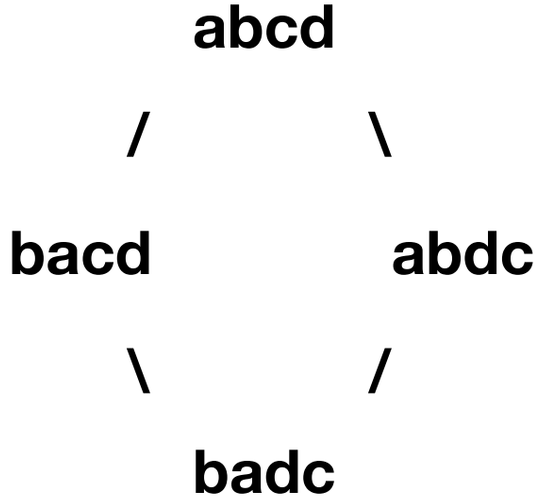
(for swap graph use inversion/unordering ;
see Labarre's cyclic way for transposition graph)

Kendall tau constraint

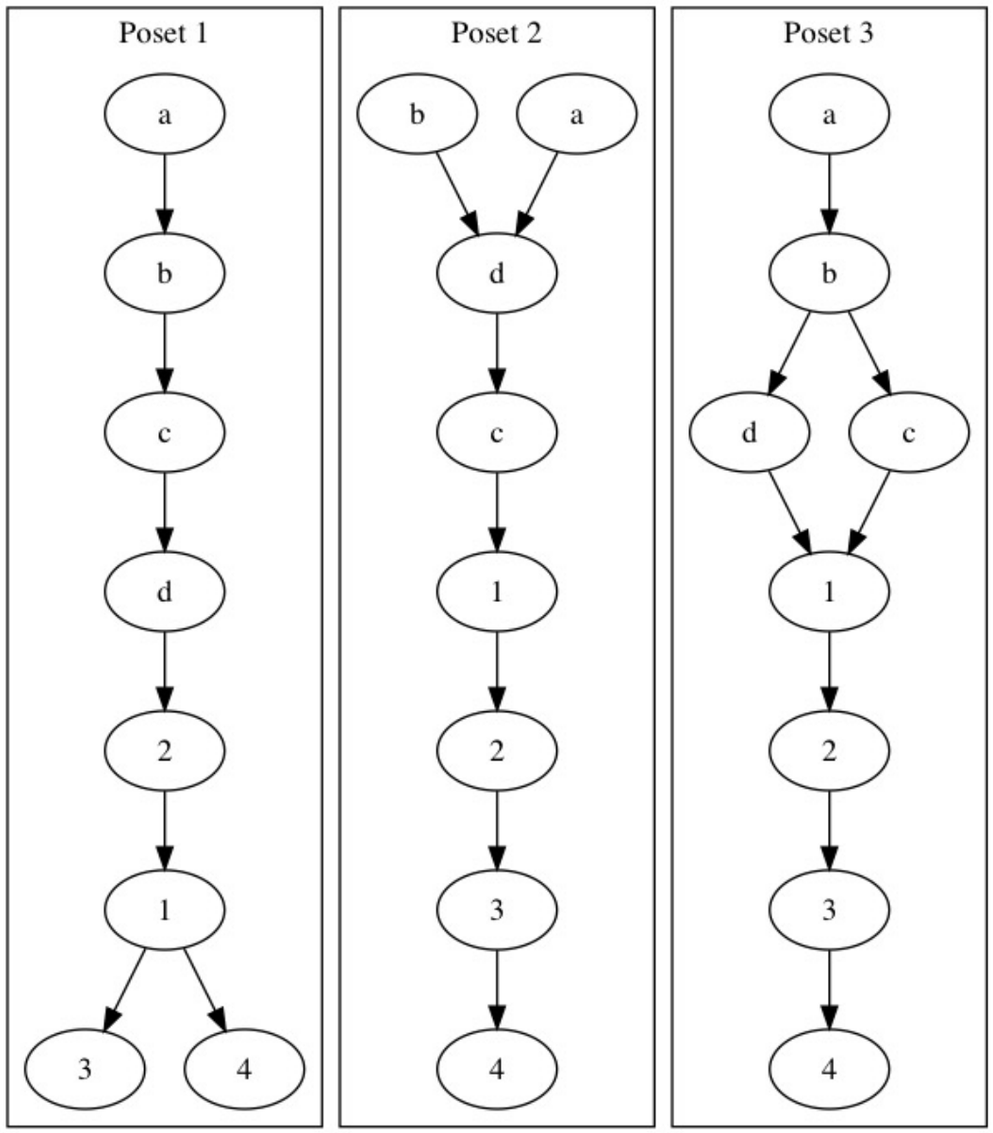
Any poset generating abcd and badc must also generate bacd and abdc



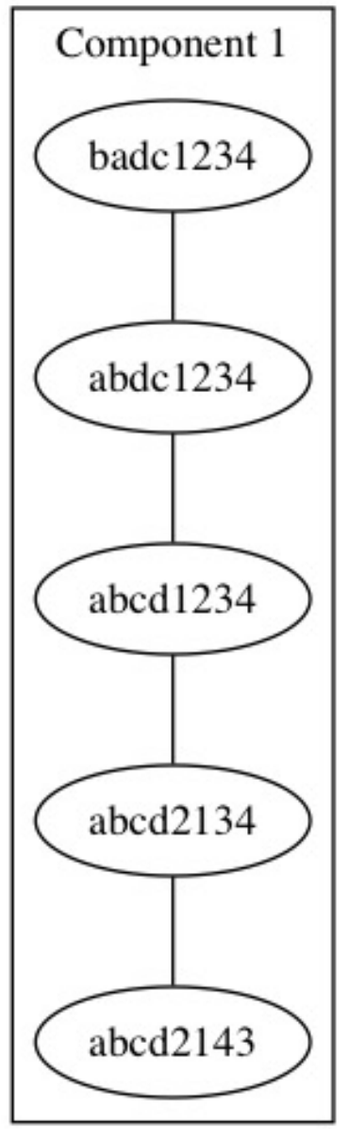
I didn't use this for experiment because encoding this takes too much time (minutes)
I think it helps when ruling out big #posets but I'm not sure because it won't terminate

Worst case result of poset count
Given enough elements, for a connected component, it is ceil(|V|/2),
regardless of how the graph is connected

consider the chain: badc1234 - abdc1234 - abcd1234 - abcd2134 - abcd2143



Cover 1 for component 1



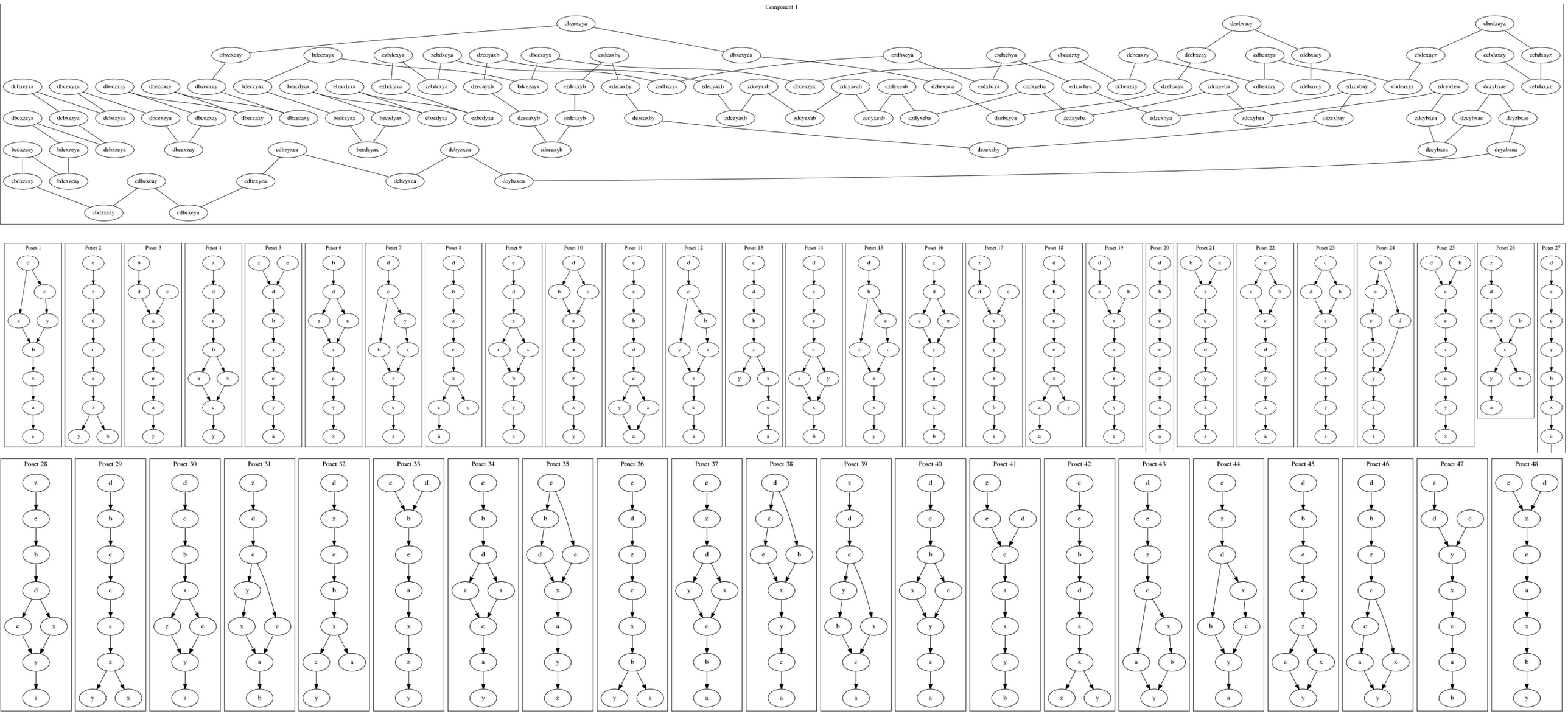
[badc1234 abdc1234 abcd1234 abcd2134 abcd2143]

sample 3000 trials from |S| = 4:
completely random (multiple components),
no timeouts, avg time = 4.2s

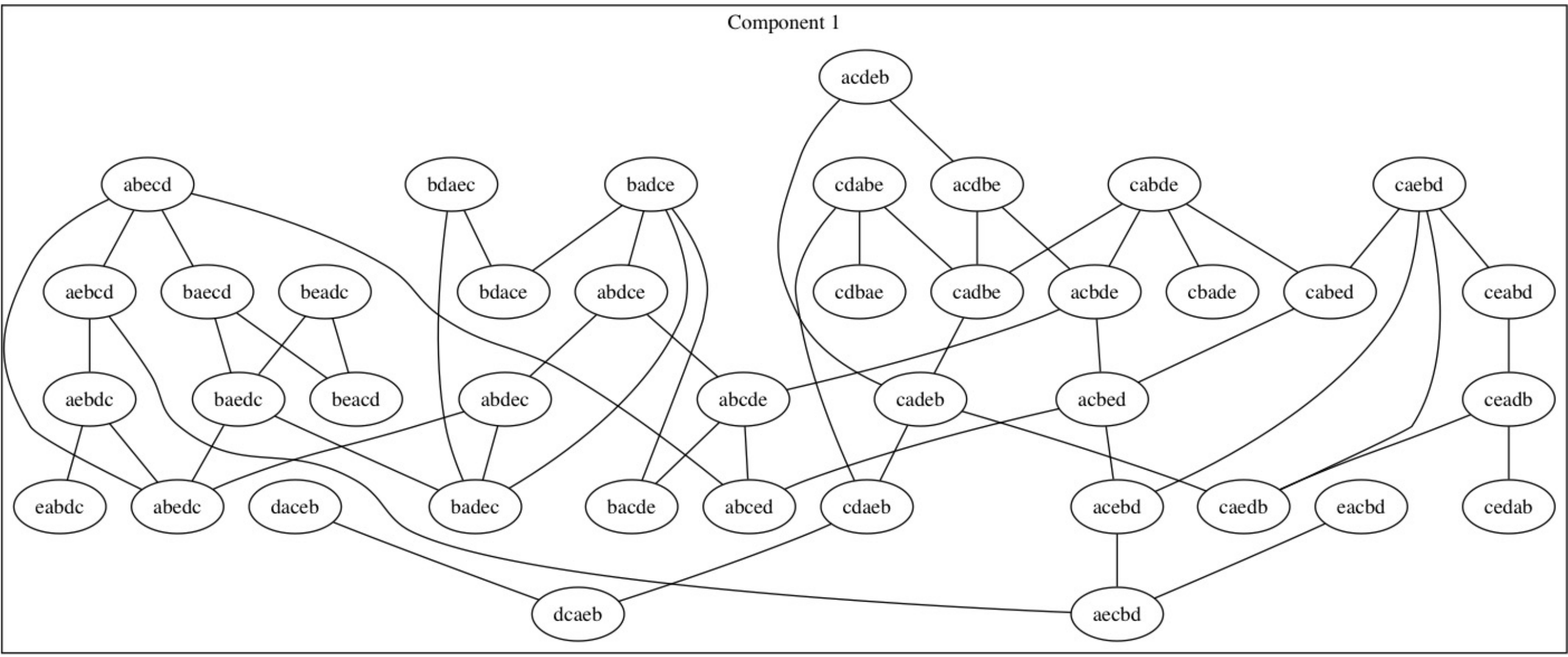
all 23 subsets(#=24) from |S| = 4:
no timeouts, avg time = 8.6s

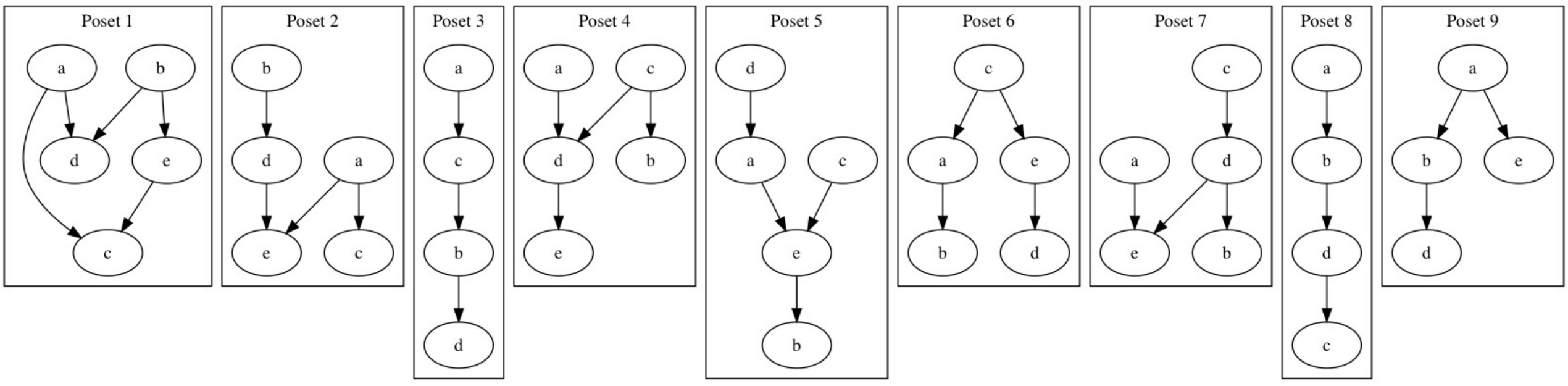
chain experiment (4 trials, |Y|=1000, |S|=8, avg=3.37 hrs)

trial 1 result:
#posets = 48
time = 15480s (4.3 hrs)

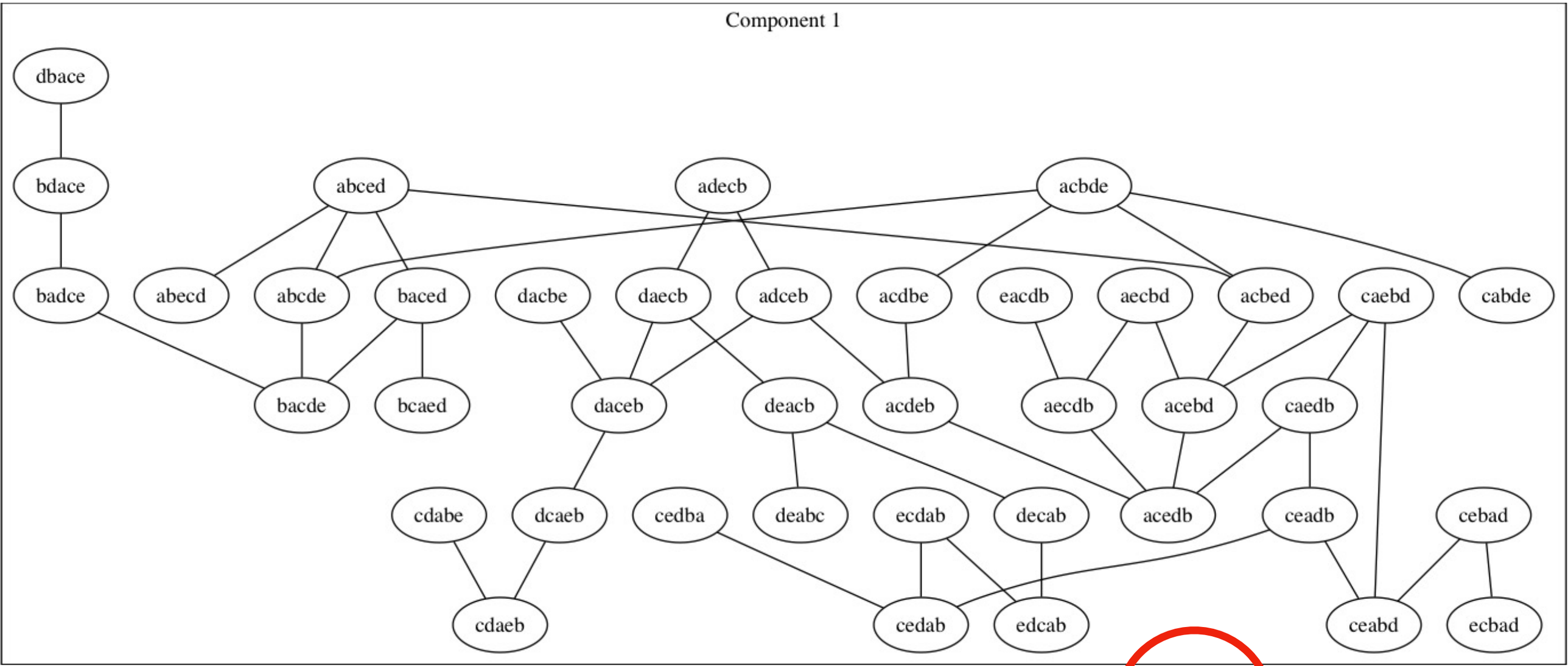


|Y| = 40, |S| = 5 CAN be easily found like: (in 38s, no timeout)





|Y| = 40, |S| = 5 CAN be hard to find like:




```
34243 rex       20   0  405M  289M 22360 R 99.8  0.0    199h python3 exp.py
```

stuck at z3 checking if 13 posets works

<= 8 : less than 1s

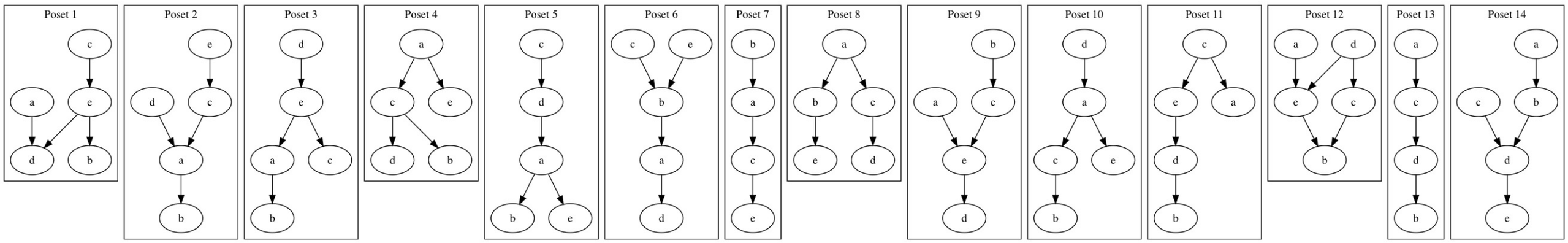
9 : 10s

10 : 16s

11 : 2153s

12 : 49746s

let 13 timeout?
14 found immediately (< 1s)



Cover 1 for component 1

but no way to know if this is minimum
but will 13 ever terminate?

later experiments found : if sat, it sat fast!
possible binary search...?

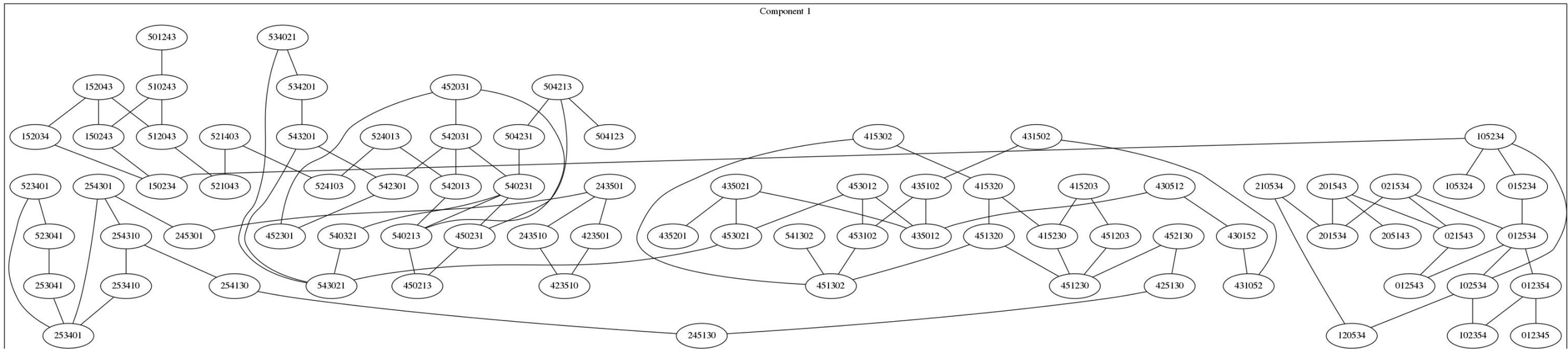
if sat then found within x seconds?

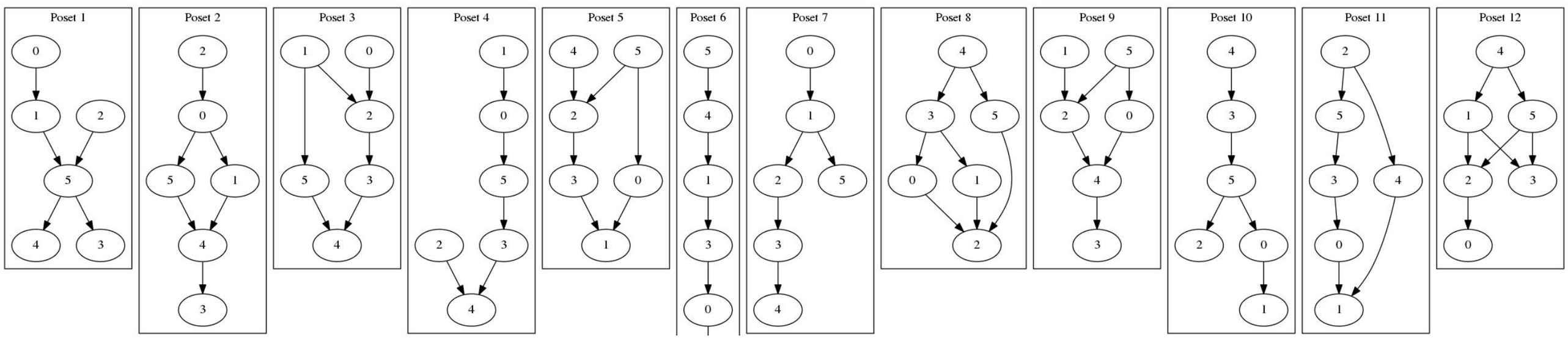
(sat but timeout) probability?

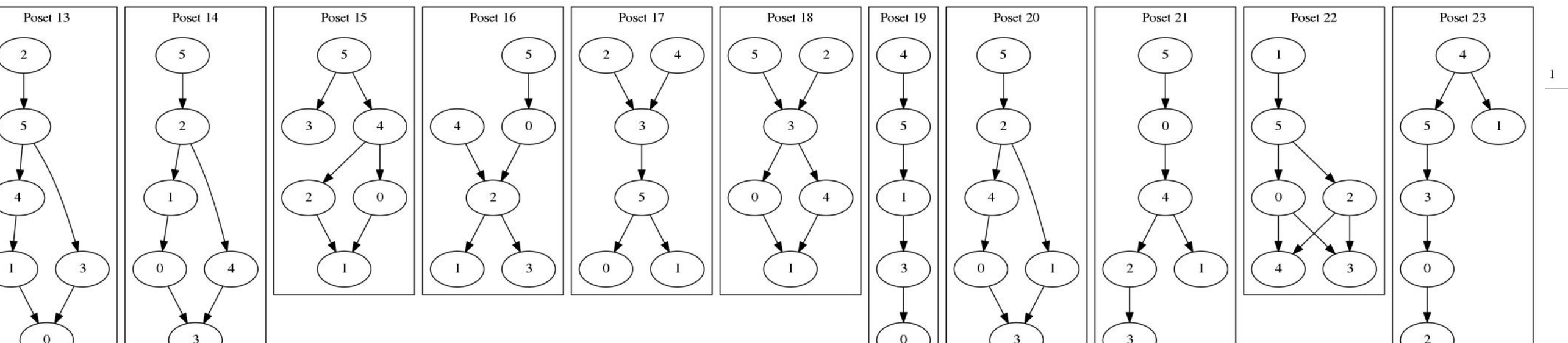
Sampling experiment:
single connected component,
1000 trials,
5 <= |omega| <= 10,
10 <= |lins| <= 100,
timeout = 5s

marked all timeouts and record (|lins|, |omega|, #posets, diameter, radius)
Still I have no idea what am I looking at...

example: trial #257

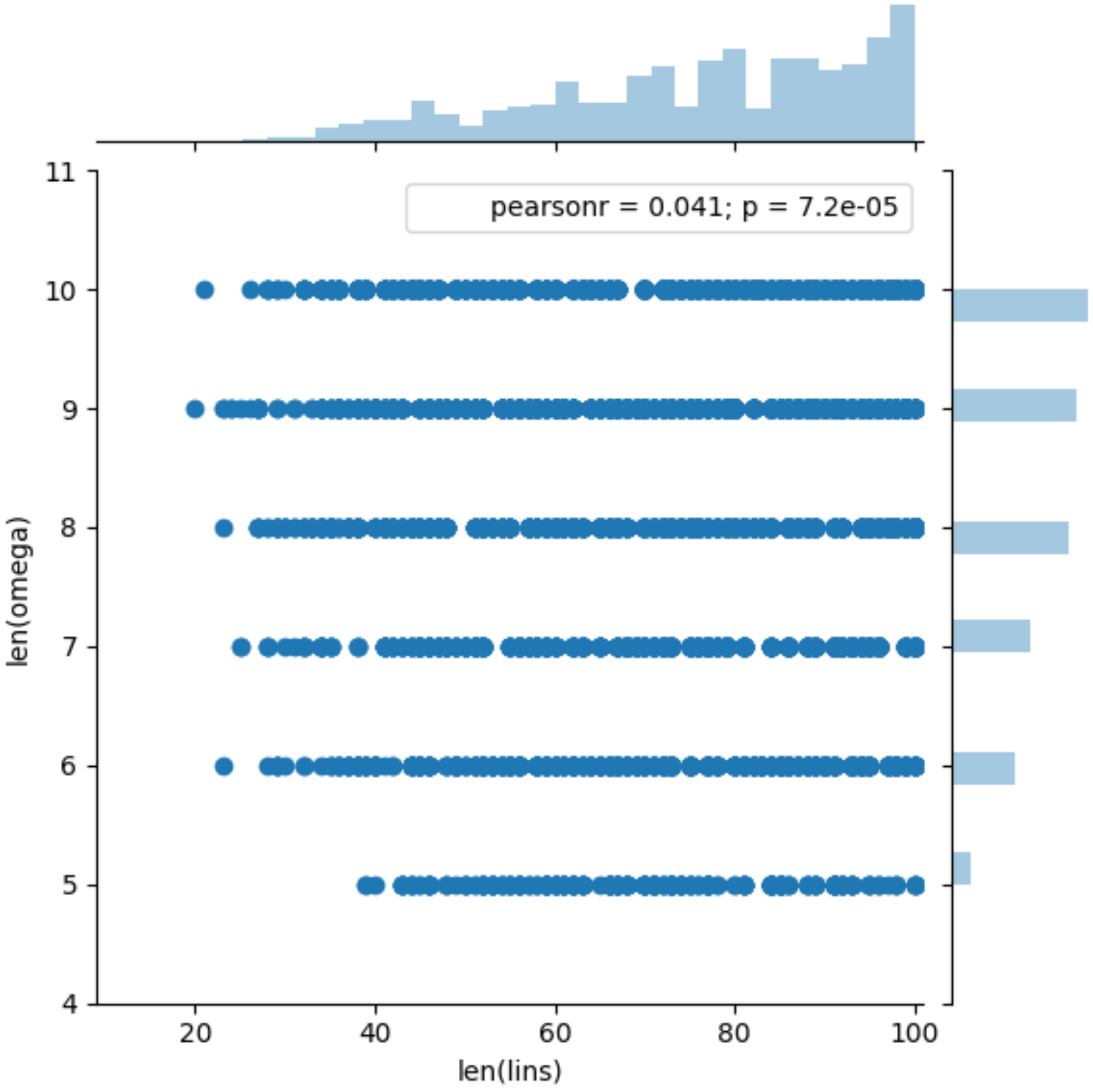


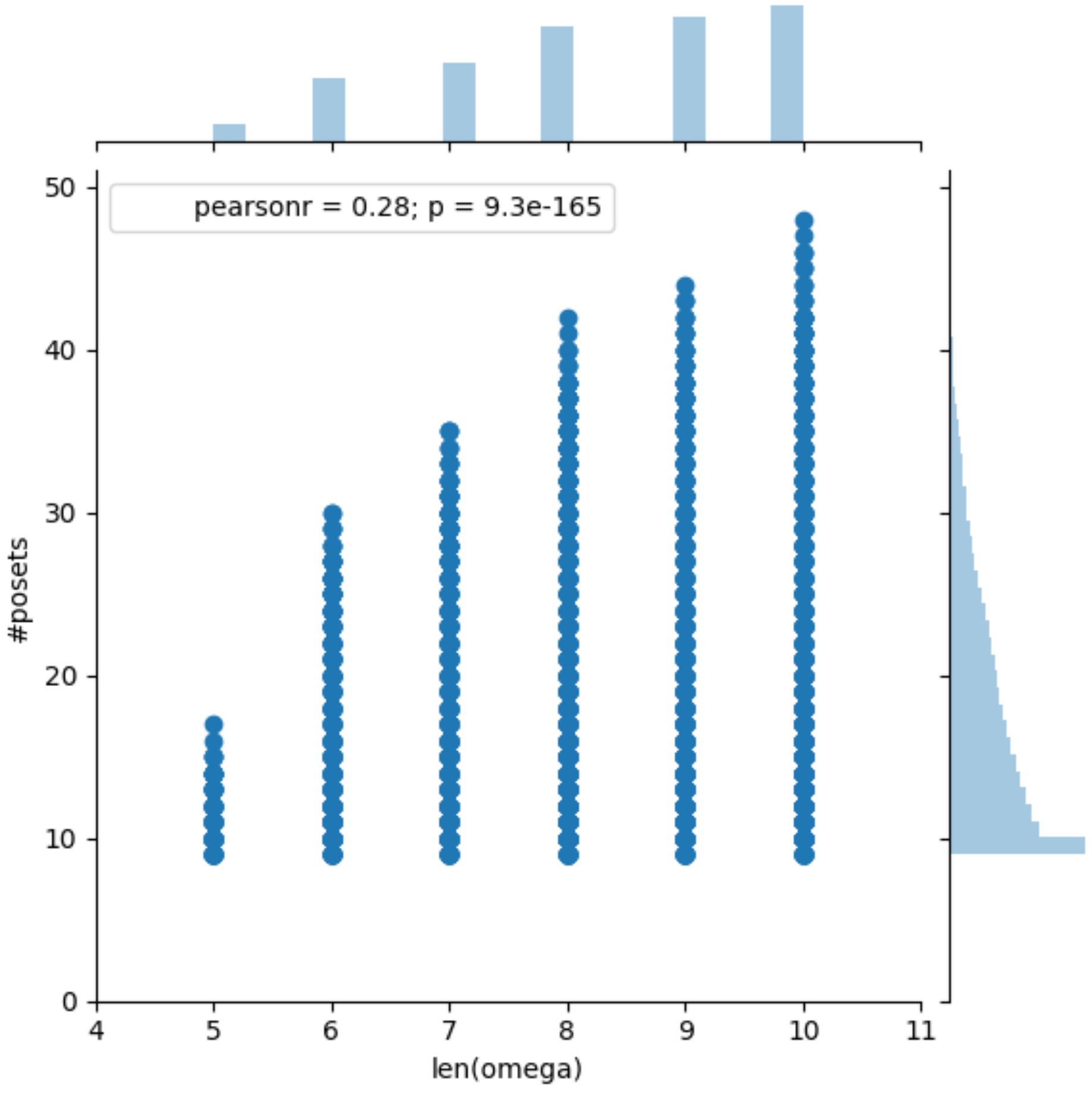


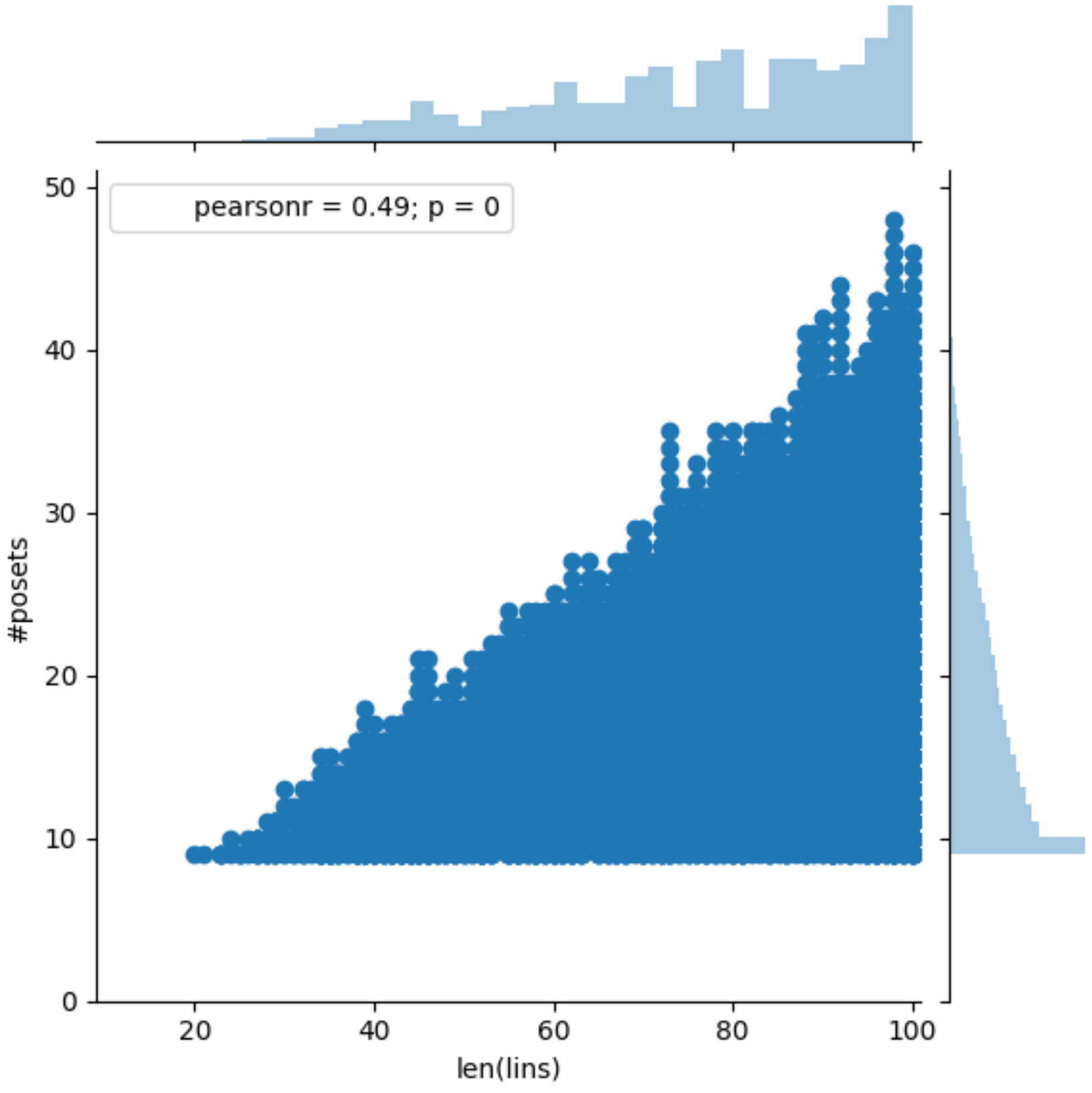


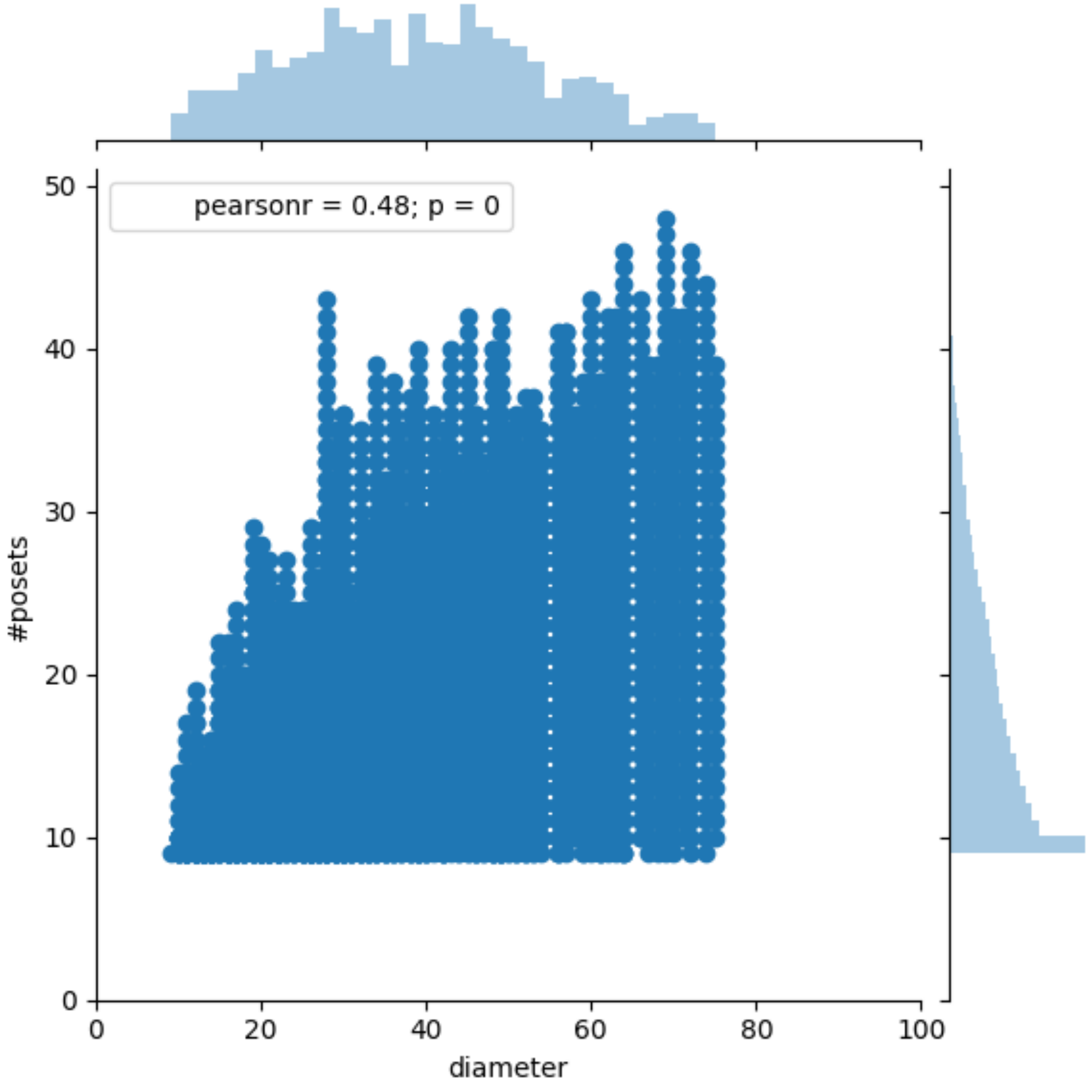
example: trial #257

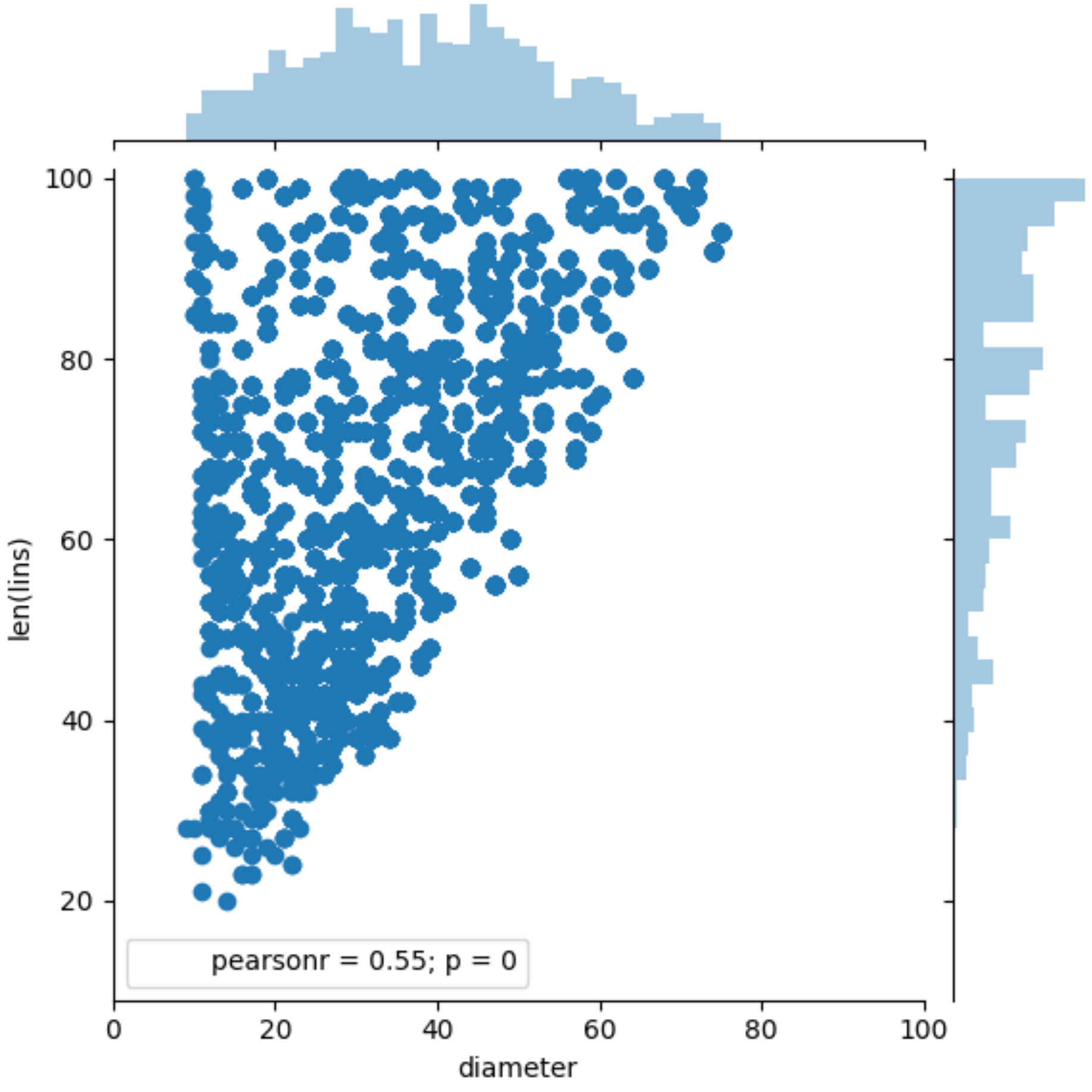
> 1 failed
> 2 failed
> 3 failed
> 4 failed
> 5 failed
> 6 failed
> 7 failed
> 8 failed
> 9 timeout
> 10 timeout
> 11 timeout
> 12 timeout
> 13 timeout
> 14 timeout
> 15 timeout
> 16 timeout
> 17 timeout
> 18 timeout
> 19 timeout
> 20 timeout
> 21 timeout
> 22 timeout
> 23 found
time = 451.0195052623749

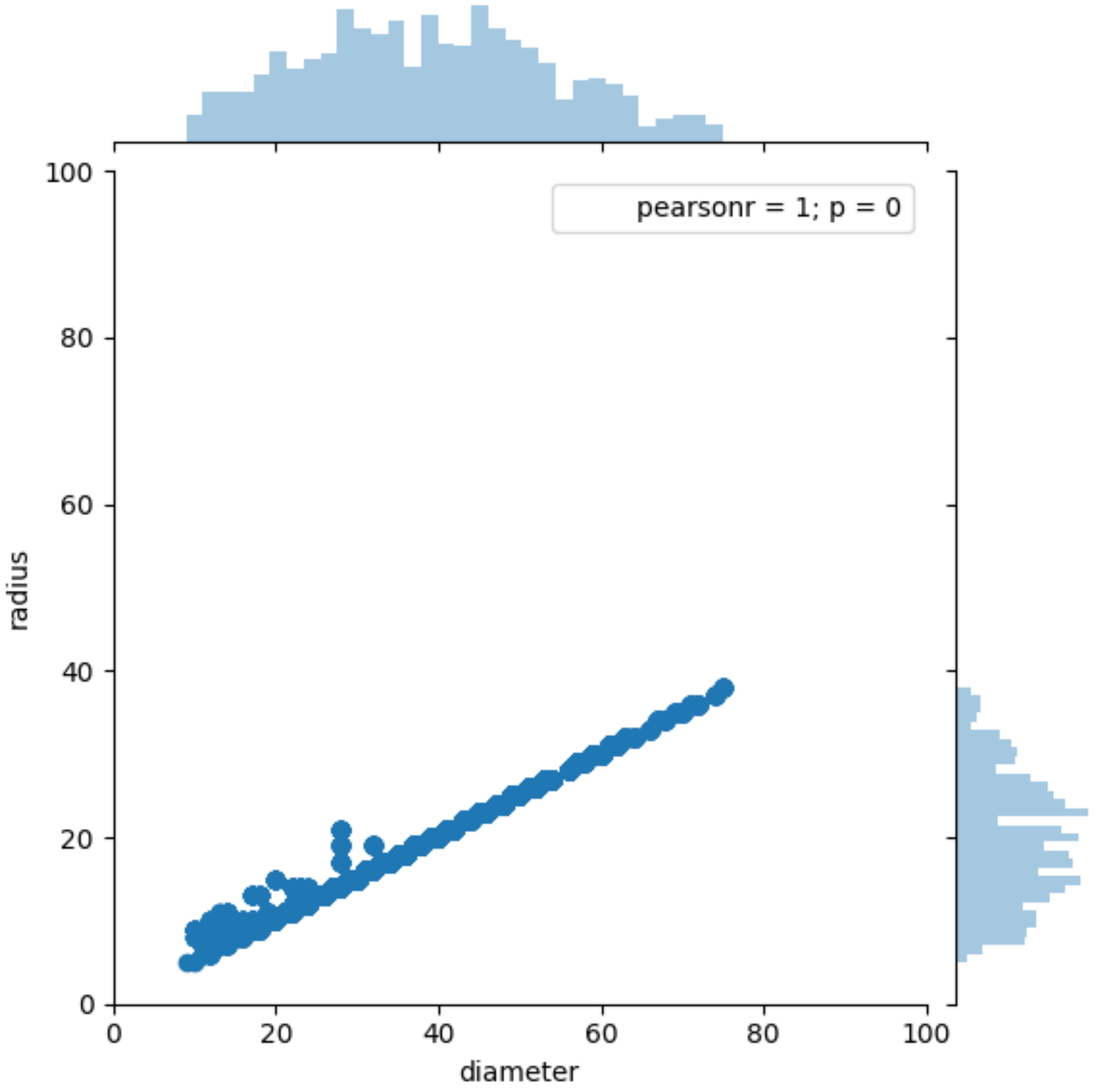

pearsonr = 0.041; p = 7.2e-05
len(omega)
len(lins)
11
10
9
8
7
6
5
4
20
40
60
80
100


pearsonr = 0.28; p = 9.3e-165
#posets
len(omega)
0
10
20
30
40
50
4
5
6
7
8
9
10
11


pearsonr = 0.49; p = 0
#posets
len(lins)
0
10
20
30
40
50
20
40
60
80
100


pearsonr = 0.48; p = 0
#posets
diameter
0
10
20
30
40
50
0
20
40
60
80
100


100
80
60
40
20
len(lins)
0
20
40
60
80
100
diameter
pearsonr = 0.55; p = 0


pearsonr = 1; p = 0
radius
diameter
0
20
40
60
80
100