COM 530500 **Network Science Homework #1**

DUE: Wednesday, October 12, 2022

No late homework will be accepted.

班級: 通訊所

姓名: 張瑋恩

學號: 111064516

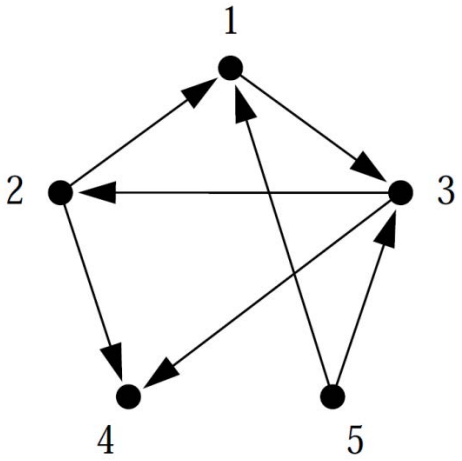

Figure 1: Network (a).

Problem 1.(10%)

(a) (5%) Write down the adjacency matrix of network (a).

(b) (5%) Write down the cocitation matrix of network (a).

Solution:

$$(a)A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \begin{cases} A_{ij} = 1, if\ there\ is\ an\ edge\ from\ j\ to\ i \\ A_{ij} = 0, otherwise \end{cases}$$

$$(b)C = \begin{bmatrix} 2 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 \\ 1 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}; C = AA^T$$ ◇

Problem 2.(10%)

(a) (5%) Write down the incidence matrix of network (b).

(b) (5%) Write down the projection matrix for the projection of network (b) onto its black vertices.

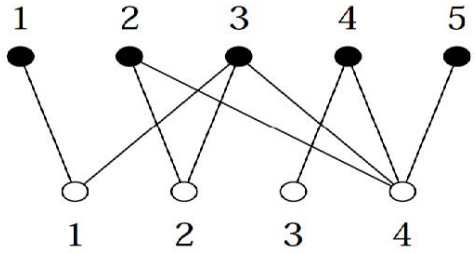

Figure 2: Network (b).

Solution:

$$(a)\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}(b)B = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 1 & 1 \\ 1 & 2 & 3 & 1 & 1 \\ 0 & 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}; P = B^T B$$

◇

Problem 3.(10%) Consider a bipartite network, with its two types of vertices. Suppose there are n_1 vertices of type 1 and n_2 vertices of type 2. Show that the mean degrees c_1 and c_2 of the two types are related by $c_2 = \frac{n_1}{n_2}c_1$.

Solution:
$c_1 = mean\ degrees\ of\ type1 \quad ; n_1 = number\ 0f\ vertices\ of\ type\ 1$
$c_2 = mean\ degrees\ of\ type2 \quad ; n_2 = number\ 0f\ vertices\ of\ type\ 2$
$c_1 \times n_1 = number\ of\ all\ vertices$
$c_2 \times n_2 = number\ of\ all\ vertices$
$\because\ c_1 \times n_1 = c_2 \times n_2$
$\therefore \quad c_2 = \frac{n_1}{n_2}c_1$ ◇

Problem 4.(20%) Given

$$A = \begin{pmatrix} 0 & 2 & -1 \\ 2 & 3 & -2 \\ -1 & -2 & 0 \end{pmatrix},$$

(a) (10%) Find all eigenvalues of matrix A.

(b) (10%) Find an orthogonal matrix U that diagonalizes A.

Solution:
$(a) \quad det(A - \lambda I) = 0 \quad ; \quad f(x) : -\lambda^3 + 3\lambda^2 + 9\lambda + 5 = 0$
$\because \lambda_1 = \lambda_2 = -1 \quad ; \quad \lambda_3 = 5$
$(b)(A - \lambda I)\vec{x} = \vec{0}$

$$\lambda_1 = \lambda_2 = -1 \quad , \begin{pmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ -1 & -2 & 1 \end{pmatrix}\vec{x_1} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Longrightarrow \vec{x_1} = c_1 \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = c_1\vec{x_1}' + c_2\vec{x_2}'$$

$\lambda_3 = 5 \quad , \begin{pmatrix} -5 & 2 & -1 \\ 2 & -2 & -2 \\ -1 & -2 & -5 \end{pmatrix} \vec{x_3} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Longrightarrow \vec{x_3} = c_3 \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = c_3 \vec{x_3}'$

$\vec{e_1} = \dfrac{\vec{x_2}'}{|\vec{x_2}'|} = \dfrac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$\vec{\phi_2} = \vec{x_1}' - \dfrac{< \vec{x_1}', \vec{e_1} >}{|e_1|} \vec{e_1} = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} - \dfrac{-\sqrt{2}}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \vec{e_2} = \dfrac{1}{\sqrt{3}} \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

$\vec{e_3} = \vec{e_1} \times \vec{e_2} = \dfrac{1}{\sqrt{6}} \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$

$U = \begin{pmatrix} e_1 & e_2 & e_3 \end{pmatrix} \quad ; diagonalizes \quad matrix \quad of \quad A = D = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 5 \end{pmatrix}$

$AU = A(\vec{e_1}, \vec{e_2}, \vec{e_3}) = (\lambda_1 e_1, \lambda_2 e_2, \lambda_3 e_3) = (\vec{e_1}, \vec{e_2}, \vec{e_3}) \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 5 \end{pmatrix} = UD$

$A = UDU^T \quad , U \quad is \quad orthogonal \quad matrix \quad of \quad A$

$Ans : U = \begin{pmatrix} e_1 & e_2 & e_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{3}} & \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \end{pmatrix} ; U \quad is \quad orthogonal \quad matrix \quad of \quad A \qquad \diamond$

Problem 5.(15%) Read the tutorial from Ping-En Lu's GitHub repository to install Python3 and python-igraph (if you need).

Paste your screenshots of "Hello, World!" of both Python 3 (5%) and python-igraph (5%), and write a brief report (5%). (For example, you can write down some problems you encountered, and how you solved them.)

Solution:

在這之前對於Python是不太熟悉的，因此一開始不會切割環境，把自己的電腦環境用的很亂，後來請教實驗室中的學長後，使用anacomda3來做環境的分割。至於Coding的部分，因為之前有打過Swift，所以很快就上手。 ◇

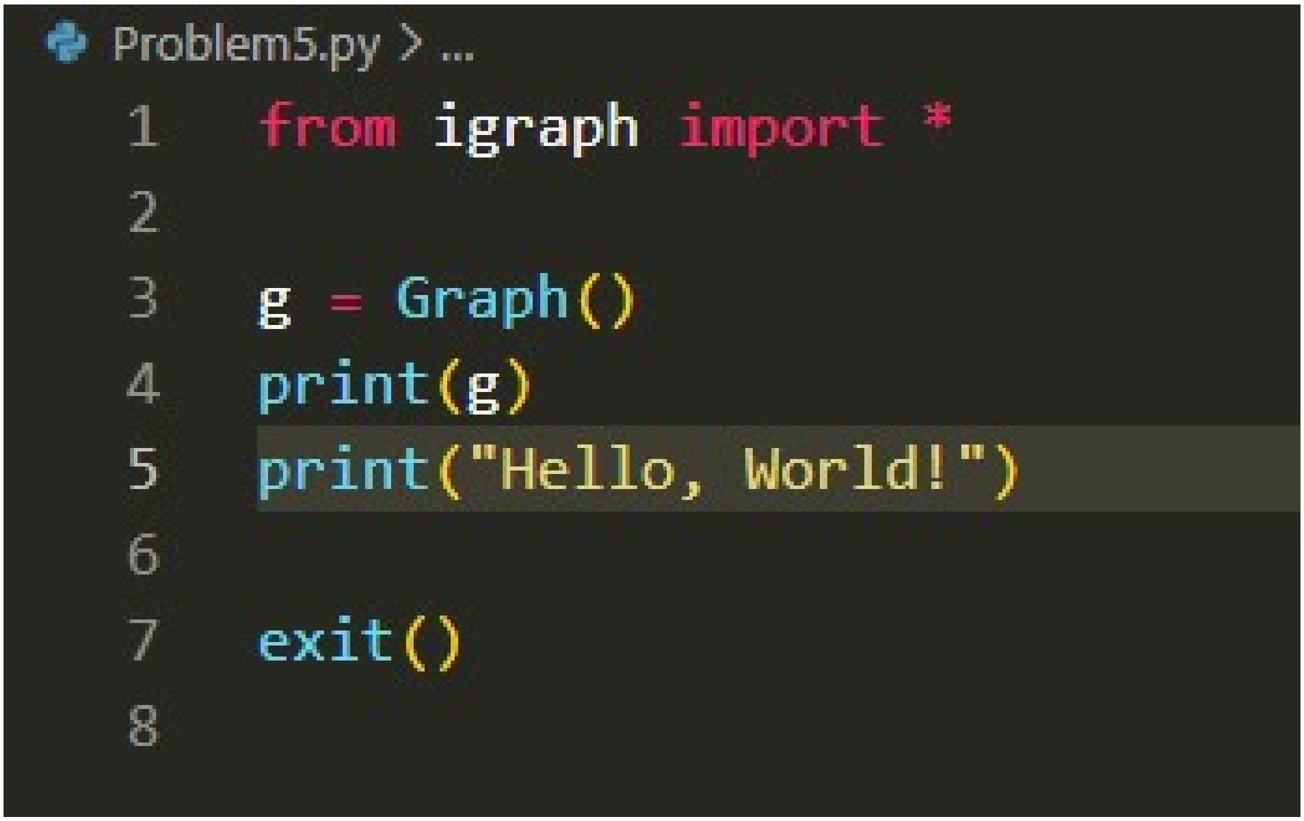

Figure 3: Coding.

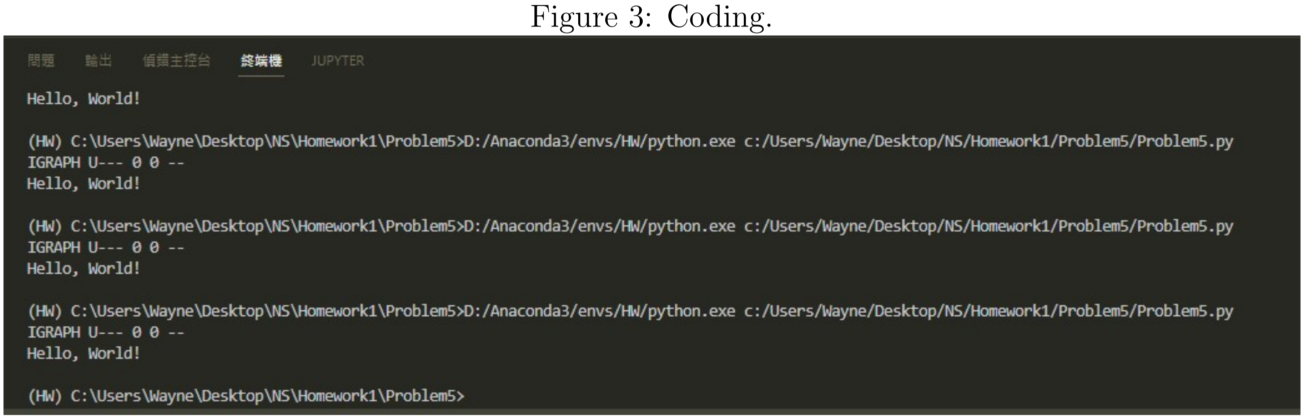

Figure 4: output result.

Problem 6.(35%) Please download the **tvshow** dataset from Ping-En Lu's GitHub repository, and find the following information from this dataset.

- Number of nodes. (5%)
- Number of edges. (5%)
- Mean degree. (5%)
- Maximum degree. (5%)
- Diameter. (5%)

You need to upload your **python source code** to eLearn, and **write a brief report (10%) including screenshots, README file, and descriptions of your code** below the solution area. There will be no points for this problem if you do not upload your python source code to iLMS.

Solution:

一開始是想用逐列掃描的方式來算，但後來發現這樣運算量太大，而且要做其他小題也會很麻煩。所以就在想有沒有比較簡單的方法，突然就想到第五題中用到的ipraph，題目會給代表應該會有用處，於是在看過官網的API介紹後，發現真的有許多的function可以用在這次的功課。也成功的做出來，不過在過程中我的diameter跟其他修課同學的不一樣，我的值是14，其他的都一樣，所以我就在想是不是我讀取的檔案方式有差別，後來也發現是讀成有向圖，所以我就再Dataframe中加入directed=False這個參數，問題也就解決。 ◇

```python
import igraph as ig                          #圖形分析資料庫
import pandas as pd                          #資料分析函式庫

csvfile = pd.read_csv('./Problem6/tvshow_edges.csv')        #讀取csv檔
g = ig.Graph.DataFrame(csvfile, directed=False)             #創建成igraph，已無向圖方式創建
ig.summary(g)                                #Summary representation of a graph.

md=(2*g.ecount()/g.vcount())                 #calculate mean degree
print("Number of nodes:%d"%g.vcount())       #Number of nodes.
print("Number of edges:%d"%g.ecount())       #item Number of edges
print("Mean degree:%s"%md)                   #item Mean degree
print("Maximum degree:%d"%g.maxdegree())     #item Maximum degree
print("Diameter:%d"%g.diameter())            #item Diameter
#print(g.farthest_points())                  #degree的路徑頭尾兩個點，加度數
#print(g.get_diameter())                     #diameter路徑
```

Figure 5: Coding.

```
問題    輸出    偵錯主控台    終端機    JUPYTER

Mean degree:8.869989722507707
Maximum degree:126
Diameter:20
(437, 813, 20)

(HW) C:\Users\Wayne\Desktop\homework1>D:/anaconda3/envs/HW/python.exe c:/Users/Wayne/Desktop/homework1/Problem6/Problem6.py
IGRAPH UN-- 3892 17261 --
+ attr: name (v)
Number of nodes:3892
Number of edges:17261
Mean degree:8.869989722507707
Maximum degree:126
Diameter:20

(HW) C:\Users\Wayne\Desktop\homework1>
```

Figure 6: output result.