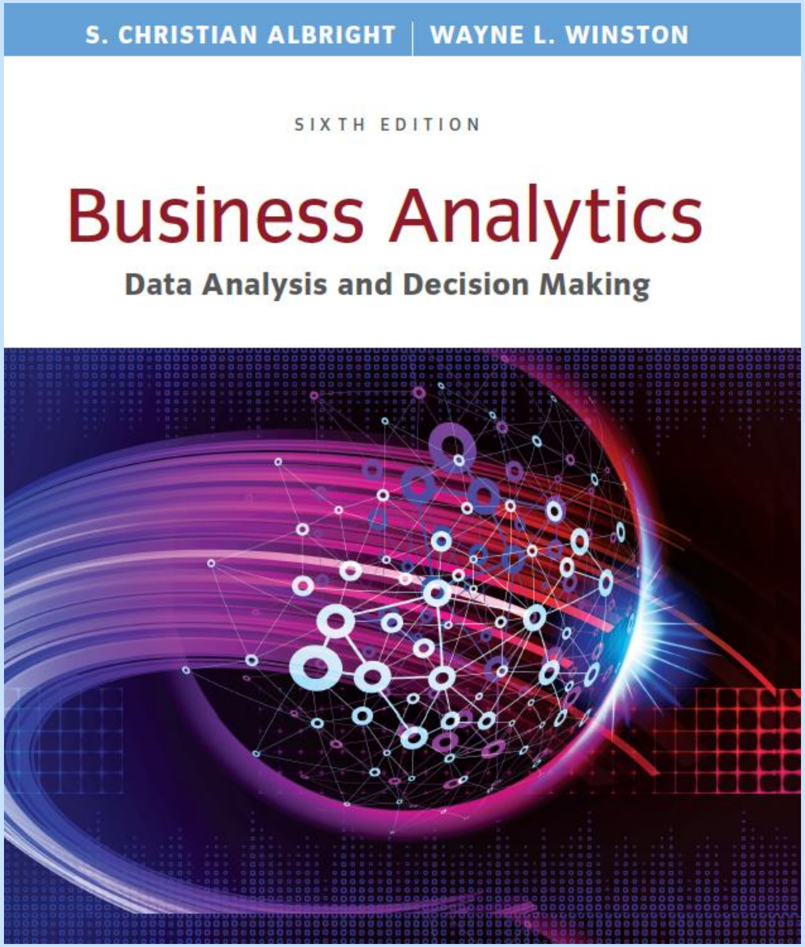


Chapter 6

Decision Making under Uncertainty

6-1 Introduction

- A formal framework for analyzing decision problems that involve uncertainty includes:
  - Criteria for choosing among alternative decisions
  - How probabilities are used in the decision-making process
  - How early decisions affect decisions made at a later stage
  - How a decision maker can quantify the value of information
  - How attitudes toward risk can affect the analysis
- A powerful graphical tool—a decision tree—guides the analysis.
  - A decision tree enables a decision maker to view all important aspects of the problem at once: the decision alternatives, the uncertain outcomes and their probabilities, the economic consequences, and the chronological order of events.

6-2 Elements of Decision Analysis

(slide 1 of 3)

- Decision analysis problems have common elements:
  1. A problem has been identified that requires a solution.
  2. A number of possible decisions have been identified.
  3. Each decision leads to a number of possible outcomes.
  4. There is uncertainty about which outcome will occur, and probabilities of the possible outcomes are assessed.
  5. For each decision and each possible outcome, a payoff is received or a cost is incurred.
  6. A "best" decision must be chosen using an appropriate decision criterion.

6-2 Elements of Decision Analysis
(slide 2 of 3)

- Identifying the problem (6-2a)
  - When something triggers the need to solve a problem, the problem that needs to be solved should be carefully identified.
- Possible decisions (6-2b)
  - The possible decisions depend on how the problem is specified.
- Possible outcomes (6-3c)
  - One of the main reasons why decision making under uncertainty is difficult is that decisions have to be made before uncertain outcomes are revealed.

6-2d Elements of Decision Analysis
(slide 3 of 3)

- Probabilities of outcomes (6-2d)
  - There is no easy way to assess the probabilities of the possible outcomes.
    - Sometimes they will be determined at least partly by historical data.
    - Other estimates will necessarily contain a heavy subjective component, such as when a new product is being introduced.
    - To complicate matters, probabilities sometimes change as more information becomes available.
- Payoffs and costs (6-2e)
  - Decisions and outcomes have consequences, either good or bad, and may be monetary or nonmonetary.

6-2f Decision Criteria

(slide 1 of 2)

- Look at the worst possible outcome for each decision and choose the decision that has the best (or least bad) of these.
- Look at the 5th percentile of the distribution of outcomes for each decision and choose the decision that has the best of these.
- Look at the best possible outcome for each decision and choose the decision that has the best of these.
- Look at the variance of the distribution of outcomes for each decision and choose the decision that has the smallest of these.
- Look at the downside risk of the distribution of outcomes for each decision and choose the decision with the smallest of these.

6-2f Decision Criteria

(slide 2 of 2)

- The **expected monetary value**, or **EMV**, for any decision is a weighted average of the possible payoffs for this decision, weighted by the probabilities of the outcomes.
  - The **expected monetary value criterion**, or **EMV criterion**, is generally regarded as the preferred criterion in most decision problems.
  - This approach assesses probabilities for each outcome of each decision and then calculates the *expected* payoff, or EMV, from each decision based on these probabilities.
  - Using this criterion, you choose the decision with the largest EMV—which is sometimes called "playing the averages."

6-2g More about the EMV Criteria

(slide 1 of 2)

- Value a decision with a given EMV the *same* as a sure monetary outcome with the same EMV.
- The EMV criterion doesn't *guarantee* good outcomes.
- The EMV criterion is easy to operationalize in a spreadsheet.
  - List the possible payoff/cost values and their probabilities, and calculate EMV with SUMPRODUCT.

	A	B	C	D	E	F	G	H
1	**Decision 1**			**Decision 2**			**Decision 3**	
2	Payoff/Cost	Probability		Payoff/Cost	Probability		Payoff/Cost	Probability
3	$50,000	0.1		$5,000	0.6		$3,000	1
4	$10,000	0.2		–$1,000	0.4			
5	–$5,000	0.7						
6								
7	EMV	$3,500		EMV	$2,600		EMV	$3,000

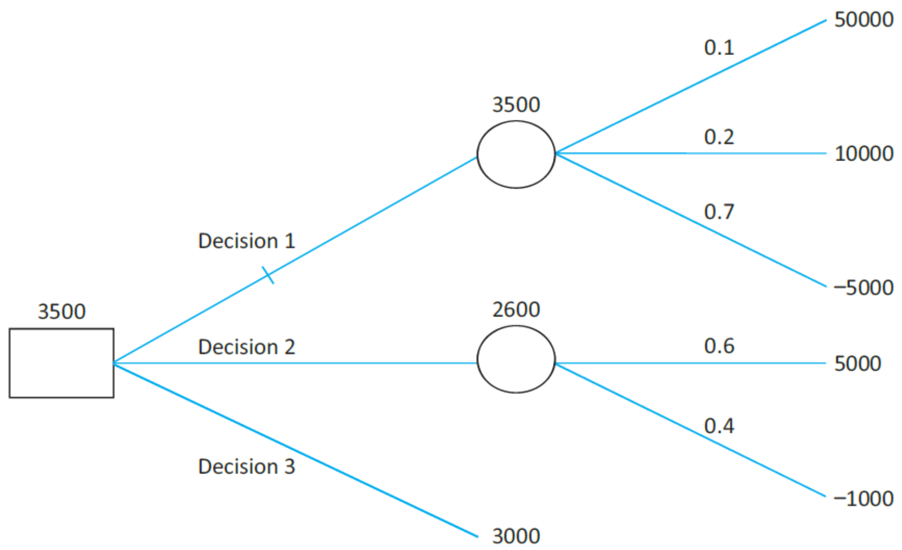
6-2g More about the EMV Criteria
(slide 2 of 2)

- The advantage to calculating EMVs in a spreadsheet is that you can easily perform sensitivity analysis on any of the inputs.
  - Here, the EMV for decision 2 is the largest of the three EMVs, so it is the best decision.

	A	B	C	D	E	F	G	H
1	**Decision 1**			**Decision 2**			**Decision 3**	
2	Payoff/Cost	Probability		Payoff/Cost	Probability		Payoff/Cost	Probability
3	\$50,000	0.1		\$5,000	0.8		\$3,000	1
4	\$10,000	0.2		–\$1,000	0.2			
5	–\$5,000	0.7						
6								
7	EMV	\$3,500		EMV	\$3,800		EMV	\$3,000

6-2h Decision Trees
(slide 1 of 4)

- A graphical tool called a **decision tree** has been developed to represent decision problems.
  - It is particularly useful for more complex decision problems.
  - It clearly shows the sequence of events (decisions and outcomes), as well as probabilities and monetary values.



Decision Trees
(slide 2 of 4)

- Decision trees are composed of *nodes* (circles, squares, and triangles) and *branches* (lines).
- The nodes represent points in time. A *decision node* (a square) represents a time when the decision maker makes a decision.
- A *probability node* (a circle) represents a time when the result of an uncertain outcome becomes known.
- An *end node* (a triangle) indicates that the problem is completed—all decisions have been made, all uncertainty has been resolved, and all payoffs and costs have been incurred.
- Time proceeds *from left to right*. Any branches leading into a node (from the left) have already occurred. Any branches leading out of a node (to the right) have not yet occurred.

Decision Trees
(slide 3 of 4)

- Branches leading out of a decision node represent the possible decisions; the decision maker can choose the preferred branch.
- Branches leading out of probability nodes represent the possible outcomes of uncertain events; the decision maker has no control over which of these will occur.
- Probabilities are listed on chance branches. These probabilities are *conditional* on the events that have already been observed (those to the left).
- Probabilities on branches leading out of any chance node must sum to 1.
- Monetary values are shown to the right of the end nodes.
- EMVs are calculated through a “folding-back” process. They are shown above the various nodes.

Decision Trees
(slide 4 of 4)

- The decision tree allows you to use the following **folding-back procedure** to find the EMVs and the optimal decision:
  - Starting from the right of the decision tree and working back to the left:
    - At each chance node, calculate an EMV—a sum of products of monetary values and probabilities.
    - At each decision node, take a maximum of EMVs to identify the optimal decision.

6-3 One-Stage Decision Problems

- In single stage decision problems, one stage is made, right now.
  - They all unfold the same way.

Example 6.1: New Product Decisions at ACME (slide 1 of 4)

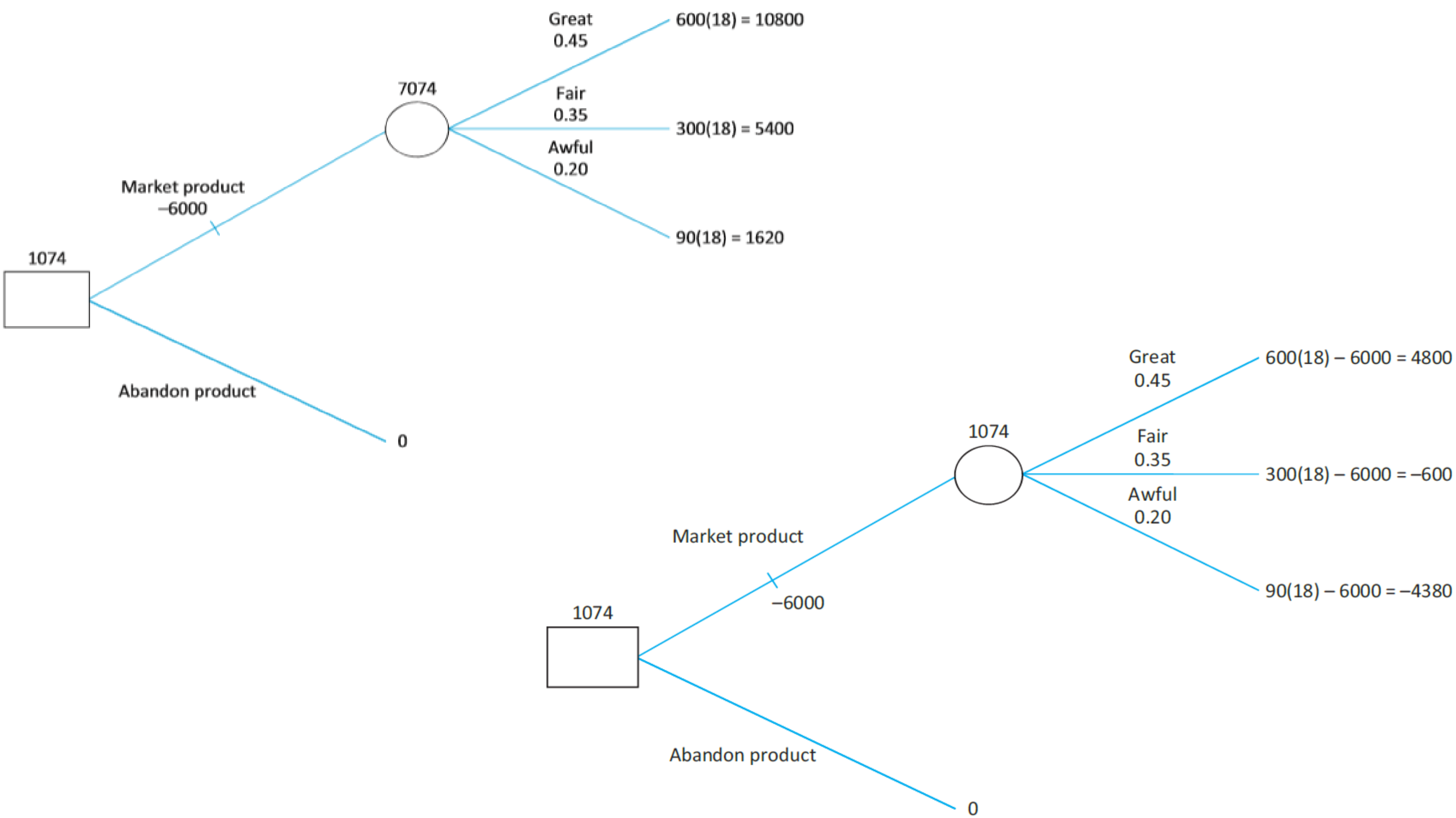
- **Objective:** To use the EMV criterion to help Acme decide whether to go ahead with the product.
- **Solution:** Acme's cost accountants estimate the monetary inputs: the fixed costs ($6,000) and the unit margin ($18).
- The uncertain sales volume is really a continuous variable but, as in many decision problems, Acme has replaced the continuum by three representative possibilities: great (45%), fair (35%) and awful (20%)
- Each sales volume is multiplied by the unit margin to obtain the net revenues.
- The formula for the EMV is the sum of the net revenues minus the fixed costs.

Example 6.1: New Product Decisions at ACME (slide 2 of 4)

	A	B	C	D	E	F	G	H	I
3	**Decision 1: Continue development and market the new product**								
4	Fixed cost	$6,000							
5	Unit margin	$18							
6									
7	Market	Probability	Sales volume	Net revenue					
8	Great	0.45	600	$10,800					
9	Fair	0.35	300	$5,400					
10	Awful	0.20	90	$1,620					
11									
12	EMV	$1,074							
13									
14	Decision 2: Stop development and abandon product								
15	No payoffs, no costs, no uncertainty								
16	EMV	$0							

All monetary values (except the unit margin in cell B5) are in $1000s, and all sales volumes are in 1000s of units.

Example 6.1: New Product Decisions at ACME (slide 3 of 4)



Example 6.1: New Product Decisions at ACME (slide 4 of 4)

- Usually, the main purpose of sensitivity analysis is to see whether the best decision changes as one or more inputs change.
- In this case, we will see whether the best decision continues to be "proceed with marketing" if the total market decreases. Specifically, we let each of the potential sales volumes decrease by the same percentage and we keep track of the EMV from marketing the product.

	J	K	L	M
3	% decrease in all sales volumes	0%		
4	EMV for decision 1	$1,074		
5				
6	Sensitivity analysis to percentage decrease in all sales volumes			
7	% decrease	EMV for decision 1		
8		$1,074		
9	5%	$720		
10	10%	$367		
11	15%	$13		
12	20%	–$341		

6-4 The PrecisionTree Add-In

(slide 1 of 2)

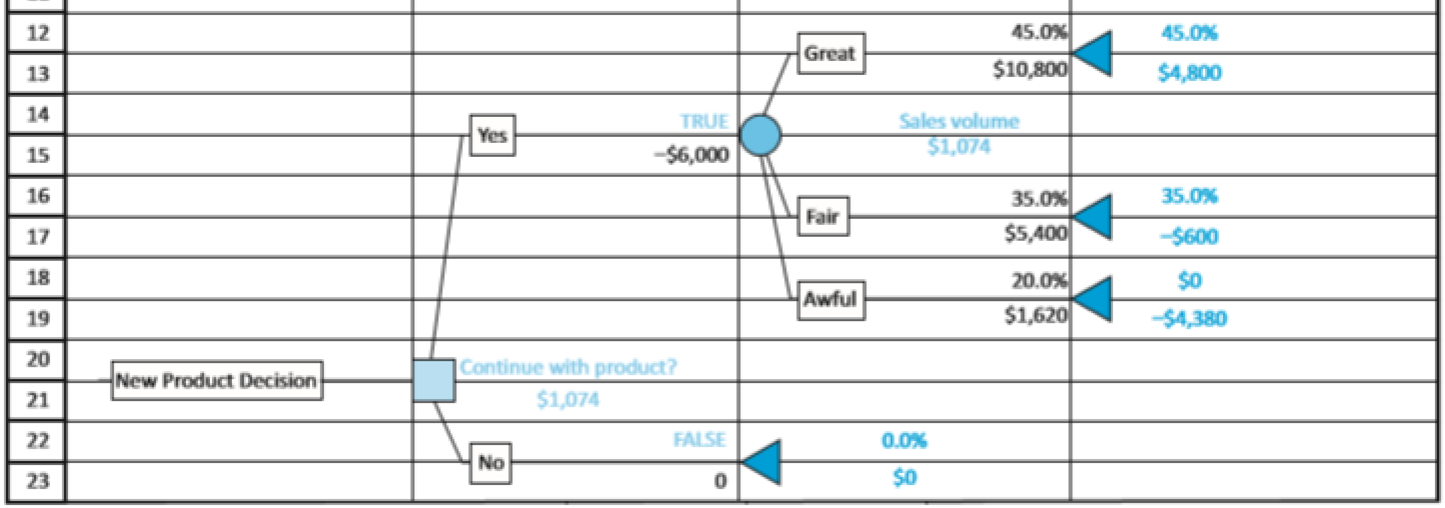
- Decision trees present a challenge for Excel®.
- **PrecisionTree**, a powerful add-in developed by Palisade Corporation, makes the process relatively straightforward.
  - It enables you to draw and label a decision tree.
  - It performs the folding-back procedure automatically.
  - It allows you to perform sensitivity analysis on key input parameters.
  - See your text for a detailed description of its use.

6-4 The PrecisionTree Add-In

(slide 2 of 2)

	A	B	C	D	E
1	**Acme single-stage new product decision**				
2					
3	**Inputs**				
4	Fixed cost	$6,000			
5	Unit margin	$18			
6					
7	Market	Probability	Sales volume	Net revenue	
8	Great	0.45	600	$10,800	
9	Fair	0.35	300	$5,400	
10	Awful	0.20	90	$1,620	
11					

All monetary values (except the unit margin in cell B5) are in $1000s, and all sales volumes are in 1000s of units.



6-5 Multistage Decision Problems
(slide 1 of 6)

- Many real-world decision problems evolve through time in stages.
- The objective is again to maximize EMV, but now we are searching for an EMV-maximizing strategy, often called a **contingency plan**, that specifies which decision to make at each stage.
  - A contingency plan tells the company which decision to make at the first stage, but the company won't know which decision to make at the second stage until the information from the first uncertain outcome is known.

Multistage Decision Problems
(slide 2 of 6)

- An important aspect of multistage decision problems is that probabilities can change through time.
  - Specifically, after you receive the information from the first-stage uncertain outcome, you might need to reassess the probabilities of future uncertain outcomes.
- Another important aspect of multistage decision problems is the value of information.
  - Sometimes the first-stage decision is to buy information that will help in making the second-stage decision. The question then is how much this information is worth.

Example 6.2: New Product Decisions with Technological Uncertainty (slide 1 of 5)

- **Objective:** To use a decision tree to find Acme's EMV-maximizing strategy for this two-stage decision problem.
- **Solution:** The probability of technological failure might be based partly on historical data (the technological failure rate of similar products in the past) but it is probably partly subjective, based on how the product's development has proceeded so far. The reason this is a two-stage decision problem is that Acme can decide right away to stop development and abandon the product, thus saving further fixed costs of development. However, if Acme decides to continue development and the product turns out to be a technological success, a second decision on whether to market the product must still be made.

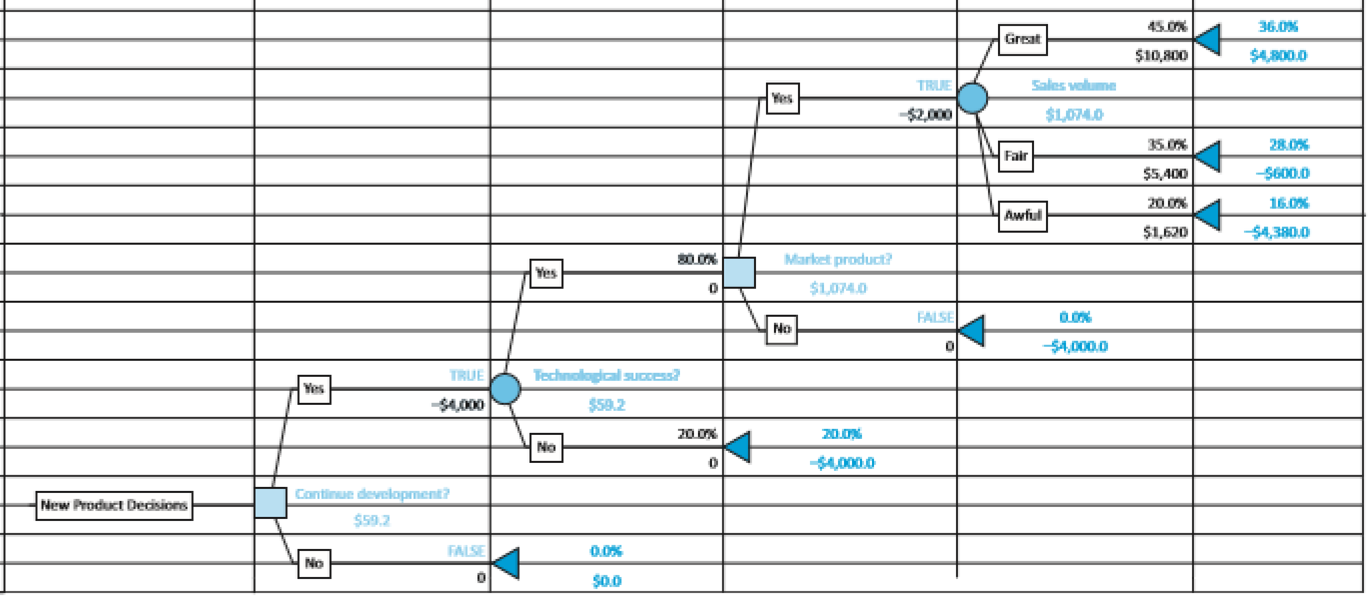
Example 6.2: New Product Decisions with Technological Uncertainty (slide 2 of 5)

- The first decision is whether to continue development.
  - If “Yes,” the fixed development cost is incurred, so it is entered on this branch.
- Then there is a probability node for the technological success or failure.
  - If it’s a failure, there are no further costs, but the fixed development cost is lost.
  - If it’s a success, Acme must decide whether to market the product. From this point, the tree is exactly like the single-stage tree, except that the fixed development cost is gone.

Example 6.2: New Product Decisions with Technological Uncertainty (slide 3 of 5)

	A	B	C	D
1	**Acme multistage new product decisions with technological uncertainty**			
2				
3	**Inputs**			
4	Probability of failure	0.2		
5	Fixed development cost	$4,000		
6	Fixed marketing cost	$2,000		
7	Unit margin	$18		
8				
9	Market	Probability	Sales volume	Net revenue
10	Great	0.45	600	$10,800
11	Fair	0.35	300	$5,400
12	Awful	0.20	90	$1,620

All monetary values (except the unit margin in cell B7) are in $1000s, and all sales volumes are in 1000s of units.



Example 6.2: New Product Decisions with Technological Uncertainty (slide 4 of 5)

- By following the TRUE branches, you can see Acme's best strategy.
  - The company should continue development, and if the product is a technological success, it should be marketed. The EMV, again the weighted average of all possible monetary outcomes with this strategy, is $59,200.
  - However, this is only the expected value, or mean, of the probability distribution of monetary outcomes. You can see the full probability distribution by requesting a risk profile from PrecisionTree (through the Decision Analysis dropdown).

Example 6.2: New Product Decisions with Technological Uncertainty (slide 5 of 5)

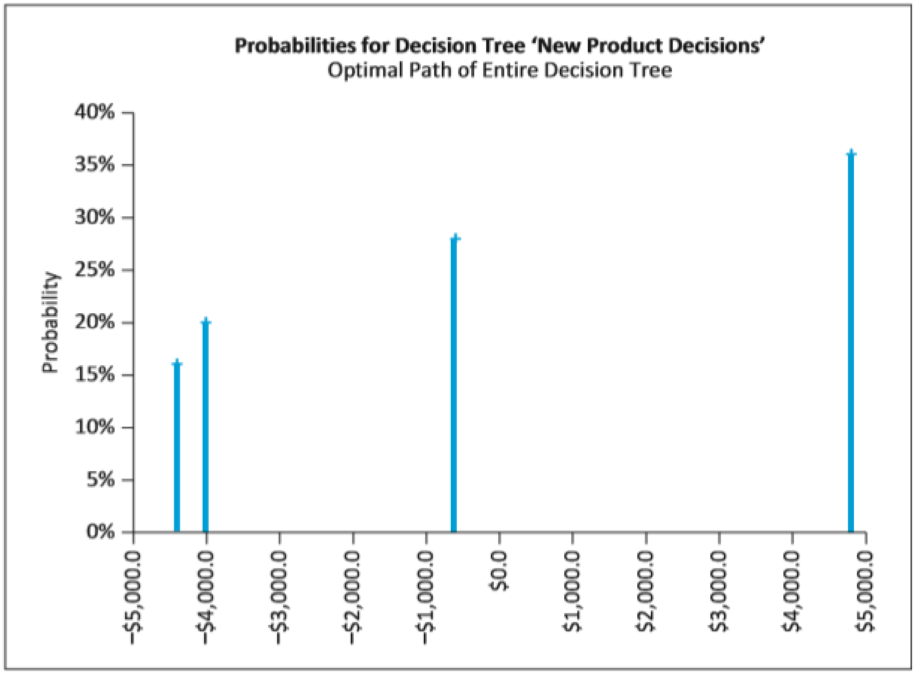


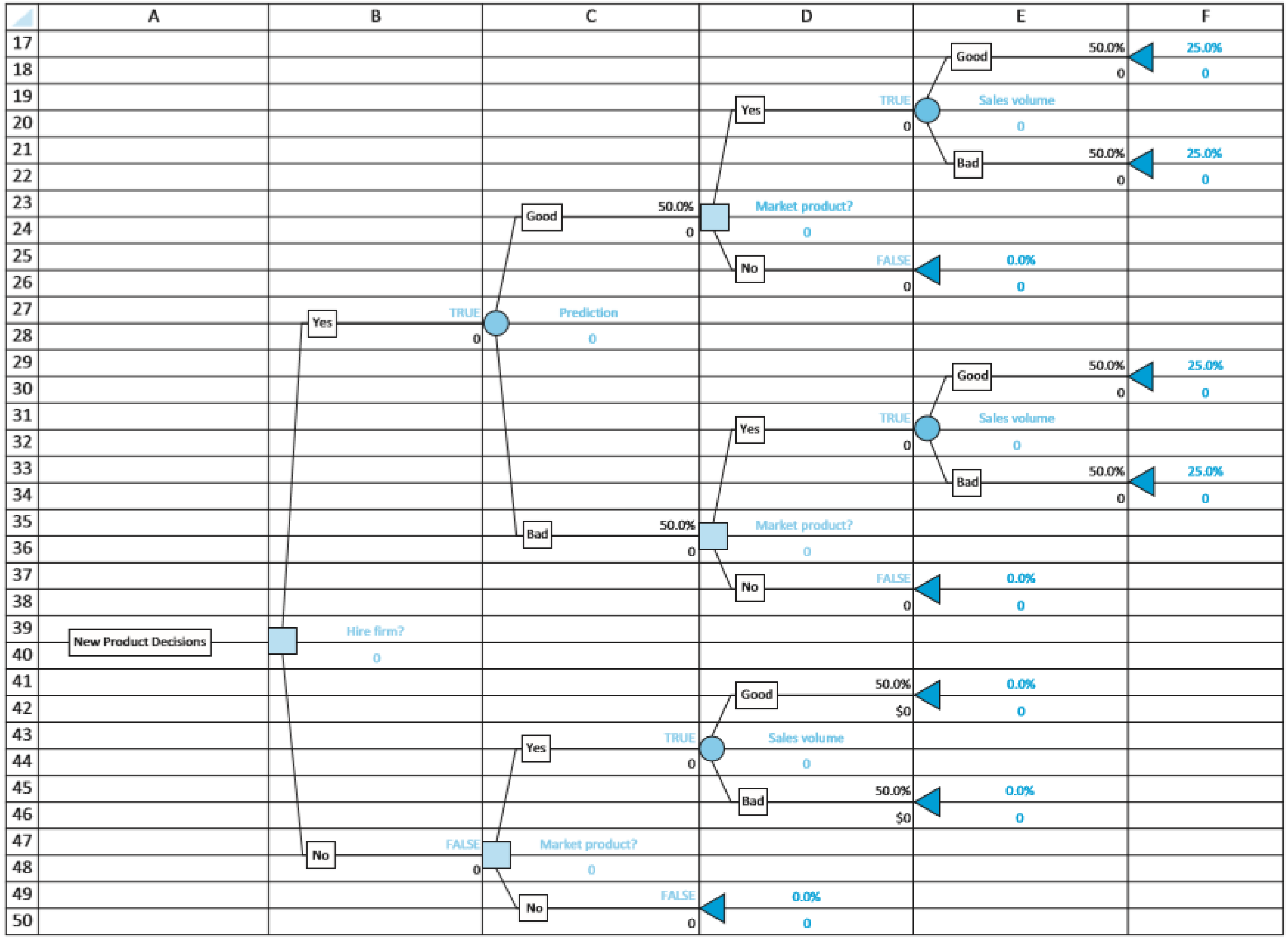
Chart Data

	Optimal Path	
	Value	Probability
#1	−$4,380.0	16.0000%
#2	−$4,000.0	20.0000%
#3	−$600.0	28.0000%
#4	$4,800.0	36.0000%

Example 6.3: New Product Decisions with Option to Buy Information (slide 1 of 10)

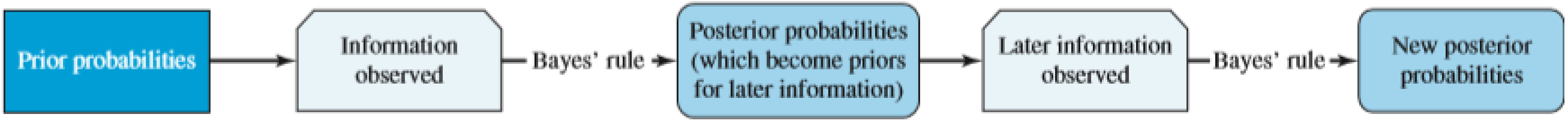
- **Objective:** To use a decision tree to see whether the marketing research firm is worth its cost and whether the product should be marketed.
- **Solution:** Acme must first decide whether to hire the marketing research firm. If it decides not to, it can then immediately decide whether to market the product. On the other hand, if it decides to hire the firm, it must then wait for the firm's prediction. After the prediction is received, Acme can then make the ultimate decision on whether to market the product. However, when making this ultimate decision, Acme should definitely take the firm's prediction into account.

Example 6.3: New Product Decisions with Option to Buy Information (slide 2 of 10)



Example 6.3: New Product Decisions with Option to Buy Information (slide 3 of 10)

- Bayes' rule: Frequency approach
  - Bayes' rule is a formal mathematical mechanism for updating probabilities as new information becomes available.
    - The original probabilities are called **prior probabilities**. Then information is observed and Bayes' rule is used to update the prior probabilities to **posterior probabilities**.
    - The actual updating mechanism can be done in two ways: with frequencies (counts) or with probabilities.



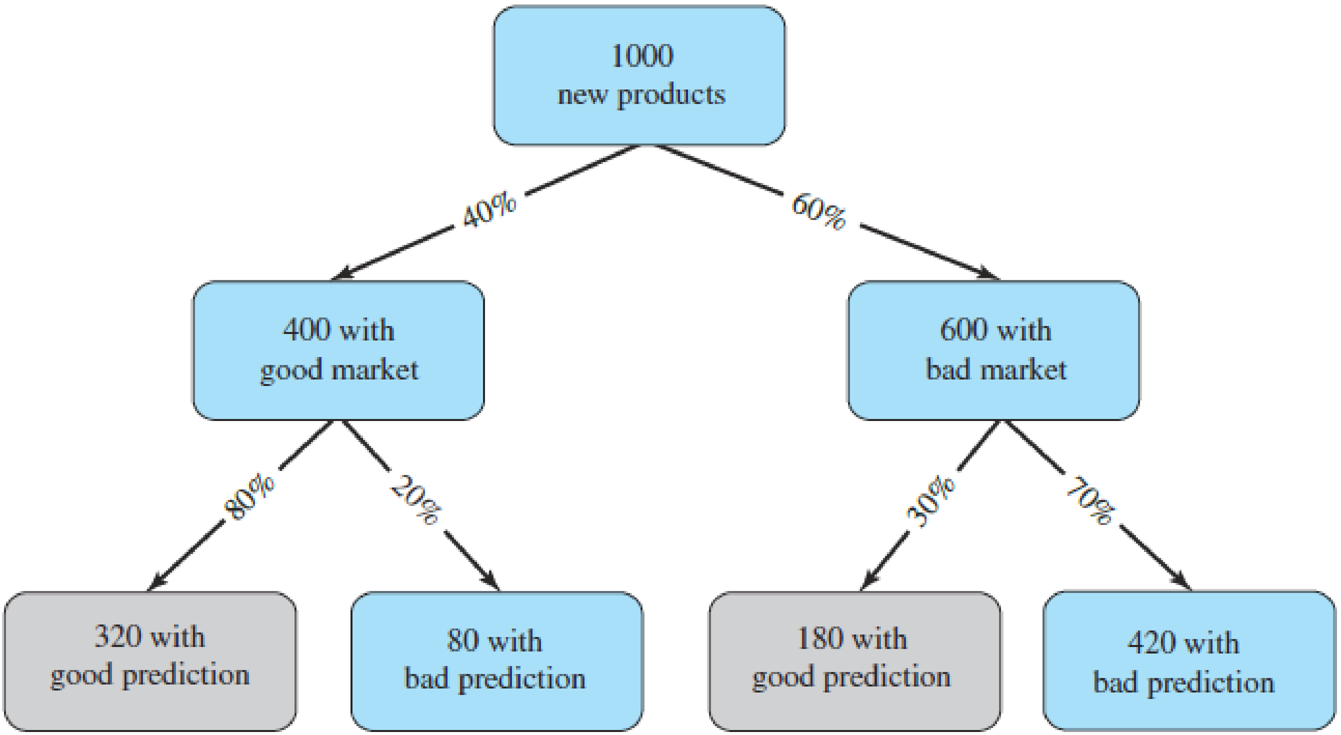
Example 6.3: New Product Decisions with Option to Buy Information (slide 4 of 10)

- The frequency approach is quite straightforward, even though it often yields surprising and unintuitive results.
  - The prior probabilities of good or bad markets are 40% and 60%. The probabilities for the accuracy of the predictions are given in Table 6.1.

Table 6.1 Prediction Accuracy of Marketing Research Firm

Actual/Predicted	Good	Bad
Good	0.8	0.2
Bad	0.3	0.7

Example 6.3: New Product Decisions with Option to Buy Information



Chance of a good market, given a good prediction = 320/(320+180) = 64%
Chance of a good market, given a bad prediction = 80/(80+420) = 16%

Chance of a good prediction = (320+180)/1000 = 50%

Example 6.3: New Product Decisions with Option to Buy Information (slide 6 of 10)

- Bayes' rule: Probability approach
  - For any possible outcome O, we let $P(O)$ be the probability of O.
  - If we want to indicate that new information, I, is available, we write the probability as $P(O \mid I)$. This is called a **conditional probability**.
  - The typical situation is that there are several outcomes such as "good market" and "bad market."
    - In general, denote these outcomes as O_1 to O_n, assuming there are n possibilities.

Example 6.3: New Product Decisions with Option to Buy Information (slide 7 of 10)

- We start with prior probabilities $P(O_1)$ to $P(O_n)$, n probabilities that sum to 1.
- Next, we observe new information, I, such as a market prediction, and we want the posterior probabilities $P(O_1 | I)$ to $P(O_n | I)$, an updated set of n probabilities that sum to 1.
- We assume that the "opposite" conditional probabilities, $P(I | O_1)$ to $P(I | O_n)$, are given. In Bayesian terminology, these are called **likelihoods**.
  - Unfortunately, these likelihoods are not what we need in the decision tree.
  - Bayes' rule is a formal rule for turning these conditional probabilities around.

Example 6.3: New Product Decisions with Option to Buy Information (slide 8 of 10)

- Bayes' rule is given by

$$P(O_i|I) = \frac{P(I|O_i)P(O_i)}{P(I|O_1)P(O_1) + \cdots + P(I|O_n)P(O_n)}$$

- The denominator in Bayes' rule is the probability *P*(*I*) of the information outcome. It is sometimes called the **law of total probability**.

$$P(I) = P(I|O_1)P(O_1) + \cdots + P(I|O_n)P(O_n)$$

- In the case where there are only two Os, labeled as O and Not O, Bayes' rule takes the following form:

$$P(O|I) = \frac{P(I|O)P(O)}{P(I|O)P(O) + P(I|\text{Not } O)P(\text{Not } O)}$$

- These formulas can all be implemented in Excel®.

Example 6.3: New Product Decisions with Option to Buy Information (slide 9 of 10)

	A	B	C	D	E	F	G	H	I
1	**Acme multistage new product decisions with an option to buy information**								
2									
3	**Inputs**								
4	Cost of market research	$150				**Bayes' rule calculations**			
5	Fixed marketing cost	$2,000				Probabilities of predictions			
6	Unit margin	$18					Good	Bad	Sum check
7							0.5	0.5	1
8	Market	Prior probability	Sales volume	Net revenue					
9	Good	0.40	600	$10,800		Posterior probabilities, given predictions			
10	Bad	0.60	100	$1,800		Actual\Predicted	Good	Bad	
11						Good	0.64	0.16	
12	Probabilities that indicate the accuracy of the predictions					Bad	0.36	0.84	
13	Actual\Predicted	Good	Bad	Sum check		Sum check	1	1	
14	Good	0.8	0.2	1					
15	Bad	0.3	0.7	1					

All monetary values (except the unit margin in cell B6) are in $1000s, and all sales volumes are in 1000s of units.

Example 6.3: New Product Decisions with Option to Buy Information (slide 10 of 10)

- The results can then be displayed in a decision tree.

Multistage Decision Problems

(slide 3 of 6)

- The value of information
  - In a decision-making context, information is usually bought to reduce the uncertainty about some outcome.
  - The **expected value of information** is the amount a firm would be willing to pay for information and is given by the formula:

EVI = EMV with (free) information – EMV without information

Multistage Decision Problems
(slide 4 of 6)

- Although the calculation of EVI is straightforward once the decision tree has been created, the decision tree itself requires a lot of probability assessments and Bayes' rule calculations.
  - Therefore, it is sometimes useful to ask how much any information could be worth, regardless of its form or accuracy. The result is called the **expected value of perfect information**, or EVPI and is given by the equation

EVPI = EMV with (free) perfect information – EMV without information

Multistage Decision Problems

(slide 5 of 6)

- A **strategy region graph** shows how the EMV varies with the conditions, for example, whether Acme hires a marketing firm.
  - This type of chart is useful for seeing whether the optimal decision *changes* over the range of the input variable.
  - It does so only if the two lines cross.

Multistage Decision Problems
(slide 6 of 6)

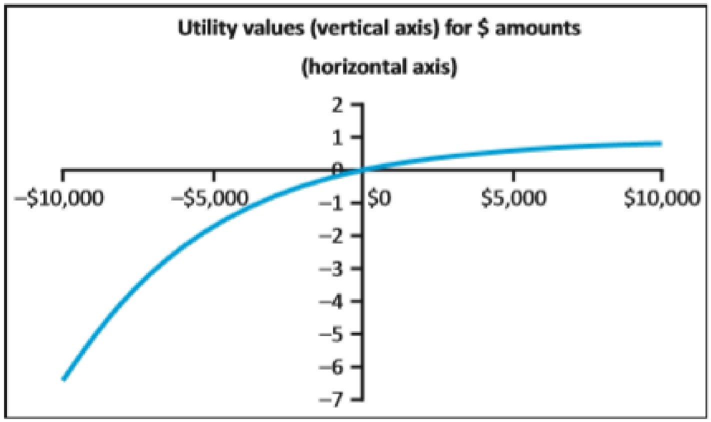
- A two-way sensitivity chart shows how the selected EMV varies as each *pair* of inputs varies simultaneously.

6-6 The Role of Risk Aversion

- Rational decision makers are sometimes willing to violate the EMV maximization criterion when large amounts of money are at stake.
  - These decision makers are willing to sacrifice some EMV to reduce risk.
- Most researchers believe that if certain basic behavioral assumptions hold, people are **expected utility** maximizers—that is, they choose the alternative with the largest expected utility.

6-6a Utility Functions
(slide 1 of 2)

- **Utility function** is a mathematical function that transforms monetary values—payoffs and costs—into *utility values*.
  - An individual's utility function specifies the individual's preferences for various monetary payoffs and costs and, in doing so, it automatically encodes the individual's attitudes toward risk.
  - Most individuals are *risk averse*, which means intuitively that they are willing to sacrifice some EMV to avoid risky gambles.
  - The resulting utility functions are shaped as shown below:



6-6a Utility Functions
(slide 2 of 2)

- Expected utility maximization
  - There are two aspects of implementing expected utility maximization in a real decision analysis.
    - First, an individual's (or company's) utility function must be assessed.
    - Second, the resulting utility function is used to find the best decision.

6-6b Exponential Utility

- Classes of ready-made utility functions have been developed to help assess people's utility functions.
- An **exponential utility** function has only one adjustable numerical parameter, called the **risk tolerance**.
  - There are straightforward ways to discover an appropriate value of this parameter for a particular individual or company, so it is relatively easy to assess.
  - An exponential utility function has the following form:

$$U(x) = 1 - e^{-x/R}$$

  - The risk tolerance for an exponential utility function is a single number that specifies an individual's aversion to risk.
    - The higher the risk tolerance, the less risk averse the individual is.

Example 6.4: New Product Decisions with Risk Aversion (slide 1 of 4)

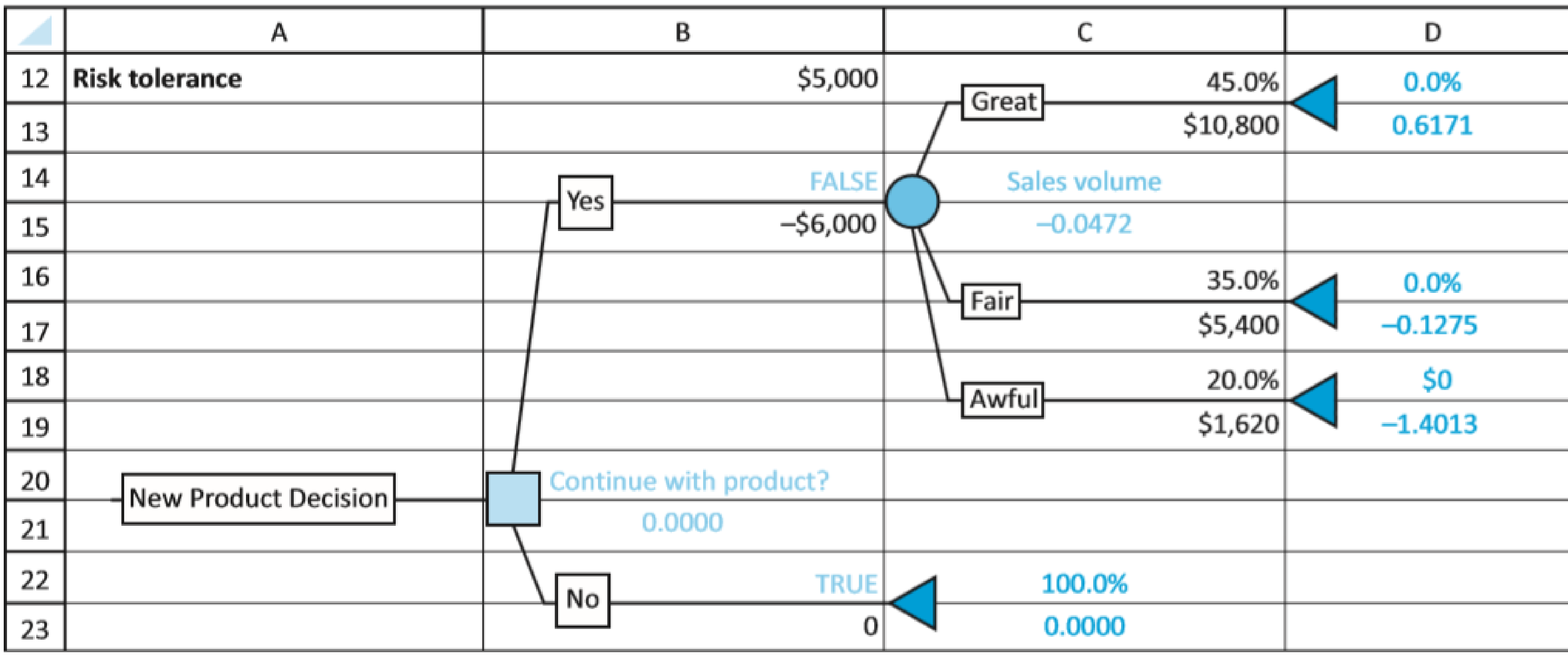
- **Objective:** To see how risk aversion affects Acme's decision-making process.
- **Solution:** Acme must first decide whether to hire the marketing research firm. If it decides not to, it can then immediately decide whether to market the product.
- On the other hand, if it decides to hire the firm, it must then wait for the firm's prediction. After the prediction is received, Acme can then make the ultimate decision on whether to market the product. However, when making this ultimate decision, Acme should definitely take the firm's prediction into account.

Example 6.4: New Product Decisions with Risk Aversion (slide 2 of 4)

	A	B	C	D	E
1	**Acme single-stage new product decision with risk aversion**				
2					
3	**Inputs**				
4	Fixed cost	$6,000			
5	Unit margin	$18			
6					
7	Market	Probability	Sales volume	Net revenue	
8	Great	0.45	600	$10,800	
9	Fair	0.35	300	$5,400	
10	Awful	0.20	90	$1,620	
11					
12	Risk tolerance	$5,000			

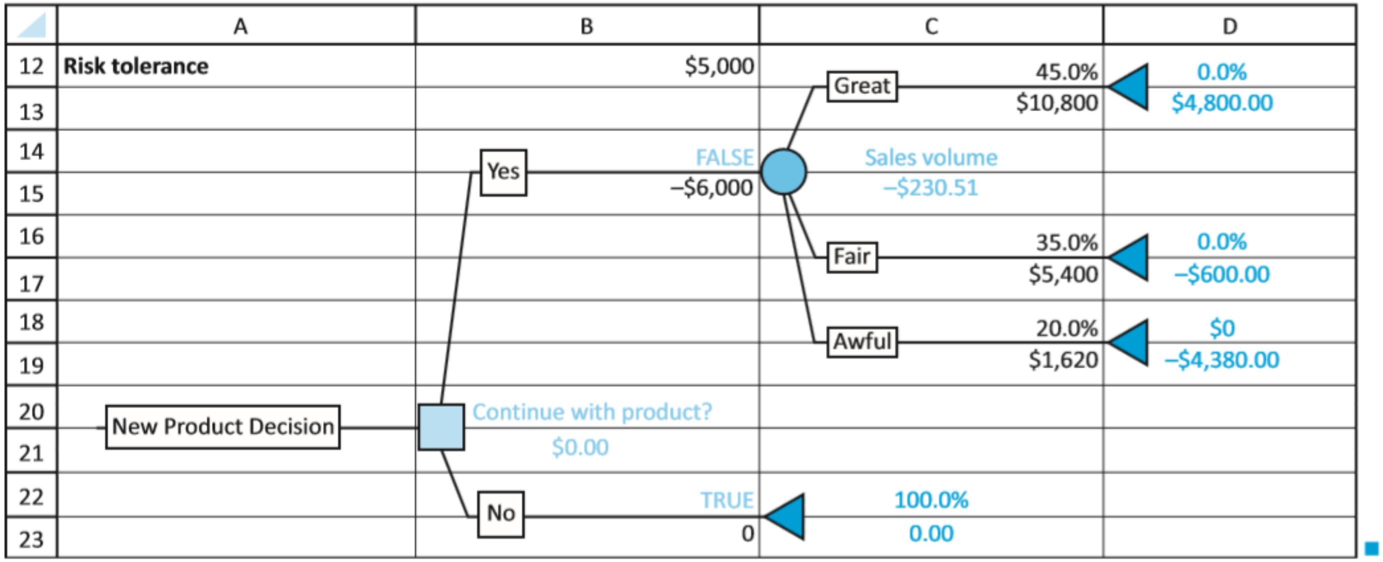
All monetary values (except the unit margin in cell B5) are in $1000s, and all sales volumes are in 1000s of units.

Example 6.4: New Product Decisions with Risk Aversion (slide 3 of 4)



Example 6.4: New Product Decisions with Risk Aversion (slide 4 of 4)

- Certainty equivalents
  - For a risk-averse person, the **certainty equivalent** of a gamble is the sure dollar amount the person would accept to avoid the gamble.
  - The person is indifferent between taking this sure amount and taking the gamble.



6-6c Is Expected Utility Maximization Used?

- Expected utility maximization is a fairly involved task.
- Theoretically, it might be interesting to researchers.
- However, in the business world, it is not used very often.
  - Risk aversion has been found to be of practical concern in only 5% to 10% of business decision analyses.
  - It is often adequate to use expected value (EMV) for most decisions.