Homework #1

Due 10:10, Oct. 13, 2021

1. Let v_1, v_2, v_3, v_4 be a set of zero-mean independent random variables with variances equal to 1, 2, 3, 4. Let x_1, x_2, x_3, x_4 be defined by

$$
\begin{aligned}
x_1 &= v_1 + v_2 + v_3 + v_4 \\
x_2 &= -v_1 + v_2 + v_3 - v_4 \\
x_3 &= v_1 - v_2 + v_3 - v_4 \\
x_4 &= v_1 + v_2 - v_3 - v_4
\end{aligned}
$$

   Which pairs of (x_i, x_j), $i, j = 1, 2, 3, 4$, are uncorrelated?

2. Let $x(n)$ and $y(n)$ be jointly stationary random sequences, where the cross-correlation between them is $r_{xy}(i) = E\{x(n)y(n+i)\}$. Show that $|r_{xy}(i)| \le \sqrt{r_{xx}(0)\, r_{yy}(0)}$, where $r_{xx}(0) = E\{x^2(n)\}$ and $r_{yy}(0) = E\{y^2(n)\}$.

3. Given that $w(n)$ is a zero-mean iid sequence and $x(n) = h(n) * w(n)$, find $r_{xx}(m)$ for each of the following cases:

   (1) $h(n) = a^n$, $n = 0, 1$; $h(n) = 0$, elsewhere.

   (2) $h(n) = a^{2n}$, $n \ge 0$; $h(n) = 0$, $n < 0$.

4. For the MA(1) process

$$x(n) = w(n) + b_1 w(n-1)$$

   show that

$$\left| \frac{r(1)}{r(0)} \right| \le 0.5$$

   and find the values of b_1 which produce equality.

5. Let $x(n)$ and $y(n)$ be both zero-mean and WSS random processes. Consider the random process $z(n)$ defined by

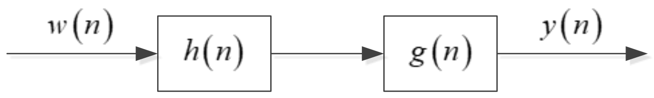
$$z(n) = x(n) + y(n)$$

   (1) Determine the autocorrelation function and the power spectral density of $z(n)$, (a) if $x(n)$ and $y(n)$ are jointly WSS; (b) if $x(n)$ and $y(n)$ are orthogonal.

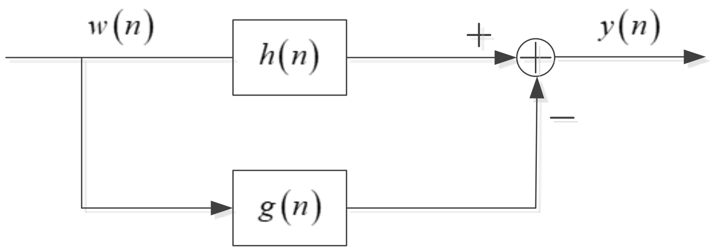
   (2) Show that if $x(n)$ and $y(n)$ are orthogonal, then the mean square of $z(n)$ is equal to the sum of the mean squares of $x(n)$ and $y(n)$.

6. Find $R_{yy}(e^{j\omega})$ for each of the following cases:

(1)



(2)



where $w(n)$ is a white signal with unit power, $h(0)=1$ $h(0)=1$, $h(1)=0.5$, $g(0)=1$, and $g(1)=-0.5$.