导出数据

初始化数据

导入函数

In[1]:=

```
Import["~/desktop/work_space/1 MMA/6 lie_bracket/SupLieAlg.m"];
?SupLieAlg`*
```

导入

Loading lie superalgebra functions ...

Out[2]=

SupLieAlg`

AnyMatchQ	EvenRootSupCQ	LieS	OddRootSupCQ	RootSupCQ
CoefSupC	f	m	PosRootSupCQ	Γ
DiagramSupC	h	n	ReadBracket	δ
e	Lie	nΓ	Root2MatrixC	ϵ

根系

[正,正]=正

[负,正]=正

打印函数

In[13]:=

```
ReadBracket[α_, β_, sα_ : 1, sβ_ : 1, sγ_ : 1] := Module[{ea, eb, ec, γ, da, db, dc},
  γ = sγ (sα α + sβ β);
  {ea, da} = If[sα == 1, {e, Γα}, {f, nΓα}];
  {eb, db} = If[sβ == 1, {e, Γβ}, {f, nΓβ}];
  {ec, dc} = If[sγ == 1, {e, Γγ}, {f, nΓγ}];
  StringForm["`8`(`1`) + (`2`) = (`3`) , [ `5`α , `6`β ] = `4`·`7`γ
\t[`9` , `10`]\n\t=`4`·`11`",
   α, β, γ, CoefSupC[α, β, sα, sβ, sγ], ea, eb, ec, If[sα == 1, "", "-"], da, db, dc]]
```

模块 如果 如果 如果 字符串形式 如果

导出数据

In[14]:=

```
Riffle[ReadBracket @@@ rootpairsppp , "\n"] // Column
```

交互插入 列

```
Print["\n------- division line -------\n\n"]
```

打印

```
Riffle[ReadBracket @@@ rootpairsnpp , "\n"] // Column
```

交互插入 列

Out[14]=

$(\epsilon_i - \epsilon_j) + (\epsilon_j - \epsilon_k) = (\epsilon_i - \epsilon_k)$, $[\ e_\alpha\ ,\ e_\beta\ ] = 1\cdot e_\gamma$

$[\alpha_i-\alpha_{1+i}-\cdots-\alpha_{-1+j}\ ,\ \alpha_j-\alpha_{1+j}-\cdots-\alpha_{-1+k}]$

$=1\cdot\alpha_i-\alpha_{1+i}-\cdots-\alpha_{-1+k}$

$(\epsilon_i - \epsilon_j) + (-\delta_k + \epsilon_j) = (-\delta_k + \epsilon_i)$, $[\ e_\alpha\ ,\ e_\beta\ ] = 1\cdot e_\gamma$

$[\alpha_i-\alpha_{1+i}-\cdots-\alpha_{-1+j}\ ,\ \alpha_j-\alpha_{1+j}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n-1+k}]$

$=1\cdot\alpha_i-\alpha_{1+i}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n-1+k}$

$(\epsilon_i - \epsilon_j) + (\delta_k + \epsilon_j) = (\delta_k + \epsilon_i)$, $[\ e_\alpha\ ,\ e_\beta\ ] = 1\cdot e_\gamma$

$[\alpha_i-\alpha_{1+i}-\cdots-\alpha_{-1+j}\ ,\ \alpha_j-\alpha_{1+j}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1+k}-\alpha_{n+k}]$

$=1\cdot\alpha_i-\alpha_{1+i}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1+k}-\alpha_{n+k}$

$(\epsilon_i - \epsilon_j) + (\epsilon_j + \epsilon_k) = (\epsilon_i + \epsilon_k)$, $[\ e_\alpha\ ,\ e_\beta\ ] = 1\cdot e_\gamma$

$[\alpha_i-\alpha_{1+i}-\cdots-\alpha_{-1+j}\ ,\ \alpha_j-\alpha_{1+j}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1}-(\alpha_n)-\cdots-\alpha_{1+k}-\alpha_k]$

$=1\cdot\alpha_i-\alpha_{1+i}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1}-(\alpha_n)-\cdots-\alpha_{1+k}-\alpha_k$

$(-\delta_j + \epsilon_i) + (2\,\delta_j) = (\delta_j + \epsilon_i)$, $[\ e_\alpha\ ,\ e_\beta\ ] = 1\cdot e_\gamma$

$[\alpha_i-\alpha_{1+i}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n-1+j}\ ,\ \alpha_{n+j}-\alpha_{n+1+j}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1+j}-\alpha_{n+j}]$

$=1\cdot\alpha_i-\alpha_{1+i}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1+j}-\alpha_{n+j}$

$(-\delta_j + \epsilon_i) + (\delta_j - \delta_k) = (-\delta_k + \epsilon_i)$, $[\ e_\alpha\ ,\ e_\beta\ ] = 1\cdot e_\gamma$

$[\alpha_i-\alpha_{1+i}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n-1+j}\ ,\ \alpha_{n+j}-\alpha_{n+1+j}-\cdots-\alpha_{n-1+k}]$

$=1\cdot\alpha_i-\alpha_{1+i}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n-1+k}$

$(-\delta_j + \epsilon_i) + (\delta_j + \delta_k) = (\delta_k + \epsilon_i)$, $[\ e_\alpha\ ,\ e_\beta\ ] = 1\cdot e_\gamma$

$[\alpha_i-\alpha_{1+i}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n-1+j}\ ,\ \alpha_{n+j}-\alpha_{n+1+j}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1+k}-\alpha_{n+k}]$

$=1\cdot\alpha_i-\alpha_{1+i}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1+k}-\alpha_{n+k}$

$(-\delta_j + \epsilon_i) + (\delta_j + \epsilon_k) = (\epsilon_i + \epsilon_k)$, $[\ e_\alpha\ ,\ e_\beta\ ] = 1\cdot e_\gamma$

$[\alpha_i-\alpha_{1+i}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n-1+j}\ ,$

$\alpha_k-\alpha_{1+k}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1+j}-\alpha_{n+j}]$

$=1\cdot\alpha_i-\alpha_{1+i}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1}-(\alpha_n)-\cdots-\alpha_{1+k}-\alpha_k$

$(\delta_i - \delta_j)$ + $(\delta_i + \delta_j)$ = $(2\,\delta_i)$, [e_α , e_β] = $2 \cdot e_\gamma$

$[\alpha_{n+i}-\alpha_{n+1+i}-\cdots-\alpha_{n-1+j}$, $\alpha_{n+i}-\alpha_{n+1+i}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1+j}-\alpha_{n+j}]$

$=2\cdot\alpha_{n+i}-\alpha_{n+1+i}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1+i}-\alpha_{n+i}$

$(\delta_i - \delta_j)$ + $(2\,\delta_j)$ = $(\delta_i + \delta_j)$, [e_α , e_β] = $1 \cdot e_\gamma$

$[\alpha_{n+i}-\alpha_{n+1+i}-\cdots-\alpha_{n-1+j}$, $\alpha_{n+j}-\alpha_{n+1+j}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1+j}-\alpha_{n+j}]$

$=1\cdot\alpha_{n+i}-\alpha_{n+1+i}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1+j}-\alpha_{n+j}$

$(\delta_i - \delta_j)$ + $(\delta_j - \delta_k)$ = $(\delta_i - \delta_k)$, [e_α , e_β] = $1 \cdot e_\gamma$

$[\alpha_{n+i}-\alpha_{n+1+i}-\cdots-\alpha_{n-1+j}$, $\alpha_{n+j}-\alpha_{n+1+j}-\cdots-\alpha_{n-1+k}]$

$=1\cdot\alpha_{n+i}-\alpha_{n+1+i}-\cdots-\alpha_{n-1+k}$

$(\delta_i - \delta_j)$ + $(\delta_j + \delta_k)$ = $(\delta_i + \delta_k)$, [e_α , e_β] = $1 \cdot e_\gamma$

$[\alpha_{n+i}-\alpha_{n+1+i}-\cdots-\alpha_{n-1+j}$, $\alpha_{n+j}-\alpha_{n+1+j}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1+k}-\alpha_{n+k}]$

$=1\cdot\alpha_{n+i}-\alpha_{n+1+i}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1+k}-\alpha_{n+k}$

$(\delta_i - \delta_j)$ + $(\delta_j + \epsilon_k)$ = $(\delta_i + \epsilon_k)$, [e_α , e_β] = $1 \cdot e_\gamma$

$[\alpha_{n+i}-\alpha_{n+1+i}-\cdots-\alpha_{n-1+j}$, $\alpha_k-\alpha_{1+k}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1+j}-\alpha_{n+j}]$

$=1\cdot\alpha_k-\alpha_{1+k}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1+i}-\alpha_{n+i}$

—————— division line ——————

$-(\epsilon_i - \epsilon_j)$ + $(\epsilon_i - \epsilon_k)$ = $(\epsilon_j - \epsilon_k)$, [f_α , e_β] = $1 \cdot e_\gamma$

$[\beta_{-1+j}-\beta_{1+i}-\cdots-\beta_i$, $\alpha_i-\alpha_{1+i}-\cdots-\alpha_{-1+k}]$

$=1\cdot\alpha_j-\alpha_{1+j}-\cdots-\alpha_{-1+k}$

$-(\epsilon_i - \epsilon_j)$ + $(-\delta_k + \epsilon_i)$ = $(-\delta_k + \epsilon_j)$, [f_α , e_β] = $1 \cdot e_\gamma$

$[\beta_{-1+j}-\beta_{1+i}-\cdots-\beta_i$, $\alpha_i-\alpha_{1+i}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n-1+k}]$

$=1\cdot\alpha_j-\alpha_{1+j}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n-1+k}$

$-(\epsilon_i - \epsilon_j)$ + $(\delta_k + \epsilon_i)$ = $(\delta_k + \epsilon_j)$, [f_α , e_β] = $1 \cdot e_\gamma$

$[\beta_{-1+j}-\beta_{1+i}-\cdots-\beta_i$, $\alpha_i-\alpha_{1+i}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1+k}-\alpha_{n+k}]$

$=1\cdot\alpha_j-\alpha_{1+j}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1+k}-\alpha_{n+k}$

$-(\epsilon_i - \epsilon_j)$ + $(\epsilon_i + \epsilon_k)$ = $(\epsilon_j + \epsilon_k)$, [f_α , e_β] = $1 \cdot e_\gamma$

$[\beta_{-1+j}-\beta_{1+i}-\cdots-\beta_i$, $\alpha_i-\alpha_{1+i}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1}-(\alpha_n)-\cdots-\alpha_{1+k}-\alpha_k]$

$=1\cdot\alpha_j-\alpha_{1+j}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1}-(\alpha_n)-\cdots-\alpha_{1+k}-\alpha_k$

$-(\epsilon_i - \epsilon_j) + (-\epsilon_j + \epsilon_k) = (-\epsilon_i + \epsilon_k)$, $[\ f_\alpha\ ,\ e_\beta\ ] = -1 \cdot e_\gamma$

$[\beta_{-1+j} - \beta_{1+i} - \cdots - \beta_i\ ,\ \alpha_k - \alpha_{1+k} - \cdots - \alpha_{-1+j}]$

$= -1 \cdot \alpha_k - \alpha_{1+k} - \cdots - \alpha_{-1+i}$

$-(-\delta_j + \epsilon_i) + (\delta_j + \epsilon_i) = (2\,\delta_j)$, $[\ f_\alpha\ ,\ e_\beta\ ] = -2 \cdot e_\gamma$

$[\beta_{n-1+j} - \beta_{n+1+i} - \cdots - \beta_{n+1} - (\beta_n) - \cdots - \beta_i\ ,$

$\alpha_i - \alpha_{1+i} - \cdots - (\alpha_n) - \alpha_{n+1} - \cdots - \alpha_{n+m-1} \Longleftarrow \alpha_{n+m} \Longrightarrow \alpha_{n+m-1} - \cdots - \alpha_{n+1+j} - \alpha_{n+j}]$

$= -2 \cdot \alpha_{n+j} - \alpha_{n+1+j} - \cdots - \alpha_{n+m-1} \Longleftarrow \alpha_{n+m} \Longrightarrow \alpha_{n+m-1} - \cdots - \alpha_{n+1+j} - \alpha_{n+j}$

$-(-\delta_j + \epsilon_i) + (-\delta_k + \epsilon_i) = (\delta_j - \delta_k)$, $[\ f_\alpha\ ,\ e_\beta\ ] = -1 \cdot e_\gamma$

$[\beta_{n-1+j} - \beta_{n+1+i} - \cdots - \beta_{n+1} - (\beta_n) - \cdots - \beta_i\ ,\ \alpha_i - \alpha_{1+i} - \cdots - (\alpha_n) - \alpha_{n+1} - \cdots - \alpha_{n-1+k}]$

$= -1 \cdot \alpha_{n+j} - \alpha_{n+1+j} - \cdots - \alpha_{n-1+k}$

$-(-\delta_j + \epsilon_i) + (\delta_k + \epsilon_i) = (\delta_j + \delta_k)$, $[\ f_\alpha\ ,\ e_\beta\ ] = -1 \cdot e_\gamma$

$[\beta_{n-1+j} - \beta_{n+1+i} - \cdots - \beta_{n+1} - (\beta_n) - \cdots - \beta_i\ ,$

$\alpha_i - \alpha_{1+i} - \cdots - (\alpha_n) - \alpha_{n+1} - \cdots - \alpha_{n+m-1} \Longleftarrow \alpha_{n+m} \Longrightarrow \alpha_{n+m-1} - \cdots - \alpha_{n+1+k} - \alpha_{n+k}]$

$= -1 \cdot \alpha_{n+j} - \alpha_{n+1+j} - \cdots - \alpha_{n+m-1} \Longleftarrow \alpha_{n+m} \Longrightarrow \alpha_{n+m-1} - \cdots - \alpha_{n+1+k} - \alpha_{n+k}$

$-(-\delta_j + \epsilon_i) + (\epsilon_i + \epsilon_k) = (\delta_j + \epsilon_k)$, $[\ f_\alpha\ ,\ e_\beta\ ] = -1 \cdot e_\gamma$

$[\beta_{n-1+j} - \beta_{n+1+i} - \cdots - \beta_{n+1} - (\beta_n) - \cdots - \beta_i\ ,$

$\alpha_i - \alpha_{1+i} - \cdots - (\alpha_n) - \alpha_{n+1} - \cdots - \alpha_{n+m-1} \Longleftarrow \alpha_{n+m} \Longrightarrow \alpha_{n+m-1} - \cdots - \alpha_{n+1} - (\alpha_n) - \cdots - \alpha_{1+k} - \alpha_k]$

$= -1 \cdot \alpha_k - \alpha_{1+k} - \cdots - (\alpha_n) - \alpha_{n+1} - \cdots - \alpha_{n+m-1} \Longleftarrow \alpha_{n+m} \Longrightarrow \alpha_{n+m-1} - \cdots - \alpha_{n+1+j} - \alpha_{n+j}$

$-(-\delta_j + \epsilon_i) + (-\delta_j + \epsilon_k) = (-\epsilon_i + \epsilon_k)$, $[\ f_\alpha\ ,\ e_\beta\ ] = -1 \cdot e_\gamma$

$[\beta_{n-1+j} - \beta_{n+1+i} - \cdots - \beta_{n+1} - (\beta_n) - \cdots - \beta_i\ ,\ \alpha_k - \alpha_{1+k} - \cdots - (\alpha_n) - \alpha_{n+1} - \cdots - \alpha_{n-1+j}]$

$= -1 \cdot \alpha_k - \alpha_{1+k} - \cdots - \alpha_{-1+i}$

$-(\delta_i - \delta_j) + (\delta_i + \delta_j) = (2\,\delta_j)$, $[\ f_\alpha\ ,\ e_\beta\ ] = 2 \cdot e_\gamma$

$[\beta_{n-1+j} - \beta_{n+1+i} - \cdots - \beta_{n+i}\ ,\ \alpha_{n+i} - \alpha_{n+1+i} - \cdots - \alpha_{n+m-1} \Longleftarrow \alpha_{n+m} \Longrightarrow \alpha_{n+m-1} - \cdots - \alpha_{n+1+j} - \alpha_{n+j}]$

$= 2 \cdot \alpha_{n+j} - \alpha_{n+1+j} - \cdots - \alpha_{n+m-1} \Longleftarrow \alpha_{n+m} \Longrightarrow \alpha_{n+m-1} - \cdots - \alpha_{n+1+j} - \alpha_{n+j}$

$-(\delta_i - \delta_j) + (2\,\delta_i) = (\delta_i + \delta_j)$, $[\ f_\alpha\ ,\ e_\beta\ ] = 1 \cdot e_\gamma$

$[\beta_{n-1+j} - \beta_{n+1+i} - \cdots - \beta_{n+i}\ ,\ \alpha_{n+i} - \alpha_{n+1+i} - \cdots - \alpha_{n+m-1} \Longleftarrow \alpha_{n+m} \Longrightarrow \alpha_{n+m-1} - \cdots - \alpha_{n+1+i} - \alpha_{n+i}]$

$= 1 \cdot \alpha_{n+i} - \alpha_{n+1+i} - \cdots - \alpha_{n+m-1} \Longleftarrow \alpha_{n+m} \Longrightarrow \alpha_{n+m-1} - \cdots - \alpha_{n+1+j} - \alpha_{n+j}$

$-(\delta_i - \delta_j) + (\delta_i - \delta_k) = (\delta_j - \delta_k)$, $[\ f_\alpha\ ,\ e_\beta\ ] = 1 \cdot e_\gamma$

$[\beta_{n-1+j} - \beta_{n+1+i} - \cdots - \beta_{n+i}\ ,\ \alpha_{n+i} - \alpha_{n+1+i} - \cdots - \alpha_{n-1+k}]$

$= 1 \cdot \alpha_{n+j} - \alpha_{n+1+j} - \cdots - \alpha_{n-1+k}$

$-(\delta_i - \delta_j) + (\delta_i + \delta_k) = (\delta_j + \delta_k)$, $[\ f_\alpha\ ,\ e_\beta\ ] = 1 \cdot e_\gamma$

Out[16]=

$[\beta_{n-1+j} - \beta_{n+1+i} - \cdots - \beta_{n+i}\ ,\ \alpha_{n+i} - \alpha_{n+1+i} - \cdots - \alpha_{n+m-1} \Longleftarrow \alpha_{n+m} \Longrightarrow \alpha_{n+m-1} - \cdots - \alpha_{n+1+k} - \alpha_{n+k}]$

$= 1 \cdot \alpha_{n+j} - \alpha_{n+1+j} - \cdots - \alpha_{n+m-1} \Longleftarrow \alpha_{n+m} \Longrightarrow \alpha_{n+m-1} - \cdots - \alpha_{n+1+k} - \alpha_{n+k}$

$-(\delta_i - \delta_j) + (\delta_i + \epsilon_k) = (\delta_j + \epsilon_k)$, $[\ f_\alpha\ ,\ e_\beta\ ] = 1 \cdot e_\gamma$

$[\beta_{n-1+j} - \beta_{n+1+i} - \cdots - \beta_{n+i}\ ,\ \alpha_k - \alpha_{1+k} - \cdots - (\alpha_n) - \alpha_{n+1} - \cdots - \alpha_{n+m-1} \Longleftarrow \alpha_{n+m} \Longrightarrow \alpha_{n+m-1} - \cdots - \alpha_{n+1+i} - \alpha_{n+i}]$

$= 1 \cdot \alpha_k - \alpha_{1+k} - \cdots - (\alpha_n) - \alpha_{n+1} - \cdots - \alpha_{n+m-1} \Longleftarrow \alpha_{n+m} \Longrightarrow \alpha_{n+m-1} - \cdots - \alpha_{n+1+j} - \alpha_{n+j}$

$-(\delta_i - \delta_j) + (-\delta_j + \epsilon_k) = (-\delta_i + \epsilon_k)$, $[\ f_\alpha\ ,\ e_\beta\ ] = -1 \cdot e_\gamma$

$[\beta_{n-1+j} - \beta_{n+1+i} - \cdots - \beta_{n+i}\ ,\ \alpha_k - \alpha_{1+k} - \cdots - (\alpha_n) - \alpha_{n+1} - \cdots - \alpha_{n-1+j}]$

$= -1 \cdot \alpha_k - \alpha_{1+k} - \cdots - (\alpha_n) - \alpha_{n+1} - \cdots - \alpha_{n-1+i}$

$-(\delta_i - \delta_j) + (-\delta_j + \delta_k) = (-\delta_i + \delta_k)$, $[\ f_\alpha\ ,\ e_\beta\ ] = -1 \cdot e_\gamma$

$[\beta_{n-1+j} - \beta_{n+1+i} - \cdots - \beta_{n+i}\ ,\ \alpha_{n+k} - \alpha_{n+1+k} - \cdots - \alpha_{n-1+j}]$

$= -1 \cdot \alpha_{n+k} - \alpha_{n+1+k} - \cdots - \alpha_{n-1+i}$

$-(\delta_i + \delta_j) + (2\,\delta_i) = (\delta_i - \delta_j)$, $[\ f_\alpha\ ,\ e_\beta\ ] = -1 \cdot e_\gamma$

$[\beta_{n+j} - \beta_{n+1+j} - \cdots - \beta_{n+m-1} \Longleftarrow \beta_{n+m} \Longrightarrow \beta_{n+m-1} - \cdots - \beta_{n+1+i} - \beta_{n+i}$

$,\ \alpha_{n+i} - \alpha_{n+1+i} - \cdots - \alpha_{n+m-1} \Longleftarrow \alpha_{n+m} \Longrightarrow \alpha_{n+m-1} - \cdots - \alpha_{n+1+i} - \alpha_{n+i}]$

$= -1 \cdot \alpha_{n+i} - \alpha_{n+1+i} - \cdots - \alpha_{n-1+j}$

$-(\delta_i + \delta_j) + (\delta_i + \delta_k) = (-\delta_j + \delta_k)$, $[\ f_\alpha\ ,\ e_\beta\ ] = -1 \cdot e_\gamma$

$[\beta_{n+j} - \beta_{n+1+j} - \cdots - \beta_{n+m-1} \Longleftarrow \beta_{n+m} \Longrightarrow \beta_{n+m-1} - \cdots - \beta_{n+1+i} - \beta_{n+i}$

$,\ \alpha_{n+i} - \alpha_{n+1+i} - \cdots - \alpha_{n+m-1} \Longleftarrow \alpha_{n+m} \Longrightarrow \alpha_{n+m-1} - \cdots - \alpha_{n+1+k} - \alpha_{n+k}]$

$= -1 \cdot \alpha_{n+k} - \alpha_{n+1+k} - \cdots - \alpha_{n-1+j}$

$-(\delta_i + \delta_j) + (\delta_i + \epsilon_k) = (-\delta_j + \epsilon_k)$, $[\ f_\alpha\ ,\ e_\beta\ ] = -1 \cdot e_\gamma$

$[\beta_{n+j} - \beta_{n+1+j} - \cdots - \beta_{n+m-1} \Longleftarrow \beta_{n+m} \Longrightarrow \beta_{n+m-1} - \cdots - \beta_{n+1+i} - \beta_{n+i}$

$,\ \alpha_k - \alpha_{1+k} - \cdots - (\alpha_n) - \alpha_{n+1} - \cdots - \alpha_{n+m-1} \Longleftarrow \alpha_{n+m} \Longrightarrow \alpha_{n+m-1} - \cdots - \alpha_{n+1+i} - \alpha_{n+i}]$

$= -1 \cdot \alpha_k - \alpha_{1+k} - \cdots - (\alpha_n) - \alpha_{n+1} - \cdots - \alpha_{n-1+j}$

$-(2\,\delta_i) + (\delta_i + \delta_k) = (-\delta_i + \delta_k)$, $[\ f_\alpha\ ,\ e_\beta\ ] = -1 \cdot e_\gamma$

$[\beta_{n+i} - \beta_{n+1+i} - \cdots - \beta_{n+m-1} \Longleftarrow \beta_{n+m} \Longrightarrow \beta_{n+m-1} - \cdots - \beta_{n+1+i} - \beta_{n+i}$

$,\ \alpha_{n+i} - \alpha_{n+1+i} - \cdots - \alpha_{n+m-1} \Longleftarrow \alpha_{n+m} \Longrightarrow \alpha_{n+m-1} - \cdots - \alpha_{n+1+k} - \alpha_{n+k}]$

$= -1 \cdot \alpha_{n+k} - \alpha_{n+1+k} - \cdots - \alpha_{n-1+i}$

$-(2\,\delta_i) + (\delta_i + \epsilon_k) = (-\delta_i + \epsilon_k)$, $[\ f_\alpha\ ,\ e_\beta\ ] = -1 \cdot e_\gamma$

$[\beta_{n+i} - \beta_{n+1+i} - \cdots - \beta_{n+m-1} \Longleftarrow \beta_{n+m} \Longrightarrow \beta_{n+m-1} - \cdots - \beta_{n+1+i} - \beta_{n+i}$

$,\ \alpha_k - \alpha_{1+k} - \cdots - (\alpha_n) - \alpha_{n+1} - \cdots - \alpha_{n+m-1} \Longleftarrow \alpha_{n+m} \Longrightarrow \alpha_{n+m-1} - \cdots - \alpha_{n+1+i} - \alpha_{n+i}]$

$= -1 \cdot \alpha_k - \alpha_{1+k} - \cdots - (\alpha_n) - \alpha_{n+1} - \cdots - \alpha_{n-1+i}$

$-(\delta_j+\epsilon_i)+(\delta_k+\epsilon_i)=(-\delta_j+\delta_k)$, $[f_\alpha , e_\beta]=1\cdot e_\gamma$

$[\beta_{n+j}-\beta_{n+1+j}-\cdots-\beta_{n+m-1}\Longleftarrow\beta_{n+m}\Longrightarrow\beta_{n+m-1}-\cdots-\beta_{n+1}-(\beta_n)-\cdots-\beta_{1+i}-\beta_i$

$,\ \alpha_i-\alpha_{1+i}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1+k}-\alpha_{n+k}]$

$=1\cdot\alpha_{n+k}-\alpha_{n+1+k}-\cdots-\alpha_{n-1+j}$

$-(\delta_j+\epsilon_i)+(\epsilon_i+\epsilon_k)=(-\delta_j+\epsilon_k)$, $[f_\alpha , e_\beta]=1\cdot e_\gamma$

$[\beta_{n+j}-\beta_{n+1+j}-\cdots-\beta_{n+m-1}\Longleftarrow\beta_{n+m}\Longrightarrow\beta_{n+m-1}-\cdots-\beta_{n+1}-(\beta_n)-\cdots-\beta_{1+i}-\beta_i$

$,\ \alpha_i-\alpha_{1+i}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1}-(\alpha_n)-\cdots-\alpha_{1+k}-\alpha_k]$

$=1\cdot\alpha_k-\alpha_{1+k}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n-1+j}$

$-(\delta_j+\epsilon_i)+(\delta_j+\epsilon_k)=(-\epsilon_i+\epsilon_k)$, $[f_\alpha , e_\beta]=-1\cdot e_\gamma$

$[\beta_{n+j}-\beta_{n+1+j}-\cdots-\beta_{n+m-1}\Longleftarrow\beta_{n+m}\Longrightarrow\beta_{n+m-1}-\cdots-\beta_{n+1}-(\beta_n)-\cdots-\beta_{1+i}-\beta_i$

$,\ \alpha_k-\alpha_{1+k}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1+j}-\alpha_{n+j}]$

$=-1\cdot\alpha_k-\alpha_{1+k}-\cdots-\alpha_{-1+i}$

$-(\epsilon_i+\epsilon_j)+(\epsilon_i+\epsilon_k)=(-\epsilon_j+\epsilon_k)$, $[f_\alpha , e_\beta]=-1\cdot e_\gamma$

$[\beta_j-\beta_{1+j}-\cdots-(\beta_n)-\beta_{n+1}-\cdots-\beta_{n+m-1}\Longleftarrow\beta_{n+m}\Longrightarrow\beta_{n+m-1}-\cdots-\beta_{n+1}-(\beta_n)-\cdots-\beta_{1+i}-\beta_i$

$,\ \alpha_i-\alpha_{1+i}-\cdots-(\alpha_n)-\alpha_{n+1}-\cdots-\alpha_{n+m-1}\Longleftarrow\alpha_{n+m}\Longrightarrow\alpha_{n+m-1}-\cdots-\alpha_{n+1}-(\alpha_n)-\cdots-\alpha_{1+k}-\alpha_k]$

$=-1\cdot\alpha_k-\alpha_{1+k}-\cdots-\alpha_{-1+j}$