Set 3– due 10 February

1) [15 points] In addition to the usual Zeeman effect, there is a quadratic Zeeman effect associated with the interaction of an atom with a constant magnetic field B,

$$\Delta E = -\frac{1}{2}\chi B^2 \tag{1}$$

arising from the $e^2\vec{A}^2/(2mc^2)$ term in the full electromagnetic Hamiltonian. Calculate this shift for the ground state of hydrogen. Express your answer in terms of the diamagnetic susceptibility χ. Recall that the wave function is $\psi(\vec{x}) = N\exp(-r/a_0)$ where a_0 is the Bohr radius and N is a normalization factor – $\int d^3x|\psi|^2 = 1$. The gauge choice $\vec{A} = \frac{1}{2}(\vec{B}\times\vec{r})$ where $\vec{B}$ is a constant, is consistent with the wave function. Other gauge choices are not.

2) [30 points] Compute the pattern of splittings (energies and eigenstates) for the Stark effect for the $n = 3$ levels of hydrogen, to lowest nonvanishing order. Evaluate the angular parts of all the matrix elements, leaving the radial integrals as free parameters. All of last semester's Wigner-Eckart and Clebsch-Gordon technology, including a table of Clebsch-Gordon coefficients, will be very useful, to deal with the 9×9 Hamiltonian matrix. In fact, very little work is needed to discover that there are only three distinct matrix elements. Evaluating them is more difficult.

3) [15 points] Complete the variational calculation with a Gaussian trial wave function of the ground state energy of the quartic oscillator, $H = p^2/(2m)+\lambda x^4$. The true energy is about $\lambda^{1/3}(\hbar^2/(2m))^{2/3}(1.0603621\ldots)$.