Midterm Examination

Dec. 1, 2021

Instructor: *Chin-Liang Wang*

1. Please describe and plot the basic configuration of adaptive filters. Also show how to apply it to channel equalization and interference cancellation applications. Explain the operations in detail. (15%)

2. Briefly describe the following measures for assessing the quality of an estimator: (a) Bias; (b) efficiency; (c) mean-squared error; (d) consistency; (e) sufficiency. (15%)

3. Consider a linear process generated by $x(n) = a_1 x(n-1) + b_0 w(n) + b_1 w(n-1)$, where $w(n)$ is a zero-mean white noise sequence with variance σ_w^2. Show that each of the following statements holds for the autocorrelation of $x(n)$: (12%)

   (a) $r(m) = \dfrac{b_0^2 + b_1^2 + 2a_1 b_1 b_0}{1-a_1^2}\sigma_w^2, \quad m = 0.$

   (b) $r(m) = \dfrac{a_1^{m-1}\sigma_w^2}{1-a_1^2}\left(a_1^2 b_1 b_0 + a_1 b_0^2 + a_1 b_1^2 + b_1 b_0\right), \quad m \geq 1.$

   (c) $r(m) = r(-m), \quad m < 0.$

4. Consider two random variables x and y with the following joint density function:

$$f(x,y) = \begin{cases} 2y, & 0 \leq y \leq x^2,\ 0 \leq x \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Find the maximum a posteriori (MAP) estimate y_{MAP}. (7%)

   (b) Find the minimum mean-squared error (MMSE) estimate y_{MMSE}. (6%)

5. Consider that an independent, identically distributed random signal $x(n)$ (with values ± 1 equally likely for each sample) is transmitted over the finite impulse response (FIR) channel $H(z) = 1 - 0.5z^{-1}$ and the output of the channel is corrupted by a zero-mean additive white Gaussian noise $v(n)$ of unit variance, as shown below. Find the second-order linear MMSE estimator $\hat{x}(n) = f_1 y(n-1) + f_2 y(n)$ to recover the desired signal $x(n)$ under an assumption that $x(n)$ and $v(n)$ are independent. (12%)