# Preparation for Quantitative Finance Interviews

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## 3 Functions of a complex variable

Reference text: [Ahl79].

The formal definition of a complex number  $z=x+yi=(x,y)\in\mathbb{C}:=\mathbb{R}^2$  is to consider the field  $(\mathbb{R}^2,+,\cdot)$  with addition and multiplication

$$(x,y) + (u,v) = (x+u,y+v),$$
  $(x,y) \cdot (u,v) = (xu - yv, xv + yu)$ 

with neutral elements  $0_{\mathbb{C}}=(0,0)$  for + and  $1_{\mathbb{C}}=(1,0)$  for  $\cdot.$ 

Note that  $\mathbb C$  cannot be ordered. However, with modulus defined as  $|z|:=\sqrt{x^2+y^2}\in [0,\infty)$  and conjugate defined as  $\overline z:=x-yi$ , we have the analogous triangle inequalities

$$|z+w| \le |z| + |w|, \qquad |z-w| \ge ||z| - |w||$$

and Cauchy-Schwarz inequality

$$\left| \sum_{j=1}^{n} \overline{z_{j}} w_{j} \right| = \left| \sum_{j=1}^{n} z_{j} w_{j} \right| \leq \sqrt{\sum_{j=1}^{n} |z_{j}|^{2}} \sqrt{\sum_{j=1}^{n} |w_{j}|^{2}}$$

mentioned earlier in §1.

A complex number z = a + bi can be expressed by *polar coordinates*:

$$a + bi = r \cos \theta + i \cdot r \sin \theta$$

where  $arg(z) := \theta \in (-\pi, \pi]$ . Upon multiplication of two complex numbers, the moduli are multiplied, and the arguments are added.

**Exercise 3.1** ([SRW19, 1.4]). Solve  $x^6 = 64$ .

*Solution.* The moduli should always be 2, and the argument  $\theta$  can be any value such that

$$6\theta = 2k\pi, \qquad k \in \mathbb{N}_0, \theta \in [0, 2\pi)$$

Therefore, there are six possible solutions:

$$2\cos(k\pi/3) + 2i\sin(k\pi/3), \qquad k \in [0, 5]$$

The extension of functions  $e^z$ ,  $\log z$ ,  $\sin z$ ,  $\cos z$ , etc. should be natural in the sense that many of the familiar properties of  $\sin$ ,  $\cos$ ,  $\exp$ ,  $\log$  are retained. We define the complex *exponential* function as

$$e^{a+bi} := e^a \cdot (\cos b + i \sin b), \qquad z = a + bi \in \mathbb{C}$$

and the complex sine and cosine functions as

$$\sin(z) = \frac{e^{iz} - e^{-iz}}{2i}, \qquad \cos(z) = \frac{e^{iz} + e^{-iz}}{2}, \qquad z \in \mathbb{C}$$

Exercise 3.2 ([SRW19, 1.1]). Calculate  $i^i$ .

*Solution.* Note that  $i = \cos(\pi/2) + i\sin(\pi/2) = e^{i\cdot\pi/2}$ . Therefore

$$i^i = \left(e^{i \cdot \pi/2}\right)^i = \boxed{e^{-\pi/2}}$$

A solution z of the equation  $e^z = w$  is called a *logarithm* of w, denoted  $z = \log w$ . Every  $w \in \mathbb{C} \setminus \{0\}$  has countably many logarithms:

$$\log(w) = \log|w| + i \cdot (\arg(w) + 2\pi n), \qquad n \in \mathbb{Z},$$

and the principal value of the logarithm of w is set for n = 0.

### 4 Ordinary differential equations

#### Solvable first-order ODEs

1. Separable:

$$x' = f(x)g(t)$$
  $\Longrightarrow$   $\int \frac{1}{f(x)} dx = \int g(t) dt + C$ 

**Exercise 4.1** ([SRW19, 1.14]). Find f(x) such that

$$f'(x) = f(x)(1 - f(x))$$

*Solution.* Let y = f(x). Then the equation is separable.

2. Homogeneous of degree k for some arbitrary value a:

$$f(at, ax) = a^k f(t, x)$$

For example, take k = 1, a = 1/t:

$$x' = f(x/t)$$

Let y = x/t, we get

$$y' \cdot t + y = f(y)$$

which reduces to case 1, a separable equation.

3. More generally, consider

$$x' = f\left(\frac{ax + bt + c}{\alpha x + \beta t + \gamma}\right)$$

where  $a, b, c, \alpha, \beta, \gamma$  are constants.

a) If  $c = \gamma = 0$ , rewrite it as

$$x' = f\left(\frac{ax/t + b}{\alpha x/t + \beta}\right)$$

which reduces to case 2.

b) If  $c, \gamma \neq 0$ , but  $a/\alpha = b/\beta = 1/k$ , we let y = ax + bt, then (noting that a is constant so a' = 0)

$$y' = a'x + ax' + b = af\left(\frac{y+c}{ky+\gamma}\right) + b$$

which reduces to case 1, a separable equation.

c) If  $c, \gamma \neq 0$  and  $a/\alpha \neq b/\beta$ , we solve the system

$$\begin{cases} ax + bt + c &= 0\\ \alpha x + \beta t + \gamma &= 0 \end{cases}$$

which must have a solution, say  $x_0, t_0$ . Take  $y = x - x_0, s = t - t_0$ , and we have

$$\frac{\mathrm{d}y}{\mathrm{d}s} = \frac{\mathrm{d}x}{\mathrm{d}t} = f\left(\frac{ax + bt + c - 0}{\alpha x + \beta t + \gamma - 0}\right) = f\left(\frac{ay + bs}{\alpha y + \beta s}\right)$$

which reduces to case 3(a).

4. Exact:

$$M(x,y) dx + N(x,y) dy = 0$$

is exact if there exists a function g(x, y) such that

$$dg = M(x, y) dx + N(x, y) dy, \qquad M(x, y) = \frac{\partial g}{\partial x}, N(x, y) = \frac{\partial g}{\partial y}$$

This happens if and only if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Then g(x,y) = C where C is some constant is the solution to this equation.

- 5. Integrating factors: if M(x, y) dx + N(x, y) dy is not exact but I(x, y) (M dx + N dy) is, I is called an *integrating factor*.
  - a) If

$$\frac{1}{N} \left( \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) =: g(x)$$

is a function of x alone, then

$$I(x,y) = \exp\left(\int g(x) \, \mathrm{d}x\right)$$

b) If

$$\frac{1}{M} \left( \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) =: h(y)$$

is a function of y alone, then

$$I(x,y) = \exp\left(-\int h(y) \,\mathrm{d}y\right)$$

Then it is reduced to case 4.

6. Linear nonhomogeneous equation:

$$x'(t) = k(t)x + a(t)$$

We multiply each side by

$$K(t) := \exp\left\{-\int_{t_0}^t k(s) \, \mathrm{d}s\right\}$$

Then

$$K(t)(x'-kx) = (Kx)'$$
  $\Longrightarrow$   $\int d(Kx) = \int K(t)a(t) dt + C$ 

which yields the result

$$x(t) = \frac{1}{K(t)} \left( \int_{t_0}^t K(s)a(s) \,\mathrm{d}s + x(t_0) \right)$$

- 7. Some nonlinear first-order ODEs:
  - a) Bernoulli:

$$x'(t) = F_t x + q(t)x^n, \qquad n \neq 0, 1$$

We let  $y = x^{1-n}$  and have

$$\frac{\mathrm{d}y}{\mathrm{d}t} = (1-n)x^{-n}x' = (1-n)x^{-n} (f(t)x + g(t)x^n) = (1-n)(fy+g)$$

b) Ricatti:

$$x'(t) = f(t)x + g(t)x^2 + h(t)$$

Suppose we already has a particular solution p(t) so that  $p' = fp + gp^2 + h$ . Subtract it from the original equation and let y = x - p:

$$x' - p' = f(x - p) + g(x + p)(x - p)$$
  
$$y' = fy + g(y + 2p)y = (f(t) + g(t)2p(t))y(t) + g(t)y^{2}(t)$$

which reduces to a Bernoulli equation.

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#### Second-order linear ODEs

We start by considering  $a_0, a_1$  real constants for

$$y''(x) + a_1y'(x) + a_0y(x) = 0$$

Solve the the characteristic function

$$\lambda^2 + a_1\lambda + a_0 = 0$$

and get two solutions  $\lambda_1, \lambda_2$ .

• Case 1: if  $\lambda_1, \lambda \in \mathbb{R}, \lambda_1 \neq \lambda_2$ :

$$y = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$$

• Case 2: if  $\lambda_{1,2} = a \pm bi, b \neq 0$  (noting that roots must be conjugate pairs):

$$y = d_1 e^{(a+bi)x} + d_2 e^{(a-bi)x} = e^{ax} \cdot (c_1 \cos(bx) + c_2 \sin(bx))$$

• Case 3: if  $\lambda_1 = \lambda_2 = \lambda$ :

$$y = (c_1 + c_2 x)e^{\lambda x}$$

**Exercise 4.2** (Baruch). Solve 4y'' - 4y' + y = 0 with initial conditions y(0) = 1, y'(0) = 0.

*Solution.* Consider  $4\lambda^2-4\lambda+1=0$  whose solutions are  $\lambda_{1,2}=1/2$ . It is a Case 3 scenario, and thus we let

$$y = (c_1 + c_2 x)e^{x/2}$$

Plugging the initial conditions, we get  $c_1 = 1, c_2 = -1/2$ .

Now, suppose  $a_1 = p(x)$ ,  $a_0 = q(x)$ , i.e. we consider

$$y''(x) + p(x)y'(x) + q(x)y(x) = 0$$

- Case 1: Suppose p,q are analytic, then we can let  $y(x) = \sum_{n=0}^{\infty} A_k x^k$  and apply the *power series method*.
- Case 2: A Cauchy-Euler equation has the form

$$t^2y'' + aty' + by = 0 \qquad \iff \qquad y'' + \frac{ay'}{t} + \frac{by}{t^2} = 0$$

We assume  $y = t^m$  and plugging in we get  $m^2 + (a-1)m + b = 0$ . Suppose this equation has two solutions  $m_{1,2}$ .

- if  $m_1 \neq m_2 \in \mathbb{R}$ ,  $y = c_1 t^{m_1} + c_2 t^{m_2}$ .
- if  $m_1 \neq m_2 \in \mathbb{C}$ , they are conjugates, so we let  $m_{1,2} = \alpha \pm \beta i$  and have

$$y = c_1 \cdot t^{\alpha} \cos(\beta \log t) + c_2 \cdot t^{\alpha} \sin(\beta \log t)$$

- if  $m_1 = m_2 = m$ , they can only be reals, and we have

$$y = c_1 \cdot t^m \log t + c_2 \cdot t^m$$

Indeed, if it is an ODE with higher order, and m becomes a root with multiplicity k, then the basis solutions are:

$$t^m, t^m \log t, t^m (\log t)^2, t^m (\log t)^3, \dots, t^m (\log t)^{k-1}$$

• Case 3: the method of Frobenius: the differential equation

$$w'' + p(z)w' + q(z)w = 0$$

has an isolated singular point at the origin if the coefficients p and q are analytic and single-valued in a disk |z| < R except at z=0 (analytic in the punctured disk). The origin is a regular singular point if p has a pole of order at most one and q a pole of order at most two there. In other words, if the origin is a regular singular point then  $p(z)=z^{-1}P(z)$  and  $q(z)=z^{-2}Q(z)$  where P and Q are analytic and single-valued in the full disk, including the origin. We'll write the standard equation with a regular singular point at the origin in the form

$$z^{2}w'' + zP(z)w' + Q(z)w = 0.$$

The functions P and Q then have power-series expansions

$$P(z) = \sum_{k=0}^{\infty} P_k z^k, \quad Q(z) = \sum_{k=0}^{\infty} Q_k z^k$$

convergent in this disk. Then we seek a solution of the form

$$w(z) = z^{\mu} \sum_{k=0}^{\infty} a_k z^k$$

Now suppose the equation is nonhomogeneous:

$$y''(x) + p(x)y'(x) + q(x)y(x) = r(x), r(x) \neq 0$$

First we solve the homogeneous equation by the methods above

$$y''(x) + p(x)y'(x) + q(x)y(x)$$

and get a basis of two solutions  $b_1(x), b_2(x)$ . We calculate the Wronskian:

$$W(x) := \begin{vmatrix} b_1 & b_2 \\ b_1' & b_2' \end{vmatrix}$$

and a particular solution p(x) is given as

$$p(x) = b_1 v_1 + b_2 v_2$$
 where  $v_1 = -\int \frac{r(x)b_2(x)}{W(x)} dx, v_2 = \int \frac{r(x)b_1(x)}{W(x)} dx$ 

Therefore, a general solution is given by

$$y(x) = C_1b_1(x) + C_2b_2(x) + p(x)$$

**Exercise 4.3** ([SRW19, 1.13]). *Solve* y'' - 4y' + 4y = 1.

Solution. Solving  $\lambda^2 - 4\lambda + 4 = 0$  gives us  $\lambda_{1,2} = 2$ . Now, since RHS = 1, a particular solution is easy to guess:  $y_p = 1/4$ . Therefore, a general solution is given by

$$y(x) = c_1 e^{2x} + c_2 e^{2x} x + 1/4$$

for some constants  $c_1, c_2$ .

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