6. Consider an adaptive FIR filter using the following steepest descent algorithm:

$$\mathbf{f}_{n+1} = \mathbf{f}_n - \alpha(\mathbf{R}\mathbf{f}_n - \mathbf{g})$$

where $\mathbf{f}_n$ denotes the nth update of the coefficient vector, α is the step size, $\mathbf{R}$ is the autocorrelation matrix of the input signal, and $\mathbf{g}$ is the cross-correlation vector between the input signal and the desired signal. Use the similarity transform $\mathbf{R}=\mathbf{Q}\Lambda\mathbf{Q}^T$ to show

$$\lim_{n\to\infty}\{\mathbf{f}_n\} \to \mathbf{f}^* \text{ (least-squares solution). (12\%)}$$

7. A causal linear shift-invariant system with input $x(n)$ and output $y(n)$ is described by the following difference equation:

$$y(n) - 0.5y(n-1) = x(n) + 2x(n-1).$$

The input signal has a power spectral density function given by $R_{xx}(e^{j\omega}) = 1/(1+0.5\cos\omega)$.

(a) What is the power spectral density function of the output signal? (6%)

(b) Is the system minimum-phase? If not, derive the spectrally equivalent minimum-phase system and the corresponding difference equation. (5%)

8. Consider a zero-mean unit variance white noise signal $w(n)$ corrupted by a single additive "echo" with amplitude β arriving after a delay of 100 samples:

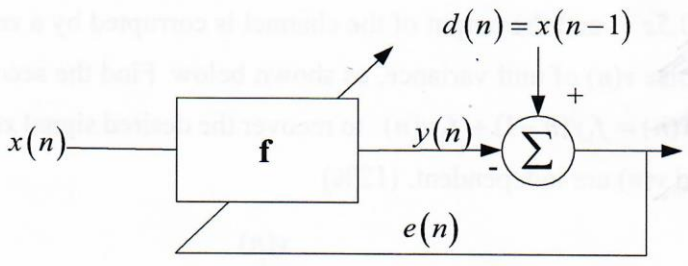
$$x(n) = w(n) + \beta w(n-100).$$

It is desired to attenuate the echo using a prediction error filter.

(a) Explain why we can use a prediction error filter to attenuate the echo. (6%)

(b) Determine the minimum order of the corresponding predictor for a successful prediction when the prediction distance is equal to 80. (4%)

9. Consider a two-tap least mean squares (LMS) adaptive FIR filter as shown in the following figure with input $x(n) = w(n) + \beta w(n-1)$, where $w(n)$ is a white Gaussian noise with unit variance.



(a) Find the least-squares solution of the filter coefficient vector **f**. (6%)

(b) Determine the convergence condition of the adaptive filter and the corresponding time constant for overall convergence. (4%)