We'll write the standard equation with a regular singular point at the origin in the form

$$z^2 w'' + zP(z)w' + Q(z)w = 0.$$

The functions P and Q then have power-series expansions

$$P(z) = \sum_{k=0}^{\infty} P_k z^k, \quad Q(z) = \sum_{k=0}^{\infty} Q_k z^k$$

convergent in this disk. Then we seek a solution of the form

$$w(z) = z^\mu \sum_{k=0}^{\infty} a_k z^k$$

Now suppose the equation is nonhomogeneous:

$$y''(x) + p(x)y'(x) + q(x)y(x) = r(x), \qquad r(x) \neq 0$$

First we solve the homogeneous equation by the methods above

$$y''(x) + p(x)y'(x) + q(x)y(x)$$

and get a basis of two solutions $b_1(x), b_2(x)$. We calculate the Wronskian:

$$W(x) := \begin{vmatrix} b_1 & b_2 \\ b_1' & b_2' \end{vmatrix}$$

and a particular solution $p(x)$ is given as

$$p(x) = b_1 v_1 + b_2 v_2 \qquad \text{where} \qquad v_1 = -\int \frac{r(x)b_2(x)}{W(x)}\,\mathrm{d}x, v_2 = \int \frac{r(x)b_1(x)}{W(x)}\,\mathrm{d}x$$

Therefore, a general solution is given by

$$y(x) = C_1 b_1(x) + C_2 b_2(x) + p(x)$$

Exercise 5.2. *Solve $y'' - 4y' + 4y = 1$.*

Solution. Solving $\lambda^2 - 4\lambda + 4 = 0$ gives us $\lambda_{1,2} = 2$. Now, since RHS $= 1$, a particular solution is easy to guess: $y_p = 1/4$. Therefore, a general solution is given by

$$y(x) = c_1 e^{2x} + c_2 e^{2x} x + 1/4$$

for some constants c_1, c_2.

6 Introduction to PDEs

Chapter 3

Numerical Methods

7 Solution of equations by iteration

Reference text: [MS03].

Nonlinear equations such as $\cos x + x^2 - 12 = 0$ do not have closed form solution, so we must use a numerical method. Intuitively, by the *intermediate value thoerem*, we know that a real valued continuous function on $[a, b]$ has at least a zero if $f(a)f(b) < 0$. This naturally leads us to the *bisection* method: given f on $[a, b]$ such that $f(a) < 0$ and $f(b) > 0$, we check whether $f((a + b)/2) > 0$ or not; in other words, we split the interval in half and repeat. However, by this method, the error goes down by a factor of 2. This is not fast enough. Fortunately we can take advantage of more information such as the values or the derivative of f at some point rather than only the signs of it.

The secant method

The *secant method* is defined by

$$x_{k+1} = x_k - f(x_k)\left(\frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})}\right), \quad k = 1, 2, \ldots$$

where x_0 and x_1 are given starting values (guesses).

Suppose that $\xi = \lim_{k\to\infty} x_k$. We say that the sequence (x_k) *converges to* ξ *with at least order* $q > 1$, if there exist a sequence (ε_k) of positive real numbers converging to 0, and $\mu > 0$, such that

$$|x_k - \xi| \le \varepsilon_k, \quad k = 0, 1, 2, \ldots, \qquad \text{and} \qquad \lim_{k\to\infty} \frac{\varepsilon_{k+1}}{\varepsilon_k^q} = \mu \tag{7.1}$$

If 7.1 holds with $\varepsilon_k = |x_k - \xi|$ for $k = 0, 1, 2, \ldots$, then the sequence (x_k) is said to *converge to* ξ *with order* q.

Chapter 4

Probability Theory

11 Combinatorics essentials

Recall the formula for *binomial coefficients* that represent the number of ways to choose an (unordered) subset of k elements from a fixed set of n elements:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Some interesting properties include:

- Pascal's triangle recurrence relation:

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

- The *binomial formula*:

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} \qquad \text{In particular, } 2^n = \sum_{k=0}^{n} \binom{n}{k}$$

- The *Chu–Vandermonde identity*:

$$\sum_{j=0}^{k} \binom{m}{j}\binom{n-m}{k-j} = \binom{n}{k}$$

Exercise 11.1 (Squarepoint)**.** *Find the number of integer triples (x, y, z) such that $x + y + z = n$ and $x \geq y \geq z > 0$. Denote this number by $f(n)$. Find $\lim_{n\to\infty} f(n)/n^2$.*

Solution. Let $y = z + k_1$ and $x = y + k_2$. with $k_1, k_2 \geq 0$. Then we have $3z + 2k_1 + k_2 = n$.

Part IV

Finance

We begin with some basic concepts. *Interest* is a key financial concept representing the cost of borrowing or the return on investing funds. *Compound interest* differs from simple interest in that it is calculated on the principal amount and the accumulated interest from previous periods. In continuous time models, such as the Black-Scholes model or for the bond yields, we use continuous compounding. In discrete time models, such as multi-step Binomial model we use discrete compounding.

For discrete compounding, the total accumulated value A, including the principal sum P plus compounded interest I, is given by the formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where: A is the final amount, P is the original principal sum, r is the nominal annual interest rate, n is the compounding frequency, t is the overall length of time the interest is applied (expressed using the same time units as r, usually years). The total compound interest generated is the final value minus the initial principal:

$$I = P\left(1 + \frac{r}{n}\right)^{nt} - P$$

For example, suppose a principal amount of $\$1,500$ is deposited in a bank paying an annual interest rate of 4.3%, compounded quarterly. Then the balance after 6 years is found by using the formula above, with $P = 1500, r = 0.043, n = 4$, and $t = 6$:

$$A = 1500 \times \left(1 + \frac{0.043}{4}\right)^{4\times 6} \approx 1938.84$$

So the amount A after 6 years is approximately $\$1,938.84$, with compound interest $1938.84 - 1500 = \$438.84$.

Continuous compounding can be thought of as making the compounding period infinitesimally small, achieved by taking the limit as n goes to infinity. The amount A after t periods of continuous compounding can be expressed in terms of the initial amount P_0 as

$$A = \lim_{n\to\infty} P_0\left(1 + \frac{r}{n}\right)^{nt} = P_0 e^{rt}$$

A *portfolio* refers to a collection of financial assets such as stocks, bonds, commodities, currencies, and cash equivalents, as well as their fund counterparts, held by an individual or an institutional investor. Portfolio *returns* refer to the gain or loss generated by a portfolio of investments over a specified period. They are typically expressed as a percentage of the portfolio's initial value. For example, if a portfolio's value increases from $\$100,000$ to $\$101,000$ in a day, the *daily return* is 1%. A monthly or yearly return is defined similarly.

Weighted average portfolio returns is a method of calculating the overall return of a portfolio by taking into account the proportion (weight) of each asset in the portfolio and its individual return. This approach gives a more accurate picture of the portfolio's performance, especially when it contains a diverse range of assets with varying returns. Such calculation is given in the following example: consider a portfolio consists of two assets A and B where asset A is worth 60% of the portfolio with a return of 8% and B is worth 40% of the portfolio with a return of 5%. The weighted average portfolio return is calculated as: $(60\% \times 8\%) + (40\% \times 5\%) = 6.8\%$.

Chapter 8

Options

Calls and *puts* are one of the most important financial instruments alongside stocks, futures and bonds. They are very similar to forward contracts, but they give to the holder of the contract the *option* either to buy or to sell as opposed to the forward contracts which require the sale is made. Therefore, the owner of the options are sure not to lose money at maturity. On the other hand, they have to pay an initial fee to obtain these contracts. Hence, their *net position at maturity* can be both positive or negative, but with a bounded downside. Options are written on a particular stock called the *underlying*.

Definition. The *European Call option* with maturity T and strike K gives its holder the option but not the obligation to buy the stock at time T for a price K, regardless the price of the stock at maturity. The future random pay-off of this option is $(S_T - K)^+$, where S_T is the future random value of the stock and $(a)^+ := \max\{a, 0\}$ for a real number a.

The product dual to the call is the put and is defined as follows.

Definition. The *European Put option* with maturity T and strike K gives its holder the option (not the obligation) to sell the stock at time T for a price K, regardless the price of the stock at maturity. The future random pay-off of this option is $(K - S_T)^+$.

Intrinsic value is the immediate payoff if an option is exercised. For a call option, it's the current price of the underlying asset minus the strike price (if positive); for a put option, it's the strike price minus the current price of the underlying asset (if positive). *Time value* is the extra value an option has due to the possibility that the market conditions might change favorably before the option expires. The longer the time until expiration, the greater the chance that this favorable change can occur, thus the higher the time value. Time value is influenced by factors like volatility of the underlying asset, time to expiration, risk-free interest rates, and dividends.

At the money (ATM), *in the money* (ITM), and *out of the money* (OTM) are terms used in options trading to describe the relationship between an option's strike price and the current price of the underlying asset. These terms help traders assess the intrinsic value and potential profitability of an option:

- At the Money (ATM):
  - An option is said to be at the money when the strike price of the option is equal to or very close to the current market price of the underlying asset.
  - For a call option, it's ATM if the strike price is equal to the market price of the underlying stock.
  - For a put option, it's ATM if the strike price is equal to the market price of the underlying stock.
  - ATM options have no intrinsic value, but they may still have time value.
- In the Money (ITM):
  - An option is in the money if exercising it would result in a profit based on the current price of the underlying asset.
  - For a call option, it's ITM if the strike price is below the market price of the underlying stock.
  - For a put option, it's ITM if the strike price is above the market price of the underlying stock.
  - ITM options have intrinsic value. For call options, it's the amount by which the stock price exceeds the strike price, and for put options, it's the amount by which the strike price exceeds the stock price.
- Out of the Money (OTM):
  - An option is out of the money if exercising it currently would not result in a profit.
  - For a call option, it's OTM if the strike price is above the market price of the underlying stock.
  - For a put option, it's OTM if the strike price is below the market price of the underlying stock.
  - OTM options do not have intrinsic value, but they may have time value based on the probability that they could become ITM before expiration.

27 Put-call parity and arbitrage

As the pay-offs of either the call or the put options are positive, one needs to pay an initial fee in order to obtain them. A nontrivial question is how to compute this initial price. First note that, for any value of S_T, we have

$$(S_T - K)^+ - (K - S_T)^+ = S_T - K$$

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