Adaptive Signal Processing HW5

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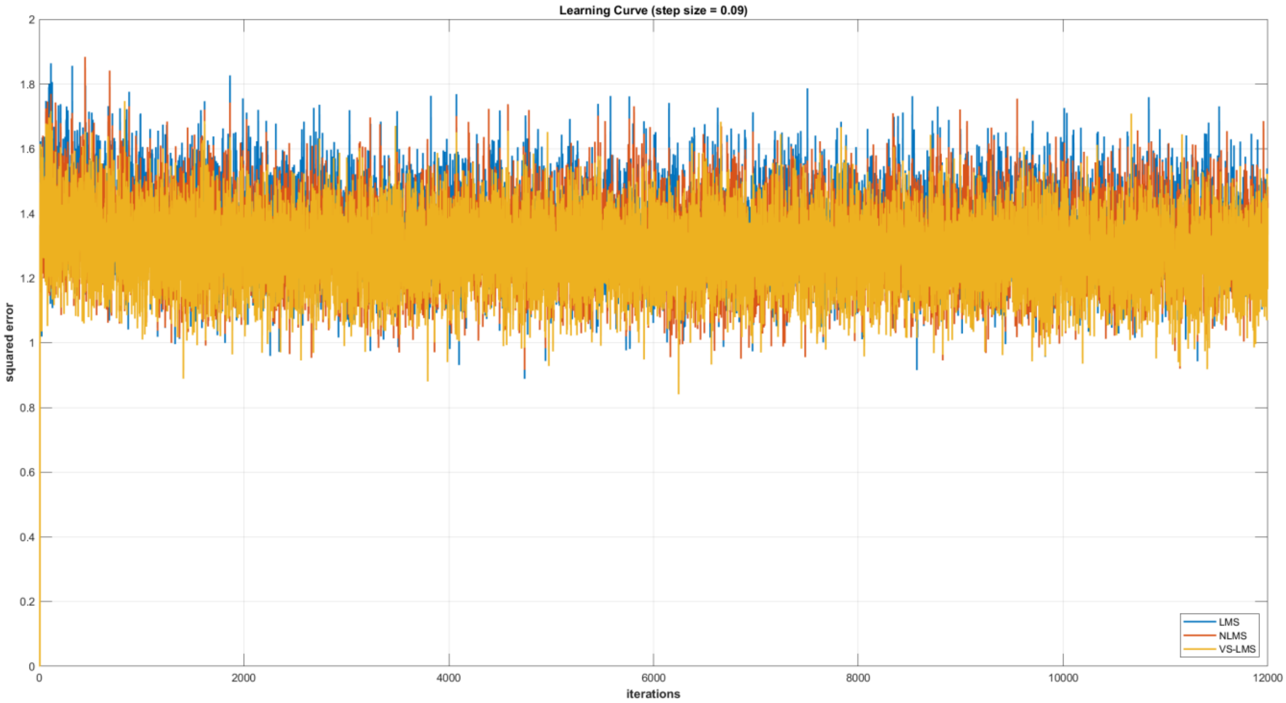
Simulation Assignments:

Generate 12,000 samples for each test data, i.e., $s(n)$, $i(n)$, and $x(n)$, in each trial. The performance of each algorithm is examined by the bit error rate (BER) and the average squared error for different averaging intervals.

(1) Plot the learning curve, i.e., $|e(n)|^2$ vs. the number of iterations for each algorithm. Note that the learning curve is obtained by averaging the results over 100 trials.



$$\boxed{0 < \alpha < \frac{2}{L \cdot E[x^2(n)]}}$$

$$E[x^2(n)] = E[i^2(n)] \cdot E[h^2(n)]$$

輸入功率 = 干擾功率 · 濾波器功率

$$= [\sigma_i^2 - \mu_i^2] \cdot 1, \quad \mu_i = 0 \ (\because i(n) \in [-1, 1])$$

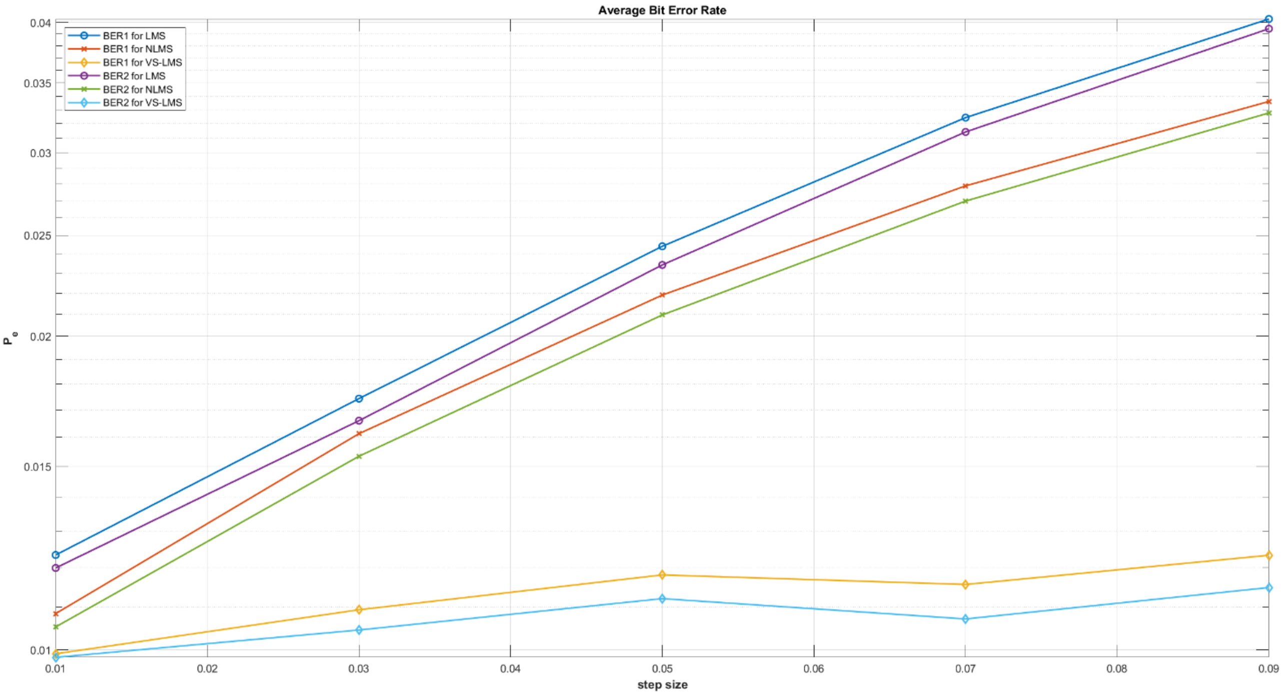
$$= \sigma^2 = \frac{(1-(-1))^2}{12} = \frac{1}{3}$$

$$\Rightarrow 0 < \alpha < \frac{2}{6 \cdot \frac{1}{3}} = 1 \quad \to \quad 0 < \alpha < 1 \ ※$$

理論上，只要 $\alpha < 1$ 都会收斂，武斷選擇 α 為 0.01 ~ 0.09，及 $\alpha_{min} = 0.0001$

(2) Calculate BERs 1 and 2 of the adaptive system for different adaptive algorithms, where BER 1 is evaluated from iterations 101 to 12000 and BER 2 is from iterations 1001 to 12000 in each trial. The final BER for each case is obtained by averaging the results over 100 trials.

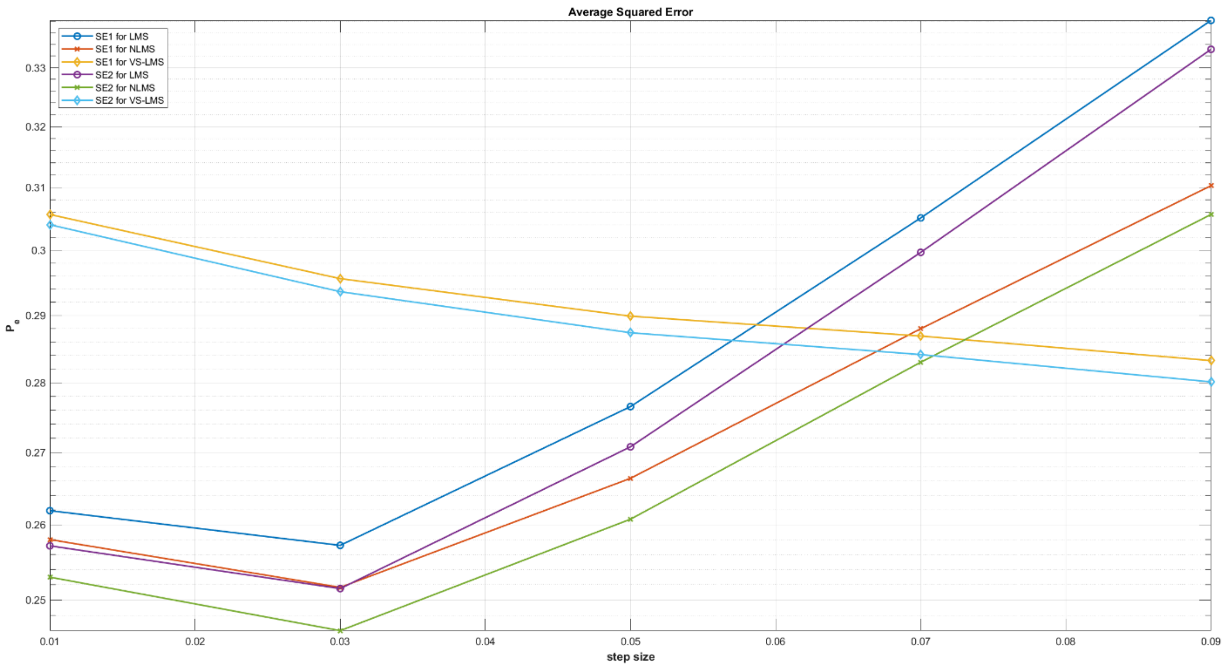
$$
\begin{cases}
\bar{P}_{e1}(i) = \dfrac{1}{12000-101} \sum_{n=101}^{12000} \{\mathrm{sgn}(e_i(n)) \neq s_i(n)\}, \ i = 1, 2, \ldots, 100 \\
\bar{P}_{e2}(i) = \dfrac{1}{12000-1001} \sum_{n=1001}^{12000} \{\mathrm{sgn}_i(e(n)) \neq s_i(n)\}, \ i = 1, 2, \ldots, 100 \\
\bar{P}_{e1} = \dfrac{1}{100} \sum_{i=1}^{100} \bar{P}_{e1}(i); \ \bar{P}_{e2} = \dfrac{1}{100} \sum_{i=1}^{100} \bar{P}_{e2}(i)
\end{cases}
$$



Algorithm	Error Rate	step size				
		0.01	0.03	0.05	0.07	0.09
LMS	BER1	0.0121	0.0176	0.0245	0.0324	0.0406
	BER2	0.0117	0.0168	0.0237	0.0314	0.0397
NLMS	BER1	0.0109	0.0161	0.0221	0.0278	0.0332
	BER2	0.0104	0.0152	0.0212	0.0269	0.0323
VS-LMS	BER1	0.0096	0.0112	0.0115	0.0117	0.0119
	BER2	0.0094	0.0108	0.0110	0.0109	0.0110

(3) Calculate the average squared error of the adaptive system for different adaptive algorithms. That is, calculate $(1/M)\sum_n [s_i(n)-e_i(n)]^2$ in the ith trial, where M is the averaging interval. The squared errors are averaged from iterations 101 to 12000 and from iterations 1001 to 12000, respectively, in each trial. The final average squared error for each case is obtained by averaging the results over 100 trials.

$$\begin{pmatrix} \bar{e}_1(i) = \dfrac{1}{12000-101}\sum_{n=101}^{12000}[s_i(n)-e_i(n)]^2, \quad i=1,2,\ldots,100 \\ \bar{e}_2(i) = \dfrac{1}{12000-1001}\sum_{n=1001}^{12000}[s_i(n)-e_i(n)]^2, \quad i=1,2,\ldots,100 \\ \bar{e}_1 = \dfrac{1}{100}\sum_{i=1}^{100}\bar{e}_1(i); \ \bar{e}_2 = \dfrac{1}{100}\sum_{i=1}^{100}\bar{e}_2(i) \end{pmatrix}$$



Algorithm	Error Rate	step size				
		0.01	0.03	0.05	0.07	0.09
LMS	ASE1	0.2618	0.2570	0.2771	0.3049	0.3380
	ASE2	0.2570	0.2512	0.2717	0.2996	0.3333
NLMS	ASE1	0.2577	0.2510	0.2676	0.2874	0.3083
	ASE2	0.2527	0.2451	0.2623	0.2825	0.3037
VS-LMS	ASE1	0.3058	0.2956	0.2889	0.2856	0.2823
	ASE2	0.3042	0.2934	0.2866	0.2828	0.2792

(4) What conclusions can you make about performance comparisons of the three algorithms from your simulation results?

若考慮 average Bit Error Rate 的話，Variable-Step size LMS Algorithm 表現最好，平均錯誤率都比 LMS 或 NLMS 來的低；若考慮 average Squared Error 的話，則是 Normalized LMS Algorithm 表現最好，在 step size = 0.01 到 0.72 的區間都比 LMS 或 NLMS 來的低。綜合考量覺得 VS-LMS 最好，因為不管 step size 取什麼大小 VS-LMS 收斂況狀都很穩定，但相對來說演算法複雜度也最高。