Code Verification and Automated Theorem Prover

(Week 4)

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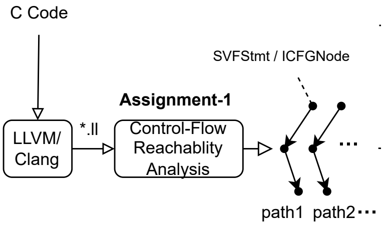
Today's class



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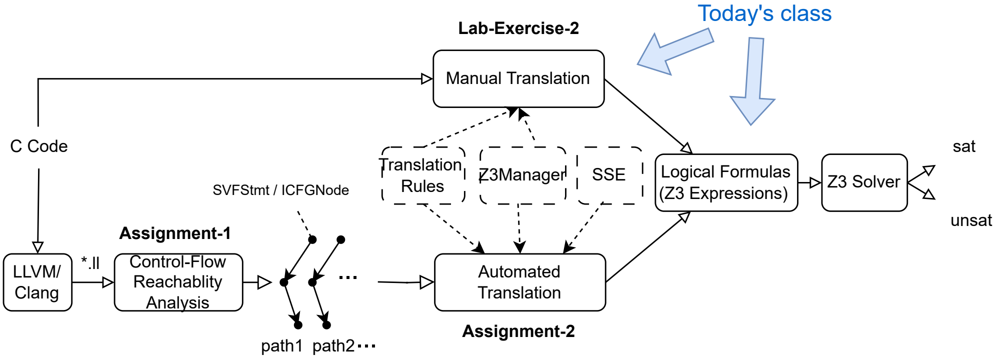

- In Lab-Exercise-2 and Assignment-2, we will conduct code verification to **prove code assertions** on top of reachability analysis (Assignment-1).
- Translating **C statements (Lab-Exercise-2)** and **SVFStmt/ICFGNode (Assignment-2)** to **logical formulas/expressions** and solve them to verify code assertions using automated theorem prover (i.e., Z3)

Formal Verification For Code

Specification $\stackrel{?}{\equiv}$ Code implementation

Formal Verification For Code

$$\begin{array}{ccc} \text{Specification} & \stackrel{?}{\equiv} & \text{Code implementation} \\ \Downarrow \text{translate} & & \Downarrow \text{translate} \\ \text{logical formulas of specification} & \stackrel{?}{\equiv} & \text{Logical formulas of code implementation.} \end{array}$$

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\text{logical formulas of specification} & \stackrel{?}{\equiv} & \text{Logical formulas of code implementation.}
\end{array}
$$

- Proving the correctness of your code given a specification (or spec) using formal methods of mathematics
- Make the connection between specifications and implementations rigid, reliable and secure by translating specification and code into logical formulas.
- The application of theorem proving tools to perform satisfiability checking of logical formulas.

Specification

- Specifications **independent of** the source code
  - Formal specification in a separate file from the source code, written in a specification language and accepted by theorem provers

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  - Formal specification in a separate file from the source code, written in a specification language and accepted by theorem provers
- Specifications **embedded in** the source code (This course)
  - `assume(expr)`: an assumed **precondition** of a program that expression `expr` always be true and uses this assumed knowledge to execute the program. `assmue` is often **optional** as many verification scenarios may not have preconditions, including Lab-Exercise-2 and Assignment-2.
  - `assert(expr)`: an expected **postcondition** embedded in the program to check that `expr` always holds for any execution, otherwise the program terminates. We use `svf_assert` in our lab/assignment as an alternative for verification purposes.

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  - `assert(expr)`: an expected **postcondition** embedded in the program to check that `expr` always holds for any execution, otherwise the program terminates. We use `svf_assert` in our lab/assignment as an alternative for verification purposes.
- Hoare logic triple $P\{prog\}Q$, represents a program expressed by a predicate (first-order) logic. It describes that when the **precondition** P is met, executing the **program** *prog* establishes the **postcondition** Q.

Hoare logic: `https://en.wikipedia.org/wiki/Hoare_logic`

Formal specifications: `https://www.hillelwayne.com/post/why-dont-people-use-formal-methods`

Pre-/Post-Conditions and Satisfiability

Prove whether the post-condition (`assert`) holds after executing the program given the pre-condition (`assume`).

```
assume(100 > x > 0);  // P
        if(x > 10) {
            y = x + 1;
        }
        else {
            y = 10;
        }
assert(y >= x + 1);  // Q
```

$\xrightarrow{\text{translate}}$ $\psi(P\{\texttt{prog}\}Q)$ $\xrightarrow{\text{feed into}}$ SAT/SMT Solver

logical formula

Will the assertion hold?

Assertions as Specifications

- In our lab and assignments, we need to verify whether the assertions (`svf_assert`) as specifications are satisfiable (expected results) or not.
- An assertion is a predicate or an expression that **always should evaluate to true** at that point during code execution.
  - help a programmer **read the code**
  - help the program **detect its own defects**
  - help catch errors earlier and **pinpoint sources of errors**

```
assert(expr);

or

svf_assert(expr);
```

$\xrightarrow{\textit{unfold}}$

```
if(expr is true){
    // continue normal execution
}
else{
    __assert_fail();
    // program failure and terminate the program
}
```

Satisfiability Solving as Logic Inference

Satisfiability solving of hoare logic triple $P\{prog\}Q$ as a logic inference problem:

- Given $P\{prog\}Q$ represented by a set of constraints (logical formulas) extracted from code, we express $P\{prog\}$ as KB **knowledge base** or **premises**, and Q is the **conclusion.** Revisit our previous example as below:

Satisfiability Solving as Logic Inference

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- Given $P\{prog\}Q$ represented by a set of constraints (logical formulas) extracted from code, we express $P\{prog\}$ as KB **knowledge base** or **premises**, and Q is the **conclusion**. Revisit our previous example as below:
  - KB : $(100 > \mathtt{x} > 0) \wedge ((\mathtt{x} > \mathtt{10} \wedge \mathtt{y} \equiv \mathtt{x} + \mathtt{1}) \vee (\mathtt{x} \leq \mathtt{10} \wedge \mathtt{y} \equiv \mathtt{10}))$
  - Q : $\mathtt{y} \geq \mathtt{x} + \mathtt{1}$
- $KB \vdash Q$?
  - Does KB semantically entail Q?
  - If all constraints in KB are true, is the assertion true?
  - Is the specification Q satisfiable given constraints from code?
- Each element (**proposition** or **predicate**) in KB can be seen as a premise and Q is the conclusion.

Propositional Logic (Statement Logic)

A **proposition** is a statement that is either true or false. Propositional logic studies the ways statements can interact with each other.

- **Propositional variables** (e.g., S) represent propositions or statements in the formal system.
- A **propositional formula** is logical formula with **propositional variables** and **logical connectives** like `and` ($\wedge$) , `or` ($\vee$), `negation` ($\neg$), `implication` ($\rightarrow$)
  - $(S_1 \wedge S_2) \rightarrow Q$. This formula means that if S_1 and S_2 are both true, then Q is true.
  - S_1 and S_2 are propositional variables. $\wedge$ and $\rightarrow$ are logical connectives.
- **Logic inference** allows certain logic formulas to be derived. These derived formulas are called **theorems** (or true propositions). The derivation can be interpreted as proof of the proposition represented by the theorem.

https://en.wikipedia.org/wiki/Propositional_calculus

http://discrete.openmathbooks.org/dmoi2/sec_propositional.html

Predicate Logic (First-Order Logic)

Predicate logic is propositional logic with predicates and quantification.

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- **Predicate logic**: is more expressive and further analyzes proposition(s) by representing their entities' properties and relations and to group entities, i.e., additionally covers predicates and quantification.

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- **Propositional logic**: boolean logic which represents statements without reflecting their structures and relations
- **Predicate logic**: is more expressive and further analyzes proposition(s) by representing their entities' properties and relations and to group entities, i.e., additionally covers predicates and quantification.
- A **predicate** P takes one or more variables/entities as input and outputs a proposition and has a truth value (either true or false).
  - A statement whose truth value is dependent on variables.
  - For example, in $P(x) : x > 5$, "x" is the variable and "$x > 5$" is the predicate. After assigning x with the value 6, $P(x)$ becomes a proposition $6 > 5$.
- A **quantifier** is applied to a set of entities
  - Universal quantifier $\forall$, meaning all, every
  - Existential quantifier $\exists$, meaning some, there exists

https://en.wikipedia.org/wiki/First-order_logic https://www.youtube.com/watch?v=ARywou8HLQk

Predicate Logic (Natural Language Example)

Consider the two statements

- "Jack got a high distinction"
- "Peter got a high distinction"

In propositional logic, these statements are viewed as being unrelated and the sub-statements/words/entities are not further analyzed.

- **Predicate logic** allows us to define a **predicate** P representing "got a high distinction" which occurs in both sentences.
- $P(x)$ is the **predicate logic statement (formula)** which accepts a name x and output as "x got a high distinction".

Predicate Logic (Code Example)

Consider these four statements

S_1:	x $>$ 20;
S_2:	x $>$ 10;
$S_2 \rightarrow Q$:	if(x $>$ 10) y = 15;
Q:	y $=$ 15;

Predicate Logic (Code Example)

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S_1:	$x > 20$;
S_2:	$x > 10$;
$S_2 \rightarrow Q$:	if($x > 10$) y = 15;
Q:	$y = 15$;

- In **propositional logic**, each statement (including its variables and constants) is viewed as one proposition. Their relations are not further analyzed.
  - Given propositions S_1 and $S_2 \rightarrow Q$ as the knowledge base KB. Does the following semantically entail $\{S_1, S_2 \rightarrow Q\} \vdash Q$ or $(S_1 \wedge S_2 \rightarrow Q) \rightarrow Q$ hold?

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- **Predicate logic** allows us to define **three predicates**: $P_1(x)$ represents x $>$ 20; $P_2(x)$ represents x $>$ 10; $Q(y)$ represents y $=$ 15 for the properties of x, y. Does the following hold using predicate logical for the inference?
  - $\{P_1(x), P_2(x) \rightarrow Q(y)\} \vdash Q(y)$ or $(P_1(x) \wedge P_2(x) \rightarrow Q(y)) \rightarrow Q(y)$

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  - $\{P_1(x), P_2(x) \rightarrow Q(y)\} \vdash Q(y)$ or $(P_1(x) \wedge P_2(x) \rightarrow Q(y)) \rightarrow Q(y)$
  - $\{$x $>$ 20, x $>$ 10 $\rightarrow$ y = 15$\} \vdash$ y = 15
  - Answer: Yes!

Satisfiability Checking (Revisit Our Example)

Given the predicate formula $\psi(P\{\texttt{prog}\}Q)$, we can verify the correctness of a program against the assertion specification Q by checking ψ's satisfiability (SAT).

```
assume(100 > x > 0);  // P
        if(x > 10) {
            y = x + 1;
        }
        else {
            y = 10;
        }
assert(y >= x + 1);  // Q
```

$\xrightarrow{\text{translate}}$ $\psi(P\{\texttt{prog}\}Q)$ $\xrightarrow{\text{feed into}}$ SAT/SMT Solver

logical formula

Satisfiability Checking for Code Verification

- $\psi(P\{\texttt{prog}\}Q)$ is satisfiable if a program *prog* is correct for all valid inputs.

$$\forall \mathtt{x}\ \forall \mathtt{y}\ \ P(\mathtt{x}) \land S_{prog}(\mathtt{x},\mathtt{y}) \rightarrow Q(\mathtt{x},\mathtt{y})$$

  - $P(x)$ is the pre-condition predicate $(\mathtt{100} > \mathtt{x} > \mathtt{0})$ over variables $\mathtt{x}$.
  - $S_{prog}(\mathtt{x},\mathtt{y})$ is the predicate representing *prog* which accepts $\mathtt{x}$ as its input, and terminates with output $\mathtt{y}$.
  - $Q(\mathtt{x},\mathtt{y})$ is the post-condition predicate $(\mathtt{y} >= \mathtt{x} + \mathtt{1})$ over variables $\mathtt{x},\mathtt{y}$.

Satisfiability Checking for Code Verification

- $\psi(P\{\texttt{prog}\}Q)$ is satisfiable if a program *prog* is correct for all valid inputs.

$$\forall \mathrm{x}\ \forall \mathrm{y}\ \ P(\mathrm{x}) \wedge S_{prog}(\mathrm{x}, \mathrm{y}) \rightarrow Q(\mathrm{x}, \mathrm{y})$$

  - $P(x)$ is the pre-condition predicate $(100 > \mathrm{x} > 0)$ over variables x.
  - $S_{prog}(\mathrm{x}, \mathrm{y})$ is the predicate representing *prog* which accepts x as its input, and terminates with output y.
  - $Q(\mathrm{x}, \mathrm{y})$ is the post-condition predicate $(\mathrm{y} >= \mathrm{x} + 1)$ over variables x, y.
- How to prove correctness for all inputs x? Search for counterexample x where ψ does not hold, that is

$$\begin{array}{ll} & \exists \mathrm{x}\ \exists \mathrm{y}\ \ \neg(P(\mathrm{x}) \wedge S_{prog}(\mathrm{x}, \mathrm{y})) \rightarrow Q(\mathrm{x}, \mathrm{y})) \\ \Rightarrow & \exists \mathrm{x}\ \exists \mathrm{y}\ \ P(\mathrm{x}) \wedge S_{prog}(\mathrm{x}, \mathrm{y}) \wedge \neg Q(\mathrm{x}, \mathrm{y}) \qquad \text{(simplification)} \end{array}$$

  Note that $P(\mathrm{x})$ is always true if a program does not have a pre-condition.

Logic formula simplification: `https://en.wikipedia.org/wiki/Logical_equivalence`

Satisfiability Checking for Code Verification

Checking whether the logical formula ψ is satisfiable by an SMT solver.

```
assume(100 > x > 0);
    if(x > 10) {
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$$\xRightarrow{\text{translate}} \underbrace{\exists \mathrm{x}\ \exists \mathrm{y}\ \ P(\mathrm{x}) \land S_{prog}(\mathrm{x}, \mathrm{y})) \land \neg Q(\mathrm{x}, \mathrm{y})}_{\text{logical formula } \psi} \xRightarrow{\text{feed into}} \text{SMT Solver}$$

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$\xrightarrow{\text{translate}}$ $\underbrace{\exists x\ \exists y\ \ P(x) \wedge S_{prog}(x, y)) \wedge \neg Q(x, y)}_{\text{logical formula } \psi}$ $\xrightarrow{\text{feed into}}$ SMT Solver

Unsatisfiable! **couterexample** `x = 10` **found**!

SMT: `https://en.wikipedia.org/wiki/Satisfiability_modulo_theories`

Translating Code into Logical Formulas

- How to extract first order logical formulas $P(\mathtt{x}) \land S_{prog}(\mathtt{x}, \mathtt{y})) \land \neg Q(\mathtt{x}, \mathtt{y})$ from code?

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- How to extract first order logical formulas $P(\mathtt{x}) \land S_{prog}(\mathtt{x},\mathtt{y})) \land \neg Q(\mathtt{x},\mathtt{y})$ from code?
- First-order logical formulas
  - The formulas of predicate logic are constructed from **propositional**, **predicate** and **object variables** by using **logical connectives** and **quantifiers** (This class)
- Translation
  - Translating `SVFStmts` of **each program path** (from Assignment-2) into a logical formula ψ, and then check the satisfiablity for each path.
  - $\forall path \in prog \quad checking(\psi_{path})$

$$\psi_{path_1}: \quad \exists \mathtt{x}\ P(\mathtt{x}) \land \big((\mathtt{x} > 10) \land (\mathtt{y} \equiv \mathtt{x}+1)\big) \land \neg Q(\mathtt{x},\mathtt{y}) \quad (\texttt{if}\ \text{branch})$$
$$\psi_{path_2}: \quad \exists \mathtt{x}\ P(\mathtt{x}) \land \big((\mathtt{x} \leq 10) \land (\mathtt{y} \equiv 10)\big) \land \neg Q(\mathtt{x},\mathtt{y}) \quad (\texttt{else}\ \text{branch})$$

Translating Code into Logical Formulas

- How to extract first-order logical formulas $P(\mathtt{x}) \wedge S_{prog}(\mathtt{x}, \mathtt{y})) \wedge \neg Q(\mathtt{x}, \mathtt{y})$ from code?
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- Translation of program paths
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  - $\forall path \in prog \quad checking(\psi_{path})$

    $\psi_{path_1}: \exists x(100 > x > 0) \wedge \big((\mathtt{x} > 10) \wedge (\mathtt{y} \equiv \mathtt{x} + 1)\big) \wedge \neg(\mathtt{y} \geq \mathtt{x} + 1)$ (`if` branch)

    $\psi_{path_2}: \exists x(100 > x > 0) \wedge \big((\mathtt{x} \leq 10) \wedge (\mathtt{y} \equiv 10)\big) \wedge \neg(\mathtt{y} \geq \mathtt{x} + 1)$ (`else` branch)
  - ψ_{path_2} : **has a counterexample** `x = 10`!!

Translating Code into Logical Formulas

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    ψ_{path_2} :$\exists x(100 > x > 0) \land ((\mathtt{x} \leq 10) \land (\mathtt{y} \equiv 10)) \land \neg(\mathtt{y} \geq \mathtt{x}+1)$ (`else` branch)
  - ψ_{path_2} : **has a counterexample** `x = 10`!!
  - **Manual translation** of C statements to logic expressions via Z3 theorem prover APIs (`Z3Mgr.h/cpp`) (Lab-Exercise-2)
  - **Automatic translation** of SVFIR to logic expressions during control-flow reachability analysis (Assignment-2)

Theorem Prover Tools

- **Interactive theorem provers** (proof assistants)
  - Formal proofs by human-machine collaboration via expressive specification languages; may not work directly on source code.
  - For example, ACL2, Coq, Isabelle, and HOL provers.
- **Automated theorem provers**
  - Proof automation (but less expressive than interactive provers); can work on real-world source code.
  - For example, Z3 and CVC.

Theorem prover tools: `https://en.wikipedia.org/wiki/Theorem_prover`

Automated Theorem Provers

A prover/solver checks if a formula $\psi(P\{\mathtt{prog}\}Q)$ is satisfiable (SAT).

- If yes, the solver returns a **model** m, a valuation of $\mathtt{x}, \mathtt{y}, \mathtt{z}$ of $\mathtt{prog}$ that satisfies ψ (i.e., m makes ψ true).
- Otherwise, the solver returns unsatisfiable (UNSAT)

SAT vs. **SMT** solvers

- **SAT** solvers accept **propositional logic** (Boolean) formulas, typically in the conjunctive normal form (CNF).
- **SMT** (satisfiability modulo theories) solvers generalize the Boolean satisfiability problem (SAT), and accept both propositional logic and more expressive **predicate logic** formulas.
  - Z3 Automated Theorem Prover, a cross-platform satisfiability modulo theories (SMT) solver developed by Microsoft (This course).

Z3: `https://github.com/Z3Prover/z3/wiki#background`

Code to Logic Expressions with Z3 Theorem Prover

(Week 5)

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Z3 Theorem Prover

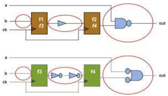
- Z3 is a Satisfiability Modulo Theories (SMT) solver from Microsoft Research.
- Targeted at solving problems in software verification and software analysis.
- Main applications are static checking, test case generation, and more ..

Hardware verification

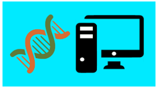
Software analysis/testing

Architecture

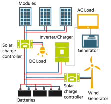
Modeling



Geometrical solving



Biological analysis

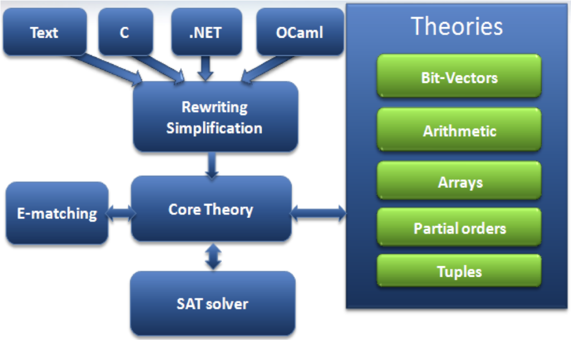


Hybrid system analysis

...

https://www.microsoft.com/en-us/research/project/z3-3/

Z3 Framework



- Z3 is an effective tool to solve **logical formulas** (Z3 expressions/constraints).
- Z3 GitHub `https://github.com/Z3Prover/z3`.
- Z3 tutorials `https://github.com/philzook58/z3_tutorial`
- Z3 slides `https://github.com/Z3Prover/z3/wiki/Slides`
- Its SMT solver supports theories such as fixed-size bit-vectors, arithmetic, extensional arrays, datatypes, uninterpreted functions, and quantifiers.
- Z3 has official APIs for **C, C++, Python, .NET**, etc.
- **Z3 solver** can find one of the feasible solutions in a set of constraints.

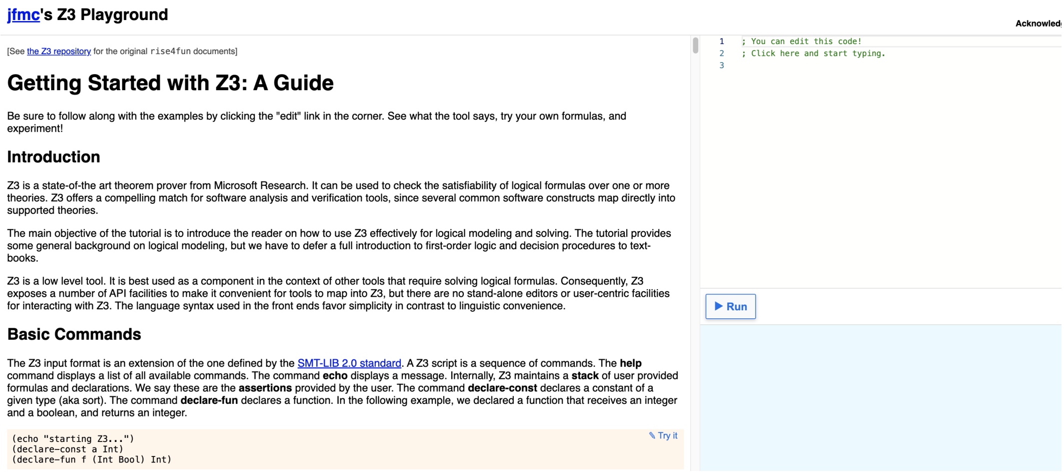
Z3 Solver and Z3 Formulas

Z3 solver accepts a first-order (predicate) logical formula ψ, and outputs one of the following results.

- `sat` if ψ is satisfiable
- `unsat` if there is a counterexample which make ψ unsatisfiable
- `unknown` if ψ is too complex and can not be solved within a time frame.

You play around and check the satisfiability of your Z3 constraints/formulas here:
`https://jfmc.github.io/z3-play` or
`https://compsys-tools.ens-lyon.fr/z3/index.php`

Z3 Playground (https://jfmc.github.io/z3-play)



Z3's Logical Formula (Constants, Check-Sat and Evaluation)

The Z3 input format (formula format) is an extension of the SMT-LIB 2.0 standard. A Z3 formula expression (`z3::expr`) has the following keywords:

- **`echo`** displays a message
- **`declare-const`** declares a constant of a given type (a.k.a `sort`)
- **`declare-fun`** declares a function
- **`assert`** adds a formula into the Z3 internal stack
- **`check-sat`** determines whether the current formulas on the Z3 stack are satisfiable or not
- **`get-model`** is used to retrieve an interpretation (one solution) that makes all formulas on the Z3 internal stack true
- **`eval`** evaluates a variable/expression produced by a model when the formulas is satisfiable.

SMT-LIB 2.0: `https://homepage.cs.uiowa.edu/~tinelli/papers/BarST-SMT-10.pdf`

Constants, Check-Sat and Evaluation (Example)

$$\psi : (\mathtt{x} > \mathtt{10}) \;\wedge\; (\mathtt{y} \equiv \mathtt{x} + \mathtt{1})$$

How to represent this formula in Z3 and feed it into Z3's solver?

Constants, Check-Sat and Evaluation (Example)

$$\psi : (x > 10) \ \wedge \ (y \equiv x + 1)$$

How to represent this formula in Z3 and feed it into Z3's solver?

```
(echo "starting Z3...")
(declare-const x Int) ;/// Declare an Int type variable "x"
(declare-const y Int) ;/// Declare an Int type variable "y"
(assert (> x 10))  ;/// Add the first part (x>10) of the conjunction into the solver
(assert (= y (+ x 1))) ;/// Add the second part (y==x+1) of the conjunction
(check-sat) ;/// Check whether added formulas are satisfiable.
(eval x) ;/// Evaluate the value of x when the formula is satisfiable
(eval y) ;/// Evaluate the value of y when the formula is satisfiable
```

Constants, Check-Sat and Evaluation (Example)

$$\psi : (x > 10) \ \wedge \ (y \equiv x + 1)$$

How to represent this formula in Z3 and feed it into Z3's solver?

```
(echo "starting Z3...")
(declare-const x Int) ;/// Declare an Int type variable "x"
(declare-const y Int) ;/// Declare an Int type variable "y"
(assert (> x 10))  ;/// Add the first part (x>10) of the conjunction into the solver
(assert (= y (+ x 1))) ;/// Add the second part (y==x+1) of the conjunction
(check-sat) ;/// Check whether added formulas are satisfiable.
(eval x) ;/// Evaluate the value of x when the formula is satisfiable
(eval y) ;/// Evaluate the value of y when the formula is satisfiable
```

Outputs of Z3's solver:

```
starting Z3...
sat /// (check-sat) result
11 /// the value of x as one satisfiable solution
12 /// the value of y as one satisfiable solution
```

Z3's Logical Formula (Uninterpreted Function)

The basic building blocks of SMT formulas are constants and uninterpreted functions.

- An uninterpreted function **has no other property** (no priori interpretation) **than its signature** (i.e., function name and arguments).
- An uninterpreted functions in first-order logic have **no side-effects** (e.g., can not change argument values and never return different values for the same input)
- **Constants** in Z3 can also be seen as **functions that take no arguments.**
- **The details and characteristics** of uninterpreted functions are **ignored.** This can **generalize and simplify** theorems and proofs.

Uninterpreted Function (Example)

```
(declare-fun f (Int) Int)    ;/// Function f accepts an Int argument and returns a Int
(assert (= (f 10) 1))    ;/// f(10) = 1
(check-sat)
```

Uninterpreted Function (Example)

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(declare-fun f (Int) Int)    ;/// Function f accepts an Int argument and returns a Int
(assert (= (f 10) 1))    ;/// f(10) = 1
(check-sat)
```

Outputs of Z3's solver:

```
sat
```

The solver returns `sat`, because `f` is an uninterpreted function (i.e., all that is known about `f` is its signature), so it is possible that `f(10) = 1`.

Uninterpreted Function (Example)

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(declare-fun f (Int) Int)    ;/// Function f accepts an Int argument and returns a Int
(assert (= (f 10) 1))    ;/// f(10) = 1
(check-sat)
```

Outputs of Z3's solver:

```
sat
```

The solver returns `sat`, because `f` is an uninterpreted function (i.e., all that is known about `f` is its signature), so it is possible that `f(10) = 1`.

```
(declare-fun f (Int) Int)    ;/// Function f accepts an Int argument and returns a Int
(assert (= (f 10) 1))    ;/// f(10) = 1
(assert (= (f 10) 2))    ;/// f(10) = 2
(check-sat)
```

Uninterpreted Function (Example)

```
(declare-fun f (Int) Int)    ;/// Function f accepts an Int argument and returns a Int
(assert (= (f 10) 1))    ;/// f(10) = 1
(check-sat)
```

Outputs of Z3's solver:

```
sat
```

The solver returns `sat`, because `f` is an uninterpreted function (i.e., all that is known about `f` is its signature), so it is possible that `f(10) = 1`.

```
(declare-fun f (Int) Int)    ;/// Function f accepts an Int argument and returns a Int
(assert (= (f 10) 1))    ;/// f(10) = 1
(assert (= (f 10) 2))    ;/// f(10) = 2
(check-sat)
```

Outputs of Z3's solver:

```
unsat
```

The solver returns `unsat`, because `f`, as an uninterpreted function, can never return different values for the same input.

Uninterpreted Function (Example)

$$\psi : \mathtt{f(x)} \equiv \mathtt{f(y)} \ \wedge \ \mathtt{x!} = \mathtt{y}$$

```
(declare-const x Int)
(declare-const y Int)
(declare-fun f (Int) Int) ;/// Function f accepts an Int argument and returns a Int
(assert (= (f x) (f y)))
(assert (not (= x y)))
(check-sat)
```

Uninterpreted Function (Example)

$$\psi : \mathtt{f}(\mathtt{x}) \equiv \mathtt{f}(\mathtt{y}) \ \wedge \ \mathtt{x}! = \mathtt{y}$$

```
(declare-const x Int)
(declare-const y Int)
(declare-fun f (Int) Int) ;/// Function f accepts an Int argument and returns a Int
(assert (= (f x) (f y)))
(assert (not (= x y)))
(check-sat)
```

Outputs of Z3's solver:

```
sat
```

An uninterpreted function can have different inputs and return the same output. For example, `f` can always return `1` regardless the value of the input argument.

Constants as Uninterpreted Function (Example)

$$\psi : (x > 10) \ \land \ (y \equiv x + 1)$$

```
(declare-fun x () Int) ;/// "x" and "y" as an uninterpreted functions
(declare-fun y () Int) ;/// Accepts no argument and return an Int
(assert (> x 10))
(assert (= y (+ x 1)))
(check-sat)
(get-model)
```

Outputs of Z3's solver:

```
sat
(
  (define-fun x () Int
    11)                   ;/// x is evaluated to be 11 for this model
  (define-fun y () Int
    12)                   ;/// y is evaluated to be 11 for this model
)
```

`(declare-const x Int)` can be seen as the syntax sugar for `(declare-fun x () Int)`.

Z3's Logical Formula (Arithmetic)

- Z3 supports majority of commonly used arithmetic operators, such as +, -, $*$, /, $<<$, $>>$, $<$, $>$, &, $|$ (The ones listed in SVFIR)
- Types of any two operands should be the same otherwise a type conversion is needed.
- Never mix types in arithmetic, and always be explicit.

```
1 (declare-const a Int)
2 (declare-const b Float32)
3 (assert (= a (+ b 1)))
4 (check-sat)
```

Outputs of Z3's solver:

```
Error: (error "line 3 column 19: Sort mismatch at argument #1 for function
(declare-fun + (Int Int) Int) supplied sort is (_ FloatingPoint 8 24)")
```

Z3's Logical Formula (`if-then-else` Expression)

- `ite(b, x, y)` represents a conditional expression, where `b` is the condition, `ite` returns `x` if `b` is evaluated true, otherwise `y` is returned
- Used for comparison or branches

```
(ite (and (= x!1 11) (= x!2 false)) 21 0)
```

The above Z3 formula evaluates (returns) 21 when x!1 is equal to 11, and x!2 is equal to false. Otherwise, it returns 0.

Z3's Logical Formula (Arrays)

Formulating a program of a mathematical theory of computation McCarthy proposed a basic theory of arrays as characterized by the **select-store** axioms.

- `(select a i)`: returns the value stored at position `i` of the array `a`;
- `(store a i v)`: returns a new array identical to `a`, but on position `i` it contains the value `v`.
- Z3 assumes that arrays are extensional over select. Z3 also enforces that if two arrays agree on all reads, then the arrays are equal.

Z3's Logical Formula (Arrays)

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- `(select a i)`: returns the value stored at position `i` of the array `a`;
- `(store a i v)`: returns a new array identical to `a`, but on position `i` it contains the value `v`.
- Z3 assumes that arrays are extensional over select. Z3 also enforces that if two arrays agree on all reads, then the arrays are equal.

The following formulas store `y` to the `x`-th position of array `a` and then load the value at `a`'s `x`-th position to `z`

```
(declare-const x Int)
(declare-const y Bool)
(declare-const z Bool)
(declare-const a (Array Int Bool))  ;/// an array of Bools with Int as the indices
(assert (= (store a x y) a))     ;/// a[x] == y
(assert (= (select a x) z))      ;/// z == a[x]
```

Z3's Logical Formula (Scopes)

Z3 maintains a global stack of declarations and assertions via **push** and **pop**

- **push**: creates a new scope by saving the current stack size.
- **pop**: removes any assertion or declaration performed between it and the matching push.

The `check-sat` command always operates on the current global stack.

Z3's Logical Formula (Scopes)

Z3 maintains a global stack of declarations and assertions via **push** and **pop**

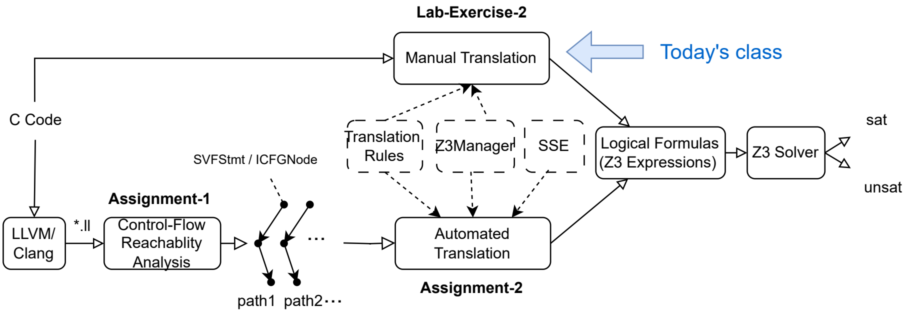
- **push**: creates a new scope by saving the current stack size.
- **pop**: removes any assertion or declaration performed between it and the matching push.

The `check-sat` command always operates on the current global stack.

```
(declare-const x Int)
(declare-const a (Array Int Int))  /// an array of Ints
(push)
(assert (= (store a 1 10) a))     ;/// a[1] == 10
(assert (= (select a 1) x))      ;/// x == a[1]
(assert (= x 20))                ;/// x == 20
(check-sat)
(pop)                            ;/// remove the three assertions
(assert (= x 10))                ;/// x == 10
(check-sat)
```

What is the output of the solver?

Today’s class



- We introduce Z3 solver, Z3 constraint format **Z3Mgr** APIs used for lab/assignment in this course.
- We learn how to manually translate C source code into logical formulas (Z3 constraints/expressions).
- Then, we will demonstrate **examples** for **manual translation** from code to Z3 constraints.

Translating Code to Z3 Formulas

We provide a Z3Mgr class (a wrapper class to manipulate Z3 APIs) to generate Z3 formulas or so-called `z3::expr`.

API	**Meanings**
`z3::expr getZ3Expr(std::string);`	Create a **variable** given a string name
`z3::expr getZ3Expr(int);`	Create a **variable** given an integer
`z3::expr getMemObjAddress(std::string);`	Create a **memory object** in program
`z3::expr getGepObjAddress(z3::expr, u32_t);`	Create a **field object** with an **offset** of an aggregate
`void addToSolver(z3::expr);`	**Add** a Z3 expression/formula to the solver
`void resetSolver();`	Clean all formulas in the the solver
`solver.check();`	**Check satisfiability** of an z3 formula
`z3::expr getEvalExpr(z3::expr);`	**Evaluate an expression** to a value based on a model.
`void printExprValues();`	**Print** the values of **all expressions** in the solver.

More details, refer to

`https://github.com/SVF-tools/Teaching-Software-Verification/wiki/SVF-APIs`

Z3Mgr::getEvalExpr

```
z3::expr Z3Mgr::getEvalExpr(z3::expr e) {
    z3::check_result res = solver.check();
    assert(res != z3::unsat && "unsatisfied constraints!");
    z3::model m = solver.get_model();
    return m.eval(e);
}
```

The `Z3Mgr::getEvalExpr` method checks if the constraints added to the Z3 solver are satisfiable. If they are, it retrieves the model that satisfies these constraints and evaluates the given complex expression `e` within this model, returning the evaluated result as one of the following:

- Boolean Expression: `is_true()` or `is_false()`
- Integer Expression: `is_numeral()`, `get_numeral_int64()`
- Real Expression: `get_numeral_double()`
- String Expression: `get_numeral_string()`

More APIs Used in Lab-Exercise-2

- `getZ3Expr(u32_t id)`: Get the z3 expression from an index
- `getZ3Expr(string name)`: Get the z3 expression from a string name.
- `getMemObjAddress(string name)`: Get the memory object address from a string name;
- `getGepObjAddress(z3::expr pointer, u32_t offset)`: Get the memory object address from a pointer and an offset
- `addToSolver(z3::expr e)`: Add an z3 expression `e` to the solver.
- `printZ3Exprs()`: Print all z3 expressions
- `printExprValues()`: Print all expressions' values after evaluation
- `getVirtualMemAddress(u32_t id)` and `isVirtualMemAddress(u32_t id)`: The `id` of an object (`ObjVar`) in SVFIR will be marked using an `AddressMask` (0x7f000000) to mimic the virtual memory address (note that this is not a physical runtime address but an abstract address)
- `getInternalID(u32_t)` will unmarsk a virtual address to get its original `ObjVar`'s `id`.
- `expr storeValue(expr loc, expr value)`: stores a `value` to address `loc`.
- `expr loadValue(expr loc)`: loads a value from address `loc`.

Translation Rules

`expr p = getZ3Expr("p")`   `expr q = getZ3Expr("q")`   `expr r = getZ3Expr("r")`

SVFStmt	C-Like form	Operations
`AddrStmt` (constant)	`p = c`	`addToSolver(p == c);`
`AddrStmt` (mem allocation)	`p = alloc`	`addToSolver(p == getMemObjAddress("alloc");)`
`CopyStmt`	`p = q`	`addToSolver(p == q);`
`LoadStmt`	$p = *q$	`addToSolver(p == loadValue(q));`
`StoreStmt`	$*p = q$	`storeValue(p, q);`
`GepStmt`	$p = \&(q \rightarrow i)$ or $p = \&q[i]$	`addToSolver(p == getGepObjAddress(q,i));`
`PhiStmt`	$r = phi(\ell_1 : p,\ \ell_2 : q)$	`if(executed from` l_1`)` `addToSolver(p==r);` `if(executed from` l_2`)` `addToSolver(q==r);`

`BranchStmt`	`if (p)` l_1 else l_2	`expr cond = getEvalExpr(p);`
		`if(cond.is_false())` `execute` l_2
		`else` `execute` l_1 `addToSolver(cond == true);`

`UnaryOPStmt`	$\neg p$	`addToSolver(!p);`
`BinaryOPStmt`	$r = p \otimes q$ BinaryOPStmt::OpCode	`addToSolver(r ==` $p \otimes q$`);`
`CmpStmt`	$r = p \odot q$ CmpStmt::Predicate	`addToSolver(r == ite(`$p \odot q$`, true, false));`
`CallPE/RetPE`	$r = f(\ldots, q, \ldots)$ $f(\ldots, p, \ldots)\{\ldots$ return $z\}$	
`CallPE`	`p = q`	`solver.push();` `addToSolver(p == q);`
`RetPE`	`p = r`	`expr ret = getEvalExpr(r);` `solver.pop();` `addToSolver(p == ret);`

Translating Code to Z3 Formulas (Scalar Example)

The target program code needs to be in **SSA form** (e.g., SVFIR).

- Top-level variables can only be defined once
  - a = 1; a = 2; $\Longrightarrow$ a1 = 1; a2 = 2;
- Memory objects can only be modified/read through top-level pointers at `StoreStmt` and `LoadStmt`.
  - `p = &a; *p = r;` The value of `a` can only be modified/read via dereferencing `p`.

Translating Code to Z3 Formulas (Scalar Example)

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- Memory objects can only be modified/read through top-level pointers at `StoreStmt` and `LoadStmt`.
  - `p = &a; *p = r;` The value of `a` can only be modified/read via dereferencing `p`.

C code:

```
int main() {

 int a;
 int b;
 a = 0;
 b = a + 1;
 assert(b>0);

}
```

→

Translator:

```
// int a;
expr a = getZ3Expr("a");
// int b;
expr b = getZ3Expr("b");
// a = 0;
addToSolver(a == 0);
// b = a+1;
addToSolver(b == (a + 1));
// assert(b > 0);
addToSolver(b > 0);
solver.check();
```

→

Z3 Formulas:

```
(declare-fun a () Int)
(declare-fun b () Int)
(assert (= a 0))
(assert (= b (+ a 1)))
(assert (> b 0))
(check-sat)
```

→ Z3's SMT solver

Translating Code to Z3 Formulas (Memory Operation Example)

- Each memory object has a unique ID and allocated with a **virtual memory address**
- In our modeling, the virtual address starts from **0x7f...... + ID** (i.e., 2130706432 + ID in decimal)
- Memory operations will be through store and load values from `loc2ValMap`, an Z3 array.

```c
int main() {
 int* p;
 int x;

 p = malloc(..);
 *p = 5;
 x = *p;
 assert(x==5);
}
```

→

```
//  int** p;
expr p = getZ3Expr("p");
//  int x;
expr x = getZ3Expr("x");
// p = malloc(..);
expr m = getMemObjAddress("malloc1");
addToSolver(p == m);
// *p = 5;
storeValue(p, getZ3Expr(5));
// x = *p;
addToSolver(x == loadValue(p));
// assert(x==5);
addToSolver(x == getZ3Expr(5));
solver.check();
```

→

```
(declare-fun p () Int)
(declare-fun loc2ValMap ()
    (Array Int Int))
(declare-fun x () Int)
(assert (= p 2130706435))
(assert (= x (select
    (store loc2ValMap 2130706435 5)
    2130706435)))
(assert (= x 5))
(check-sat)
```

C code | Translator | Z3 Formulas

What's next?

- (1) Understand Z3 formula format in the slides
- (2) Understand `Z3Mgr` class in the GitHub Repository of Software-Security-Analysis
- (3) Start working on the Quiz-2 on WebCMS and Lab-Exercise-2 by implementing a manual translation from code to Z3 formulas using `Z3Mgr` and `Z3Examples` in for code assertion verification.