# Package 'agop'

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Title Aggregation Operators and Preordered Sets

**Description** Tools supporting multi-criteria decision making, including variable number of criteria, by means of aggregation operators and preordered sets. Possible applications include, but are not limited to, scientometrics and bibliometrics.

URL http://www.rexamine.com/resources/agop/

BugReports http://github.com/Rexamine/agop/issues

ByteCompile TRUE

Type Package

**Depends** R (>= 2.12.0), base, stats, grDevices, graphics, Matrix,igraph

License LGPL (>= 3)

Suggests testthat

**Collate** 'agop-package.R' 'visualization.R' 'preorders.R' 'agops-impact.R' 'agops-classical.R' 'distrib-pareto2.R' 'distrib-pareto2-estimators.R' 'distrib-pareto2-ftest.R'

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agop-package

Aggregation Operators Package for R

### Description

Index

"The process of combining several numerical values into a single representative one is called aggregation, and the numerical function performing this process is called aggregation function. This simple definition demonstrates the size of the field of application of aggregation: applied mathematics (e.g. probability, statistics, decision theory), computer science (e.g. artificial intelligence, operation research), as well as many applied fields (economics and finance, pattern recognition and image processing, data fusion, multicriteria decision making, automated reasoning etc.). Although history of aggregation is probably as old as mathematics (think of the arithmetic mean), its existence has reminded underground till only recent (...)." (Grabisch et al, 2009, p. xiii)

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#### **Details**

**agop** is an open source (LGPL 3) package for R, to which anyone can contribute. It started as a fork of the **CITAN** package (Gagolewski, 2011).

For more infrmation refer to the Package Vignette. Its most recent version is available at http://github.com/Rexamine/agop/raw/master/inst/doc/agop-Tutorial.pdf.

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closure\_total\_fair

Total Closure of Adjacency Matrix [Fair Totalization]

### **Description**

Fair totalization: for each pair (x,y) s.t. not xRy and not xRy let from now on xRy and yRx

### Usage

closure\_total\_fair(B)

#### **Arguments**

В

object of class igraph or a square 0-1 matrix of class Matrix or matrix

#### **Details**

If you want a total preorder, call closure\_transitive.

#### Value

object of class Matrix

### See Also

Other binary\_relations: closure\_transitive, de\_transitive, get\_equivalence\_classes, get\_incomparable\_pairs, get\_independent\_sets, is\_reflexive, is\_total, is\_transitive, pord\_weakdom, rel\_graph

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closure\_transitive

Transitive Closure of Adjacency Matrix

### Description

This may be slow for large graphs.

### Usage

```
closure_transitive(B)
```

### **Arguments**

В

object of class igraph or a square 0-1 matrix of class Matrix or matrix

#### Value

object of class Matrix

#### See Also

Other binary\_relations: closure\_total\_fair, de\_transitive, get\_equivalence\_classes, get\_incomparable\_pairs, get\_independent\_sets, is\_reflexive, is\_total, is\_transitive, pord\_weakdom, rel\_graph

de\_transitive

De-transitivitize Graph

### **Description**

Useful for draving Hasse diagrams.

#### Usage

```
de_transitive(B)
```

### **Arguments**

В

object of class igraph or a square 0-1 matrix of class Matrix or matrix

### Value

object of class Matrix

### See Also

```
Other binary_relations: closure_total_fair, closure_transitive, get_equivalence_classes, get_incomparable_pairs, get_independent_sets, is_reflexive, is_total, is_transitive, pord_weakdom, rel_graph
```

get\_equivalence\_classes

Get All Equivalence Classes of a Total Binary Relation

### **Description**

Note that we assume that B is total, reflexive and transitive.

#### **Usage**

```
get_equivalence_classes(B)
```

### **Arguments**

В

object of class igraph or a square 0-1 matrix of class Matrix or matrix

### Value

list of integer vectors; each list element defines an equivalence class by listing vertices' numbers; each vector is ordered by the outdegrees of their nodes (they are the same in each class)

#### See Also

Other binary\_relations: closure\_total\_fair, closure\_transitive, de\_transitive, get\_incomparable\_pairs, get\_independent\_sets, is\_reflexive, is\_total, is\_transitive, pord\_weakdom, rel\_graph

```
get_incomparable_pairs
```

Get Incomparable Pairs in an Adjacency Matrix

### **Description**

A pair (x,y) is incomparable iff not xRy and not xRy

#### **Usage**

```
get_incomparable_pairs(B)
```

### **Arguments**

В

object of class igraph or a square 0-1 matrix of class Matrix or matrix

### **Details**

See also get\_independent\_sets of how to generate all maximal independent sets.

get\_independent\_sets 7

### Value

integer matrix with two columns (indices of incomparable elements, not that these are pairs, and not sets: you'll get (i,j) and (j,i))

#### See Also

Other binary\_relations: closure\_total\_fair, closure\_transitive, de\_transitive, get\_equivalence\_classes, get\_independent\_sets, is\_reflexive, is\_total, is\_transitive, pord\_weakdom, rel\_graph

get\_independent\_sets Get All Maximal Independent Sets

### **Description**

The function generates vectors of indices  $S_j = \{i_1, ..., i_{k_j}\}$  such that all pairs from  $S_j$  are incomparable (A pair (i,i') is incomparable iff not iRi' and not i'Ri, see also get\_incomparable\_pairs.

#### Usage

get\_independent\_sets(B)

### **Arguments**

В

object of class igraph or a square 0-1 matrix of class Matrix or matrix

#### **Details**

Note that we assume that B is transitive. Loops are not taken into account at all.

### Value

list of integer vectors; each list element defines an independent set of vertices numbers

### See Also

Other binary\_relations: closure\_total\_fair, closure\_transitive, de\_transitive, get\_equivalence\_classes, get\_incomparable\_pairs, is\_reflexive, is\_total, is\_transitive, pord\_weakdom, rel\_graph

8 index\_g

index\_g

Egghe's g-index

### **Description**

Given a sequence of n non-negative numbers  $x=(x_1,\ldots,x_n)$ , where  $x_i \geq x_j \geq 0$  for  $i \leq j$ , the g-index (Egghe, 2006) for x is defined as

$$G(x) = \max\{i = 1, \dots, n : \sum_{i=1}^{i} x_i \ge i^2\}$$

if  $n \ge 1$  and  $x_1 \ge 1$ , or G(x) = 0 otherwise.

### Usage

```
index_g(x) index_g(x) # same as index_g(x), deprecated alias index_g_z(x)
```

### **Arguments**

Χ

a non-negative numeric vector

### **Details**

index.g is a (deprecated) alias for index\_g.

Note that index\_g is not a zero-insensitive impact function, see Examples section. index\_g\_zi is its zero-sensitive variant: it assumes that the aggregated vector is padded with zeros.

The h-index is the same as the discrete Sugeno integral of x w.r.t. the counting measure (cf. Torra, Narukawa, 2008).

If non-increasingly sorted vector is given, the function is O(n).

For historical reasons, this function is also available via its alias, index.h [but its usage is deprecated].

### Value

a single numeric value

#### References

Egghe L., Theory and practise of the g-index, Scientometrics 69(1), 131-152, 2006. Torra V., Narukawa Y., The h-index and the number of citations: Two fuzzy integrals. IEEE Transactions on Fuzzy Systems 16(3), 2008, 795-797.

index\_h

### See Also

Other impact\_functions: index\_h, index\_lp, index\_maxprod, index\_rp, index\_w, index.h, index.lp, index.rp

### **Examples**

```
sapply(list(c(9), c(9,0), c(9,0,0), c(9,0,0,0)), index_g) # not a zero-sensitive agop
```

index\_h

Hirsch's h-index

### Description

Given a sequence of n non-negative numbers  $x=(x_1,\ldots,x_n)$ , where  $x_i \geq x_j \geq 0$  for  $i \leq j$ , the h-index (Hirsch, 2005) for x is defined as

$$H(x) = \max\{i = 1, \dots, n : x_i \ge i\}$$

if  $n \ge 1$  and  $x_1 \ge 1$ , or H(x) = 0 otherwise.

### Usage

```
index_h(x)
```

index.h(x) # same as  $index_h(x)$ , deprecated alias

### **Arguments**

Х

a non-negative numeric vector

### **Details**

If non-increasingly sorted vector is given, the function is O(n).

For historical reasons, this function is also available via its alias, index.h [but its usage is deprecated].

See index\_rp and owmax for natural generalizations.

### Value

a single numeric value

### References

Hirsch J.E., An index to quantify individual's scientific research output, Proceedings of the National Academy of Sciences 102(46), 16569-16572, 2005.

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### See Also

Other impact\_functions: index\_g, index\_g\_zi, index\_lp, index\_maxprod, index\_rp, index\_w, index.g, index.lp, index.rp

### Examples

index\_lp

*The*  $l_p$ *-index* 

### **Description**

Given a sequence of n non-negative numbers  $x=(x_1,\ldots,x_n)$ , where  $x_i\geq x_j$  for  $i\leq j$ , the  $l_p$ -index for  $p=\infty$  equals to

$$l_p(x) = \arg \max_{(i,x_i), i=1,...,n} \{ix_i\}$$

if  $n \ge 1$ , or  $l_{\infty}(x) = 0$  otherwise. Note that if  $(i, x_i) = l_{\infty}(x)$ , then

$$MAXPROD(x) = prod(l_{\infty}(x)) = ix_i,$$

where MAXPROD is the index proposed in (Kosmulski, 2007), see index\_maxprod.

For the definition of the  $l_p$ -index for  $p < \infty$  we refer to (Gagolewski, Grzegorzewski, 2009a).

### Usage

```
index_lp(x, p = Inf, projection = prod)
index.lp(x, p = Inf, projection = prod) # deprecated
  alias
```

#### Arguments

```
x a non-negative numeric vector \mathbf{p} \qquad \qquad \text{index order, } p \in [1,\infty]; \text{ defaults } \infty \text{ (Inf)}.
```

projection function

index\_maxprod 11

#### **Details**

The  $l_p$ -index, by definition, is not an impact function, as it produces 2 numeric values. Thus, it should be projected to one dimension. However, you may set projection to identity to obtain the 2-dimensional index

If non-increasingly sorted vector is given, the function is O(n).

For historical reasons, this function is also available via its alias, index.lp [but its usage is deprecated].

#### Value

```
result of projection(c(i, x_i))
```

#### References

Gagolewski M., Grzegorzewski P., A geometric approach to the construction of scientific impact indices, Scientometrics, 81(3), 2009a, pp. 617-634.

Gagolewski M., Debski M., Nowakiewicz M., Efficient algorithms for computing "geometric" scientific impact indices, Research Report of Systems Research Institute, Polish Academy of Sciences RB/1/2009, 2009b.

Kosmulski M., MAXPROD - A new index for assessment of the scientific output of an individual, and a comparison with the h-index, Cybermetrics, 11(1), 2007.

#### See Also

```
Other impact_functions: index_g, index_g_zi, index_h, index_maxprod, index_rp, index_w, index.g, index.h, index.rp
```

### **Examples**

index\_maxprod

Kosmulski's MAXPROD-index

### Description

Given a sequence of n non-negative numbers  $x=(x_1,\ldots,x_n)$ , where  $x_i \geq x_j \geq 0$  for  $i \leq j$ , the MAXPROD-index (Kosmulski, 2007) for x is defined as

```
MAXPROD(x) = \max\{ix_i : i = 1, ..., n\}
```

#### Usage

```
index_maxprod(x)
```

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### Arguments

Х

a non-negative numeric vector

#### **Details**

If non-increasingly sorted vector is given, the function is O(n).

MAXPROD index is the same as the discrete Shilkret integral of x w.r.t. the counting measure.

See index\_lp for a natural generalization.

#### Value

a single numeric value

### References

Kosmulski M., MAXPROD - A new index for assessment of the scientific output of an individual, and a comparison with the h-index, Cybermetrics 11(1), 2007.

### See Also

Other impact\_functions: index\_g, index\_g\_zi, index\_h, index\_lp, index\_rp, index\_w, index\_g, index.h, index.lp, index.rp

index\_rp

*The r\_p-index* 

### **Description**

Given a sequence of n non-negative numbers  $x=(x_1,\ldots,x_n)$ , where  $x_i\geq x_j$  for  $i\leq j$ , the  $r_p$ -index for  $p=\infty$  equals to

$$r_p(x) = \max_{i=1,...,n} \{\min\{i, x_i\}\}$$

if  $n \geq 1$ , or  $r_{\infty}(x) = 0$  otherwise. That is, it is equivalent to a particular OWMax operator, see ownax.

For the definition of the  $r_p$ -index for  $p < \infty$  we refer to (Gagolewski, Grzegorzewski, 2009).

### Usage

```
index_rp(x, p = Inf)
index.rp(x, p = Inf) # same as index_rp(x, p), deprecated
  alias
```

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### **Arguments**

x a non-negative numeric vector

p index order,  $p \in [1, \infty]$ ; defaults  $\infty$  (Inf).

#### **Details**

Note that if  $x_1, \ldots, x_n$  are integers, then

$$r_{\infty}(x) = H(x),$$

where H is the h-index (Hirsch, 2005) and

$$r_1(x) = W(x),$$

where W is the w-index (Woeginger, 2008), see index\_h and index\_w.

If non-increasingly sorted vector is given, the function is O(n).

For historical reasons, this function is also available via its alias, index.rp [but its usage is deprecated].

#### Value

a single numeric value

#### References

Gagolewski M., Grzegorzewski P., A geometric approach to the construction of scientific impact indices, Scientometrics, 81(3), 2009, pp. 617-634.

Hirsch J.E., An index to quantify individual's scientific research output, Proceedings of the National Academy of Sciences 102(46), 16569-16572, 2005.

Woeginger G.J., An axiomatic characterization of the Hirsch-index, Mathematical Social Sciences, 56(2), 224-232, 2008.

### See Also

Other impact\_functions: index\_g, index\_g\_zi, index\_h, index\_lp, index\_maxprod, index\_w, index.g, index.h, index.lp

### **Examples**

```
x <- runif(100, 0, 100);
index.rp(x);  # the r_oo-index
floor(index.rp(x));  # the h-index
index.rp(floor(x), 1);  # the w-index</pre>
```

index\_w

index\_w

Woeginger's w-index

### **Description**

Given a sequence of n non-negative numbers  $x=(x_1,\ldots,x_n)$ , where  $x_i\geq x_j\geq 0$  for  $i\leq j$ , the w-index (Woeginger, 2008) for x is defined as

$$W(x) = \max\{i = 1, \dots, n : x_j \ge i - j + 1, \forall j = 1, \dots, i\}$$

### Usage

 $index_w(x)$ 

### **Arguments**

Χ

a non-negative numeric vector

### **Details**

If non-increasingly sorted vector is given, the function is O(n).

See index\_rp for a natural generalization.

### Value

a single numeric value

#### References

Woeginger G. J., An axiomatic characterization of the Hirsch-index. Mathematical Social Sciences 56(2), 2008, 224-232.

### See Also

Other impact\_functions: index\_g, index\_g\_zi, index\_h, index\_lp, index\_maxprod, index\_rp, index.g, index.h, index.lp, index.rp

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is\_reflexive

Check if Given Adjacency Matrix is Reflexive

### **Description**

A binary relation R is reflexive, iff for all x we have xRx. The function just checks whether all elements on the diagonal of B are non-zeros.

### Usage

```
is_reflexive(B)
```

### **Arguments**

В

object of class igraph or a square 0-1 matrix of class Matrix or matrix

### Value

single logical value

#### See Also

Other binary\_relations: closure\_total\_fair, closure\_transitive, de\_transitive, get\_equivalence\_classes, get\_incomparable\_pairs, get\_independent\_sets, is\_total, is\_transitive, pord\_weakdom, rel\_graph

is\_total

Check if Given Adjacency Matrix is Total

### Description

A binary relation R is total, iff for all x, y we have xRy or yRx.

### Usage

```
is_total(B)
```

### **Arguments**

В

object of class igraph or a square 0-1 matrix of class Matrix or matrix

### Value

single logical value

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### See Also

Other binary\_relations: closure\_total\_fair, closure\_transitive, de\_transitive, get\_equivalence\_classes, get\_incomparable\_pairs, get\_independent\_sets, is\_reflexive, is\_transitive, pord\_weakdom, rel\_graph

is\_transitive

Check if Given Adjacency Matrix is Transitive

### **Description**

A binary relation R is transitive, iff for all x, y, z we have xRy and  $yRz \Rightarrow xRz$ 

### Usage

```
is_transitive(B)
```

### **Arguments**

В

object of class igraph or a square 0-1 matrix of class Matrix or matrix

#### Value

single logical value

### See Also

Other binary\_relations: closure\_total\_fair, closure\_transitive, de\_transitive, get\_equivalence\_classes, get\_incomparable\_pairs, get\_independent\_sets, is\_reflexive, is\_total, pord\_weakdom, rel\_graph

owa

WAM and OWA Operators

### **Description**

Computes the Weghted Arithmetic Mean or the Ordered Weighted Averaging aggregation operator.

### Usage

```
owa(x, w = rep(1/length(x), length(x)))
wam(x, w = rep(1/length(x), length(x)))
```

owmax 17

### Arguments

x numeric vector to be aggregated

w numeric vector of the same length as x, with elements in [0,1], and such that  $\sum_{i} w_{i} = 1$ ; weights

#### **Details**

The OWA operator is given by

$$\mathsf{OWA}_{\mathtt{w}}(\mathtt{x}) = \sum_{i=1}^n w_i x_{\{i\}}$$

where  $x_{\{i\}}$  denotes the i-th greatest value in x.

The WAM operator is given by

$$\mathsf{WAM}_{\mathtt{w}}(\mathtt{x}) = \sum_{i=1}^{n} w_i x_i$$

If the elements of w does not sum up to 1, then they are normalized and a warning is generated.

Both functions return the ordinary arithmetic mean by default. Special cases of OWA include the trimmed mean (cf. mean) and winsorized mean.

There is a strong connection between the OWA operators and the Choquet integrals.

### Value

single numeric value

### References

Yager R.R., On ordered weighted averaging aggregation operators in multicriteria decision making, IEEE Transactions on Systems, Man, and Cybernetics 18(1), 1988, pp. 183-190.

### See Also

Other aggregation\_operators: owmax, owmin, wmax, wmin

owmax

WMax, WMin, OWMax, and OWMin Operators

### **Description**

Computes the (Ordered) Weighted Maximum/Minimum.

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### Usage

### **Arguments**

x numeric vector to be aggregated

w numeric vector of the same length as x; weights

### **Details**

The OWMax operator is given by

$$\mathsf{OWMax}_{\mathtt{w}}(\mathtt{x}) = \bigvee_{i=1}^n w_i \wedge x_{\{i\}}$$

where  $x_{\{i\}}$  denotes the *i*-th greatest value in x.

The OWMin operator is given by

$$\mathsf{OWMin}_{\mathtt{w}}(\mathtt{x}) = \bigwedge_{i=1}^n w_i \vee x_{\{i\}}$$

The WMax operator is given by

$$\mathsf{WMax}_{\mathtt{w}}(\mathtt{x}) = \bigvee_{i=1}^n w_i \wedge x_i$$

The WMin operator is given by

$$\mathsf{WMin}_{\mathtt{w}}(\mathtt{x}) = \bigwedge_{i=1}^{n} w_i \vee x_i$$

OWMax and WMax return the greatest value in x by default, and OWMin and WMin - the smallest value in x.

Note that e.g. in the case of OWMax operator the aggregation w.r.t. w gives the same result as that of w.r.t. sort(w). Moreover, classically, it is assumed that if we agregate vectors with elements in [a,b], then the largest weight should be equal to b.

There is a strong connection between the OWMax/OWMin operators and the Sugeno integrals. Additionally, it may be shown that the OWMax and OWMin classes are equivalent.

Moreover, index\_h for integer data is a particular OWMax operator.

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### Value

single numeric value

#### References

Dubois D., Prade H., Testemale C., Weighted fuzzy pattern matching, Fuzzy Sets and Systems 28, 1988, pp. 313-331.

#### See Also

Other aggregation\_operators: owa, wam

pareto2\_estimate\_mle Parameter Estimation in the Pareto-II Distribution (MLE)

### **Description**

Finds the maximum likelihood estimator of the type II Pareto distribution's shape parameter k and, if not given explicitly, scale parameter s.

#### **Usage**

```
pareto2_estimate_mle(x, s = NA_real_, smin = 1e-04,
  smax = 20, tol = .Machine$double.eps^0.25)
```

### **Arguments**

| X    | a non-negative numeric vector                                    |
|------|--|
| S    | a-priori known scale parameter, $s>0$ or NA if unknown (default) |
| smin | lower bound for the scale parameter to look for                  |
| smax | upper bound for the scale parameter to look for                  |
| tol  | the desired accuracy (convergence tolerance)                     |

### **Details**

Note that if s is not given, then the maximum of the likelihood function may not exist for some input vectors. This estimator may have large mean squared error. Consider using pareto2\_estimate\_mmse. For known s, the estimator is unbiased.

#### Value

a numeric vector with the following named components:

- k estimated parameter of shape
- s estimated (or known, see the s argument) parameter of scale

or c(NA, NA) if the maximum of the likelihood function could not be found.

### See Also

Other Pareto2: dpareto2, pareto2\_estimate\_mmse, pareto2\_test\_f, ppareto2, qpareto2, rpareto2

pareto2\_estimate\_mmse Parameter Estimation in the Pareto-II Distribution (MMSE)

### **Description**

Finds the MMS estimator of the type II Pareto distribution parameters using the Bayesian method (and the R code) developed by Zhang and Stevens (2009).

### Usage

```
pareto2_estimate_mmse(x)
```

### Arguments

Х

a non-negative numeric vector

### Value

a numeric vector with the following named components:

- k estimated parameter of shape,
- s estimated parameter of scale.

### References

Zhang J., Stevens M.A., A New and Efficient Estimation Method for the Generalized Pareto Distribution, Technometrics 51(3), 2009, 316-325.

### See Also

Other Pareto2: dpareto2, pareto2\_estimate\_mle, pareto2\_test\_f, ppareto2, qpareto2, rpareto2

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| pareto2_test_f Two-Sample F-test For Equality of Shape Pareto Distributions | Parameters for Type II- |  |
|---|-------------------------|--|
|---|-------------------------|--|

### **Description**

Performs F-test for equality of shape parameters of two samples from the Pareto type-II distributions with known and equal scale parameters, s > 0.

### Usage

```
pareto2_test_f(x, y, s,
   alternative = c("two.sided", "less", "greater"),
   significance = NULL)
```

### **Arguments**

| X            | a non-negative numeric vector   |
|--------------|---|
| у            | a non-negative numeric vector   |
| S            | the known scale parameter, $s>0$  |
| alternative  | indicates the alternative hypothesis and must be one of "two.sided" (default), "less", or "greater" |
| significance | significance level, $0 < \text{significance} < 1$ or NULL. See the Value section for details        |

### **Details**

Given two samples  $(X_1,...,X_n)$  i.i.d.  $P2(k_x,s)$  and  $(Y_1,...,Y_m)$  i.i.d.  $P2(k_y,s)$  this test verifies the null hypothesis  $H_0: k_x = k_y$  against two-sided or one-sided alternatives, depending on the value of alternative. It bases on test statistic  $T(X,Y) = \frac{n\sum_{i=1}^m \log(1+Y_i/m)}{m\sum_{i=1}^n \log(1+X_i/n)}$  which, under  $H_0$ , has the Snedecor's F distribution with (2m,2n) degrees of freedom.

Note that for  $k_x < k_y$ , then X dominates Y stochastically.

### Value

If significance is not NULL, then the list of class power. htest with the following components is passed as a result:

- statistic the value of the test statistic.
- result either FALSE (accept null hypothesis) or TRUE (reject).
- alternative a character string describing the alternative hypothesis.
- method a character string indicating what type of test was performed.
- data.name a character string giving the name(s) of the data.

Otherwise, the list of class htest with the following components is passed as a result:

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- statistic the value of the test statistic.
- p.value the p-value of the test.
- alternative a character string describing the alternative hypothesis.
- method a character string indicating what type of test was performed.
- data.name a character string giving the name(s) of the data.

#### See Also

Other Pareto2: dpareto2, pareto2\_estimate\_mle, pareto2\_estimate\_mmse, ppareto2, qpareto2, rpareto2

plot\_producer

Draws a Graphical Representation of a Given Vector

### **Description**

Draws a step function that represents given numeric vector with elements in  $[0, \infty]$ .

### Usage

```
plot_producer(x,
   type = c("left.continuous", "right.continuous", "curve"),
   extend = FALSE, add = FALSE, pch = 1, col = 1, lty = 1,
   lwd = 1, cex = 1, col.steps = col, lty.steps = 2,
   lwd.steps = 1, xlab = "", ylab = "", main = "",
   xmarg = 10, xlim = c(0, length(x) * 1.2),
   ylim = c(0, max(x)), ...)
```

#### **Arguments**

```
non-negative numeric vector
Х
type
                  character; type of the graphical 'left.continuous' (the default) or 'right.continuous'
                  for step functions and 'curve' for a continuous step curve
                  logical; should the plot be extended infinitely to the right? Defaults to FALSE
extend
add
                  logical; indicates whether to start a new plot, FALSE by default
pch,col,lty,lwd,cex,xmarg
                  graphical parameters
col.steps,lty.steps,lwd.steps
                  graphical parameters, used only for type of 'left.continuous' and 'right.continuous'
                  only
ylim,xlim,xlab,ylab,main,...
                  additional graphical parameters, see plot.default
```

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### **Details**

In **agop**, a given vector  $x = (x_1, \dots, x_n)$  can be represented by a step function defined for  $0 \le y < n$  and given by:

$$\pi(y) = x_{(n-\lfloor y+1\rfloor+1)}$$

(for type == 'right.continuous') or for  $0 < y \le n$ 

$$\pi(y) = x_{(n-\lfloor y \rfloor + 1)}$$

(for type == 'left.continuous', the default) or by a curve joining the points  $(0,x_{(n)})$ ,  $(1,x_{(n)})$ ,  $(1,x_{(n-1)})$ ,  $(2,x_{(n-1)})$ , ...,  $(n,x_{(1)})$ . Here,  $x_{(i)}$  denotes the i-th smallest value in x.

In bibliometrics, a step function of one of the two above-presented types is called a citation function.

For historical reasons, this function is also available via its alias, plot.citfun [but its usage is deprecated].

#### Value

nothing interesting

### **Examples**

```
john_s <- c(11,5,4,4,3,2,2,2,2,1,1,1,0,0,0,0)
plot_producer(john_s, main="Smith, John", col="red")</pre>
```

pord\_weakdom

Weak Dominance Relation (Preorder)

#### **Description**

```
x \le y iff nx \le ny AND for all i = 1, ..., n x_{(n-i+1)} \le y_{(m-i+1)}, where nx = length(x) and ny = length(y).
```

### Usage

```
pord_weakdom(x, y)
```

### **Arguments**

x numeric vector y numeric vector

### **Details**

This function accepts only vectors with nonnegative elements.

#### Value

single logical value; whether  $x \le y$ 

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### See Also

Other binary\_relations: closure\_total\_fair, closure\_transitive, de\_transitive, get\_equivalence\_classes, get\_incomparable\_pairs, get\_independent\_sets, is\_reflexive, is\_total, is\_transitive, rel\_graph

rel\_graph

Create Adjacency Matrix of Given Binary Relation

### **Description**

Note that adjacency matrix can also be conceived as a directed graph (DAG). ret[i,j] iff  $i \le j$ 

### Usage

```
rel_graph(x, pord, ...)
```

### Arguments

x list with elements to compare, preferrably named pord function with 2 arguments, returning boolean value additional arguments passed to pord

### Value

```
square 0-1 Matrix (of class Matrix)
```

### See Also

Other binary\_relations: closure\_total\_fair, closure\_transitive, de\_transitive, get\_equivalence\_classes, get\_incomparable\_pairs, get\_independent\_sets, is\_reflexive, is\_total, is\_transitive, pord\_weakdom

rpareto2

Pareto Type-II (Lomax) Distribution

### Description

Density, cumulative distribution function, quantile function, and random generation for the Pareto Type-II (Lomax) distribution with shape parameter k > 0 and scale parameter s > 0.

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### Usage

```
rpareto2(n, k = 1, s = 1)

ppareto2(q, k = 1, s = 1, lower.tail = TRUE)

qpareto2(p, k = 1, s = 1, lower.tail = TRUE)

dpareto2(x, k = 1, s = 1)
```

### **Arguments**

| p vector of probabilities $\begin{array}{ll} \text{n} & \text{integer; number of observations} \\ \text{k} & \text{vector of shape parameters, } k>0 \\ \text{s} & \text{vector of scale parameters, } s>0 \\ \\ \text{lower.tail} & \text{logical; if TRUE (default), probabilities are } P(X\leq x) \text{, and } P(X>x) \text{ otherwise} \\ \end{array}$ | x,q        | vector of quantiles  |
|--|------------|--|
| k vector of shape parameters, $k>0$ s vector of scale parameters, $s>0$  | p          | vector of probabilities  |
| s vector of scale parameters, $s > 0$  | n          | integer; number of observations  |
| •  | k          | vector of shape parameters, $k > 0$  |
| lower.tail logical; if TRUE (default), probabilities are $P(X \leq x)$ , and $P(X > x)$ otherwise  | S          | vector of scale parameters, $s > 0$  |
|  | lower.tail | logical; if TRUE (default), probabilities are $P(X \leq x)$ , and $P(X > x)$ otherwise |

### **Details**

If 
$$X\sim P2(k,s)$$
, then  ${\rm supp}\,X=[0,\infty).$  The c.d.f. for  $x\ge 0$  is given by 
$$F(x)=1-s^k/(s+x)^k$$
 and the density by 
$$f(x)=ks^k/(s+x)^{k+1}.$$

### Value

numeric vector; dpareto2 gives the density, ppareto2 gives the cumulative distribution function, qpareto2 calculates the quantile function, and rpareto2 generates random deviates.

### See Also

Other Pareto2: pareto2\_estimate\_mle, pareto2\_estimate\_mmse, pareto2\_test\_f

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