A Guide to the agop 0.01-devel Package for R

Aggregation Operators in R

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May 22, 2013

The package, as well as this tutorial, is still in its early days – any suggestions are welcome!

Contents

1 Getting started

intro..... aggregation.... [16]

R is a free, open sourced software environment for statistical computing and graphics, which includes an implementation of a very powerful and quite popular high-level language called S. It runs on all major operating systems, i.e. Windows, Linux, and MacOS X. To install R and/or find some information on the S language please visit R Project's Homepage at www.R-project.org. Perhaps you may also wish to install RStudio, a convenient development environment for R. It is available at www.rsudio.org.

agop is an Open Source (licensed under GNU LGPL 3) package for R $\geq$ 2.12 to which anyone can contribute. It started as a fork of the CITAN (Citation Analysis Toolpack) package for R.

Each session with agop should be preceded by a call to:

```
library("agop") # Load the package
```

To view the main page of the manual we type:

```
library(help="agop")
```

For more information please visit the package's homepage [15]. In case of any problems, comments, or suggestions feel free to contact the authors. Good luck!

2 Theoretical Background

Let $\mathbb{I} = [a, b]$, possibly with $a = -\infty$ or $b = \infty$ (in many practical situations we choose $\mathbb{I} = [0, 1]$ or $\mathbb{I} = [0, \infty]$).

Moreover, let $\mathbb{I}^{1,2,\cdots} = \bigcup_{n=1}^{\infty} \mathbb{I}^n$ (set of vectors with elements in $\mathbb{I}$ – of any length).

Notational convention: For $\mathbf{x}, \mathbf{y} \in \mathbb{I}^n$ we write $\mathbf{x} \leq \mathbf{y}$ iff for all i it holds $x_i \leq y_i$.

All binary arithmetic operations on vectors $\mathbf{x}, \mathbf{y} \in \mathbb{I}^n$ are performed element-wise, e.g. $\mathbf{x}+\mathbf{y} = (x_1+y_1, \ldots, x_n+y_n) \in \mathbb{I}^n$. Similarly: $-$, $\cdot$, $/$, $\wedge$ (min), $\vee$ (max), etc. Additionally, each function of one variable $\mathsf{f} : \mathbb{I} \to \mathbb{I}$ can be extended to the vector space: we write $\mathsf{f}(\mathbf{x}) = (\mathsf{f}(x_1), \ldots, \mathsf{f}(x_n))$.

Let $(n * c) = (c, \ldots, c) \in \mathbb{I}^n$.

Let $x_{(i)}$ denote the ith order statistic, i.e. the ith smallest value in $\mathbf{x}$. Moreover, for convenience, let $x_{\{i\}} = x_{|\mathbf{x}|-i+1}$ denote the ith greatest value in $\mathbf{x}$.

2.1 Aggregation Operators and Their Basic Properties

Definition 1. $\mathsf{F} : \mathbb{I}^{1,2,\cdots} \to \mathbb{I}$ is called an *(extended) aggregation operator* (cf. [16]) if it is at least *nondecreasing* in each variable, i.e. for all n and $\mathbf{x}, \mathbf{y} \in \mathbb{I}^n$ if $\mathbf{x} \leq \mathbf{y}$, then $\mathsf{F}(\mathbf{x}) \leq \mathsf{F}(\mathbf{y})$.

Note that each aggregation operator is a mapping into $\mathbb{I}$, thus for all n we have $\inf_{\mathbf{x}\in\mathbb{I}^n} \mathsf{F}(\mathbf{x}) \geq a$ and $\sup_{\mathbf{x}\in\mathbb{I}^n} \mathsf{F}(\mathbf{x}) \leq b$. By nondecreasingness, however, these conditions reduce to $\mathsf{F}(n * a) \geq a$ and $\mathsf{F}(n * b) \leq b$.

Definition 2. We call F *symmetric* if ....

Definition 3. We call F *idempotent* if ....

Definition 4. We call F *additive* if ....

Definition 5. We call F *minitive* if ....

Definition 6. We call F *maxitive* if ....

Definition 7. We call F *modular* if ....

2.2 Impact Functions and The Producers Assessment Problem

.....

Let $\mathbb{I} = [0, \infty]$ represent the set of values that some a priori chosen paper quality measure may take. These may of course be non-integers, for example when we consider citations normalized with respect to the number of papers' authors.

It is widely accepted, see e.g. (Woeginger, [**?**, **?**, **?**]; Rousseau, [**?**]; Quesada, [**?**, **?**]; Gagolewski, Grzegorzewski, [**?**]; Franceschini, Maisano, [**?**]), that each aggregation operator $\mathsf{J} : \mathbb{I}^{1,2,\cdots} \to \mathbb{I}$ to be applied in the impact assessment process should at least be:

(a) nondecreasing in each variable (additional citations received by a paper or an improvement of its quality measure does not result in a decrease of the authors' overall evaluation),

(b) arity-monotonic (by publishing a new paper we never decrease the overall valuation of the entity),

(c) symmetric (independent of the order of elements' presentation, i.e. we may always assume that we aggregate vectors that are already sorted).

Conditions (a) and (b) imply that each impact function is able – at least potentially – to describe two "dimensions" of the author's output quality: (a) his/her ability to write eagerly-cited or highly-valuated papers and (b) his/her overall productivity.

More formally, condition (a) holds if and only if for each n and $\mathbf{x}, \mathbf{y} \in \mathbb{I}^n$ such that $(\forall i)$ $x_i \leq y_i$ we have $\mathsf{J}(\mathbf{x}) \leq \mathsf{J}(\mathbf{y})$. On the other hand, axiom (b) is fulfilled iff for any $\mathbf{x} \in \mathbb{I}^{1,2,\cdots}$ and $y \in \mathbb{I}$ it holds $\mathsf{J}(\mathbf{x}) \leq \mathsf{J}(x_1, \ldots, x_n, y)$. Lastly, requirement (c) holds iff for all n and $\mathbf{x} \in \mathbb{I}^n$ we have $\mathsf{J}(\mathbf{x}) = \mathsf{J}(x_{\{1\}}, \ldots, x_{\{n\}})$, where $x_{\{i\}}$ denotes the ith largest value from $\mathbf{x}$, i.e. its $(n-i+1)$th order statistic.

3 Predefined Classes of Aggregation Operators in agop

3.1 A Note on Representing Numeric Data and Applying Functions in R

Generally, in our implementation we most often deal with numeric vectors. Recall how we create them in R:

```
(x1 <- c(5, 2, 3, 1, 0, 0))
## [1] 5 2 3 1 0 0
class(x1)
## [1] "numeric"
(x2 <- rep(10, 3))
## [1] 10 10 10
(x3 <- 10:1) # the same as seq(10, 1)
##  [1] 10  9  8  7  6  5  4  3  2  1
(x4 <- seq(1, 5, length.out=6))
## [1] 1.0 1.8 2.6 3.4 4.2 5.0
(x5 <- seq(1, 5, by=1.25))
## [1] 1.00 2.25 3.50 4.75
```

Sometimes we will store the vectors of the same length in a matrix (column-/row-wise.... col/rownames....) `apply()`....

```
expertopinions <- matrix(c(
      6,7,2,3,1, # this will be the first COLUMN
      8,3,2,1,9, # 2nd
      4,2,4,1,6  # 3rd
   ),
   ncol=3,
   dimnames=list(NULL, c("A", "B", "C")) # only column names set
)
class(expertopinions)
```

```
## [1] "matrix"
```

```
print(expertopinions)   # or print(authors)
```

```
##      A B C
## [1,] 6 8 4
## [2,] 7 3 2
## [3,] 2 2 4
## [4,] 3 1 1
## [5,] 1 9 6
```

```
apply(expertopinions, 2, mean) # on each COLUMN apply the mean() function
```

```
##   A   B   C
## 3.8 4.6 3.4
```

...or in a list, especially when they are not of the same length.... `lapply()`.... `sapply()`..... possibly named elements...

```
authors <- list(
   "John S." = c(7,6,2,1,0),
   "Kate F." = c(9,8,7,6,4,1,1,0)
)
class(authors)
```

```
## [1] "list"
```

```
str(authors)   # or print(authors)
```

```
## List of 2
##  $ John S.: num [1:5] 7 6 2 1 0
##  $ Kate F.: num [1:8] 9 8 7 6 4 1 1 0
```

```
index_h(authors[[1]]) # the h-index (see below) for 1st author
```

```
## [1] 2
```

```
sapply(authors, index_h) # calculate the h-index for all vectors in a list
```

```
## John S. Kate F.
##       2       4
```

```
index_h(authors) # index_h() expects an numeric vector on input
```

```
## Error:  argument 'x' should be a numeric vector (or an object coercible to)
```

3.2 ...

weighted average, OWA (L-statistics), qL
WMin, OWMin, qI
WMax, OWMax (S-statistics), qS
OM3

3.3 Bibliometric Impact Indices

Below we assume that $\mathbb{I} = [0, \infty]$.

The h-index. Given a sequence $\mathbf{x} = (x_1, \ldots, x_n) \in \mathbb{I}^{1,2,\cdots}$, the *Hirsch index* [17] of $\mathbf{x}$ is defined as $\mathsf{H}(x) = \max\{i = 1, \ldots, n : x_{\{i\}} \geq i\}$ if $n \geq 1$ and $x_{\{1\}} \geq 1$, or $\mathsf{H}(x) = 0$ otherwise. It may be shown that the h-index is an zero-insensitive OM3 aggeration operator, see [5], with:

$$\mathsf{H}(\mathbf{x}) = \bigvee_{i=1,\ldots,n}^{n} i \wedge \lfloor x_{\{i\}} \rfloor.$$

Interpretation: "an author has h-index of H if H of his/her n most cited papers have at least H citations each, and the other $n - H$ papers are cited no more that H times each". The h-index may also be expressed as a Sugeno integral [19] w.r.t. to a counting measure, cf. [20].

agop implementation: `index_h()`.

```
index_h(c(6,5,4,2,1,0,0,0,0,0,0))
## [1] 3
index_h(c(-1,3,4,2)) # only for x>=0
## Error:  all elements in ‘x‘ should be in [0,Inf]
```

The g-index. Egghe's g-index [4]

The index is interesting from the computational point of view – it is calculated on the nondecreasing vector of cumulative sums, `cumsum(sort(x, decreasing=TRUE))`

The w-index.

The MAXPROD**-index.**

The r_p-indices.

The l_p-indices.

4 Visualization

4.1 Depicting producers

The `plot_producer()` function may be used to draw a graphical representation of a given numeric vector, i.e. what is sometimes called a citation function in scientometrics.

A given vector $\mathbf{x} = (x_1, \ldots, x_n)$ can be represented by a step function defined for $0 \leq y < n$ and given by:

$$\pi(y) = x_{(n-\lfloor y+1 \rfloor+1)}.$$

This function may be obtained by setting `type == 'right.continuous'` argument in `plot_producer()`. Recall that $x_{(i)}$ denotes i-th smallest value in $\mathbf{x}$.

On the other hand, for `type == 'left.continuous'` (the default), we get

$$\pi(y) = x_{(n-\lfloor y \rfloor+1)}$$

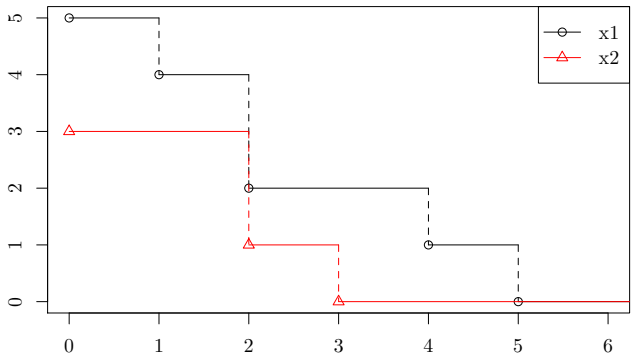
for $0 < y \leq n$.

Moreover, this function may depict the curve joining the sequence of points $(0, x_{(n)}), (1, x_{(n)}), (1, x_{(n-1)}), (2, x_{(n-1)}), \ldots, (n, x_{(1)})$.

The `plot_producer()` function behaves much like the well-known R's `plot.default()` and allows for passing all its graphical parameters.

For example, let us depict the state of two given producers, $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$.

```
x1 <- c(5, 4, 2, 2, 1)
x2 <- c(3, 3, 1, 0, 0, 0, 0)
plot_producer(x1, extend=TRUE)
plot_producer(x2, add=TRUE, col=2, pch=2, extend=TRUE)
legend('topright', c('x1', 'x2'), col=c(1, 2), lty=1, pch=c(1, 2))
```



5 Pre-orders

.....

Let us consider the following relation on $\mathbb{I}^{1,2,\cdots}$. For any $\mathbf{x} \in \mathbb{I}^n$ and $\mathbf{y} \in \mathbb{I}^m$ we write $\mathbf{x} \trianglelefteq \mathbf{y}$ if and only if $n \leq m$ and $x_{\{i\}} \leq y_{\{i\}}$ for all $i \in \min\{n, m\}$. Of course, $\trianglelefteq$ is a pre-order – it would have been a partial order, if we had defined it on the set of *sorted* vectors.

In other words, we say that an author X is (weakly) dominated by an author Y, if X has no more papers than Y and each the ith most cited paper of X has no more citations than the ith most cited paper of Y. Not that the $m-n$ least cited Y's papers are not taken into account here. Most importantly, however, there exist pairs of vectors that are *incomparable* with respect to $\trianglelefteq$ (see the illustration below).

This pre-order in agop as `pord_weakdom()`.

```
c(pord_weakdom(5:1, 10:1), pord_weakdom(10:1, 5:1)) # 5:1 <= 10:1
## [1]  TRUE FALSE
c(pord_weakdom(3:1, 5:4),  pord_weakdom(5:1, 3:1))  # 3:1 ?? 5:4
## [1] FALSE FALSE
```

We have the following result (Gagolewski, Grzegorzewski, [**?**]). Let $\mathsf{F} \in \mathcal{E}(\mathbb{I})$. Then F is symmetric, nondecreasing in each variable and arity-monotonic if and only if for any $\mathbf{x}, \mathbf{y}$ if $\mathbf{x} \trianglelefteq \mathbf{y}$, then $\mathsf{F}(\mathbf{x}) \leq \mathsf{F}(\mathbf{y})$. Therefore, the class of impact functions may be equivalently defined as all the aggregation operators that are nondecreasing with respect to this preorder.

Additionally, we will write $\mathbf{x} \triangleleft \mathbf{y}$ if $\mathbf{x} \trianglelefteq \mathbf{y}$ and $\mathbf{x} \neq \mathbf{y}$ (strict dominance).

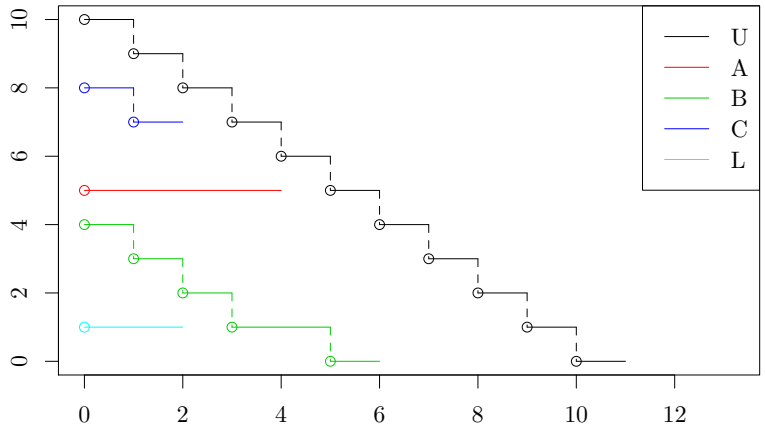
Example. Let us consider the 5 following vectors.

```
ex1 <- list(
   U = 10:0,           # some upper bound
   A = c(5,5,5,5),     # moderate productivity & quality
```

```
   B = c(4,3,2,1,1,0), # high productivity
   C = c(8,7),         # high quality
   L = c(1,1)          # some lower bound
)
```

Plot of "citation" curves:

```
for (i in seq_along(ex1))
plot_producer(ex1[[i]], add=(i>1), col=i)
legend("topright", legend=names(ex1), col=1:length(ex1), lty=1)
```



get adjacency matrix for $(\{A, B, C, L, U\}, \trianglelefteq)$....

```
ord <- rel_graph(ex1, pord_weakdom, disable.check=TRUE)
print(ord)
```

```
## 5 x 5 sparse Matrix of class "dgCMatrix"
##   U A B C L
## U 1 . . . .
## A 1 1 . . .
## B 1 . 1 . .
## C 1 . . 1 .
## L 1 1 1 1 1
```

```
is_reflexive(ord)  # is reflexive
```

```
## [1] TRUE
```

```
is_transitive(ord) # is transitive
```

```
## [1] TRUE
```

```
is_total(ord)      # not a total preorder...
```

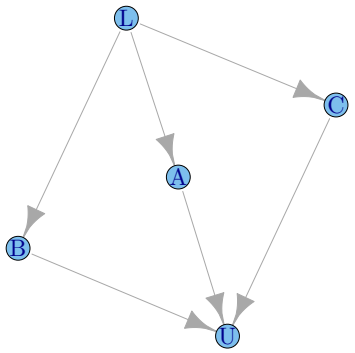
```
## [1] FALSE
```

We see that we have $A??B$, $A??C$, $B??C$ (no pair from $\{A, B, C\}$ is comparable w.r.t. $\trianglelefteq$).

To draw the Hasse diagram, it will be good to de-transitivize the graph (for æsthetic reasons)....

```
require(igraph)
hasse <- graph.adjacency(de_transitive(ord))
set.seed(1234567) # igraph's draving facilities are far from perfect
```

```
plot(hasse, layout=layout.fruchterman.reingold(hasse, dim=2))
```

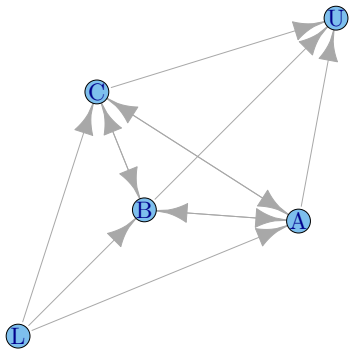


$(\{A, B, C, L, U\}, \trianglelefteq)$ is not totally ordered, let's apply fair totalization (set $x \trianglelefteq y$ and $y \trianglelefteq x$ whenever $\neg(x \trianglelefteq y$ or $y \trianglelefteq x)$ + calculate transitive closure

```
ord_total <- closure_transitive(closure_total_fair(ord)) # a total preorder
print(ord_total)
```

```
## 5 x 5 sparse Matrix of class "dgCMatrix"
##   U A B C L
## U 1 . . . .
## A 1 1 1 1 .
## B 1 1 1 1 .
## C 1 1 1 1 .
## L 1 1 1 1 1
```

```
hasse <- graph.adjacency(de_transitive(ord_total))
set.seed(1234)
plot(hasse, layout=layout.fruchterman.reingold(hasse, dim=2))
```



Thus, we've obtained $L \prec (A \simeq B \simeq C) \prec U$.

6 Aggregation Operators from the Probabilistic Perspective

Theory of aggregation looks on the aggregation operators from the algebraic/calculus perspective. Of course, we should always be interested in their probabilistic properties, e.g. in i.i.d. RVs models.

In such case we assume that input data are in fact realizations of some random samples.

In probability, an aggregation operator is simply called a *statistic* (formalism.....)

6.1 Stochastic Models

Pareto2.....

6.2 Stochastic Properties of Aggregation Operators

OWA, L-statistics

OWMax, S-statistics

h-index and its distribution

7 NEWS/CHANGELOG

```
agop package NEWS

*******************************************************************************

0.01 /under development/

* initial release
```

Acknowledgments. This document has been generated with LaTeX, knitr and the tikzDevice package for R. Their authors' wonderful work is fully appreciated.

The contribution of Marek Gagolewski was partially supported by the European Union from resources of the European Social Fund, Project PO KL "Information technologies: Research and their interdisciplinary applications", agreement UDA-POKL.04.01.01-00-051/10-00 (March-June 2013), and by FNP START Scholarship from the Foundation for Polish Science (2013).

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