Poset Paper

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Abstract. hello world

1 Introduction

Intro part here

2 Preliminaries

A partial order is a binary relation $\leq$ that is reflexive, antisymmetric, and transitive. A set $\mathcal{S}$ with a partial order is called a partially ordered set(poset). A linear order $<$ is a partial order that is total. If $\leq = \emptyset$, we call $\leq$ the discreet order. A linear extension $<$ of a partial order $\leq$ is an extension of $\leq$ that is total. Let $\mathcal{E}(\leq)$ be set of all linear extensions of $\leq$. We wrtie $P \sqsubseteq L$ iff L extends P and L is a linear order.

For a poset P and $x, y \in \mathcal{P}$, let it be that $x \prec_P y \triangleq x \leq_P y \wedge \nexists z \in \mathcal{P}, x \leq_P z \wedge z \leq_P y$; $x \parallel_P y \triangleq x \nleq_P y \wedge y \nleq_P x$, and, for linear extension $L, L' \in \mathcal{E}(P)$, that $L \leftrightarrow_{x,y} L' \triangleq x \prec_L y \wedge y \prec_{L'} x$.

Let the swap graph $\mathcal{G}(P)$ of poset P be the undirected graph (V, E) such that $V \triangleq \mathcal{E}(\leq)$ and $E \triangleq \{(L, L') | \exists x, y \in P, L \leftrightarrow_{x,y} L'\}$.

The Hasse diagram of a poset P is a directed acyclic graph constructed with transitivity removed. If L is a linear order, we shall consider it in string form of linearizations. For a poset P with $\leq$ let $\mathcal{L}(P)$ be $\mathcal{E}(\leq)$ in string form; then, it is a subset of the permutations of elements of P; that is, $\mathcal{L}(P) \subseteq \mathfrak{S}(\mathcal{P})$. Note that in the case of discreet order, $\mathcal{L}(P) = \mathfrak{S}(\mathcal{P})$, and that for any poset, $\mathcal{L}(P)$ is the set of topological sort, in string form, of the Hasse diagram.

The poset cover problem finds, given a set of linearizations Υ, a set of posets $\mathcal{C}$, called a cover, such that $\Upsilon = \bigcup_{P \in C} \mathcal{L}(P)$ and that $|\mathcal{C}|$ is minimal. This problem is proved to be NP-complete.(cite)

3 Related Theorems

Theorem 1. *Permutations and Nerode*

Theorem 2. *(Corollary of Szpilrajn extension theorem) For a partial order $\leq$, $\leq = \bigcap_{< \in \mathcal{E}(\leq)} <$.*

Theorem 3. *(Heath and Nema) If $x \parallel y$ for $\leq$ and there is $<\in \mathcal{E}(\leq)$ such that $x \prec y$ for $<$, then there is $<'\in \mathcal{E}(\leq)$ such that $y \prec x$ for $<'$.*

Theorem 4. *(Pruesse? and Ruskey) Swap graphs are connected*

Theorem 5. *Swap graphs are connected by Kendall tau paths*

Definition 1. *(Poset cover problem) A poset cover $\mathcal{C}$ of a set posets on S for a set of linearizations Υ satisfies the following: (1) $\forall L \in \Upsilon \exists P \in \mathcal{C}, P \sqsubseteq L$; (2) $\forall L \in \mathfrak{S}(\mathcal{S}) - \Upsilon \forall P \in \mathcal{C}, P \not\sqsubseteq L$.*

4 SAT Encoding Part

5 Exp Part

6 Conclusions

7 References