Abstract Interpretation for Code Analysis and Verification

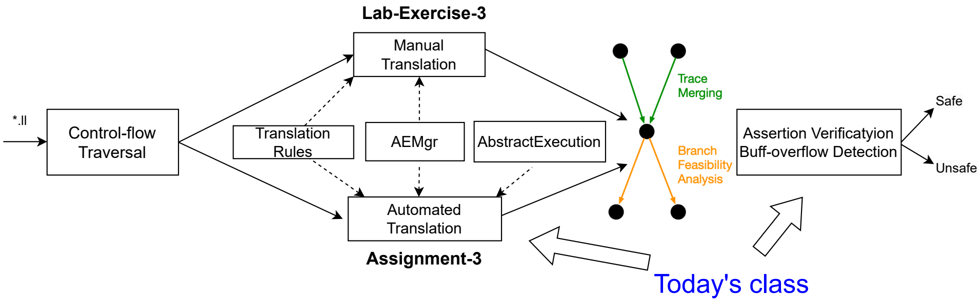
(Week 9)

Yulei Sui

School of Computer Science and Engineering

University of New South Wales, Australia

Today's class



Topological Order

Analysis Order of Nodes on Control-Flow Graph

? How to analyze a program **free of loop**?

✓ Analyze each node **once** adhering to the **topological order** on the acyclic control-flow graph of the program.

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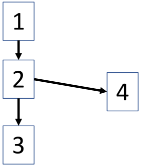
A **topological order** of a graph $G(V, E)$ is a linear ordering of its nodes such that for every directed edge $a \rightarrow b$, node a always precedes node b in the ordering.

- Must be a **direct acyclic graph** (DAG) and has at least one topo ordering.
- The ordering respects the **direction of edges.**

Example of topological order:



acyclic graph G

1 2 3 4 ✓

1 2 4 3 ✓

1 3 2 4 ✗

Valid/invalid topological order

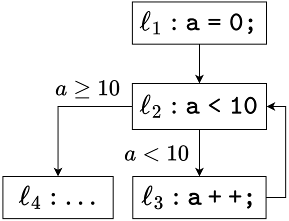
Weak Topological Order

Analysis Order of Nodes on Control-Flow Graph

❓ How to analyze a program **containing loops**?

✔ We can analyze a program containing loops adhering to the **weak topological order** (WTO) on its control flow graph.

What is the weak topological order?



Control Flow Graph

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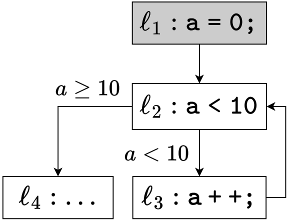
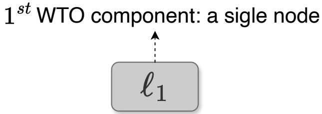


Control Flow Graph



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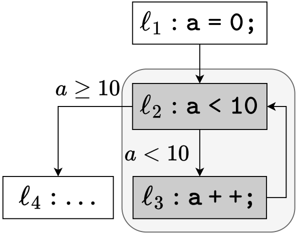

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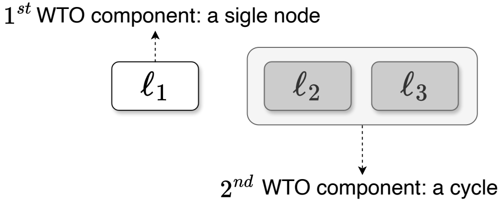

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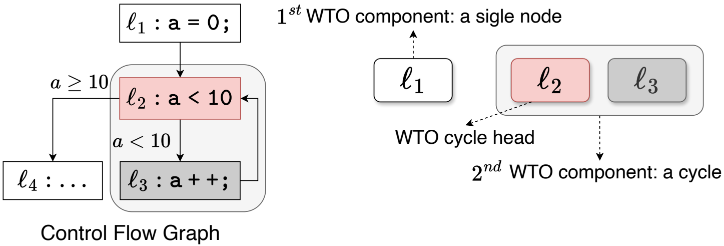

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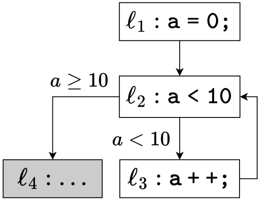
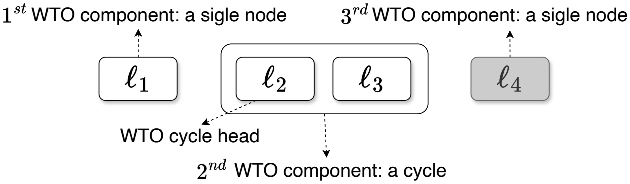


Control Flow Graph



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Analysis Order of Nodes on Control-Flow Graph

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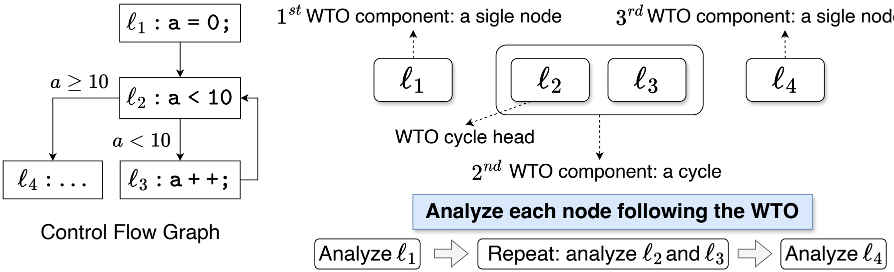

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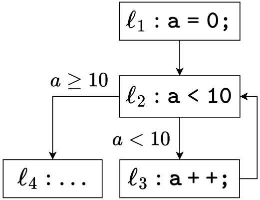
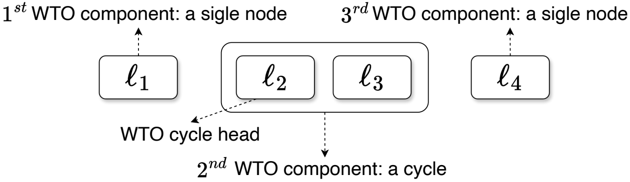


Control Flow Graph



Analyze each node following the WTO

Analyze ℓ_1 ⇨ Repeat: analyze ℓ_2 and ℓ_3 ⇨ Analyze ℓ_4

Perform widening when analyzing the cycle heads (i.e., ℓ_2)

WTO, Widening and Narrowing

Why Weak Topological Order?

- Handling cyclic dependencies
- Efficient fixed-point computation

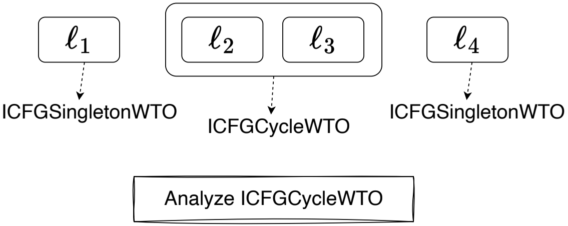
Why Widening?

- Over-approximation
- Prevent non-termination

Why Narrowing?

- Refine precision after widening converges
- The specific conditions or constraints used for narrowing:
  - Loop exit conditions (this course)
  - Type constraints (8-bit integer ranging from [-128, 127])
  - Bounds from arithmetic operations If `x = y + z`, and $\mathtt{y} \in [1, 5]$ and $\mathtt{z} \in [2, 3]$, then $\mathtt{x} \in [3, 8]$. If widening gives [1, 10], narrowing can refine this to [3, 8].
  - User-specification (assertions and guard conditions)

Widening and Narrowing



Algorithm 1: Abstract execution guided by WTO (part 2)

```
Function handleCycleWTO(cycle):
    cycle_head := cycle→head()→node() ;
    increasing := true ;
    cur_iter := 0 ;
    while true do
        cur_iter++;
        if cur_iter ≥ Options :: WidenDelay() then
            prev_head_state := postAbsTrace[cycle_head];
            handleSingletonWTO(cycle→head());
            cur_head_state := postAbsTrace[cycle_head];
            if increasing then
                postAbsTrace[cycle_head] := prev_head_state.widen(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    increasing := false;
                    continue;
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                postAbsTrace[cycle_head] := prev_head_state.narrow(cur_head_state) ;
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                    break ;
        else
            handleSingletonWTO(cycle→head());
        handleWTOComponents(cycle→getWTOComponents());
```
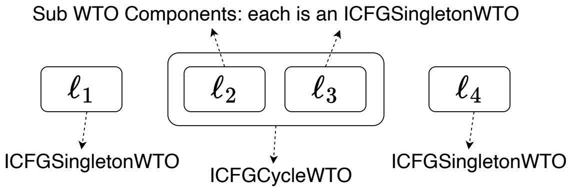

Widening and Narrowing



Analyze ICFGCycleWTO

Algorithm 2: Abstract execution guided by WTO (part 2)

```
Function handleCycleWTO(cycle):
    cycle_head := cycle→head()→node() ;
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    while true do
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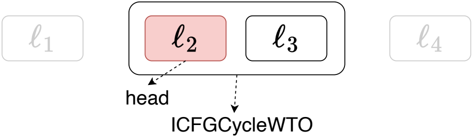
Weak Topological Order



Get ICFGCycleWTO head ℓ_2

Algorithm 3: Abstract execution guided by WTO (part 2)

```
Function handleCycleWTO(cycle):
    cycle_head := cycle→head()→node() ;
    increasing := true ;
    cur_iter := 0 ;
    while true do
        cur_iter++;
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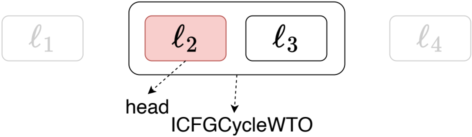
Weak Topological Order



Handle ICFGCycleWTO head ℓ_2

Algorithm 4: Abstract execution guided by WTO (part 2)

```
Function handleCycleWTO(cycle):
    cycle_head := cycle→head()→node() ;
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    cur_iter := 0 ;
    while true do
        cur_iter++;
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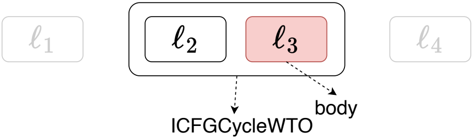
Widening and Narrowing



Handle ICFGCycleWTO body ℓ_3

Algorithm 5: Abstract execution guided by WTO (part 2)

```
Function handleCycleWTO(cycle):
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    while true do
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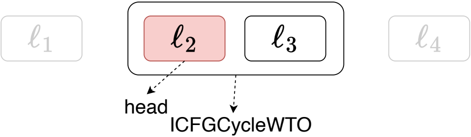
```

Note: getIWTOcomponents returns Cycle WTO body, i.e., ℓ_3

Widening and Narrowing



When $cur\_iter \geq Options :: WidenDelay()$

perform widening on ℓ_2

Algorithm 6: Abstract execution guided by WTO (part 2)

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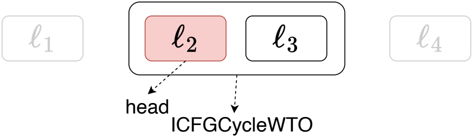
Widening and Narrowing



Widening reaches a fixpoint

Algorithm 7: Abstract execution guided by WTO (part 2)

```
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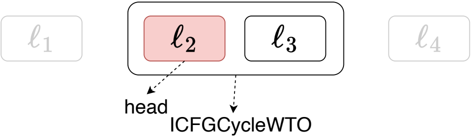
Widening and Narrowing



Perform narrowing on ℓ_2

Algorithm 8: Abstract execution guided by WTO (part 2)

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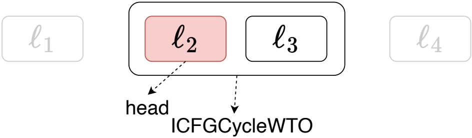
Widening and Narrowing



Narrowing reaches a fixpoint, Finish!

Algorithm 9: Abstract execution guided by WTO (part 2)

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Abstract Interpretation on SVFIR

Week 9

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University of New South Wales, Australia

Abstract Interpretation on Pointer-Free SVFIR

Interval Domain

- For simplicity, let's first consider abstract execution on a pointer-free language.
- This means there are no operations for memory allocation (like $\mathtt{p} = \mathtt{alloc_o}$) or for indirect memory accesses (such as `p = *q` or `*p = q`).
- Here are the pointer-free SVFSTMTs and their C-like forms:

SVFSTMT	**C-Like form**
CONSSTMT	$\ell : \mathtt{p} = \mathtt{c}$
COPYSTMT	$\ell : \mathtt{p} = \mathtt{q}$
BINARYSTMT	$\ell : \mathtt{r} = \mathtt{p} \otimes \mathtt{q}$
PHISTMT	$\ell : \mathtt{r} = \mathtt{phi}(\mathtt{p_1}, \mathtt{p_2}, \ldots, \mathtt{p_n})$
SEQUENCE	$\ell_1; \ell_2$
BRANCHSTMT	ℓ_1 : if$(x < c)$ then ℓ_2 else ℓ_3

Abstract Interpretation on Pointer-Free SVFIR

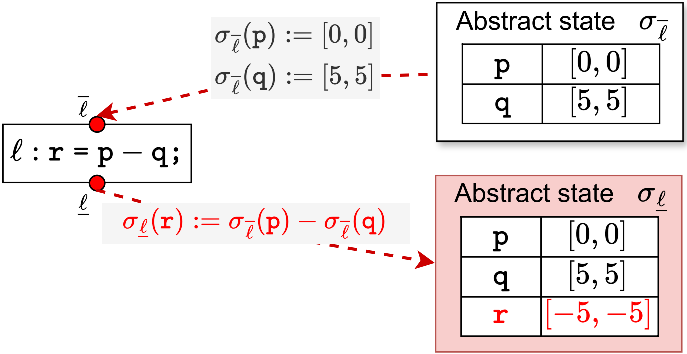
Interval Domain

Let's use the *Interval* abstract domain to update σ based on the following rules for different SVFSTMT:

SVFSTMT	**C-Like form**	**Abstract Execution Rule**
CONSSTMT	$\ell : \mathtt{p} = \mathtt{c}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := [\mathtt{c}, \mathtt{c}]$
COPYSTMT	$\ell : \mathtt{p} = \mathtt{q}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \sigma_{\overline{\ell}}(\mathtt{q})$
BINARYSTMT	$\ell : \mathtt{r} = \mathtt{p} \otimes \mathtt{q}$	$\sigma_{\underline{\ell}}(r) := \sigma_{\overline{\ell}}(p) \hat{\otimes} \sigma_{\overline{\ell}}(q)$
PHISTMT	$\ell : \mathtt{r} = \mathtt{phi}(\mathtt{p}_1, \mathtt{p}_2, \ldots, \mathtt{p}_n)$	$\sigma_{\underline{\ell}}(r) := \bigsqcup_{i=1}^{n} \sigma_{\overline{\ell}}(p_i)$
SEQUENCE	$\ell_1 ; \ell_2$	$\forall v \in \mathbb{V}, \sigma_{\overline{\ell_2}}(v) \sqsupseteq \sigma_{\underline{\ell_1}}(v)$
BRANCHSTMT	ℓ_1 : if($x < c$) then ℓ_2 else ℓ_3	$\sigma_{\overline{\ell_2}}(x) := \sigma_{\underline{\ell_1}}(x) \sqcap [-\infty, c-1],\ \text{if } \sigma_{\underline{\ell_1}}(x) \sqcap [-\infty, c-1] \neq \bot$ $\sigma_{\overline{\ell_3}}(x) := \sigma_{\underline{\ell_1}}(x) \sqcap [c, +\infty],\ \text{if } \sigma_{\underline{\ell_1}}(x) \sqcap [c, +\infty] \neq \bot$

Abstract Interpretation on BINARYSTMT

SVFSTMT	C-Like form	Abstract Execution Rule
BINARYSTMT	$\ell : \mathtt{r} = \mathtt{p} \otimes \mathtt{q}$	$\sigma_{\underline{\ell}}(r) := \sigma_{\overline{\ell}}(p) \hat{\otimes} \sigma_{\overline{\ell}}(q)$

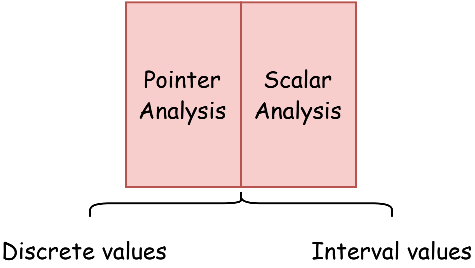


Abstract Interpretation in the Presence of Pointers

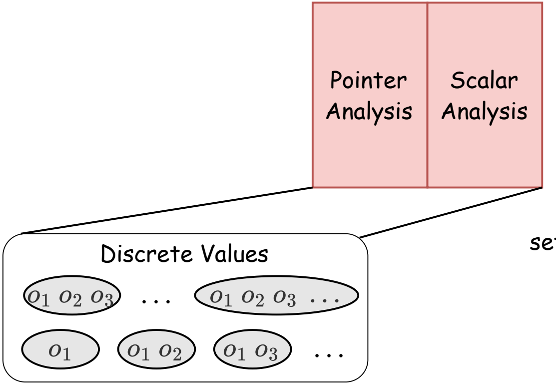
- SVFIR in the presence of pointers contain pointer-related statements including ADDRSTMT, GEPSTMT, LOADSTMT and STORESTMT.
- Abstract interpretation needs to be performed on **a combined domain of intervals and addresses**.

SVFSTMT	**C-Like form**
CONSSTMT	$\ell : \mathtt{p} = \mathtt{c}$
COPYSTMT	$\ell : \mathtt{p} = \mathtt{q}$
BINARYSTMT	$\ell : \mathtt{r} = \mathtt{p} \otimes \mathtt{q}$
PHISTMT	$\ell : \mathtt{r} = \mathtt{phi}(\mathtt{p_1}, \mathtt{p_2}, \ldots, \mathtt{p_n})$
SEQUENCE	$\ell_1 ; \ell_2$
BRANCHSTMT	ℓ_1 : if$(x < c)$ then ℓ_2 else ℓ_3
ADDRSTMT	$\ell : \mathtt{p} = \mathtt{alloc}$
GEPSTMT	$\ell : \mathtt{p} = \&(\mathtt{q} \rightarrow \mathtt{i})$ or $\mathtt{p} = \&\mathtt{q}[\mathtt{i}]$
LOADSTMT	$\ell : \mathtt{p} = *\mathtt{q}$
STORESTMT	$\ell : *\mathtt{p} = \mathtt{q}$

Combined Analysis



Combined Analysis Using Discrete Values



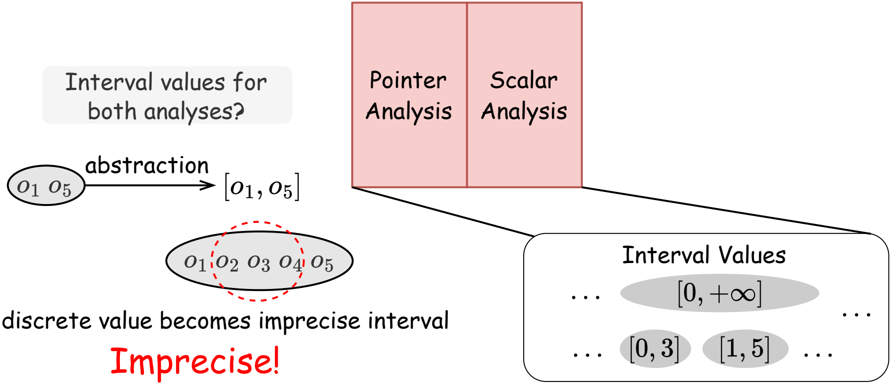
Discrete values for both analyses?

{1, 2, 3, ..., infinite numbers}

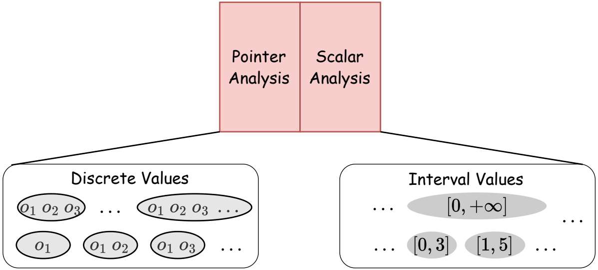
set of all possible values with inputs

Unscalable!

Combined Analysis Using Interval Values



Abstract Interpretation Over a Combined Domain

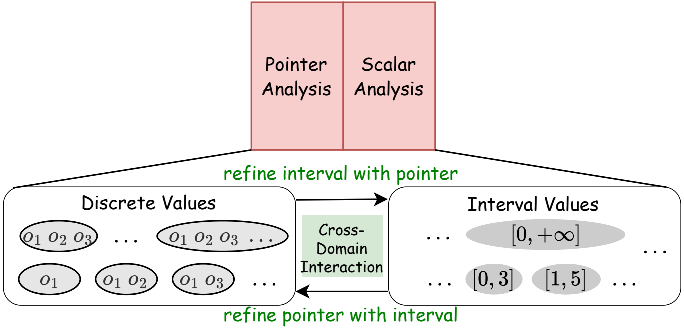



```
p = malloc(m*sizeof(int)); // p points to an array of size m
q = malloc(n*sizeof(int)); // q points to an array of size n
```

```
m = r[i];
```

- The discrete values for points-to set of `p`, `q` depend on interval values of `m` and `n`.
- The interval value of `m` depends on the pointer aliasing between `p`, `q` and `&r[i]`.
- Cyclic dependency between two domains requiring a bi-directional refinement. (variables highlighted in blue and red denote the discrete values and interval values dependent),

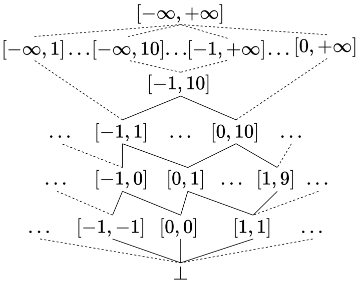
Abstract Interpretation Over a Combined Domain



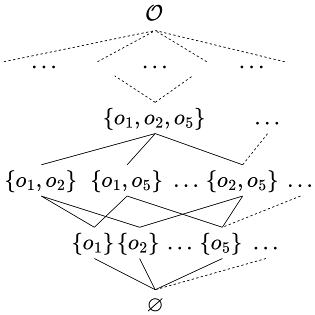
We require **a combination of interval and memory address domains** to precisely and efficiently perform abstract execution on SVFIR in the presence of pointers.

Abstract Interpretation over Interval and MemAddress Domains

A Combined Domain of Intervals and Discrete Memory Addresses



Interval domain for scalar variables



MemAddress domain for discrete memory address values

SVF Program Variables (SVFVar)

Program Variables	Domain	Meanings
`SVFVar`	$\mathbb{V} = \mathbb{P} \cup \mathbb{O}$	Program Variables
`ValVar`	$\mathbb{P}$	Top-level variables (scalars and pointers)
`ObjVar`	$\mathbb{O} = \mathbb{S} \cup \mathbb{G} \cup \mathbb{H} \cup \mathbb{C}$	Memory Objects (constant data, stack, heap, global) (function objects are considered as global objects)
`FIObjVar`	$o \in (\mathbb{S} \cup \mathbb{G} \cup \mathbb{H})$	A single (base) memory object
`GepObjVar`	$o_i \in (\mathbb{S} \cup \mathbb{G} \cup \mathbb{H}) \times \mathbb{P}$	i-th subfield/element of an (aggregate) object
`ConstantData`	$\mathbb{C}$	Constant data (e.g., numbers and strings)
Program Statement	$\ell \in \mathbb{L}$	Statements labels

Abstract Trace for The Combined Domain

- For top-level variables $\mathbb{P}$, we use $\sigma \in \mathbb{L} \times \mathbb{P} \rightarrow \mathit{Interval} \times \mathit{MemAddress}$ to track the memory addresses or interval values of these variables.
- For memory objects $\mathbb{O}$, we use $\delta \in \mathbb{L} \times \mathbb{O} \rightarrow \mathit{Interval} \times \mathit{MemAddress}$ to track their abstract values

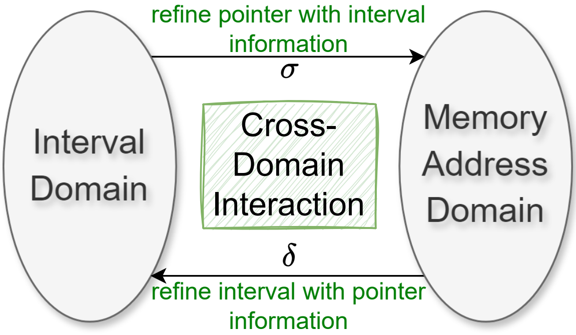
	Notation	Domain	Data Structure Implementation
Abstract trace	σ	$\mathbb{L} \times \mathbb{P} \rightarrow \mathit{Interval} \times \mathit{MemAddress}$	*preAbsTrace*, *postAbsTrace*
	δ	$\mathbb{L} \times \mathbb{O} \rightarrow \mathit{Interval} \times \mathit{MemAddress}$	
Abstract state	σ_L	$\mathbb{P} \rightarrow \mathit{Interval} \times \mathit{MemAddress}$	*AbstractState.varToAbsVal*
	δ_L	$\mathbb{O} \rightarrow \mathit{Interval} \times \mathit{MemAddress}$	*AbstractState.addrToAbsVal*
Abstract value	$\sigma_L(p)$	$\mathit{Interval} \times \mathit{MemAddress}$	*AbstractValue*
	$\delta_L(o)$		

- *Interval* is used for tracking the interval value of **scalar variables** $\mathbb{P}$.
- *MemAddress* is used for tracking the memory addresses of **memory address variables** $\mathbb{O}$.

Implementation of Abstract Trace and State in Assignment-3

- For a program point L, the abstract state is an instance of class *AEState*, consisting of:
  - Top-level variable, *varToAbsVal* : $\sigma_L \in \mathbb{P} \rightarrow \mathit{Interval} \times \mathit{MemAddress}$
  - Memory object, *addrToAbsVal* : $\delta_L \in \mathit{MemAddress} \rightarrow \mathit{Interval} \times \mathit{MemAddress}$
- The abstract trace has two maps, *preAbsTrace* and *postAbsTrace*, which maintains abstract states before and after each `ICFGNode` respectively.
  - For an `ICFGNode` ℓ, *preAbsTrace*(ℓ) retrieves the abstract state $\langle \sigma_{\overline{\ell}}, \delta_{\overline{\ell}} \rangle$, and *postAbsTrace*$(\ell)$ represents $\langle \sigma_{\underline{\ell}}, \delta_{\underline{\ell}} \rangle$.
  - For each abstract state $\langle \sigma_{\overline{\ell}}, \delta_{\overline{\ell}} \rangle$ we use `as[varId]` to operate $\sigma_{\underline{\ell}}$ and use `storeValue` and `loadValue` to operate $\delta_{\underline{\ell}}$.
  - Each variable's `AbstractValue` (e.g., `as[VarId]`) is initialized as $\perp$ in an `AbstractState` before assigned a new value.
  - Each `AbstractValue` (e.g., `as[VarId]`) is a 2-element tuple consisting of an interval `as[VarId].getInterval()` and an address set `as[Varid].getAddrs()`.
  - Print out `SVFVars` and their `AbstractValues` in an `AbstractState` by invoking `as.printAbstractState()`

Abstract Trace for The Combined Domain



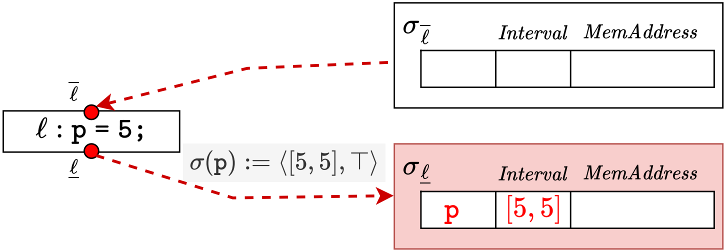
Abstract Execution Rules on SVFIR in the Presence of Pointers

Now let's use the *Interval* $\times$ *MemAddress* abstract domain to update σ and δ based on the following rules for different SVFSTMT:

SVFSTMT	C-Like form	Abstract Execution Rule
CONSSTMT	$\ell : \mathtt{p} = \mathtt{c}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \langle[\mathtt{c}, \mathtt{c}], \bot\rangle$
COPYSTMT	$\ell : \mathtt{p} = \mathtt{q}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \sigma_{\overline{\ell}}(\mathtt{q})$
BINARYSTMT	$\ell : \mathtt{r} = \mathtt{p} \otimes \mathtt{q}$	$\sigma_{\underline{\ell}}(r) := \sigma_{\overline{\ell}}(p) \hat{\otimes} \sigma_{\overline{\ell}}(q)$
PHISTMT	$\ell : \mathtt{r} = \mathtt{phi}(\mathtt{p}_1, \mathtt{p}_2, \ldots, \mathtt{p}_n)$	$\sigma_{\underline{\ell}}(r) := \bigsqcup_{i=1}^{n} \sigma_{\overline{\ell}}(p_i)$
BRANCHSTMT	ℓ_1 : if$(x < c)$ then ℓ_2 else ℓ_3	$\sigma_{\overline{\ell_2}}(x) := \sigma_{\underline{\ell_1}}(x) \sqcap [-\infty, c-1]$, if $\sigma_{\underline{\ell_1}}(x) \sqcap [-\infty, c-1] \neq \bot$ $\sigma_{\overline{\ell_3}}(x) := \sigma_{\underline{\ell_1}}(x) \sqcap [c, +\infty]$, if $\sigma_{\underline{\ell_1}}(x) \sqcap [c, +\infty] \neq \bot$
SEQUENCE	$\ell_1; \ell_2$	$\delta_{\overline{\ell_2}} \sqsupseteq \delta_{\underline{\ell_1}}, \sigma_{\overline{\ell_2}} \sqsupseteq \sigma_{\underline{\ell_1}}$
ADDRSTMT	$\ell : \mathtt{p} = \mathtt{alloc}_{\mathtt{o_i}}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \langle \top, \{o_i\} \rangle$
GEPSTMT	$\ell : \mathtt{p} = \&(\mathtt{q} \rightarrow \mathtt{i})$ or $\mathtt{p} = \&\mathtt{q}[\mathtt{i}]$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \bigsqcup_{\mathtt{o} \in \gamma(\sigma_{\overline{\ell}}(\mathtt{q}))} \bigsqcup_{j \in \gamma(\sigma_{\overline{\ell}}(\mathtt{i}))} \langle \top, \{\mathtt{o.fld}_j\} \rangle$
LOADSTMT	$\ell : \mathtt{p} = *\mathtt{q}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \bigsqcup_{o \in \{o \mid o \in \sigma_{\overline{\ell}}(q)\}} \delta_{\overline{\ell}}(o)$
STORESTMT	$\ell : *\mathtt{p} = \mathtt{q}$	$\delta_{\underline{\ell}} := (\{o \mapsto \sigma_{\overline{\ell}}(\mathtt{q}) \mid o \in \gamma(\sigma_{\overline{\ell}}(\mathtt{p}))\} \sqcup \delta_{\underline{\ell}})$

Abstract Execution on ConsStmt

SVFStmt	C-Like form	Abstract Execution Rule
ConsStmt	$\ell : \texttt{p} = \texttt{c}$	$\sigma_{\underline{\ell}}(\texttt{p}) := \langle[\texttt{c}, \texttt{c}], \bot\rangle$



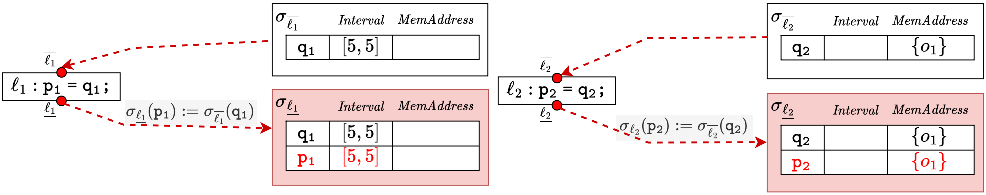
Algorithm 10: Abstract Execution Rule for ConsStmt

Function *updateStateOnAddr(addr)*:

```
node = addr→getICFGNode();
as = getAbsStateFromTrace(node);
initObjVar(as, SVFUtil :: cast⟨ObjVar⟩(addr→getRHSVar()));
as[addr→getLHSVarID()] = as[addr→getRHSVarID()];
```

Abstract Execution on COPYSTMT

SVFSTMT	C-Like form	Abstract Execution Rule
COPYSTMT	$\ell : \texttt{p} = \texttt{q}$	$\sigma_{\underline{\ell}}(\texttt{p}) := \sigma_{\overline{\ell}}(\texttt{q})$

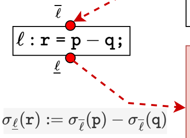


Algorithm 11: Abstract Execution Rule for COPYSTMT

```
Function updateStateOnCopy(copy):
    node = copy→getICFGNode();
    as = getAbsStateFromTrace(node);
    lhs = copy→getLHSVarID();
    rhs = copy→getRHSVarID();
    as[lhs] = as[rhs];
```

Abstract Execution on BINARYSTMT

SVFSTMT	C-Like form	Abstract Execution Rule
BINARYSTMT	$\ell : \mathtt{r} = \mathtt{p} \otimes \mathtt{q}$	$\sigma_{\underline{\ell}}(r) := \sigma_{\overline{\ell}}(p) \hat{\otimes} \sigma_{\overline{\ell}}(q)$



$\sigma_{\overline{\ell}}$	*Interval*	*MemAddress*
p	$[0, 0]$	
q	$[5, 5]$	

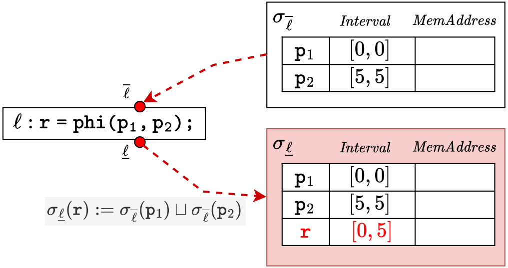
$\sigma_{\underline{\ell}}$	*Interval*	*MemAddress*
p	$[0, 0]$	
q	$[5, 5]$	
r	$[-5, -5]$	

Algorithm 12: Abstract Execution Rule for BINARYSTMT

```
Function updateStateOnBinary(binary):
    node = binary→getICFGNode();
    as = getAbsStateFromTrace(node);
    op0 = binary→getOpVarID(0);
    op1 = binary→getOpVarID(1);
    res = binary→getResID();
    as[res] = as[op0] ⊗̂ as[op1]
```

Abstract Execution on PHISTMT

SVFSTMT	C-Like form	Abstract Execution Rule
PHISTMT	$\ell : \mathtt{r} = \mathtt{phi}(\mathtt{p_1}, \mathtt{p_2}, \ldots, \mathtt{p_n})$	$\sigma_{\underline{\ell}}(r) := \bigsqcup_{i=1}^{n} \sigma_{\overline{\ell}}(p_i)$

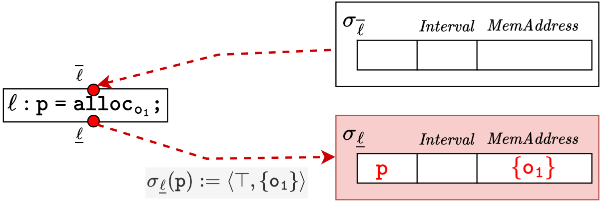


Algorithm 13: Abstract Execution Rule for PHISTMT

```
Function updateStateOnPhi(phi):
    node = phi→getICFGNode();
    as = getAbsStateFromTrace(node);
    res = phi→getResID();
    rhs = AbstractValue();
    for i = 0; i < phi→getOpVarNum(); i++ do
        curId = phi→getOpVarID(i);
        rhs.join_with(as[curId])
    as[res] = rhs
```

Abstract Execution on AddrStmt

SVFStmt	C-Like form	Abstract Execution Rule
AddrStmt	$\ell : \mathtt{p} = \mathtt{alloc}_{\mathtt{o_i}}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \langle \top, \{o_i\} \rangle$



Algorithm 14: Abstract Execution Rule for AddrStmt

```
Function updateStateOnAddr(addr):
    node = addr→getICFGNode();
    as = getAbsStateFromTrace(node);
    initObjVar(as, SVFUtil :: cast⟨ObjVar⟩(addr→getRHSVar()));
    as[addr→getLHSVarID()] = as[addr→getRHSVarID()];
```

Abstract Execution on GepStmt

SVFStmt	C-Like form	Abstract Execution Rule
GepStmt	$\ell : \mathtt{p} = \&(\mathtt{q} \rightarrow \mathtt{i})$ or $\mathtt{p} = \&\mathtt{q}[\mathtt{i}]$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \bigsqcup_{o\in\gamma(\sigma_{\overline{\ell}}(\mathtt{q}))} \bigsqcup_{j\in\gamma(\sigma_{\overline{\ell}}(\mathtt{i}))} \langle \top, \{\mathtt{o.fld}_j\} \rangle$

$\overline{\ell}$

ℓ : `p = &(q[i]);`

$\underline{\ell}$

$\sigma_{\underline{\ell}}(\mathtt{p}) := \langle \top, \{\mathtt{o_1.fld_1}\} \rangle \sqcup \langle \top, \{\mathtt{o_1.fld_2}\} \rangle$

$\sigma_{\overline{\ell}}$

	Interval	MemAddress
q		$\{\mathtt{o_1}\}$
i	$[1, 2]$	

$\sigma_{\underline{\ell}}$

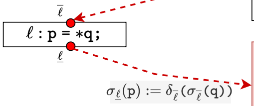
	Interval	MemAddress
q		$\{\mathtt{o_1}\}$
i	$[1, 2]$	
p		$\{\mathtt{o_1.fld_1}, \mathtt{o_1.fld_2}\}$

Algorithm 15: Abstract Execution Rule for GepStmt

```
Function updateStateOnGep(gep):
    node = gep→getICFGNode();
    as = getAbsStateFromTrace(node);
    rhs = gep→getRHSVarID();
    lhs = gep→getLHSVarID();
    as[lhs] = as.getGepObjAddrs(rhs, as.getElementIndex(gep));;
```

Abstract Execution on LOADSTMT

SVFSTMT	C-Like form	Abstract Execution Rule
LOADSTMT	$\ell : \texttt{p} = *\texttt{q}$	$\sigma_{\underline{\ell}}(\texttt{p}) := \bigsqcup_{o \in \{o \mid o \in \sigma_{\overline{\ell}}(q)\}} \delta_{\overline{\ell}}(o)$



$\sigma_{\overline{\ell}}$	*Interval*	*MemAddress*
q		$\{o_1\}$

$\delta_{\overline{\ell}}$	*Interval*	*MemAddress*
o_1	$[5, 5]$	

$\sigma_{\underline{\ell}}$	*Interval*	*MemAddress*
q		$\{o_1\}$
p	$[5, 5]$	

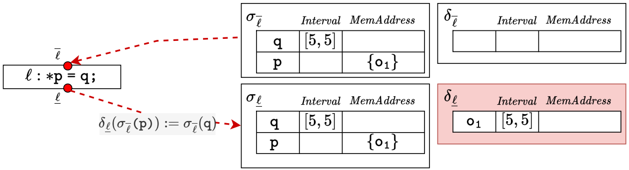
$\delta_{\underline{\ell}}$	*Interval*	*MemAddress*
o_1	$[5, 5]$	

Algorithm 16: Abstract Execution Rule for LOADSTMT

```
Function updateStateOnLoad(load):
    node = load→getICFGNode();
    as = getAbsStateFromTrace(node);
    rhs = load→getRHSVarID();
    lhs = load→getLHSVarID();
    as[lhs] = as.loadValue(rhs)
```

Abstract Execution on STORESTMT

SVFSTMT	C-Like form	Abstract Execution Rule
STORESTMT	$\ell : *\mathtt{p} = \mathtt{q}$	$\delta_{\underline{\ell}} := (\{o \mapsto \sigma_{\overline{\ell}}(\mathtt{q}) \mid o \in \gamma(\sigma_{\overline{\ell}}(\mathtt{p}))\} \sqcup \delta_{\underline{\ell}})$



Algorithm 17: Abstract Execution Rule for STORESTMT

Function *updateStateOnStore(store)*:

```
node = store→getICFGNode();
as = getAbsStateFromTrace(node);
rhs = store→getRHSVarID();
lhs = store→getLHSVarID();
as.storeValue(lhs, as[rhs])
```

Overall Algorithm of Abstract Interpretation

Algorithm 1: Analyse from main function

```
Function analyse() // driver function to start the analysis:
    initWTO();
    handleGlobalNode();
    if getSVFFunction (main) then
        wto := funcToWTO[main];
        handleWTOComponents(wto → getWTOComponents());
```

Algorithm 2: Handle WTO components

```
Function handleWTOComponents (wtoComps):
    for wtoNode ∈ wtoComps do
        if node = SVFUtil :: dyn_cast⟨ICFGSingletonWTO⟩(wtoNode) then
            handleSingletonWTO(node)
        else if cycle = SVFUtil :: dyn_cast⟨ICFGCycleWTO⟩(wtoNode) then
            handleCycleWTO(cycle)
        else
            assert(false&&"unknownWTOtype!")
```

Algorithm 3: Handle Singleton WTO

```
Function handleSingletonWTO (singletonWTO):
    node := singletonWTO → node();
    feasible := mergeStatesFromPredecessors(node, preAbsTrace[node]);
    if feasible then
        postAbsTrace[node] := preAbsTrace[node];
    else
        return;
    foreach stmt ∈ node → getSVFStmts() do
        updateAbsState(stmt);
        bufOverflowDetection(stmt);
    if callnode = SVFUtil :: dyn_cast⟨CallICFGNode⟩(node) then
        funName := callnode → getCallSite() → getCallee() → getName()
        if funName == "OVERFLOW"&&funName == "svf_assert" then
            // Handle svf_assert and OVERFLOW stub function for
            correctness validation;
            handleStubFunctions(callnode);
        else
            // Does not analyze recursive functions in this course;
            handleCallSite(callnode);
```

Overall Algorithm of Abstract Interpretation

Algorithm 4: Handle Cycle WTO

```
Function handleCycleWTO (cycle):
    feasible := mergeStatesFromPredecessors(cycle_head, preAbsTrace[cycle_head]);
    increasing := true;
    if !feasible then
        return;
    else
        cur_iter := 0;
        while true do
            if cur_iter >= Options.WidenDelay() then
                prev_head_as := postAbsTrace[cycle_head];
                handleSingletonWTO(cycle.head());
                cur_head_as := postAbsTrace[cycle_head];
                if increasing then
                    postAbsTrace[cycle_head] := prev_head_as.widening(cur_head_as);
                    if postAbsTrace[cycle_head] == prev_head_as then
                        increasing := false;
                        Continue;
                else
                    postAbsTrace[cycle_head] := prev_head_as.narrowing(cur_head_as);
                    if postAbsTrace[cycle_head] == prev_head_as then
                        Break;
            else
                handleSingletonWTO(cycle.head());
            cur_iter ++;
```

An Example: Abstract Trace σ for Top-level Variables

```c
extern void assert(int);

int main(){
    int a = 0;
    while(a < 10) {
        a++;
    }
    assert(a == 10);
    return 0;
}
```

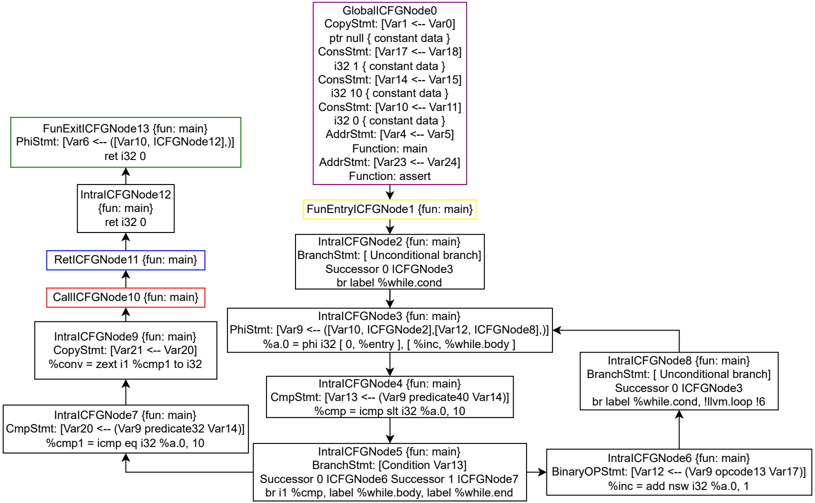
Source Code

Compile to LLVM IR

```llvm
define dso_local i32 @main() {
entry:
  br label %while.cond
while.cond:
  %a.0 = phi i32 [ 0, %entry ], [ %inc, %while.body ]
  %cmp = icmp slt i32 %a.0, 10
  br i1 %cmp, label %while.body, label %while.end
while.body:
  %inc = add nsw i32 %a.0, 1
  br label %while.cond,
while.end:
  %cmp1 = icmp eq i32 %a.0, 10
  %conv = zext i1 %cmp1 to i32
  call void @assert(i32 noundef %conv)
  ret i32 0
}
```

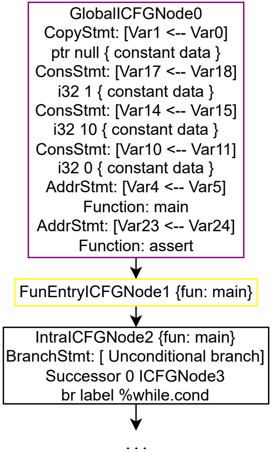
LLVM IR

An Example: Abstract Trace σ for Top-level Variables



ICFG

An Example: Abstract Trace σ for Top-level Variables



ICFG

Algorithm 5: Abstract execution guided by WTO

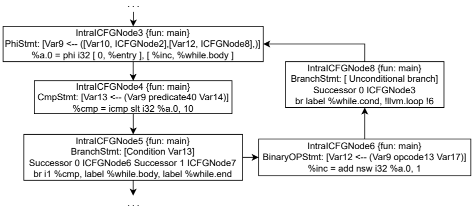
Function `handleStatement`(ℓ)**:**
  tmpAS := *preAbsTrace*[ℓ];
  if ℓ *is* ConsStmt *or* AddrStmt **then**
    `updateStateOnAddr`(ℓ);
  else if ℓ *is* CopyStmt **then**
    `updateStateOnCopy`(ℓ);
  . . . ;

postAbsTrace[ℓ_0].*varToAbsVal* :

SVFVar	AbstractValue$\langle$*Interval*, *MemAddress*$\rangle$
Var0	$\langle\perp, \{0x7f00\}\rangle$
Var1	$\langle\perp, \{0x7f00\}\rangle$
Var18	$\langle[1, 1], \perp\rangle$
Var17	$\langle[1, 1], \perp\rangle$
Var14	$\langle[10, 10], \perp\rangle$
Var15	$\langle[10, 10], \perp\rangle$
Var10	$\langle[0, 0], \perp\rangle$
Var11	$\langle[0, 0], \perp\rangle$
	...

Print out the table via `as.printAbstractState()`. The `AbstractValue` can **either be an interval or addresses**, but not both!

An Example: Abstract Trace σ for Top-level Variables



ICFG

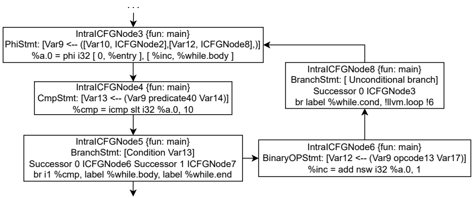
$postAbsTrace[\ell_3].varToAbsVal$:

SVFVar	AbstractValue$\langle Interval, MemAddress\rangle$
	...
$Var10$	$\langle[0,0],\perp\rangle$
$Var9$	$\langle[0,0],\perp\rangle$
	...

Algorithm 12: Handle Cycle WTO

```
Function handleCycleWTO(cycle):
    cycle_head := cycle→head()→node() ;
    increasing := true ;
    cur_iter := 0 ;
    while true do
        if cur_iter ≥ Options :: WidenDelay() then
            prev_head_state := postAbsTrace[cycle_head];
            handleICFGNode(cycle→head());
            cur_head_state := postAbsTrace[cycle_head];
            if increasing then
                postAbsTrace[cycle_head] := prev_head_state.widen(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    increasing := false;
                    continue;
            else
                postAbsTrace[cycle_head] := prev_head_state.narrow(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    break ;
        else
            handleICFGNode(cycle→head());
        handleWTOComponents(cycle→getWTOComponents());
        cur_iter++ // cur_iter ≡ 1, Options :: WidenDelay() ≡ 3;
```

An Example: Abstract Trace σ for Top-level Variables



ICFG

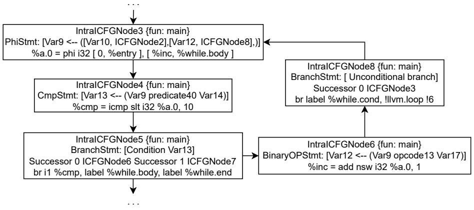
$postAbsTrace[\ell_8].varToAbsVal$:

SVFVar	AbstractValue$\langle Interval, MemAddress \rangle$
	...
$Var10$	$\langle [0,0], \bot \rangle$
$Var9$	$\langle [0,0], \bot \rangle$
$Var12$	$\langle [1,1], \bot \rangle$
	...

Algorithm 12: Handle Cycle WTO

```
Function handleCycleWTO(cycle):
    cycle_head := cycle→head()→node() ;
    increasing := true ;
    cur_iter := 0 // cur_iter ≡ 1, Options :: WidenDelay() ≡ 3;
    while true do
        if cur_iter ≥ Options :: WidenDelay() then
            prev_head_state := postAbsTrace[cycle_head];
            handleICFGNode(cycle→head());
            cur_head_state := postAbsTrace[cycle_head];
            if increasing then
                postAbsTrace[cycle_head] := prev_head_state.widen(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    increasing := false;
                    continue;
            else
                postAbsTrace[cycle_head] := prev_head_state.narrow(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    break ;
        else
            handleICFGNode(cycle→head());
        handleWTOComponents(cycle→getWTOComponents());
        cur_iter++ // cur_iter ≡ 2, Options :: WidenDelay() ≡ 3;
```

An Example: Abstract Trace σ for Top-level Variables



ICFG

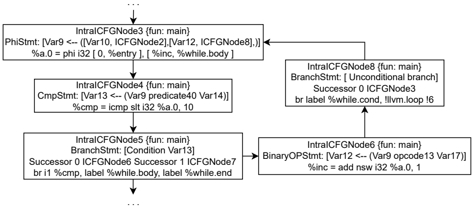
$postAbsTrace[\ell_3].varToAbsVal$:

SVFVar	AbstractValue⟨*Interval*, *MemAddress*⟩
	...
Var9	$\langle[0,1],\perp\rangle$
Var12	$\langle[1,1],\perp\rangle$
	...

Algorithm 12: Handle Cycle WTO

```
Function handleCycleWTO(cycle):
    cycle_head := cycle→head()→node() ;
    increasing := true ;
    cur_iter := 0 ;
    while true do
        if cur_iter ≥ Options :: WidenDelay() then
            prev_head_state := postAbsTrace[cycle_head];
            handleICFGNode(cycle→head());
            cur_head_state := postAbsTrace[cycle_head];
            if increasing then
                postAbsTrace[cycle_head] := prev_head_state.widen(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    increasing := false;
                    continue;
            else
                postAbsTrace[cycle_head] := prev_head_state.narrow(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    break ;
        else
            handleICFGNode(cycle→head());
        handleWTOComponents(cycle→getWTOComponents());
        cur_iter++ // cur_iter ≡ 2,  Options :: WidenDelay() ≡ 3;
```

An Example: Abstract Trace σ for Top-level Variables



ICFG

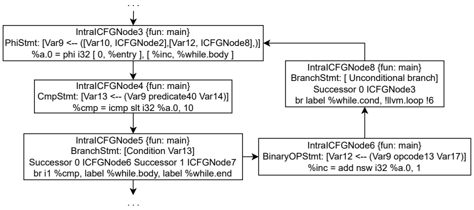
$postAbsTrace[\ell_8].varToAbsVal$:

SVFVar	AbstractValue$\langle Interval, MemAddress \rangle$
	...
$Var9$	$\langle [0, 1], \perp \rangle$
$Var12$	$\langle [1, 2], \perp \rangle$
	...

Algorithm 12: Handle Cycle WTO

```
Function handleCycleWTO(cycle):
    cycle_head := cycle→head()→node() ;
    increasing := true ;
    cur_iter := 0 ;
    while true do
        if cur_iter ≥ Options :: WidenDelay() then
            prev_head_state := postAbsTrace[cycle_head];
            handleICFGNode(cycle→head());
            cur_head_state := postAbsTrace[cycle_head];
            if increasing then
                postAbsTrace[cycle_head] := prev_head_state.widen(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    increasing := false;
                    continue;
            else
                postAbsTrace[cycle_head] := prev_head_state.narrow(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    break ;
        else
            handleICFGNode(cycle→head());
        handleWTOComponents(cycle→getWTOComponents());
        cur_iter++ // cur_iter ≡ 2, Options :: WidenDelay() ≡ 3;
```

An Example: Abstract Trace σ for Top-level Variables



ICFG

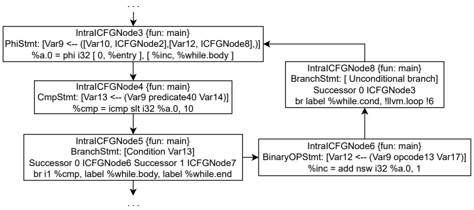
$postAbsTrace[\ell_3].varToAbsVal$:

SVFVar	$\langle Interval, MemAddress \rangle$
...	
Var9	$\langle [0, +\infty], \perp \rangle$
Var12	$\langle [1, 2], \perp \rangle$
...	

Algorithm 12: Handle Cycle WTO

```
Function handleCycleWTO(cycle):
    cycle_head := cycle→head()→node() ;
    increasing := true ;
    cur_iter := 0 ;
    while true do
        if cur_iter ≥ Options :: WidenDelay() then
            prev_head_state := postAbsTrace[cycle_head];
            handleICFGNode(cycle→head());
            cur_head_state := postAbsTrace[cycle_head];
            if increasing then
                postAbsTrace[cycle_head] := prev_head_state.widen(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    increasing := false;
                    continue;
            else
                postAbsTrace[cycle_head] := prev_head_state.narrow(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    break ;
        else
            handleICFGNode(cycle→head());
        handleWTOComponents(cycle→getWTOComponents());
        cur_iter++ // cur_iter ≡ 3,  Options :: WidenDelay ≡ 3;
```

An Example: Abstract Trace σ for Top-level Variables



ICFG

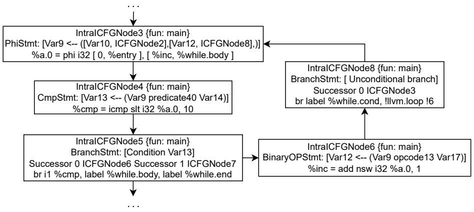
$postAbsTrace[\ell_8].varToAbsVal$:

SVFVar	AbstractValue$\langle Interval, MemAddress \rangle$
...	
$Var9$	$\langle [0, 9], \bot \rangle$
$Var12$	$\langle [1, 10], \bot \rangle$
...	

Algorithm 12: Handle Cycle WTO

Function `handleCycleWTO`(*cycle*):
  cycle_head := cycle→head()→node() ;
  increasing := true ;
  cur_iter := 0 ;
  while *true* **do**
    if $cur\_iter \geq Options :: WidenDelay()$ **then**
      $prev\_head\_state := postAbsTrace[cycle\_head]$;
      `handleICFGNode`(*cycle→head()*);
      $cur\_head\_state := postAbsTrace[cycle\_head]$;
      if *increasing* **then**
        $postAbsTrace[cycle\_head] := prev\_head\_state.widen(cur\_head\_state)$;
        if $postAbsTrace[cycle\_head] == prev\_head\_state$ **then**
          increasing := false;
          continue;
      else
        $postAbsTrace[cycle\_head] := prev\_head\_state.narrow(cur\_head\_state)$;
        if $postAbsTrace[cycle\_head] == prev\_head\_state$ **then**
          break ;
    else
      `handleICFGNode`(*cycle→head()*);
    `handleWTOComponents`(*cycle→getWTOComponents()*) // $cur\_iter \equiv 3$;
    cur_iter++ // $cur\_iter \equiv 3$, $Options :: WidenDelay \equiv 3$;

An Example: Abstract Trace σ for Top-level Variables



ICFG

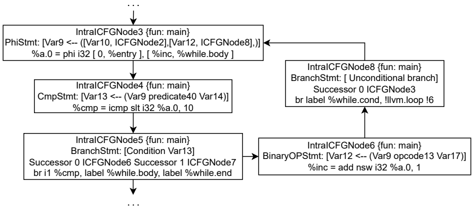
$postAbsTrace[\ell_3].varToAbsVal$:

SVFVar	$\langle Interval, MemAddress \rangle$
...	
Var9	$\langle [0, +\infty], \perp \rangle$
Var12	$\langle [1, 10], \perp \rangle$
...	

Algorithm 12: Handle Cycle WTO

```
Function handleCycleWTO(cycle):
    cycle_head := cycle→head()→node() ;
    increasing := true ;
    cur_iter := 0;
    while true do
        if cur_iter ≥ Options :: WidenDelay() then
            prev_head_state := postAbsTrace[cycle_head];
            handleICFGNode(cycle→head());
            cur_head_state := postAbsTrace[cycle_head];
            if increasing then
                postAbsTrace[cycle_head] := prev_head_state.widen(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    increasing := false;
                    continue;
            else
                postAbsTrace[cycle_head] := prev_head_state.narrow(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    break ;
        else
            handleICFGNode(cycle→head());
        handleWTOComponents(cycle→getWTOComponents());
        cur_iter++ // cur_iter ≡ 4,  Options :: WidenDelay ≡ 3;
```

An Example: Abstract Trace σ for Top-level Variables



ICFG

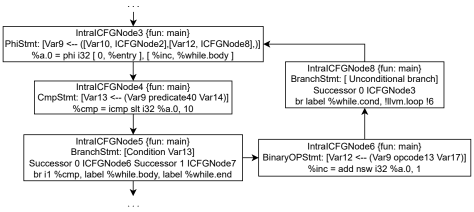
$postAbsTrace[\ell_3].varToAbsVal$:

SVFVar	$\langle Interval, MemAddress \rangle$
	...
Var9	$\langle [0, 10], \bot \rangle$
Var12	$\langle [1, 10], \bot \rangle$
	...

Algorithm 12: Handle Cycle WTO

```
Function handleCycleWTO(cycle):
    cycle_head := cycle→head()→node() ;
    increasing := true ;
    cur_iter := 0 ;
    while true do
        if cur_iter ≥ Options :: WidenDelay() then
            prev_head_state := postAbsTrace[cycle_head];
            handleICFGNode(cycle→head()) // increasing ≡ false;
            cur_head_state := postAbsTrace[cycle_head];
            if increasing then
                postAbsTrace[cycle_head] := prev_head_state.widen(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    increasing := false;
                    continue;
            else
                postAbsTrace[cycle_head] := prev_head_state.narrow(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    break ;
        else
            handleICFGNode(cycle→head());
        handleWTOComponents(cycle→getWTOComponents());
        cur_iter++ // cur_iter ≡ 5, Options :: WidenDelay ≡ 3;
```

An Example: Abstract Trace σ for Top-level Variables



ICFG

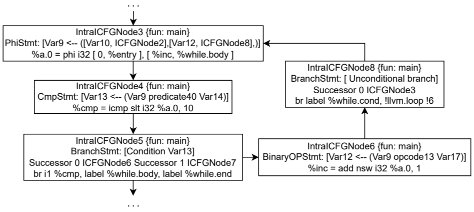
$postAbsTrace[\ell_8].varToAbsVal$:

SVFVar	$\langle Interval, MemAddress \rangle$
...	
Var9	$\langle [0, 9], \perp \rangle$
Var12	$\langle [1, 10], \perp \rangle$
...	

Algorithm 12: Handle Cycle WTO

```
Function handleCycleWTO(cycle):
    cycle_head := cycle→head()→node() ;
    increasing := true ;
    cur_iter := 0 ;
    while true do
        if cur_iter ≥ Options :: WidenDelay() then
            prev_head_state := postAbsTrace[cycle_head];
            handleICFGNode(cycle→head());
            cur_head_state := postAbsTrace[cycle_head];
            if increasing then
                postAbsTrace[cycle_head] := prev_head_state.widen(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    increasing := false;
                    continue;
            else
                postAbsTrace[cycle_head] := prev_head_state.narrow(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    break ;
        else
            handleICFGNode(cycle→head());
        handleWTOComponents(cycle→getWTOComponents()) // cur_iter ≡ 5;
        cur_iter++ // cur_iter ≡ 5, Options :: WidenDelay ≡ 3;
```

An Example: Abstract Trace σ for Top-level Variables



ICFG

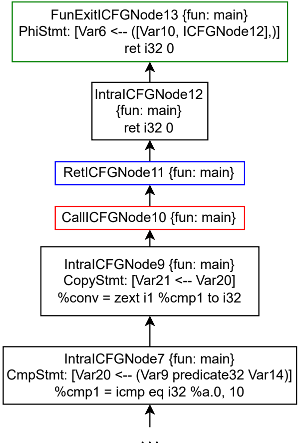
$postAbsTrace[\ell_3].varToAbsVal$:

SVFVar	$\langle Interval, MemAddress \rangle$
...	
Var9	$\langle [0, 10], \perp \rangle$
Var12	$\langle [1, 10], \perp \rangle$
...	

Algorithm 12: Handle Cycle WTO

```
Function handleCycleWTO(cycle):
    cycle_head := cycle→head()→node() ;
    increasing := true ;
    cur_iter := 0 ;
    while true do
        if cur_iter ≥ Options :: WidenDelay() then
            prev_head_state := postAbsTrace[cycle_head];
            handleICFGNode(cycle→head()) // increasing ≡ false;
            cur_head_state := postAbsTrace[cycle_head];
            if increasing then
                postAbsTrace[cycle_head] := prev_head_state.widen(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    increasing := false;
                    continue;
            else
                postAbsTrace[cycle_head] := prev_head_state.narrow(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    break ;
        else
            handleICFGNode(cycle→head());
        handleWTOComponents(cycle→getWTOComponents());
        cur_iter++ // cur_iter ≡ 6, Options :: WidenDelay ≡ 3;
```

An Example: Abstract Trace σ for Top-level Variables



ICFG

Algorithm 13: Abstract execution guided by WTO

Function `handleStatement`(ℓ)**:**
    tmpAS := *preAbsTrace*[ℓ];
    if ℓ *is* BINARYSTMT **then**
        `updateStateOnBinary`(ℓ);
    ...;

postAbsTrace[ℓ_7].*varToAbsVal* :

SVFVar	$\langle$*Interval*, *MemAddress*$\rangle$
...	
Var9	$\langle [10, 10], \perp \rangle$
Var20	$\langle [1, 1], \perp \rangle$
...	