Friday 3rd May, 2019

Definition 1. *Let S be the program states in program P. A path is a sequence $\pi = s_1 s_2 s_3 ...$ permitted by P. A variable v a function $v : S \mapsto \mathbb{R}$. Every non-empty finite path $\pi = s_1 ... s_i$ determines a probe state p_π such that we take $v(p_\pi) = v(s_i)$ for any v. Henceforth a path is taken to contain a mix of normal program states and probe states. Write p instead of p_π since there can be no confusion. For example, let*

$$\pi = s_1 \cdot p_{s_1} \cdot s_2 \cdot s_3 \cdot p_{s_1 p_{s_1} s_2 s_3} \cdot p_{s_1 p_{s_1} s_2 s_3 p_{s_1 p_{s_1} s_2 s_3}}$$

write

$$\pi = s_1 \cdot p \cdot s_2 \cdot s_3 \cdot p \cdot p$$

Definition 2. *The* probed value $\mathbb{P}[\![v \mid \pi]\!]$ *of variable v regarding path π is:*

- *$v(p)$ where p is the last(?) probe state in π and*
- *undefined if there is no probe state in π.*

Definition 3. *The* expected probed value $\mathbb{EP}[\![v \mid \mathit{\Pi}]\!]$ *of variable v regarding paths $\mathit{\Pi}$ is:*

$$\sum_{\pi \in \mathit{\Pi}} \mathsf{Pr}(\pi)\, \mathbb{P}[\![v \mid \pi]\!]$$

Generality

For a program P without probe states and variable v, insert probe states immediately after `init` and just before `sink`:

$$\begin{aligned} \mathbb{ER}[\![\mathsf{F}\,\mathtt{sink} \mid P^v]\!] &= \sum_{\pi \in \mathsf{F}\,\mathtt{sink}} \mathsf{Pr}(\pi)\, \mathsf{R}(\pi) \\ &= \sum_{\pi \in \mathsf{F}\,\mathtt{sink}} \mathsf{Pr}(\pi)\, \mathbb{P}[\![v \mid \pi]\!] = \mathbb{EP}[\![v \mid \mathsf{F}\,\mathtt{sink}]\!] \end{aligned}$$

and

$$\begin{aligned} \mathbb{LER}[\![\mathsf{F}\,\mathtt{sink} \mid P^v]\!] &= \mathbb{ER}[\![\mathsf{F}\,\mathtt{sink} \mid P^v]\!] + \mathsf{Pr}(\neg\,\mathsf{F}\,\mathtt{sink} \mid P^v) \\ &= \mathbb{ER}[\![\mathsf{F}\,\mathtt{sink} \mid P^v]\!] + \sum_{\pi \in \neg\,\mathsf{F}\,\mathtt{sink}} \mathsf{Pr}(\pi) \\ &= \mathbb{ER}[\![\mathsf{F}\,\mathtt{sink} \mid P^v]\!] + \sum_{\pi \in \neg\,\mathsf{F}\,\mathtt{sink}} \mathsf{Pr}(\pi)\, \mathbb{P}[\![1 \mid \pi]\!] \\ &= \mathbb{EP}[\![v \mid \mathsf{F}\,\mathtt{sink}]\!] + \mathbb{EP}[\![1 \mid \neg\,\mathsf{F}\,\mathtt{sink}]\!] \end{aligned}$$

What should $\mathbb{E}\mathbb{P}[\![x \mid \top]\!]$ be for:

```
x = 1
Probe
x = 2
Probe
x = 3
```

Listing 1: We'd want $\mathbb{E}\mathbb{P}[\![x \mid \top]\!] = 2$ and that compels the definition of $\mathbb{P}$ to take the *last* probe state.

```
while true:
    x = 1
    Probe
```

Listing 2: We'd want $\mathbb{E}\mathbb{P}[\![x \mid \top]\!] = 1$ but the probe state occurs infinitely often so there is no *last* probe state.

```
while true:
    x = 1 [0.5] x = 2
    Probe
```

Listing 3: We'd want $\mathbb{E}\mathbb{P}[\![x \mid \top]\!] = 1.5$ and even if we take not the last probe state but every probed value weighted by its path's probability, we would count in too much: $0.5 \cdot 1 + 0.5 \cdot 2 + 0.25 \cdot 1 + 0.25 \cdot 2 + 0.25 \cdot 1 + 0.25 \cdot 2 + \ldots$.

```
x = 1
while true:
    x = x · 0.5
    Probe
```

Listing 4: We'd want $\mathbb{E}\mathbb{P}[\![x \mid \top]\!] = 0$ but I have no idea how would this work since $\sum_i 0.5^i = \frac{0.5}{1-0.5} = 1$. For these two I don't think I know enough to properly handle them in a meaningful way. I'm still reading on semantics.