Lambda' Algorithm

For f

$$
\begin{aligned}
f(x+a_k) &= \bigvee_{f(a+a_k)=1}^{m} a \\
\mathscr{M}(f(x+a_k)) &= \bigvee_{f(a+a_k)=1}^{m} \mathscr{M}(a) \\
&= \bigvee_{f(a+a_k)=1}^{m} \{ \bigwedge_{a[i]=1} x_i \} \\
&= \bigvee_{f(a+a_k)=1}^{m} \mathscr{T}'_k \\
\mathscr{M}_{a_k}(f) &= (\bigvee_{f(a+a_k)=1}^{m} \mathscr{T}'_k)(x+a_k)
\end{aligned}
$$

For $S'_i = \{v_n + a_i \mid n\}$

$$
\begin{aligned}
\mathrm{M_{DNF}}(S'_i) &= \{\mathrm{M_{TERM}}(v_n+a_i) \mid (v_n+a_i) \in S'_i\} \\
&= \{ \bigwedge_{(v_n+a_i)[j]=1} x_j \mid (v_n+a_i) \in S'_i\} \\
\bigvee_{(v_n+a_i)\in S'_i} \mathrm{M_{DNF}}(S'_i) &= \bigvee_{(v_n+a_i)\in S'_i} \bigwedge_{(v_n+a_i)[j]=1} x_j \\
H_i &= (\bigvee_{(v_n+a_i)\in S_i} \mathrm{M_{DNF}}(S'_i))(x+a_i)
\end{aligned}
$$

Proposition A'

Let f be a boolean function. If a is an assignment such that $f(a) = 1$, then for the minterm DNF $\bigvee_{i=1}^{m} T_i$ of f there is a term T_a such that $\mathscr{M}(T_a) = \mathrm{M}_{\mathrm{TERM}}(a)$.

Proof Assume

1. v^δ is a positive counterexample.
2. I^δ is non-empty.
3. $\mathrm{M}_{\mathrm{TERM}}(v^\delta + a_i) \in \mathscr{T}_i' - \mathrm{M}_{\mathrm{DNF}}(S_i'^{\delta-1})$ for $i \in I^\delta$.
4. $\mathrm{M}_{\mathrm{DNF}}(S_i^\delta) \subseteq \mathscr{T}_i$ for $i = 1..t$.
5. $H_i^\delta \to \mathscr{M}_{a_i}(f)$ for $i = 1..t$.

Then induction is as follows.

c. Let $i \in I^{\delta+1}$. Since (b), $H_i^\delta(v^{\delta+1}) = 0$. By definition,

$$H_i^\delta(v^{\delta+1}) = (\bigvee_{v_n \in S_i'^\delta} \mathrm{M}_{\mathrm{DNF}}(S_i'^\delta))(v^{\delta+1} + a_i) = 0$$

so

$$\mathrm{M}_{\mathrm{TERM}}(v^{\delta+1} + a_i) \notin \mathrm{M}_{\mathrm{DNF}}(S_i'^\delta)$$

By (a),

$$f(v^{\delta+1}) = f((v^{\delta+1} + a_i) + a_i) = 1$$

By Proposition A', there is a term T in the minterm DNF of $f(x + a_i)$ such that $\mathscr{M}(T) = \mathrm{M}_{\mathrm{TERM}}(v^{\delta+1} + a_i)$. Therefore

$$\mathrm{M}_{\mathrm{TERM}}(v^{\delta+1} + a_i) \in \mathscr{T}_i'$$

This is the crux of the proof.

e. By definition,

$$H_i^{\delta+1}(x) = (\bigvee_{(\widehat{v_n} + a_i) \in S_i^{\delta+1}} \mathrm{M}_{\mathrm{DNF}}(S_i^{\delta+1}))(x + a_i)$$

and

$$\mathscr{M}_{a_k}(f) = (\bigvee_{n=1}^{s} \mathscr{T}_k)(x + a_k)$$

By (d), $\mathrm{M}_{\mathrm{DNF}}(S_i^{\delta+1}) \subseteq \mathscr{T}_i$, so

$$\bigvee_{(\widehat{v_n}+a_i)\in S_i^{\delta+1}} \mathrm{M}_{\mathrm{DNF}}(S_i^{\delta+1}) \to \bigvee_{n=1}^{s} \mathscr{T}_k$$

Therefore

$$H_i^{\delta+1}(x) \to \mathscr{M}_{a_k}(f)$$