Set 2– due 3 February

1) [15 points] Consider a two-state system with Hamiltonian

$$H = \begin{pmatrix} \epsilon_1 & \lambda\Delta \\ \lambda\Delta & \epsilon_2 \end{pmatrix} \tag{1}$$

As you did in set 1, first find the energy eigenvalues and eigenfunctions exactly. Then, assume that the system is almost degenerate, that $\epsilon_2 - \epsilon_1 \equiv \epsilon << \lambda|\Delta|$. Show that the exact result you just found is close to the perturbative answer you would have, but using degenerate state perturbation theory, namely treating

$$H_0 = \begin{pmatrix} \epsilon_1 & \lambda\Delta \\ \lambda\Delta & \epsilon_1 \end{pmatrix} \tag{2}$$

as the zeroth order term and

$$H_1 = \begin{pmatrix} 0 & 0 \\ 0 & \epsilon_2 - \epsilon_1 \end{pmatrix}. \tag{3}$$

as the perturbation. Solve for both energies and states. This is another example about the use of degenerate-state perturbation theory.

2) [10 points] Find the eigen-energies of

$$H = \begin{pmatrix} E_1 & b & 0 \\ b & E_1 & c \\ 0 & c & E_2 \end{pmatrix} \tag{4}$$

through second order in the small parameters b, c, using degenerate state perturbation theory.

3) [35 points] Consider the Hamiltonian for a quantum mechanical pendulum,

$$H = -\frac{\hbar^2}{2ml^2}\frac{\partial^2}{\partial\theta^2} + mgl(1 - \cos\theta), \tag{5}$$

in perturbation theory. (a) [15 points] For g large, find the energies of all the states of the pendulum assuming that the problem is mostly a harmonic oscillator, and the θ^4 term in the expansion of the cosine is a perturbation. (It's an exponentially small error to extend the limits of θ to $\pm\infty$.) (b) [15 points] For g small, treat the whole potential as a perturbation and calculate the energies of all the states of the system to lowest nontrivial order in the potential. (c) [5 points] From your answers to (a) and (b), describe more carefully what "g large" and "g small" mean (i.e., what is the dimensionless small parameter describing the perturbative expansions)?