

Quantitative Auditing of AI Fairness with differentially private Synthetic Data

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Ensuring fairness in AI systems is crucial, but auditing these systems often requires sensitive data, which raises security and privacy concerns. To address this, we introduce a framework that utilizes differentially private synthetic data for fairness evaluations. This approach allows auditors to assess AI systems without exposing sensitive information. Through experiments on datasets like Adult, COMPAS, and Diabetes, we compare fairness metrics of synthetic and real data. Our results ... (to be continued)

CCS Concepts: • **General and reference** → **Metrics**; • **Security and privacy** → **Usability in security and privacy**; *privacy-preserving protocols*; • **Information systems** → *Data mining*; *Information systems applications*; • **Computing methodologies** → *Machine learning*; *Artificial intelligence*; • **Mathematics of computing** → *Contingency table analysis*.

Additional Key Words and Phrases: AI, fairness, auditing, differential privacy, synthetic data

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1 Introduction

Fairness in machine learning has become an important topic as AI systems become widely used. Biased AI systems could result in amplified unfairness. Ensuring fairness in AI systems is crucial to prevent these biases from being exacerbated by AI systems. One approach is to audit the fairness of existing AI systems. Quantitative fairness auditing is a process of evaluating the fairness of AI systems with a range of fairness metrics. Many metrics have been proposed to help evaluate the fairness of AI systems.

Auditing can evaluate the fairness of AI systems, sometimes revealing cases of unfairness in them. For example, investigative journalists at ProPublica found that the COMPAS system, which is used to predict recidivism in the criminal justice system in the United States, is biased against certain demographic groups. This highlights the need for fairness auditing conducted by third parties.

To conduct audits, auditors may rely on access to real datasets containing sensitive information, raising significant security and privacy concerns. This reliance invites data security attacks on auditors, making them targets of unauthorized access. Analysis on real data runs the risk of data inference attacks, where confidential information may be deduced from seemingly harmless or aggregated data. Various technologies have been developed to mitigate these risks, such as differential privacy, where the idea is that whether or not any individual is in a dataset has a limited impact.

To mitigate these risks, the use of synthetic data has been proposed. Synthetic data are fake data that is artificially generated from real data. The principle of this technology is to preserve some statistical properties of the real data while protecting the privacy of the individuals in the real data. Among the generation techniques, differentially private synthetic data offers a way to investigate the data properties under differential privacy guarantees.

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We propose a novel auditing framework that leverages differentially private synthetic data to evaluate the fairness of AI systems. Our framework lets the auditors assess the fairness of AI systems without exposing sensitive information. Differentially private synthetic data is a technique that generates synthetic data from real data that preserves some statistical properties of the real data while ensuring differential privacy. This technology enables auditing without exposing sensitive information. However, synthetic data also introduces new challenges. Whether or not it accurately preserves the fairness properties of the real data is a critical question.

Our work examines the capabilities of existing differentially private synthetic data generation technologies in preserving fairness properties. Through empirical experiments on multiple real datasets, we aim to determine the effectiveness of differentially private synthetic data in this respect.

2 Related Work

Fairness in machine learning has become a crucial area of research[4]. Various fairness measures have been proposed to quantify[61] the fairness of AI models[8, 14, 15, 15, 24, 29, 46, 63]. We consider these fairness measures in this work.

Auditing of AI models is an important step in ensuring that these models are fair[22]. Different tools have been developed to check the fairness of AI models[7, 9, 51]. We use a tool produced from our previous work[62].

Differential privacy has risen to become the standard of privacy protection in many data analyses[25]. It offers a formal framework for quantifying privacy guarantees when releasing information derived from sensitive data[18, 19]. We use differential privacy in this work to protect the privacy of individuals in the data in our auditing framework.

Different flavors of differential privacy have been proposed over the years[16], such as Gaussian differential privacy[17], Pufferfish differential privacy[28], Bayesian differential privacy[54], and Rényi differential privacy[40]. We consider Rényi differential privacy specifically in this work.

The generation of synthetic data[33, 47] under differential privacy constraints allows for data sharing without compromising privacy[53]. There have been several techniques developed for differentially private synthetic data generation[1, 10, 11, 20, 49, 60] in recent years. We use the technique that won the 2018 NIST differential privacy Synthetic Data Challenge competition[38].

There have also been some works that aim to alter the properties of synthetic data, such as introducing bias to them[5, 26]. In this work, we aim to preserve the properties of the original data. Thus, we do not consider these works.

While helpful, there are some technical limitations[13, 23, 52, 59] and ethical concerns[58] with the use of synthetic data. Synthetic data may not always preserve properties of real data. This may cause inaccuracies in downstream tasks such as fairness evaluation. Our work examines these limitations.

3 Preliminaries

Let $\prod_i x_i$ denote the Cartesian product of x_i s.

An *attribute* is a symbol A . The set of all attributes is \mathcal{A} . An *attribute value* is a symbol a . The set of all attribute values is Ω . An *attribute value space* is a function $\sigma : \mathcal{A} \rightarrow 2^\Omega$ specifying the set of valid values that attributes can take on. A *row* with respect to σ is a function $r : \mathcal{A} \rightarrow \Omega$ where $r(A) \in \sigma(A)$. The set of all rows is \mathcal{R} . A *database* is a tuple $D = (\mathcal{A}_D, \Omega_D, \sigma_D, \mathcal{R}_D)$ where $\mathcal{A}_D \subseteq \mathcal{A}$ is the set of attributes in D , $\Omega_D \subseteq \Omega$ is the set of attribute values in D , $\sigma_D : \mathcal{A}_D \rightarrow 2^{\Omega_D}$ is the function specifying the valid values that each attribute can take on in D , and $\mathcal{R}_D \subseteq \mathcal{R}$ is the set of rows with respect to σ_D in D with $r_D : \mathcal{A}_D \rightarrow \Omega_D$ for all $r_D \in \mathcal{R}_D$ and $r_D(A) \in \sigma_D(A)$ for all $A \in \mathcal{A}_D$. The set of all databases is \mathcal{D} .

Let $X \in \mathcal{X}$ be a discrete random variable and $p(x) := \Pr[X = x]$. Its *marginal Shannon entropy* is $H(X) := -\sum_{x \in \mathcal{X}} p(x) \log p(x)$. It quantifies the level of uncertainty of X . Let $Y \in \mathcal{Y}$ be another discrete random variable. Their *joint Shannon entropy* is $H(X, Y) := -\sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}} p(x, y) \log p(x, y)$.

Let X, Y be discrete random variables. Their *mutual information* is $I(X; Y) := H(X) + H(Y) - H(X, Y)$. It quantifies the level of dependence between X and Y .

Let $\{X_i\}$ be a set of discrete random variables indexed by a graph $G = (V, E)$, where V represents the random variables X_i s and E represents dependencies between these random variables. A *Markov random field* is a probability distribution over X_i s, such that each random variable X_i , given its neighborhood in G , is conditionally independent of all other variables. Since edges represent dependencies, cliques in a Markov random field represent groups of variables that are all mutually dependent. As a machine learning model, there has been much development in the estimation and inference of Markov random fields[30, 41].

3.1 Fairness Measures

For fairness measures[46, 62], let Y denote the ground truth of an outcome, let \hat{Y} denote the predicated result of an outcome, let S denote the protected attribute, and let ϵ denote some threshold. Y, \hat{Y}, S are binary.

Fairness measures can be broadly categorized into independence, separation, and sufficiency, which are defined by conditional independence in Table 1. $X \perp Y | Z$ denotes the conditional independence between X and Y conditioning on Z .

Table 1. Fairness categories.

Category	Definition
Independence	$S \perp \hat{Y}$
Separation	$S \perp \hat{Y} Y$
Sufficiency	$S \perp Y \hat{Y}$

These categories can be expanded into forms of probability. For example, the definition of separation is expanded to

$$\begin{aligned} P[\hat{Y} = 1 | S = 1, Y = 1] &= P[\hat{Y} = 1 | S \neq 1, Y = 1] \\ P[\hat{Y} = 1 | S = 1, Y = 0] &= P[\hat{Y} = 1 | S \neq 1, Y = 0] \end{aligned}$$

The definition can be relaxed. Its relaxation, for some parameter ϵ , is

$$\begin{aligned} |P[\hat{Y} = 1 | S = 1, Y = 1] - P[\hat{Y} = 1 | S \neq 1, Y = 1]| &\leq \epsilon \\ |P[\hat{Y} = 1 | S = 1, Y = 0] - P[\hat{Y} = 1 | S \neq 1, Y = 0]| &\leq \epsilon \end{aligned}$$

which is also the definition of a fairness measure called equalized odds.

We consider in this work various fairness measures listed in Table 2.

3.2 Rényi differential privacy

A *randomized mechanism* is a randomized algorithm $M : \mathcal{D} \rightarrow \mathbb{R}^p$ that takes a database and, after introducing noise, outputs some results.

Table 2. Fairness measures.

Category	Fairness Measure	Definition
Independence	Demographic Parity	$ P[\hat{Y} = 1 S = 1] - P[\hat{Y} = 1 S \neq 1] \leq \epsilon$
Separation	Equalized Odds (False Positive)	$ P[\hat{Y} = 1 S = 1, Y = 0] - P[\hat{Y} = 1 S \neq 1, Y = 0] \leq \epsilon$
	Equalized Odds (True Positive)	$ P[\hat{Y} = 1 S = 1, Y = 1] - P[\hat{Y} = 1 S \neq 1, Y = 1] \leq \epsilon$
Sufficiency	Conditional Use Accuracy Equality (True Positive)	$ P[Y = 1 S = 1, \hat{Y} = 1] - P[Y = 1 S \neq 1, \hat{Y} = 1] \leq \epsilon$
	Conditional Use Accuracy Equality (True Negative)	$ P[Y = 0 S = 1, \hat{Y} = 0] - P[Y = 0 S \neq 1, \hat{Y} = 0] \leq \epsilon$
N/A	Overall Accuracy Equality	$ P[Y = \hat{Y} S = 1] - P[Y = \hat{Y} S \neq 1] \leq \epsilon$

Let D_1, D_2 be two databases. They are *neighbors*, denoted $D_1 \sim D_2$, if $|\mathcal{R}_{D_1} \Delta \mathcal{R}_{D_2}| = 1$ where Δ denotes symmetric difference; that is, one database contains an extra row than the other. In other words, they differ in exactly one row.

Definition 3.1 (Gaussian Mechanism[19]). Let $f : \mathcal{D} \rightarrow \mathbb{R}^p$ be a function. The Gaussian Mechanism M adds independent and identically distributed Gaussian noise with mean 0 and standard deviation σ to each component of the p -dimensional vector output of $f(D)$

$$M(D) = f(D) + \mathcal{N}(0_p, \sigma^2 \mathbb{I}_p)$$

where \mathcal{N} is a multivariate normal distribution with mean vector 0_p and covariance matrix $\sigma^2 \mathbb{I}_p$ where \mathbb{I}_p is the identity matrix.

Definition 3.2 (Rényi Differential Privacy (RDP)). Let P_X denote the probability distribution associated with the random vector \mathbf{X} . A randomized mechanism M satisfies (α, γ) -RDP for $\alpha \geq 1$ and $\gamma \geq 1$ if, for all databases $D_1 \sim D_2$, we have

$$D_\alpha(P_{M(D_1)} \| P_{M(D_2)}) \leq \gamma$$

where $D_\alpha(P_1 \| P_2)$ is the Rényi divergence[32, 56, 57] of order α between probability distributions P_1, P_2 over x

$$D_\alpha(P_1 \| P_2) := \frac{1}{\alpha - 1} \log \int P_1(x)^\alpha P_2(x)^{1-\alpha} dx$$

THEOREM 3.3 (RDP OF THE GAUSSIAN MECHANISM[21, 40]). The Gaussian Mechanism satisfies $(\alpha, \alpha \frac{\Delta_f^2}{2\sigma^2})$ -RDP, where Δ_f denotes the sensitivity[19] of f , which is defined as the maximum L^2 -norm difference in the output of f

$$\Delta_f := \max_{D_1 \sim D_2} \|f(D_1) - f(D_2)\|_2$$

3.3 Differentially Private Synthetic Data

Let $C \subseteq \mathcal{A}$ be a subset of attributes. Let $\Omega_C = \prod_{i \in C} \Omega_i$. Let x be a row and x_C denote the restriction of x to C . A *marginal*[3, 38] of C on database D is a function $\mu_D : \Omega_C \rightarrow \mathbb{N}_0$ such that $\mu_D(t) = \sum_{x \in \mathcal{D}} \delta_{t, x_C}$ where δ is the Kronecker function; that is, it is a lookup table of the counts of each possible combination of attribute values. We call marginals of $|C| = n$ attributes n -way marginals.

The task of differentially private synthetic data[35, 36, 44, 55] is, given a database D , adding some noise to marginals of D such that it satisfies some differential privacy guarantees and outputting another database D' , such that the L^1 -norm errors between marginals of D and D' is small; that is, their marginals $\mu_D, \mu_{D'}$ are similar.

For example, suppose we have a database with attributes sex and race. The 2-way marginals of the original database and the synthetic database are shown in Table 3. The marginals of the synthetic data is supposed to be similar to that of the original database.

Table 3. Example marginals.

(a) Marginal of original data.		(b) Marginal of synthetic data.	
Attributes	Count	Attributes	Count
Male,White	24	Male,White	22
Female,White	33	Female,White	35
Male,Black	13	Male,Black	10
Female,Black	47	Female,Black	46

4 Motivation

Our auditing framework is tripartite. It consists of three parties: the data provider, the model maker, and the third-party auditor.

The data provider is responsible for supplying the raw datasets which should originate from trustworthy sources, such as government agencies like a census bureau.

The model maker develops AI models. These are AI companies or research labs specialized in training and optimizing AI models.

The third-party auditor acts as an evaluator, using our tool to audit the AI models for fairness issues by combining both the datasets and the models. These may be investigative journalists or regulatory bodies.

In the framework of our previous work[62], after obtaining real data from the data provider, the 3rd party auditor holds onto the real data for performing fairness audits, and it supposedly retains it indefinitely for the possibility of any future audits. However, this practice raises both security and privacy concerns.

For security, it creates a point of vulnerability of unauthorized access, and the auditor is now a target of data security attacks. The auditor may not have the necessary resources to defend against these threats. A breach at the auditor's end could result in compromises of individuals' sensitive information.

As for privacy, on the other hand, it introduces risks of information leakage. Release of analyses on real data may inadvertently reveal sensitive information by data inference attacks. Attackers can exploit data patterns in outputs or combine outputs with external data to infer sensitive information.

Thus, we introduce a new framework where the auditor generates synthetic data based on real data upon retrieval of the real data, and then holds onto the synthetic data and discards the real data, preventing all further security and privacy violations. The third-party still retains the ability to audit all incoming future models as needed.

5 Methodology

We employed the tools of the winner of the 2018 NIST differential privacy Synthetic Data Challenge competition[43] by [34, 37–39] and the fairness checker tool from our previous research[62].

This research is conducted in Python Jupyter notebooks and is publicly available.

We shall illustrate the workflow of our methodology with an example of the COMPAS dataset, which contains attributes such as sex and race with the prediction target attribute being recidivism.

5.1 Data Synthesis

The synthesis framework is three-fold, namely, select-measure-generate[35, 36]. We first select the important marginals to preserve, measure them by adding differential privacy noise, and then generate synthetic data.

Underneath the hood, the tool employs a Markov random field. The select step corresponds to marking cliques in a Markov random field, and the generate step corresponds to sampling from the fitted Markov random field.

By default, all 1-way marginals are selected to preserve the quantity of each attribute element. We can further preserve correlations by adding n -way marginals. For example, if we want to preserve the relationship between sex and race, we may add the clique (sex,race).

In a perfect world where all correlation information is to be preserved, we may wish to make a complete graph. However, this is intractable as the complexity of the problem would skyrocket. Furthermore, algorithms for Markov random field favor graphs with specific shapes, such as trees.

To circumvent this complexity explosion, instead, [38] devised a technique where the mutual information of all the database attribute pairs is calculated, and then a maximum spanning tree is identified with edge weights being the mutual information to obtain a skeleton tree-shaped Markov random field. The marginals are then selected based on the edges of this tree.

In the COMPAS example, we may select the marginal (sex,race).

For the competition, [38] further manually added certain cliques based on his investigation of the competition dataset. For example, they manually added the clique (sex,city,income). In addition, they would add some edges based on some sophisticated heuristics tailored to that particular dataset.

We do not consider these additional heuristics for the generality of our approach. Meanwhile, his other approach where the auditor does access the real dataset does not fit our framework.

While synthetic data generation methods often leverage domain knowledge to improve accuracy, we chose to employ a vanilla approach with only the maximum spanning tree method without incorporating additional heuristics. This decision was made due to two main factors: first, the need for more specific domain knowledge about the dataset makes it difficult to identify meaningful dependencies; second, prediction targets may differ from dataset to dataset, which means that tailored heuristics could not be effectively generalized. By avoiding dataset-specific adjustments, our method remains general and applicable to a wide range of datasets. Furthermore, the bare tree structure makes the method more tractable and computationally efficient.

For the measure step, we followed the examples provided in the tool’s repository. Gaussian noises are added to the selected marginals. Half of the privacy budget $\epsilon = 1$ is spent on all 1-way marginals and the other half on the selected cliques. These marginals are then fed to the tool to fit the Markov random field. By [38], this procedure satisfies $(\alpha, \frac{\alpha}{2\sigma^2})$ -RDP for all $\alpha \geq 1$.

In the COMPAS example, the selected the marginals are perturbed by Gaussian noise, as shown in Table 4.

5.2 Fairness Checking

After synthesizing the dataset, we used the fairness checker from [62] to compute the fairness measures of any incoming AI model.

The fairness checker is an open-sourced public domain Python package that computes various fairness measures, such as those mentioned in Table 2.

The checker is designed to be user-friendly and agnostic to the underlying AI model. It is also designed to be easily extensible to accommodate new fairness measures.

The checker simply iterates through the given database D and computes the results based on some given predicates on the rows r_i s, and finally outputs the fairness measure values.

Table 4. Example marginals.

(a) Original marginal.		(b) Perturbed marginal.	
Attributes	Count	Attributes	Count
Female,African-American	1537	Female,African-American	1545.7885
Female,Asian	8	Female,Asian	4.3236
Female,Caucasian	1465	Female,Caucasian	1444.7303
Female,Hispanic	215	Female,Hispanic	208.2250
Female,Native American	15	Female,Native American	26.7479
Female,Other	143	Female,Other	150.8175
Male,African-American	8254	Male,African-American	8296.6274
Male,Asian	63	Male,Asian	92.1107
Male,Caucasian	4621	Male,Caucasian	4643.0108
Male,Hispanic	1236	Male,Hispanic	1274.0831
Male,Native American	42	Male,Native American	28.9542
Male,Other	717	Male,Other	710.3473

Protected groups S , predicted outcomes \hat{Y} , and ground truths Y are all formulated as these predicates. These are straightforward logical boolean expressions. Specifically, they are given as Python functions that output boolean values.

In the COMPAS example, if the sensitive attribute is sex and the protected group is female, the protected group predicate would be $S := r(\text{sex}) = \text{Female}$. This can be easily implemented in Python as a comparison function.

The interpretation of the resulting fairness measure values is dependent on the third-party auditors. The auditors may have different thresholds ϵ for different fairness measures or different AI models.

6 Experiments

To test the viability of our method, we compare the metrics computed from the synthetic dataset against those of the original dataset. We used various datasets with fairness concerns mentioned in [46].

We looked at several publicly available datasets, such as Adult[6, 48], COMPAS[31, 45], and Diabetes[27, 42]. These datasets are known to have fairness issues[46].

The fairness checker evaluates datasets based on multiple fairness metrics, such as demographic parity and equalized odds. By comparing these measures between the synthetic and original datasets, we aim to ensure that the synthetic data preserves the fairness properties of the original data. The comparison process is three-fold. It goes as follows.

The dataset is first processed so it can be fed into the synthetic data generator. Some marginals are selected according to their mutual information, as described in the Section 5, and the synthetic data generator model is fitted to the original data according to the selected marginals. Then the generator is run 100 times to obtain 100 sets of synthetic data.

Next, several AI models are extracted from various real-life authors from Kaggle. They are finetuned to perform well on the original dataset. For one, a random forest model is finetuned by searching hyperparameters settings[12]. Another random forest model is finetuned with over-sampling methods[50]. Also, a logistic regression model is finetuned by performing principal component analysis[2].

The AI models and both the original dataset and synthetic datasets are then fed to the fairness checker. Sensitive attributes are identified based on manual examination with common sense or by referring to [46]. Then, applicable fairness measures are computed using the checker for both the original and the synthetic.

6.1 Adult Income Dataset

The Adult[6, 48] dataset comes from the 1994 census in the United States and contains about 30000 individuals. The dataset has 14 attributes, such as sex, age, and education. The task is to predict whether an individual earns more than \$50,000 a year. We set the sensitive attribute as sex and the protected group as female. The results are shown in Table 5.

Table 5. Fairness measures experiment results of the Adult dataset. Average difference is 0.0775, and fit time is 17m1s.

Measure	Original	Synthetic	Difference
Demographic Parity	0.1933	0.0114	0.1818
Overall Accuracy Equality	0.0250	0.0169	0.0080
Equalized Odds (False Positive)	0.0175	0.0060	0.0115
Equalized Odds (True Positive)	0.0114	0.0303	0.0189
Conditional Use Accuracy Equality (True Positive)	0.1941	0.0315	0.1626
Conditional Use Accuracy Equality (True Negative)	0.1102	0.0299	0.0803

6.2 COMPAS Dataset

The COMPAS[31, 45] dataset comes from an investigative report by ProPublica of the COMPAS criminal recidivism assessment system and contains about 7000 individuals. The dataset has 10 applicable attributes, such sex, age, and priors count. The task is to predict whether an individual will re-offend. We set the sensitive attribute as sex and the protected group as female. The results are shown in Table 6.

Table 6. Fairness measures experiment results of the COMPAS dataset. Average difference is 0.0727, and fit time is 33s.

Measure	Original	Synthetic	Difference
Demographic Parity	0.1310	0.0809	0.0501
Overall Accuracy Equality	0.0079	0.0146	0.0067
Equalized Odds (False Positive)	0.0249	0.0802	0.0553
Equalized Odds (True Positive)	0.0177	0.0825	0.0648
Conditional Use Accuracy Equality (True Positive)	0.1709	0.0551	0.1157
Conditional Use Accuracy Equality (True Negative)	0.1695	0.0256	0.1439

6.3 Diabetes Dataset

The Diabetes[27, 42] dataset comes from the hospital readmission data published in the 1994 AI in Medicine journal and contains about 100000 individuals. The dataset has 8 attributes, such as gender, age, and smoking history. The task is to predict whether an individual will be readmitted. We set the sensitive attribute as gender and the protected group as female. The results are shown in Table 7.

7 Evaluation

first list formulas

count vars

theyre not part of marginals

Table 7. Fairness measures experiment results of the Diabetes dataset. Average difference is 0.0078, and fit time is 1m24s.

Measure	Original	Synthetic	Difference
Demographic Parity	0.0135	0.0015	0.0119
Overall Accuracy Equality	0.0077	0.0011	0.0065
Equalized Odds (False Positive)	0.0000	0.0010	0.0010
Equalized Odds (True Positive)	0.0086	0.0094	0.0008
Conditional Use Accuracy Equality (True Positive)	0.0133	0.0059	0.0074
Conditional Use Accuracy Equality (True Negative)	0.0216	0.0019	0.0197

they cant be computed

we can say less about fairness with these with less info

auditor need to know theyre aproxing scores

auditor need to keep in mind that these scores are for datasets and we preserve these scores thats it. scores may not be accurate. it doesnt mean its less easy to test fairness

7.1 Positive Results

it works!

7.2 Negative Results

independence

$$P[\hat{Y}|S] - P[\hat{Y}|\bar{S}] = \frac{P[\hat{Y} \cap S]}{P[S]} - \frac{P[\hat{Y} \cap \bar{S}]}{P[\bar{S}]}$$

separation

$$P[\hat{Y}|S, Y] - P[\hat{Y}|\bar{S}, Y] = \frac{P[\hat{Y} \cap S \cap Y]}{P[S \cap Y]} - \frac{P[\hat{Y} \cap \bar{S} \cap Y]}{P[\bar{S} \cap Y]}$$

sufficiency

$$P[Y|S, \hat{Y}] - P[Y|\bar{S}, \hat{Y}] = \frac{P[Y \cap S \cap \hat{Y}]}{P[S \cap \hat{Y}]} - \frac{P[Y \cap \bar{S} \cap \hat{Y}]}{P[\bar{S} \cap \hat{Y}]}$$

accuracy equality

$$P[Y = \hat{Y}|S] - P[Y = \hat{Y}|\bar{S}] = \frac{P[Y = \hat{Y} \cap S]}{P[S]} - \frac{P[Y = \hat{Y} \cap \bar{S}]}{P[\bar{S}]}$$

8 Conclusion

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