Homework #3

Due 10:10, Nov. 12, 2021

1. Consider *M* measurements of a known signal $s(n)$ embedded in a zero-mean iid Gaussian noise $w(n)$ with variance σ_w^2 as

$$x(n) = As(n) + w(n); n = 0,1,\ldots,M-1$$

where *A* is an amplitude parameter to be estimated.

(1) Find the ML estimate.

(2) Find the MAP estimate if *A* is Gaussian distributed with mean μ_A and variance σ_A^2.

(3) Find the MAP estimate if the *A* has the following distribution:

$$f(A) = \begin{cases} \frac{A}{\sigma_A^2} e^{-\frac{A^2}{2\sigma_A^2}}; & A \geq 0 \\ 0; & A < 0 \end{cases}$$

2. Consider a signal model given by

$$x(n) = A + w(n), \quad n = 0,1,\ldots,M-1$$

where *A* follows an uniform distribution between $-A_0$ and A_0, with zero mean and variance σ_A^2, $w(n)$ is i.i.d. Gaussian with zero-mean and variance σ_w^2, and *A* and $w(n)$ are independent. Find the linear MMSE estimator of *A*. (Note that σ_A^2 is viewed as a known value, so you can directly apply this term to your answer if necessary.)

3. Consider a zero-mean unit-variance white noise signal $w(n)$ corrupted by a single additive "echo" with amplitude *a* arriving after a delay of 50 samples. The resulting signal can be expressed as

$$x(n) = w(n) + aw(n-50).$$

Try to attenuate the first echo, $aw(n-50)$, using a prediction error filter.

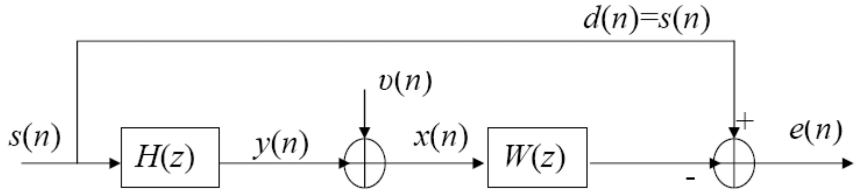
(1) Find a prediction filter of order 15 and the corresponding prediction error for each of the following cases: (i) prediction distance $n_0 = 30$, (ii) prediction distance $n_0 = 40$, and (iii) prediction distance $n_0 = 50$.

(2) Determine the range of n_0 for "successful prediction."

4. Consider the input data $x(n) = w(n)$ is a zero-mean iid sequence with variance σ_w^2, and $d(n) = h(n) * w(n)$. Show that, if $h(n)$ is causal and shift-invariant, then the *L*-point MMSE filter is $\mathbf{f}^* = \left[h(0) \quad h(1) \quad \cdots \quad h(L-1)\right]^T$ with error energy $J_{\min} = \sum_{i=L}^{\infty} h^2(i)\sigma_w^2$.

5. Consider a three-point system with impulse response $h(0) = 1$, $h(1) = -1$, and $h(2) = 0.25$.
   (1) Set up and solve the $L = 2$ zero-delay causal least-squares inverse filter for this system.
   (2) Find the associated error energy $J_{\min}$.
   (3) Show that $J_{\min}$ is lower than that for the filter obtained by simply truncating the long-division expansion of $1/H(z)$ to two terms, i.e., $1/H(z) = 1 + az^{-1} + bz^{-2} + \cdots$ and truncate $1/H(z)$ to $H'(z) = 1 + az^{-1}$.

6. Consider the following optimal equalizer design



   where the input signals $s(n)$ and the noise $v(n)$ are modeled as the white zero-mean random signals with variances of $\sigma_s^2 = 1$ and $\sigma_v^2 = 0.1$, respectively.
   (1) If $H(z) = 1 - 2z^{-1}$, find the optimal 2-tap linear MMSE equalizer $W(z) = w_0 + w_1 z^{-1}$?
   (2) If $H(z) = a + bz^{-1} + az^{-2}$ and $W(z) = w_0 + w_1 z^{-1} + w_2 z^{-2} + w_1 z^{-3} + w_0 z^{-4}$, how many units of time delay you have to insert in the link between $s(n)$ and $d(n)$ so that the designed equalizer achieves the best performance?