Abstract Interpretation for Code Analysis and Verification

(Week 9)

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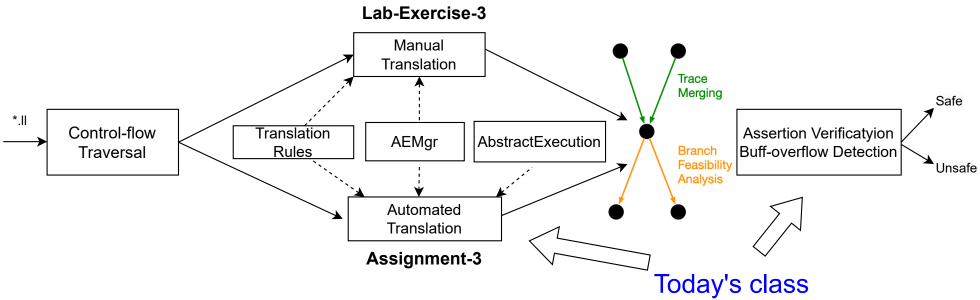
Today’s class



Topological Order

Analysis Order of Nodes on Control-Flow Graph

? How to analyze a program **free of loop**?

✔ Analyze each node **once** adhering to the **topological order** on the acyclic control-flow graph of the program.

Topological Order

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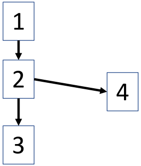
? How to analyze a program **free of loop**?

✔ Analyze each node **once** adhering to the **topological order** on the acyclic control-flow graph of the program.

A **topological order** of a graph $G(V, E)$ is a linear ordering of its nodes such that for every directed edge $a \rightarrow b$, node a always precedes node b in the ordering.

- Must be a **direct acyclic graph** (DAG) and has at least one topo ordering.
- The ordering respects the **direction of edges**.

Example of topological order:



acyclic graph G

Valid/invalid topological order

How About Analyzing Loops?

- **Topological Order** can only be used for directed acyclic graphs (DAGs).
- **Weak Topological Order (WTO)** is a relaxation of the more stringent topological order for graphs with loops.
  - **Cycles Permitted**: allows for cycles within the graph.
  - **Hierarchical Decomposition**: A graph is decomposed into a hierarchical structure where each node or a strongly connected component (SCC) can contain subnodes.
  - **Weak Topological Order or Partial Order**: In a WTO, nodes and SCCs are arranged in a partial order (not enumerating possible infinite loop paths). This order respects the dependencies in a way that allows for iterative analysis.
  - We will practice loop handling using WTO in `Assignment-3`. Function recursions will not be handled in this Assignment.

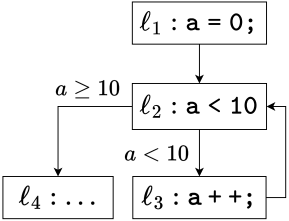
Weak Topological Order

Analysis Order of Nodes on Control-Flow Graph

? How to analyze a program **containing loops**?

✔ We can analyze a program containing loops adhering to the **weak topological order** (WTO) on its control flow graph.

What is the weak topological order?



Control Flow Graph

Weak Topological Order

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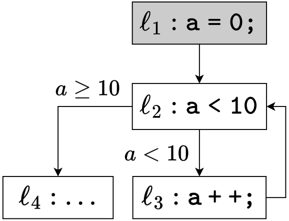
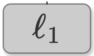


Control Flow Graph

1^{st} WTO component: a sigle node



Weak Topological Order

Analysis Order of Nodes on Control-Flow Graph

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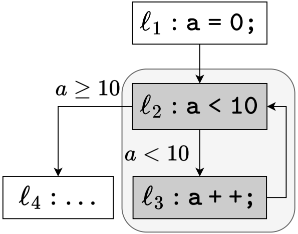
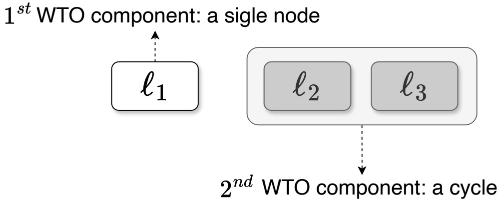


Control Flow Graph



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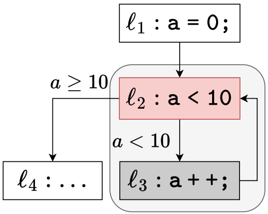
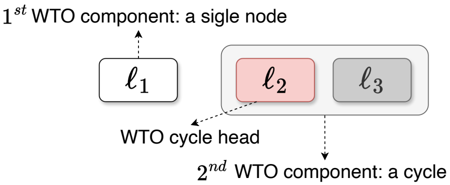


Control Flow Graph



Weak Topological Order

Analysis Order of Nodes on Control-Flow Graph

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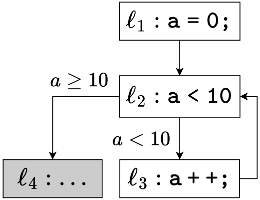

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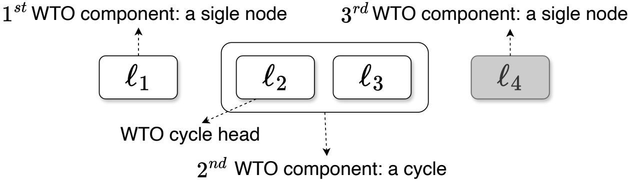

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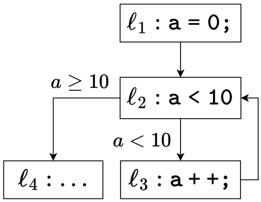
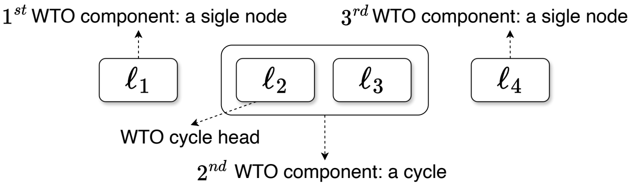


Control Flow Graph



Analyze each node following the WTO

Analyze ℓ_1 ⇒ Repeat: analyze ℓ_2 and ℓ_3 ⇒ Analyze ℓ_4

Weak Topological Order

Analysis Order of Nodes on Control-Flow Graph

❓ How to analyze a program **containing loops**?

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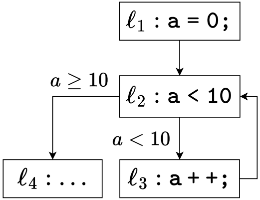
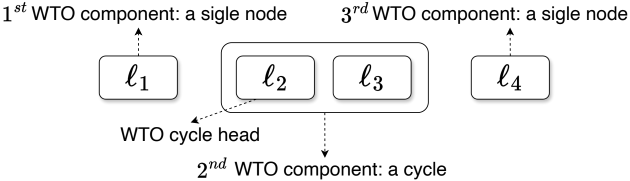
What is the weak topological order?



Control Flow Graph



Analyze each node following the WTO

Analyze ℓ_1 ⇨ Repeat: analyze ℓ_2 and ℓ_3 ⇨ Analyze ℓ_4

Perform widening when analyzing the cycle heads (i.e., ℓ_2)

WTO, Widening and Narrowing

Why Weak Topological Order (WTO)?

- Handling cyclic dependencies
- Efficient fixed-point computation

Why Widening?

- Over-approximation
- Prevent non-termination

Why Narrowing?

- Refine precision after widening converges
- The specific conditions or constraints used for narrowing:
  - Loop exit conditions (this course)
  - Type constraints (8-bit integer ranging from [-128, 127])
  - Bounds from arithmetic operations If `x = y + z`, and $\mathtt{y} \in [1, 5]$ and $\mathtt{z} \in [2, 3]$, then $\mathtt{x} \in [3, 8]$. If widening gives [1, 10], narrowing can refine this to [3, 8].
  - User-specification (assertions and guard conditions)

Overall Algorithm of Abstract Interpretation in Assignment-3

Algorithm 1: Analyse from main function

```
Function analyse() // driver function to start the analysis:
    initWTO();
    handleGlobalNode();
    if getSVFFunction (main) then
        wto := funcToWTO[main];
        handleWTOComponents(wto → getWTOComponents());
```

Algorithm 2: Handle WTO components

```
Function handleWTOComponents (wtoComps):
    for wtoNode ∈ wtoComps do
        if node = SVFUtil :: dyn_cast⟨ICFGSingletonWTO⟩(wtoNode) then
            handleSingletonWTO(node)
        else if cycle = SVFUtil :: dyn_cast⟨ICFGCycleWTO⟩(wtoNode) then
            handleCycleWTO(cycle)
        else
            assert(false&&"unknownWTOtype!")
```

Algorithm 3: Handle Singleton WTO

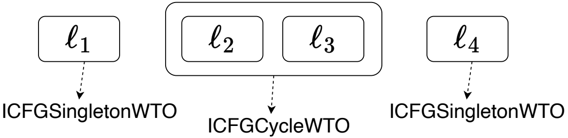
```
Function handleSingletonWTO(singletonWTO):
    node := singletonWTO → node();
    feasible := mergeStatesFromPredecessors(node, preAbsTrace[node]);
    if feasible then
        postAbsTrace[node] := preAbsTrace[node];
    else
        return;
    foreach stmt ∈ node → getSVFStmts() do
        updateAbsState(stmt);
        bufOverflowDetection(stmt);
    if callnode = SVFUtil :: dyn_cast⟨CallICFGNode⟩(node) then
        funName := callnode → getCallSite() → getCallee() → getName()
        if funName == "OVERFLOW" &&funName == "svf_assert" then
            // Handle svf_assert and OVERFLOW stub function for
            correctness validation;
            handleStubFunctions(callnode);
        else
            // Does not analyze recursive functions in this course;
            handleCallSite(callnode);
```

Overall Algorithm of Abstract Interpretation in Assignment-3

Algorithm 4: Handle Cycle WTO

```
Function handleCycleWTO (cycle):
    feasible := mergeStatesFromPredecessors(cycle_head, preAbsTrace[cycle_head]);
    increasing := true;
    if !feasible then
        return;
    else
        cur_iter := 0;
        while true do
            if cur_iter >= Options.WidenDelay() then
                prev_head_as := postAbsTrace[cycle_head];
                handleSingletonWTO(cycle.head());
                cur_head_as := postAbsTrace[cycle_head];
                if increasing then
                    postAbsTrace[cycle_head] := prev_head_as.widening(cur_head_as);
                    if postAbsTrace[cycle_head] == prev_head_as then
                        increasing := false;
                        Continue;
                else
                    postAbsTrace[cycle_head] := prev_head_as.narrowing(cur_head_as);
                    if postAbsTrace[cycle_head] == prev_head_as then
                        Break;
            else
                handleSingletonWTO(cycle.head());
            cur_iter++;
```

Widening and Narrowing



Analyze ICFGCycleWTO

Algorithm 12: Handle Cycle WTO

```
Function handleCycleWTO(cycle):
    ... ;
    cycle_head := cycle→head()→node() ;
    increasing := true ;
    cur_iter := 0 ;
    while true do
        if cur_iter ≥ Options :: WidenDelay() then
            prev_head_state := postAbsTrace[cycle_head];
            handleSingletonWTO(cycle→head());
            cur_head_state := postAbsTrace[cycle_head];
            if increasing then
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                if postAbsTrace[cycle_head] == prev_head_state then
                    increasing := false;
                    continue;
            else
                postAbsTrace[cycle_head] := prev_head_state.narrow(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    break ;
        else
            handleSingletonWTO(cycle→head());
        handleWTOComponents(cycle→getWTOComponents());
        cur_iter++;
```
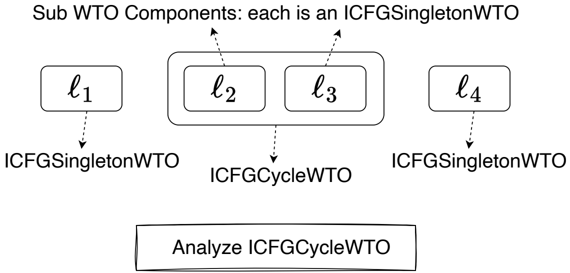

Widening and Narrowing



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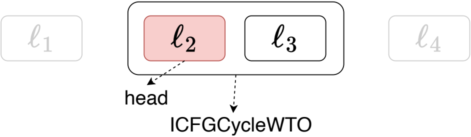
Weak Topological Order



Get ICFGCycleWTO head ℓ_2

Algorithm 12: Handle Cycle WTO

```
Function handleCycleWTO(cycle):
    cycle_head := cycle→head()→node() ;
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    while true do
        if cur_iter ≥ Options :: WidenDelay() then
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            cur_head_state := postAbsTrace[cycle_head];
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        else
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        cur_iter++;
```
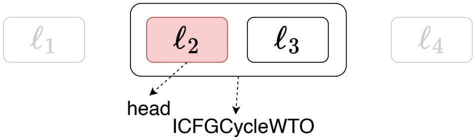

Weak Topological Order



Handle ICFGCycleWTO head ℓ_2

Algorithm 12: Handle Cycle WTO

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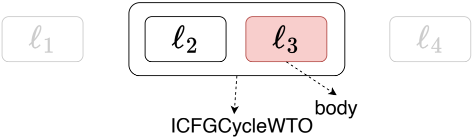
Widening and Narrowing



Handle ICFGCycleWTO body ℓ_3

Algorithm 12: Handle Cycle WTO

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    while true do
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            prev_head_state := postAbsTrace[cycle_head];
            handleSingletonWTONode(cycle→head());
            cur_head_state := postAbsTrace[cycle_head];
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                if postAbsTrace[cycle_head] == prev_head_state then
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                    continue;
            else
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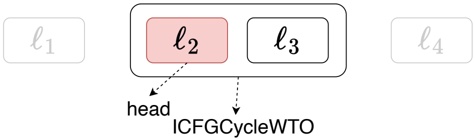
                if postAbsTrace[cycle_head] == prev_head_state then
                    break ;
        else
            handleSingletonWTONode(cycle→head());
        handleWTOComponents(cycle→getWTOComponents());
        cur_iter++;
```

Note: getIWTOcomponents returns Cycle WTO body, i.e., ℓ_3

Widening and Narrowing



When $cur\_iter \geq Options :: WidenDelay()$
perform widening on ℓ_2

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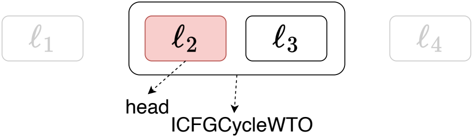
Widening and Narrowing



Widening reaches a fixpoint

Algorithm 12: Handle Cycle WTO

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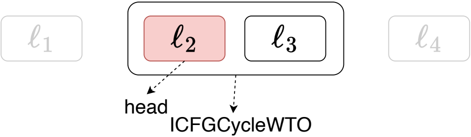
Widening and Narrowing



Perform narrowing on ℓ_2

Algorithm 12: Handle Cycle WTO

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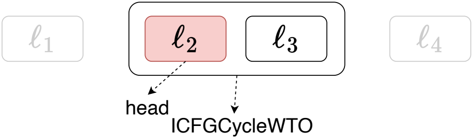
Widening and Narrowing



Narrowing reaches a fixpoint, Finish!

Algorithm 12: Handle Cycle WTO

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```

Abstract Interpretation on SVFIR

Week 9

Yulei Sui

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Abstract Interpretation on Pointer-Free SVFIR

Interval Domain

- For simplicity, let's first consider abstract execution on a pointer-free language.
- This means there are no operations for memory allocation (like $\texttt{p} = \texttt{alloc}_{\texttt{o}}$) or for indirect memory accesses (such as `p = *q` or `*p = q`).
- Here are the pointer-free SVFSTMTs and their C-like forms:

SVFSTMT	**C-Like form**
CONSSTMT	$\ell : \texttt{p} = \texttt{c}$
COPYSTMT	$\ell : \texttt{p} = \texttt{q}$
BINARYSTMT	$\ell : \texttt{r} = \texttt{p} \otimes \texttt{q}$
PHISTMT	$\ell : \texttt{r} = \texttt{phi}(\texttt{p}_1, \texttt{p}_2, \ldots, \texttt{p}_n)$
SEQUENCE	$\ell_1 ; \ell_2$
BRANCHSTMT	ℓ_1 : if$(x < c)$ then ℓ_2 else ℓ_3

Abstract Interpretation on Pointer-Free SVFIR

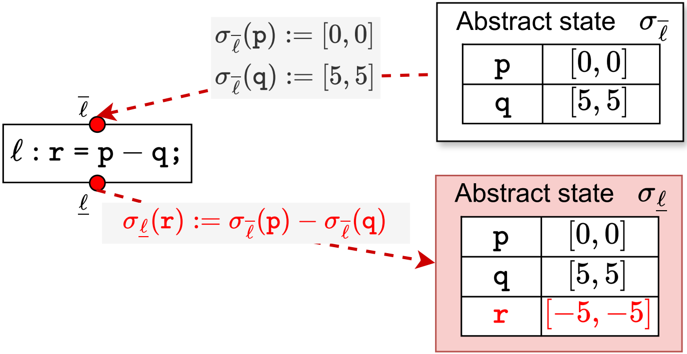
Interval Domain

Let's use the *Interval* abstract domain to update σ based on the following rules for different SVFStmt:

SVFStmt	C-Like form	Abstract Execution Rule
ConsStmt	$\ell : \mathtt{p} = \mathtt{c}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := [\mathtt{c}, \mathtt{c}]$
CopyStmt	$\ell : \mathtt{p} = \mathtt{q}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \sigma_{\overline{\ell}}(\mathtt{q})$
BinaryStmt	$\ell : \mathtt{r} = \mathtt{p} \otimes \mathtt{q}$	$\sigma_{\underline{\ell}}(r) := \sigma_{\overline{\ell}}(p) \hat{\otimes} \sigma_{\overline{\ell}}(q)$
PhiStmt	$\ell : \mathtt{r} = \mathtt{phi}(\mathtt{p}_1, \mathtt{p}_2, \ldots, \mathtt{p}_n)$	$\sigma_{\underline{\ell}}(r) := \bigsqcup_{i=1}^{n} \sigma_{\overline{\ell}}(p_i)$
Sequence	$\ell_1; \ell_2$	$\forall v \in \mathbb{V}, \sigma_{\overline{\ell_2}}(v) \sqsupseteq \sigma_{\underline{\ell_1}}(v)$
BranchStmt	ℓ_1 : if($x < c$) then ℓ_2 else ℓ_3	$\sigma_{\overline{\ell_2}}(x) := \sigma_{\underline{\ell_1}}(x) \sqcap [-\infty, c-1]$, if $\sigma_{\underline{\ell_1}}(x) \sqcap [-\infty, c-1] \neq \bot$ $\sigma_{\overline{\ell_3}}(x) := \sigma_{\underline{\ell_1}}(x) \sqcap [c, +\infty]$, if $\sigma_{\underline{\ell_1}}(x) \sqcap [c, +\infty] \neq \bot$

Abstract Interpretation on BINARYSTMT

SVFSTMT	C-Like form	Abstract Execution Rule
BINARYSTMT	$\ell : \mathtt{r} = \mathtt{p} \otimes \mathtt{q}$	$\sigma_{\underline{\ell}}(r) := \sigma_{\overline{\ell}}(p) \hat{\otimes} \sigma_{\overline{\ell}}(q)$

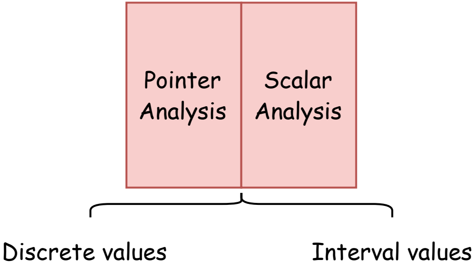


Abstract Interpretation in the Presence of Pointers

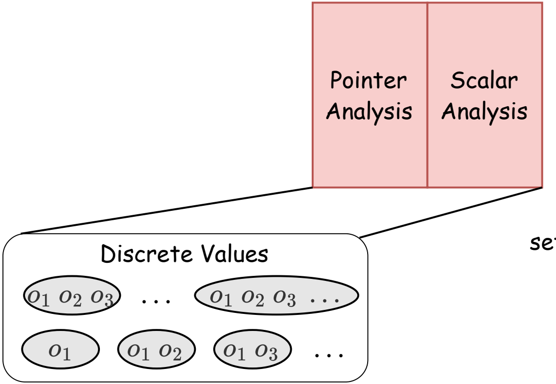
- SVFIR in the presence of pointers contain pointer-related statements including ADDRSTMT, GEPSTMT, LOADSTMT and STORESTMT.
- Abstract interpretation needs to be performed on **a combined domain of intervals and addresses.**

SVFSTMT	**C-Like form**
CONSSTMT	$\ell : \texttt{p} = \texttt{c}$
COPYSTMT	$\ell : \texttt{p} = \texttt{q}$
BINARYSTMT	$\ell : \texttt{r} = \texttt{p} \otimes \texttt{q}$
PHISTMT	$\ell : \texttt{r} = \texttt{phi}(\texttt{p}_1, \texttt{p}_2, \ldots, \texttt{p}_n)$
SEQUENCE	$\ell_1 ; \ell_2$
BRANCHSTMT	ℓ_1 : if$(x < c)$ then ℓ_2 else ℓ_3
ADDRSTMT	$\ell : \texttt{p} = \texttt{alloc}$
GEPSTMT	$\ell : \texttt{p} = \&(\texttt{q} \rightarrow \texttt{i})$ or $\texttt{p} = \&\texttt{q}[\texttt{i}]$
LOADSTMT	$\ell : \texttt{p} = *\texttt{q}$
STORESTMT	$\ell : *\texttt{p} = \texttt{q}$

Combined Analysis



Combined Analysis Using Discrete Values



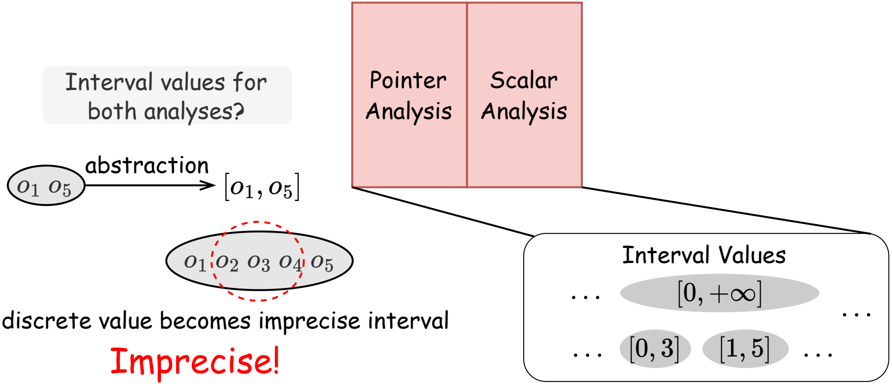
Discrete values for both analyses?

{1, 2, 3, ..., infinite numbers}

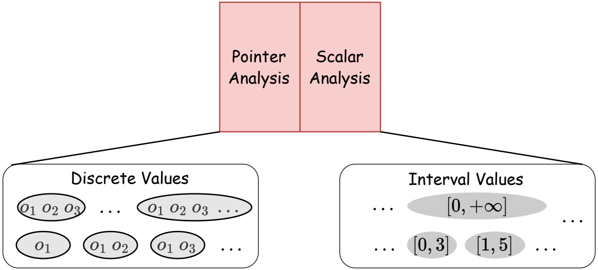
set of all possible values with inputs

Unscalable!

Combined Analysis Using Interval Values



Abstract Interpretation Over a Combined Domain

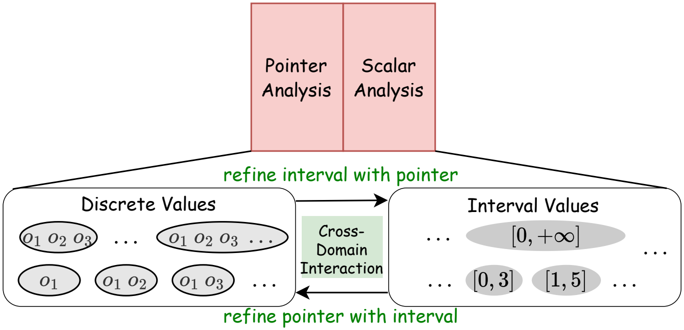


p = malloc(m*sizeof(int)); // p points to an array of size m
q = malloc(n*sizeof(int)); // q points to an array of size n

m = r[i];

- The discrete values for points-to set of `p`, `q` depend on interval values of `m` and `n`.
- The interval value of `m` depends on the pointer aliasing between `p`, `q` and `&r[i]`.
- Cyclic dependency between two domains requiring a bi-directional refinement. (variables highlighted in blue and red denote the discrete values and interval values dependent),

Abstract Interpretation Over a Combined Domain

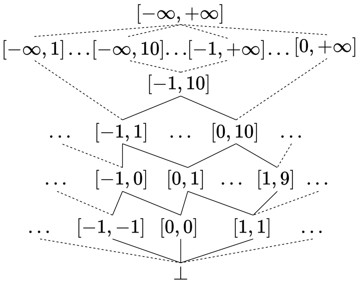


We require **a combination of interval and memory address domains** to precisely and efficiently perform abstract execution on SVFIR in the presence of pointers.

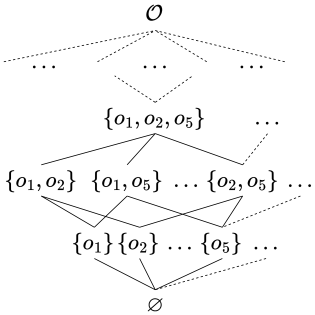
Precise Sparse Abstract Execution via Cross-Domain Interaction, ICSE 2024

Abstract Interpretation over Interval and MemAddress Domains

A Combined Domain of Intervals and Discrete Memory Addresses



Interval domain for scalar variables



MemAddress domain for discrete memory address values

SVF Program Variables (SVFVar)

Program Variables	Domain	Meanings
`SVFVar`	$\mathbb{V} = \mathbb{P} \cup \mathbb{O}$	Program Variables
`ValVar`	$\mathbb{P}$	Top-level variables (scalars and pointers)
`ObjVar`	$\mathbb{O} = \mathbb{S} \cup \mathbb{G} \cup \mathbb{H} \cup \mathbb{C}$	Memory Objects (constant data, stack, heap, global) (function objects are considered as global objects)
`FIObjVar`	$o \in (\mathbb{S} \cup \mathbb{G} \cup \mathbb{H})$	A single (base) memory object
`GepObjVar`	$o_i \in (\mathbb{S} \cup \mathbb{G} \cup \mathbb{H}) \times \mathbb{P}$	i-th subfield/element of an (aggregate) object
`ConstantData`	$\mathbb{C}$	Constant data (e.g., numbers and strings)
Program Statement	$\ell \in \mathbb{L}$	Statements labels

Abstract Trace for The Combined Domain

- For top-level variables $\mathbb{P}$, we use $\sigma \in \mathbb{L} \times \mathbb{P} \rightarrow \mathit{Interval} \times \mathit{MemAddress}$ to track the memory addresses or interval values of these variables.
- For memory objects $\mathbb{O}$, we use $\delta \in \mathbb{L} \times \mathbb{O} \rightarrow \mathit{Interval} \times \mathit{MemAddress}$ to track their abstract values

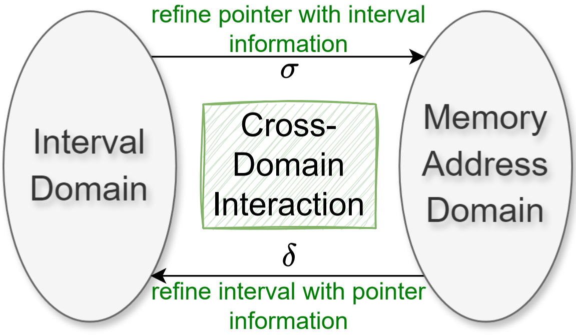
	Notation	Domain	Data Structure Implementation
Abstract trace	σ	$\mathbb{L} \times \mathbb{P} \rightarrow \mathit{Interval} \times \mathit{MemAddress}$	*preAbsTrace*, *postAbsTrace*
	δ	$\mathbb{L} \times \mathbb{O} \rightarrow \mathit{Interval} \times \mathit{MemAddress}$	
Abstract state	σ_L	$\mathbb{P} \rightarrow \mathit{Interval} \times \mathit{MemAddress}$	*AbstractState.varToAbsVal*
	δ_L	$\mathbb{O} \rightarrow \mathit{Interval} \times \mathit{MemAddress}$	*AbstractState.addrToAbsVal*
Abstract value	$\sigma_L(p)$	$\mathit{Interval} \times \mathit{MemAddress}$	*AbstractValue*
	$\delta_L(o)$		

- *Interval* is used for tracking the interval value of **scalar variables** $\mathbb{P}$.
- *MemAddress* is used for tracking the memory addresses of **memory address variables** $\mathbb{O}$.

Implementation of Abstract Trace and State in Assignment-3

- For a program point L, the abstract state is an instance of class *AEState*, consisting of:
  - Top-level variable, *varToAbsVal* : $\sigma_L \in \mathbb{P} \rightarrow Interval \times MemAddress$
  - Memory object, *addrToAbsVal* : $\delta_L \in MemAddress \rightarrow Interval \times MemAddress$
- The abstract trace has two maps, *preAbsTrace* and *postAbsTrace*, which maintains abstract states before and after each `ICFGNode` respectively.
  - For an `ICFGNode` ℓ, *preAbsTrace*(ℓ) retrieves the abstract state $\langle \sigma_{\overline{\ell}}, \delta_{\overline{\ell}} \rangle$, and *postAbsTrace*$(\ell)$ represents $\langle \sigma_{\underline{\ell}}, \delta_{\underline{\ell}} \rangle$.
  - For each abstract state $\langle \sigma_{\overline{\ell}}, \delta_{\overline{\ell}} \rangle$ we use `as[varId]` to operate $\sigma_{\underline{\ell}}$ and use `storeValue` and `loadValue` to operate $\delta_{\underline{\ell}}$.
  - Each variable's `AbstractValue` (e.g., `as[VarId]`) is initialized as $\perp$ in an `AbstractState` before assigned a new value.
  - Each `AbstractValue` (e.g., `as[VarId]`) is a 2-element tuple consisting of an interval `as[VarId].getInterval()` and an address set `as[Varid].getAddrs()`.
  - Print out `SVFVars` and their `AbstractValues` in an `AbstractState` by invoking `as.printAbstractState()`

Abstract Trace for The Combined Domain



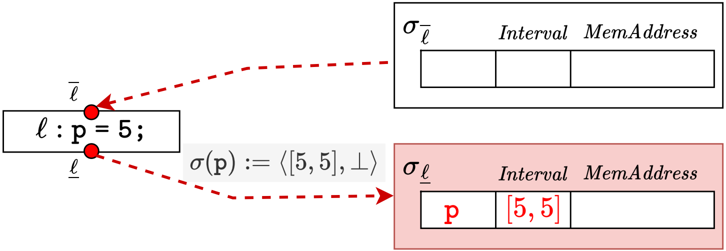
Abstract Execution Rules on SVFIR in the Presence of Pointers

Now let's use the *Interval* $\times$ *MemAddress* abstract domain to update σ and δ based on the following rules for different SVFSTMT:

SVFSTMT	**C-Like form**	**Abstract Execution Rule**
CONSSTMT	$\ell : \texttt{p} = \texttt{c}$	$\sigma_{\underline{\ell}}(\texttt{p}) := \langle[\texttt{c},\texttt{c}], \bot\rangle$
COPYSTMT	$\ell : \texttt{p} = \texttt{q}$	$\sigma_{\underline{\ell}}(\texttt{p}) := \sigma_{\overline{\ell}}(\texttt{q})$
BINARYSTMT	$\ell : \texttt{r} = \texttt{p} \otimes \texttt{q}$	$\sigma_{\underline{\ell}}(r) := \sigma_{\overline{\ell}}(p) \hat{\otimes} \sigma_{\overline{\ell}}(q)$
CMPSTMT	$\ell : \texttt{r} = \texttt{p} \odot \texttt{q}$	$\sigma_{\underline{\ell}}(r) := \sigma_{\overline{\ell}}(p) \hat{\odot} \sigma_{\overline{\ell}}(q)$
PHISTMT	$\ell : \texttt{r} = \texttt{phi}(\texttt{p}_1, \texttt{p}_2, \ldots, \texttt{p}_n)$	$\sigma_{\underline{\ell}}(r) := \bigsqcup_{i=1}^{n} \sigma_{\overline{\ell}}(p_i)$
BRANCHSTMT	ℓ_1 : if($x < c$) then ℓ_2 else ℓ_3	$\sigma_{\overline{\ell_2}}(x) := \sigma_{\underline{\ell_1}}(x) \sqcap [-\infty, c-1]$, if $\sigma_{\underline{\ell_1}}(x) \sqcap [-\infty, c-1] \neq \bot$ $\sigma_{\overline{\ell_3}}(x) := \sigma_{\underline{\ell_1}}(x) \sqcap [c, +\infty]$, if $\sigma_{\underline{\ell_1}}(x) \sqcap [c, +\infty] \neq \bot$
SEQUENCE	$\ell_1; \ell_2$	$\delta_{\overline{\ell_2}} \sqsupseteq \delta_{\underline{\ell_1}}, \sigma_{\overline{\ell_2}} \sqsupseteq \sigma_{\underline{\ell_1}}$
ADDRSTMT	$\ell : \texttt{p} = \texttt{alloc}_{\texttt{o}_\texttt{i}}$	$\sigma_{\underline{\ell}}(\texttt{p}) := \langle\bot, \{o_i\}\rangle$
GEPSTMT	$\ell : \texttt{p} = \&(\texttt{q} \rightarrow \texttt{i})$ or $\texttt{p} = \&\texttt{q}[\texttt{i}]$	$\sigma_{\underline{\ell}}(\texttt{p}) := \bigsqcup_{\texttt{o} \in \gamma(\sigma_{\overline{\ell}}(\texttt{q}))} \bigsqcup_{j \in \gamma(\sigma_{\overline{\ell}}(\texttt{i}))} \langle\bot, \{\texttt{o.fld}_j\}\rangle$
LOADSTMT	$\ell : \texttt{p} = {*}\texttt{q}$	$\sigma_{\underline{\ell}}(\texttt{p}) := \bigsqcup_{o \in \{o \mid o \in \sigma_{\overline{\ell}}(q)\}} \delta_{\overline{\ell}}(o)$
STORESTMT	$\ell : {*}\texttt{p} = \texttt{q}$	$\delta_{\underline{\ell}} := (\{o \mapsto \sigma_{\overline{\ell}}(\texttt{q}) \mid o \in \gamma(\sigma_{\overline{\ell}}(\texttt{p}))\} \sqcup \delta_{\underline{\ell}})$

Abstract Interpretation on ConsStmt

SVFStmt	C-Like form	Abstract Execution Rule
ConsStmt	$\ell : p = c$	$\sigma_{\underline{\ell}}(p) := \langle [c, c], \perp \rangle$



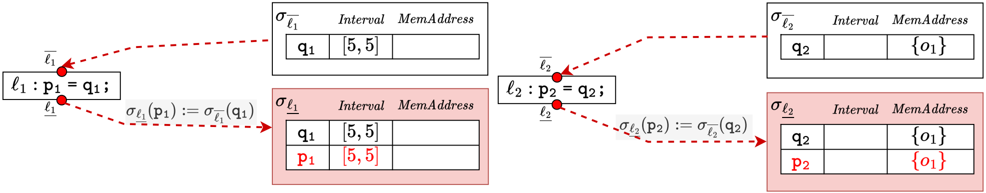
Algorithm 13: Abstract Execution Rule for ConsStmt

Function *updateStateOnAddr(addr)*:

```
node = addr→getICFGNode();
as = getAbsStateFromTrace(node);
initObjVar(as, SVFUtil :: cast⟨ObjVar⟩(addr→getRHSVar()));
as[addr→getLHSVarID()] = as[addr→getRHSVarID()];
```

Abstract Interpretation on CopyStmt

SVFStmt	C-Like form	Abstract Execution Rule
CopyStmt	$\ell : \mathtt{p} = \mathtt{q}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \sigma_{\overline{\ell}}(\mathtt{q})$

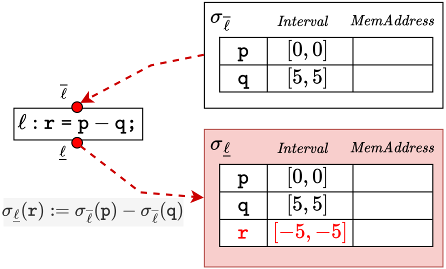


Algorithm 14: Abstract Execution Rule for CopyStmt

```
Function updateStateOnCopy(copy):
    node = copy→getICFGNode();
    as = getAbsStateFromTrace(node);
    lhs = copy→getLHSVarID();
    rhs = copy→getRHSVarID();
    as[lhs] = as[rhs];
```

Abstract Interpretation on BINARYSTMT

SVFSTMT	C-Like form	Abstract Execution Rule
BINARYSTMT	$\ell : \mathtt{r} = \mathtt{p} \otimes \mathtt{q}$	$\sigma_{\underline{\ell}}(r) := \sigma_{\overline{\ell}}(p) \hat{\otimes} \sigma_{\overline{\ell}}(q)$



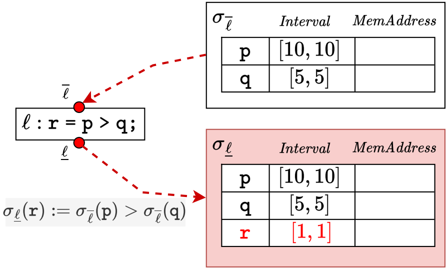
Algorithm 15: Abstract Execution Rule for BINARYSTMT

```
Function updateStateOnBinary(binary):
    node = binary→getICFGNode();
    as = getAbsStateFromTrace(node);
    op0 = binary→getOpVarID(0);
    op1 = binary→getOpVarID(1);
    res = binary→getResID();
    as[res] = as[op0]⊗̂as[op1]
```

Operands `op0` and `op1` are assumed to be properly initialized (no uninitialized variables or randomization).

Abstract Interpretation on CMPSTMT

SVFSTMT	C-Like form	Abstract Execution Rule
CMPSTMT	$\ell : \mathtt{r} = \mathtt{p} \odot \mathtt{q}$	$\sigma_{\underline{\ell}}(r) := \sigma_{\overline{\ell}}(p) \hat{\otimes} \sigma_{\overline{\ell}}(q)$



Algorithm 16: Abstract Execution Rule for CMPSTMT

Function *updateStateOnCmp(cmp)*:

```
node = cmp→getICFGNode();
as = getAbsStateFromTrace(node);
op0 = cmp→getOpVarID(0);
op1 = cmp→getOpVarID(1);
res = cmp→getResID();
as[res] = as[op0] ⊙̂ as[op1]
```

Operands `op0` and `op1` are assumed to be properly initialized (no uninitialized variables or randomization).

Abstract Interpretation on PHISTMT

SVFSTMT	C-Like form	Abstract Execution Rule
PHISTMT	$\ell : \mathtt{r} = \mathtt{phi}(\mathtt{p_1}, \mathtt{p_2}, \ldots, \mathtt{p_n})$	$\sigma_{\underline{\ell}}(r) := \bigsqcup_{i=1}^{n} \sigma_{\overline{\ell}}(p_i)$

$\overline{\ell}$

ℓ : **r = phi(p$_1$, p$_2$);**

$\underline{\ell}$

$\sigma_{\underline{\ell}}(\mathtt{r}) := \sigma_{\overline{\ell}}(\mathtt{p_1}) \sqcup \sigma_{\overline{\ell}}(\mathtt{p_2})$

$\sigma_{\overline{\ell}}$

	Interval	*MemAddress*
p$_1$	$[0,0]$	
p$_2$	$[5,5]$	

$\sigma_{\underline{\ell}}$

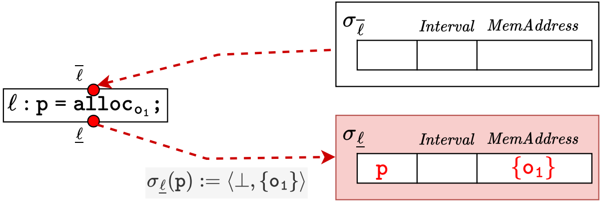
	Interval	*MemAddress*
p$_1$	$[0,0]$	
p$_2$	$[5,5]$	
r	$[0,5]$	

Algorithm 17: Abstract Execution Rule for PHISTMT

```
Function updateStateOnPhi(phi):
    node = phi→getICFGNode();
    as = getAbsStateFromTrace(node);
    res = phi→getResID();
    rhs = AbstractValue();
    for i = 0; i < phi→getOpVarNum(); i++ do
        curId = phi→getOpVarID(i);
        rhs.join_with(as[curId])
    as[res] = rhs
```

Abstract Interpretation on AddrStmt

SVFStmt	C-Like form	Abstract Execution Rule
AddrStmt	$\ell : \texttt{p} = \texttt{alloc}_{\texttt{o}_\texttt{i}}$	$\sigma_{\underline{\ell}}(\texttt{p}) := \langle \bot, \{o_i\} \rangle$



Algorithm 18: Abstract Execution Rule for AddrStmt

```
Function updateStateOnAddr(addr):
    node = addr → getICFGNode();
    as = getAbsStateFromTrace(node);
    initObjVar(as, SVFUtil :: cast⟨ObjVar⟩(addr → getRHSVar()));
    as[addr → getLHSVarID()] = as[addr → getRHSVarID()];
```

Abstract Interpretation on GEPSTMT

SVFSTMT	C-Like form	Abstract Execution Rule
GEPSTMT	$\ell : \texttt{p} = \&(\texttt{q} \to \texttt{i})$ or $\texttt{p} = \&\texttt{q}[\texttt{i}]$	$\sigma_{\underline{\ell}}(\texttt{p}) := \bigsqcup_{\texttt{o} \in \gamma(\sigma_{\overline{\ell}}(\texttt{q}))} \bigsqcup_{j \in \gamma(\sigma_{\overline{\ell}}(\texttt{i}))} \langle \bot, \{\texttt{o.fld}_j\} \rangle$

$\overline{\ell}$

`ℓ : p = &(q[i]);`

$\underline{\ell}$

$\sigma_{\underline{\ell}}(\texttt{p}) := \langle \bot, \{\texttt{o}_1.\texttt{fld}_1\} \rangle \sqcup \langle \bot, \{\texttt{o}_1.\texttt{fld}_2\} \rangle$

$\sigma_{\overline{\ell}}$

	Interval	*MemAddress*
q		$\{\texttt{o}_1\}$
i	$[1, 2]$	

$\sigma_{\underline{\ell}}$

	Interval	*MemAddress*
q		$\{\texttt{o}_1\}$
i	$[1, 2]$	
p		$\{\texttt{o}_1.\texttt{fld}_1, \texttt{o}_1.\texttt{fld}_2\}$

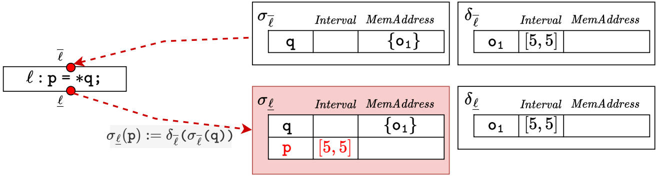
Algorithm 19: Abstract Execution Rule for GEPSTMT

Function *updateStateOnGep(gep)*:

```
node = gep→getICFGNode();
as = getAbsStateFromTrace(node);
rhs = gep→getRHSVarID();
lhs = gep→getLHSVarID();
as[lhs] = as.getGepObjAddrs(rhs, as.getElementIndex(gep));;
```

Abstract Interpretation on LoadStmt

SVFStmt	C-Like form	Abstract Execution Rule
LoadStmt	$\ell : \mathtt{p} = *\mathtt{q}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \bigsqcup_{o \in \{o \mid o \in \sigma_{\overline{\ell}}(q)\}} \delta_{\overline{\ell}}(o)$



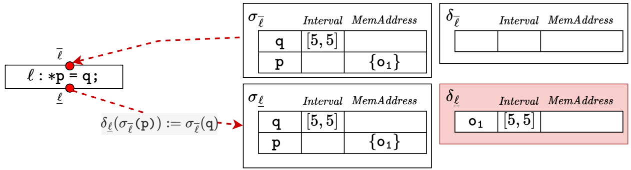
Algorithm 20: Abstract Execution Rule for LoadStmt

Function *updateStateOnLoad(load)*:

```
node = load→getICFGNode();
as = getAbsStateFromTrace(node);
rhs = load→getRHSVarID();
lhs = load→getLHSVarID();
as[lhs] = as.loadValue(rhs)
```

Abstract Interpretation on STORESTMT

SVFSTMT	C-Like form	Abstract Execution Rule
STORESTMT	$\ell : *p = q$	$\delta_{\underline{\ell}} := (\{o \mapsto \sigma_{\overline{\ell}}(q) \mid o \in \gamma(\sigma_{\overline{\ell}}(p))\} \sqcup \delta_{\underline{\ell}})$



Algorithm 21: Abstract Execution Rule for STORESTMT

```
Function updateStateOnStore(store):
    node = store→getICFGNode();
    as = getAbsStateFromTrace(node);
    rhs = store→getRHSVarID();
    lhs = store→getLHSVarID();
    as.storeValue(lhs, as[rhs])
```

An Example: Abstract Trace σ for Top-level Variables

```c
extern void assert(int);

int main(){
    int a = 0;
    while(a < 10) {
        a++;
    }
    assert(a == 10);
    return 0;
}
```

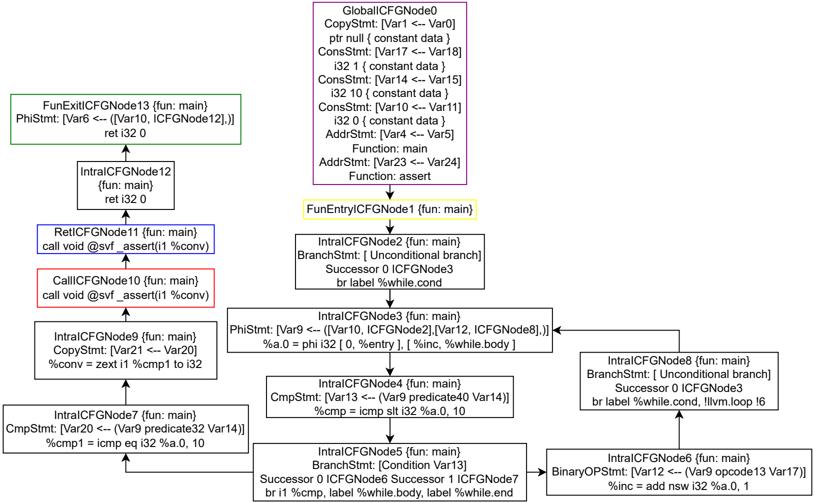
Source Code

Compile to LLVM IR ⇨

```llvm
define dso_local i32 @main() {
entry:
  br label %while.cond
while.cond:
  %a.0 = phi i32 [ 0, %entry ], [ %inc, %while.body ]
  %cmp = icmp slt i32 %a.0, 10
  br i1 %cmp, label %while.body, label %while.end
while.body:
  %inc = add nsw i32 %a.0, 1
  br label %while.cond,
while.end:
  %cmp1 = icmp eq i32 %a.0, 10
  %conv = zext i1 %cmp1 to i32
  call void @assert(i32 noundef %conv)
  ret i32 0
}
```

LLVM IR

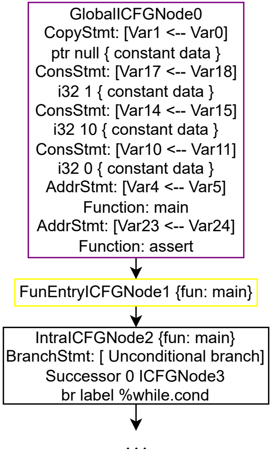
An Example: Abstract Trace σ for Top-level Variables



ICFG

An Example: Abstract Trace σ for Top-level Variables

Before Entering Loop



ICFG

Algorithm 22: Abstract execution guided by WTO

Function `handleStatement`(ℓ)**:**
- *tmpAS* := *preAbsTrace*[ℓ];
- **if** ℓ *is* ConsStmt *or* AddrStmt **then**
  - `updateStateOnAddr`(ℓ);
- **else if** ℓ *is* CopyStmt **then**
  - `updateStateOnCopy`(ℓ);
- ...;

postAbsTrace[*ICFGNode0*].*varToAbsVal* :

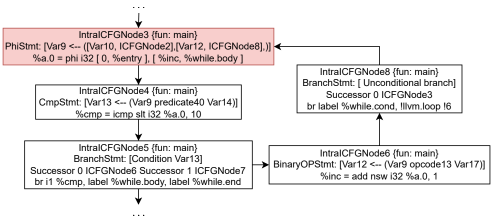
`SVFVar`	`AbstractValue`$\langle$*Interval*, *MemAddress*$\rangle$
Var0	$\langle \bot, \{0x7f00\} \rangle$
Var1	$\langle \bot, \{0x7f00\} \rangle$
Var18	$\langle [1, 1], \bot \rangle$
Var17	$\langle [1, 1], \bot \rangle$
Var14	$\langle [10, 10], \bot \rangle$
Var15	$\langle [10, 10], \bot \rangle$
Var10	$\langle [0, 0], \bot \rangle$
Var11	$\langle [0, 0], \bot \rangle$

...

Print out the table via `as.printAbstractState()`. The `AbstractValue` can **either be an interval or addresses**, but not both!

An Example: Abstract Trace σ for Top-level Variables

Widen Delay Phase (cur_iter is 0)



ICFG

$postAbsTrace[ICFGNode3].varToAbsVal$:

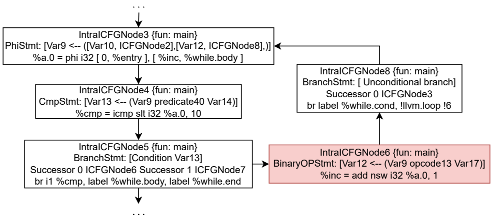
SVFVar	AbstractValue⟨*Interval*, *MemAddress*⟩
...	
Var10	$\langle[0,0],\bot\rangle$
Var9	$\langle[0,0],\bot\rangle$
...	

Algorithm 12: Handle Cycle WTO

```
Function handleCycleWTO(cycle):
    cycle_head := cycle→head()→node() ;
    increasing := true ;
    cur_iter := 0 ;
    while true do
        // cur_iter ≡ 0, Options :: WidenDelay() ≡ 2
        if cur_iter ≥ Options :: WidenDelay() then
            prev_head_state := postAbsTrace[cycle_head];
            handleSingletonWTO(cycle→head());
            cur_head_state := postAbsTrace[cycle_head];
            if increasing then
                postAbsTrace[cycle_head] := prev_head_state.widen(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    increasing := false;
                    continue;
            else
                postAbsTrace[cycle_head] := prev_head_state.narrow(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    break ;
        else
            handleSingletonWTO(cycle→head());
        handleWTOComponents(cycle→getWTOComponents());
        cur_iter++;
```

An Example: Abstract Trace σ for Top-level Variables

Widen Delay Phase (cur_iter is 0)



ICFG

$postAbsTrace[ICFGNode6].varToAbsVal$:

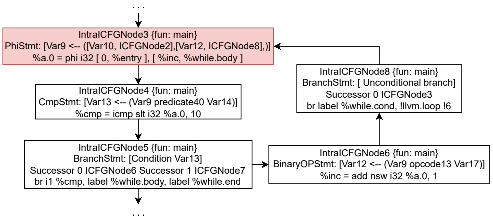
SVFVar	AbstractValue$\langle Interval, MemAddress\rangle$
	...
$Var10$	$\langle [0,0], \perp \rangle$
$Var9$	$\langle [0,0], \perp \rangle$
$Var12$	$\langle [1,1], \perp \rangle$
	...

Algorithm 12: Handle Cycle WTO

```
Function handleCycleWTO(cycle):
    cycle_head := cycle→head()→node() ;
    increasing := true ;
    cur_iter := 0 ;
    while true do
        // cur_iter ≡ 0, Options :: WidenDelay() ≡ 2;
        if cur_iter ≥ Options :: WidenDelay() then
            prev_head_state := postAbsTrace[cycle_head];
            handleSingletonWTO(cycle→head());
            cur_head_state := postAbsTrace[cycle_head];
            if increasing then
                postAbsTrace[cycle_head] := prev_head_state.widen(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    increasing := false;
                    continue;
            else
                postAbsTrace[cycle_head] := prev_head_state.narrow(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    break ;
        else
            handleSingletonWTO(cycle→head());
        handleWTOComponents(cycle→getWTOComponents());
        cur_iter++;
```

An Example: Abstract Trace σ for Top-level Variables

Widen Delay Phase (cur_iter is 1)



ICFG

$postAbsTrace[ICFGNode3].varToAbsVal$:

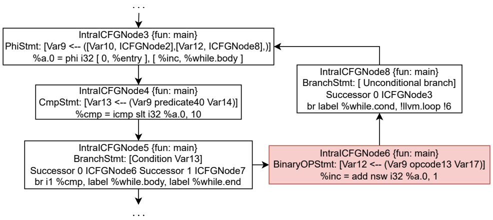
SVFVar	AbstractValue⟨*Interval*, *MemAddress*⟩
...	
Var9	$\langle [0, 1], \perp \rangle$
Var12	$\langle [1, 1], \perp \rangle$
...	

Algorithm 12: Handle Cycle WTO

Function handleCycleWTO(*cycle*):
  cycle_head := cycle→head()→node() ;
  increasing := true ;
  cur_iter := 0 ;
  while *true* **do**
    // *cur_iter* ≡ 1, *Options* :: *WidenDelay*() ≡ 2;
    if *cur_iter* ≥ *Options* :: *WidenDelay*() **then**
      prev_head_state := *postAbsTrace*[*cycle_head*];
      handleSingletonWTO(*cycle*→*head*());
      cur_head_state := *postAbsTrace*[*cycle_head*];
      if *increasing* **then**
        postAbsTrace[*cycle_head*] := *prev_head_state.widen*(*cur_head_state*) ;
        if *postAbsTrace*[*cycle_head*] == *prev_head_state* **then**
          increasing := false;
          continue;
      else
        postAbsTrace[*cycle_head*] := *prev_head_state.narrow*(*cur_head_state*) ;
        if *postAbsTrace*[*cycle_head*] == *prev_head_state* **then**
          break ;
    else
      handleSingletonWTO(*cycle*→*head*());
    handleWTOComponents(*cycle*→*getWTOComponents*());
    cur_iter++ ;

An Example: Abstract Trace σ for Top-level Variables

Widen Delay Phase (cur_iter is 1)



ICFG

$postAbsTrace[ICFGNode6].varToAbsVal$:

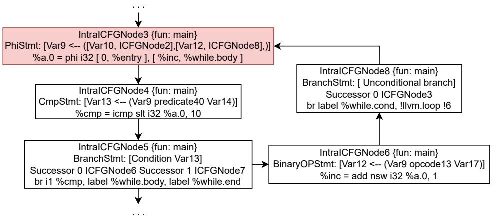
SVFVar	AbstractValue⟨*Interval*, *MemAddress*⟩
...	
Var9	$\langle [0,1], \perp \rangle$
Var12	$\langle [1,2], \perp \rangle$
...	

Algorithm 12: Handle Cycle WTO

```
Function handleCycleWTO(cycle):
    cycle_head := cycle→head()→node() ;
    increasing := true ;
    cur_iter := 0 ;
    while true do
        // cur_iter ≡ 1, Options :: WidenDelay() ≡ 2;
        if cur_iter ≥ Options :: WidenDelay() then
            prev_head_state := postAbsTrace[cycle_head];
            handleSingletonWTO(cycle→head());
            cur_head_state := postAbsTrace[cycle_head];
            if increasing then
                postAbsTrace[cycle_head] := prev_head_state.widen(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    increasing := false;
                    continue;
            else
                postAbsTrace[cycle_head] := prev_head_state.narrow(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    break ;
        else
            handleSingletonWTO(cycle→head());
        handleWTOComponents(cycle→getWTOComponents());
        cur_iter++ ;
```

An Example: Abstract Trace σ for Top-level Variables

Widen Phase (cur_iter is 2)



ICFG

$postAbsTrace[ICFGNode3].varToAbsVal$:

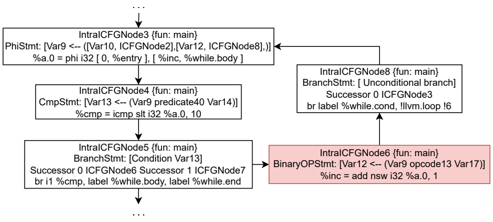
SVFVar	$\langle Interval, MemAddress\rangle$
	...
Var9	$\langle[0, +\infty], \perp\rangle$
Var12	$\langle[1, +\infty], \perp\rangle$
	...

Algorithm 12: Handle Cycle WTO

```
Function handleCycleWTO(cycle):
    cycle_head := cycle→head()→node() ;
    increasing := true ;
    cur_iter := 0 ;
    while true do
        // cur_iter ≡ 2,  Options :: WidenDelay() ≡ 2
        if cur_iter ≥ Options :: WidenDelay() then
            prev_head_state := postAbsTrace[cycle_head];
            handleSingletonWTO(cycle→head());
            cur_head_state := postAbsTrace[cycle_head];
            if increasing then
                postAbsTrace[cycle_head] := prev_head_state.widen(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    increasing := false;
                    continue;
            else
                postAbsTrace[cycle_head] := prev_head_state.narrow(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    break ;
        else
            handleSingletonWTO(cycle→head());
        handleWTOComponents(cycle→getWTOComponents());
        cur_iter++ ;
```

An Example: Abstract Trace σ for Top-level Variables

Widen Phase (cur_iter is 2)



ICFG

$postAbsTrace[ICFGNode6].varToAbsVal$:

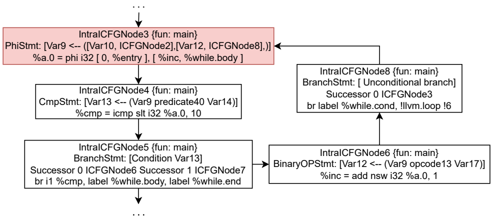
`SVFVar`	`AbstractValue`$\langle Interval, MemAddress\rangle$
...	
$Var9$	$\langle [0,9], \perp \rangle$
$Var12$	$\langle [1,10], \perp \rangle$
...	

Algorithm 12: Handle Cycle WTO

```
Function handleCycleWTO(cycle):
    cycle_head := cycle→head()→node() ;
    increasing := true ;
    cur_iter := 0 ;
    while true do
        // cur_iter ≡ 2,  Options :: WidenDelay() ≡ 2
        if cur_iter ≥ Options :: WidenDelay() then
            prev_head_state := postAbsTrace[cycle_head];
            handleSingletonWTO(cycle→head());
            cur_head_state := postAbsTrace[cycle_head];
            if increasing then
                postAbsTrace[cycle_head] := prev_head_state.widen(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    increasing := false;
                    continue;
            else
                postAbsTrace[cycle_head] := prev_head_state.narrow(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    break ;
        else
            handleSingletonWTO(cycle→head());
        handleWTOComponents(cycle→getWTOComponents())
        cur_iter++ ;
```

An Example: Abstract Trace σ for Top-level Variables

Widen Phase Fixed Point



ICFG

$postAbsTrace[ICFGNode3].varToAbsVal$:

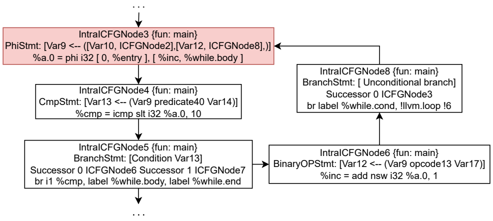
SVFVar	$\langle Interval, MemAddress \rangle$
	...
Var9	$\langle [0, +\infty], \perp \rangle$
Var12	$\langle [1, +\infty], \perp \rangle$
	...

Algorithm 12: Handle Cycle WTO

```
Function handleCycleWTO(cycle):
    cycle_head := cycle→head()→node() ;
    increasing := true ;
    cur_iter := 0;
    while true do
        // cur_iter ≡ 3, Options :: WidenDelay() ≡ 2
        if cur_iter ≥ Options :: WidenDelay() then
            prev_head_state := postAbsTrace[cycle_head];
            handleSingletonWTO(cycle→head());
            cur_head_state := postAbsTrace[cycle_head];
            if increasing then
                postAbsTrace[cycle_head] := prev_head_state.widen(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    increasing := false;
                    continue;
            else
                postAbsTrace[cycle_head] := prev_head_state.narrow(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    break ;
        else
            handleSingletonWTO(cycle→head());
        handleWTOComponents(cycle→getWTOComponents());
        cur_iter++ ;
```

An Example: Abstract Trace σ for Top-level Variables

Narrow Phase



ICFG

$postAbsTrace[ICFGNode3].varToAbsVal$:

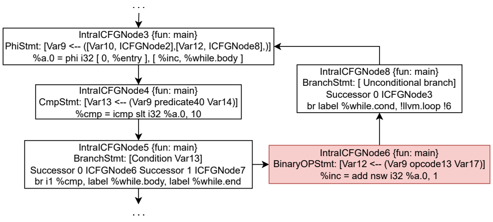
SVFVar	$\langle Interval, MemAddress \rangle$
	...
$Var9$	$\langle [0, 10], \perp \rangle$
$Var12$	$\langle [1, 10], \perp \rangle$
	...

Algorithm 12: Handle Cycle WTO

```
Function handleCycleWTO(cycle):
    cycle_head := cycle→head()→node() ;
    increasing := true ;
    cur_iter := 0 ;
    while true do
        // cur_iter ≡ 3,  Options :: WidenDelay() ≡ 2
        if cur_iter ≥ Options :: WidenDelay() then
            prev_head_state := postAbsTrace[cycle_head];
            handleSingletonWTO(cycle→head()) // increasing ≡ false;
            cur_head_state := postAbsTrace[cycle_head];
            if increasing then
                postAbsTrace[cycle_head] := prev_head_state.widen(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    increasing := false;
                    continue;
            else
                postAbsTrace[cycle_head] := prev_head_state.narrow(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    break ;
        else
            handleSingletonWTO(cycle→head());
        handleWTOComponents(cycle→getWTOComponents());
        cur_iter++ ;
```

An Example: Abstract Trace σ for Top-level Variables

Narrow Phase



ICFG

$postAbsTrace[ICFGNode6].varToAbsVal$:

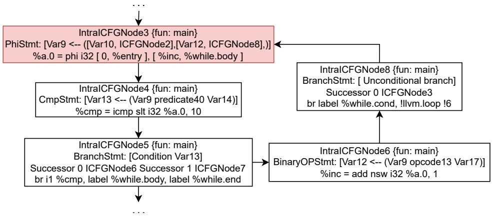
SVFVar	$\langle Interval, MemAddress \rangle$
	...
$Var9$	$\langle [0, 9], \perp \rangle$
$Var12$	$\langle [1, 10], \perp \rangle$
	...

Algorithm 12: Handle Cycle WTO

```
Function handleCycleWTO(cycle):
    cycle_head := cycle→head()→node() ;
    increasing := true ;
    cur_iter := 0 ;
    while true do
        // cur_iter ≡ 3,  Options :: WidenDelay() ≡ 2
        if cur_iter ≥ Options :: WidenDelay() then
            prev_head_state := postAbsTrace[cycle_head];
            handleSingletonWTO(cycle→head());
            cur_head_state := postAbsTrace[cycle_head];
            if increasing then
                postAbsTrace[cycle_head] := prev_head_state.widen(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    increasing := false;
                    continue;
            else
                postAbsTrace[cycle_head] := prev_head_state.narrow(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    break ;
        else
            handleSingletonWTO(cycle→head());
        handleWTOComponents(cycle→getWTOComponents()) ;
        cur_iter++;
```

An Example: Abstract Trace σ for Top-level Variables

Narrow Phase Fixed Point



ICFG

$postAbsTrace[ICFGNode3].varToAbsVal$:

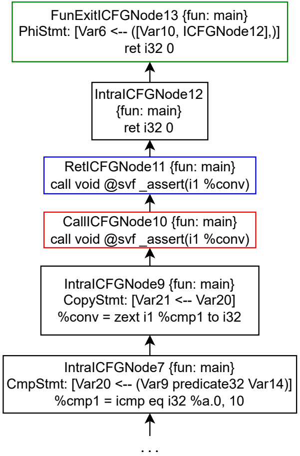
SVFVar	$\langle Interval, MemAddress \rangle$
	...
Var9	$\langle [0, 10], \perp \rangle$
Var12	$\langle [1, 10], \perp \rangle$
	...

Algorithm 12: Handle Cycle WTO

```
Function handleCycleWTO(cycle):
    cycle_head := cycle→head()→node() ;
    increasing := true ;
    cur_iter := 0 ;
    while true do
        // cur_iter ≡ 4, Options :: WidenDelay() ≡ 2
        if cur_iter ≥ Options :: WidenDelay() then
            prev_head_state := postAbsTrace[cycle_head];
            handleSingletonWTO(cycle→head()) // increasing ≡ false;
            cur_head_state := postAbsTrace[cycle_head];
            if increasing then
                postAbsTrace[cycle_head] := prev_head_state.widen(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    increasing := false;
                    continue;
            else
                postAbsTrace[cycle_head] := prev_head_state.narrow(cur_head_state) ;
                if postAbsTrace[cycle_head] == prev_head_state then
                    break ;
        else
            handleSingletonWTO(cycle→head());
        handleWTOComponents(cycle→getWTOComponents());
        cur_iter++ ;
```

An Example: Abstract Trace σ for Top-level Variables

After Exiting Loop



ICFG

Algorithm 13: Abstract execution guided by WTO

Function `handleStatement`(ℓ)**:**
    tmpAS := *preAbsTrace*[ℓ];
    if ℓ *is* CmpStmt **then**
        `updateStateOnCmp`(ℓ);
    . . . ;

postAbsTrace[*ICFGNode7*].*varToAbsVal* :

SVFVar	$\langle$*Interval*, *MemAddress*$\rangle$
	. . .
Var9	$\langle[10, 10], \perp\rangle$
Var20	$\langle[1, 1], \perp\rangle$
	. . .