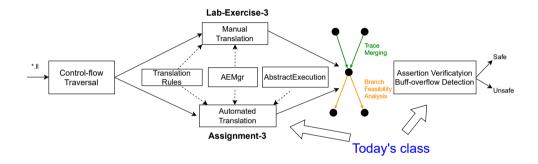
Abstract Interpretation for Code Analysis and Verification (Week 9)

Yulei Sui

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Today's class



Topological Order

- ? How to analyze a program free of loop?
- ✓ Analyze each node once adhering to the topological order on the acyclic control-flow graph of the program.

Topological Order

Analysis Order of Nodes on Control-Flow Graph

- ? How to analyze a program free of loop?
- ✓ Analyze each node once adhering to the topological order on the acyclic control-flow graph of the program.

A **topological order** of a graph G(V, E) is a linear ordering of its nodes such that for every directed edge $a \to b$, node a always precedes node b in the ordering.

- Must be a direct acyclic graph (DAG) and has at least one topo ordering.
- The ordering respects the **direction of edges**.

Example of topological order:



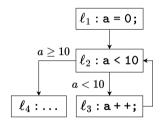
How About Analyzing Loops?

- Topological Order can only be used for directed acyclic graphs (DAGs).
- Weak Topological Order (WTO) is a relaxation of the more stringent topological order for graphs with loops.
 - Cycles Permitted: allows for cycles within the graph.
 - Hierarchical Decomposition: A graph is decomposed into a hierarchical structure where each node or a strongly connected component (SCC) can contain subnodes.
 - Weak Topological Order or Partial Order: In a WTO, nodes and SCCs are arranged in a partial order (not enumerating possible infinite loop paths). This order respects the dependencies in a way that allows for iterative analysis.
 - We will practice loop handling using WTO in Assignment-3. Function recursions will not be handled in this Assignment.

Analysis Order of Nodes on Control-Flow Graph

- ? How to analyze a program containing loops?
- ✓ We can analyze a program containing loops adhering to the weak topological order (WTO) on its control flow graph.

What is the weak topological order?

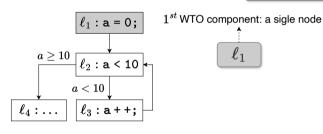


Control Flow Graph

Analysis Order of Nodes on Control-Flow Graph

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What is the weak topological order?

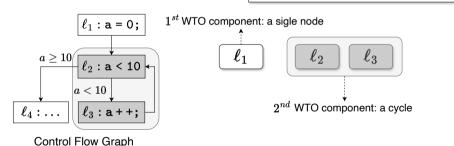


Control Flow Graph

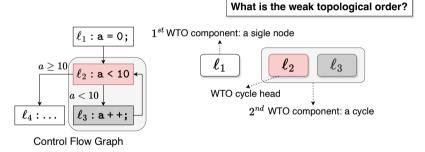
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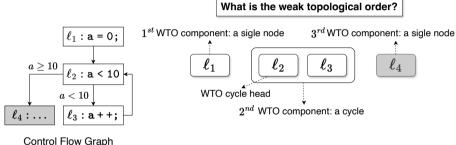
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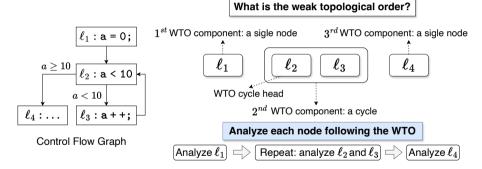
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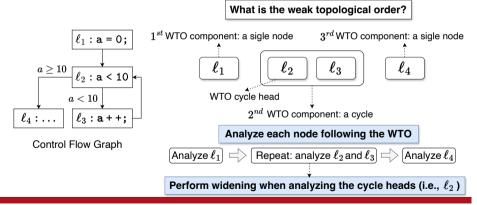
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- ? How to analyze a program containing loops?
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WTO, Widening and Narrowing

Why Weak Topological Order?

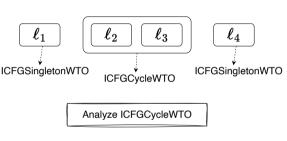
- Handling cyclic dependencies
- Efficient fixed-point computation

Why Widening?

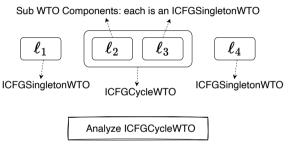
- Over-approximation
- Prevent non-termination

Why Narrowing?

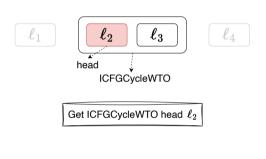
- Refine precision after widening converges
- The specific conditions or constraints used for narrowing:
 - Loop exit conditions (this course)
 - Type constraints (8-bit integer ranging from [-128, 127])
 - Bounds from arithmetic operations If x = y + z, and $y \in [1, 5]$ and $z \in [2, 3]$, then $x \in [3, 8]$. If widening gives [1, 10], narrowing can refine this to [3, 8].
 - User-specification (assertions and guard conditions)



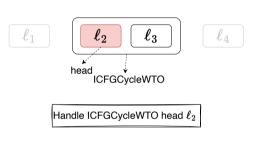
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Algorithm 12: Handle Cycle WTO
  Function handleCycleWTO(cycle):
      cvcle\_head := cvcle \rightarrow head() \rightarrow node() :
      increasing := true :
      cur_iter := 0 :
      while true do
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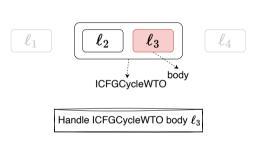
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          cur iter++:
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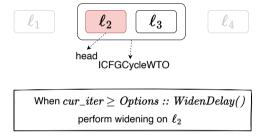
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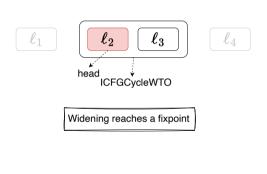
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Note: getIWTOcomponents returns Cycle WTO body, i.e., ℓ_3

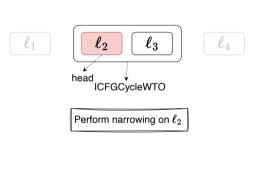


Algorithm 12: Handle Cycle WTO

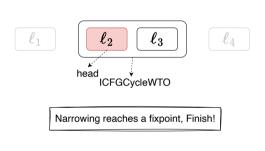
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Abstract Interpretation on SVFIR

Week 9

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Abstract Interpretation on Pointer-Free SVFIR

Interval Domain

- For simplicity, let's first consider abstract execution on a pointer-free language.
- This means there are no operations for memory allocation (like p = alloc_o) or for indirect memory accesses (such as p = *q or *p = q).
- Here are the pointer-free SVFSTMTs and their C-like forms:

SVFSTMT	C-Like form
ConsStmt	$\ell: p = c$
COPYSTMT	$\ell: \mathtt{p} = \mathtt{q}$
BINARYSTMT	$\ell:\mathtt{r}=\mathtt{p}\otimes\mathtt{q}$
РніЅтмт	$\ell: \mathtt{r} = \mathtt{phi}(\mathtt{p_1}, \mathtt{p_2}, \ldots, \mathtt{p_n})$
SEQUENCE	$\ell_1; \ell_2$
BRANCHSTMT	ℓ_1 : if($x < c$) then ℓ_2 else ℓ_3

Abstract Interpretation on Pointer-Free SVFIR

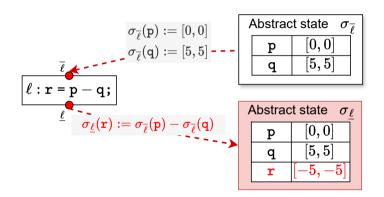
Interval Domain

Let's use the *Interval* abstract domain to update σ based on the following rules for different SVFSTMT:

SVFSTMT	C-Like form	Abstract Execution Rule
CONSSTMT	$\mid \ell : p = c$	$\mid \ \sigma_{\underline{\ell}}(\mathtt{p}) := [\mathtt{c},\mathtt{c}]$
COPYSTMT	$\mid \ \ell : \mathtt{p} = \mathtt{q}$	$\mid \sigma_{\underline{\ell}}(\mathtt{p}) := \sigma_{\overline{\ell}}(\mathtt{q})$
BINARYSTMT	$\mid \ \ell : \mathtt{r} = \mathtt{p} \otimes \mathtt{q}$	$\mid \ \sigma_{\underline{\ell}}(r) := \sigma_{\overline{\ell}}(ho) \hat{\otimes} \sigma_{\overline{\ell}}(q)$
РніЅтмт	$\big \ \ell: \mathtt{r} = \mathtt{phi}(\mathtt{p}_1,\mathtt{p}_2,\ldots,\mathtt{p}_n)$	$\mid \sigma_{\underline{\ell}}(r) := \bigsqcup_{i=1}^n \sigma_{\overline{\ell}}(p_i)$
SEQUENCE	$\ell_1;\ell_2$	$\mid \ orall v \in \mathbb{V}, \sigma_{\overline{\ell_2}}(v) \sqsupseteq \sigma_{\underline{\ell_1}}(v)$
BRANCHSTMT	$\ell_1: if(x < c) then \ell_2 else \ell_3$	$\sigma_{\overline{\ell_2}}(x) := \sigma_{\underline{\ell_1}}(x) \sqcap [-\infty, c-1], \text{ if } \sigma_{\ell_1}(x) \sqcap [-\infty, c-1] \neq \bot$ $\sigma_{\overline{\ell_3}}(x) := \sigma_{\underline{\ell_1}}(x) \sqcap [c, +\infty], \text{ if } \sigma_{\underline{\ell_1}}(x) \sqcap [c, +\infty] \neq \bot$

Abstract Interpretation on BINARYSTMT

SVFSTMT	C-Like form	Abstract Execution Rule
BINARYSTMT	$\ell: \mathtt{r} = \mathtt{p} \otimes \mathtt{q}$	$\sigma_{\underline{\ell}}(r) := \sigma_{\overline{\ell}}(p) \hat{\otimes} \sigma_{\overline{\ell}}(q)$

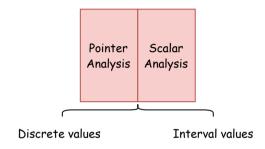


Abstract Interpretation in the Presence of Pointers

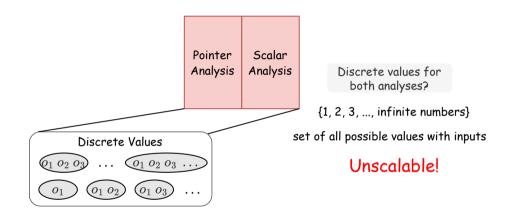
- SVFIR in the presence of pointers contain pointer-related statements including ADDRSTMT, GEPSTMT, LOADSTMT and STORESTMT.
- Abstract interpretation needs to be performed on a combined domain of intervals and addresses.

SVFSTMT	C-Like form
CONSSTMT	$\ell: p = c$
COPYSTMT	$\ell: \mathtt{p} = \mathtt{q}$
BINARYSTMT	$\ell: \mathtt{r} = \mathtt{p} \otimes \mathtt{q}$
РніЅтмт	$\ell: \mathtt{r} = \mathtt{phi}(\mathtt{p_1},\mathtt{p_2},\ldots,\mathtt{p_n})$
SEQUENCE	$\ell_1; \ell_2$
BRANCHSTMT	ℓ_1 : if($x < c$) then ℓ_2 else ℓ_3
A DDR S TMT	$\ell: \mathtt{p} = \mathtt{alloc}$
GEPSTMT	$\ell: \mathtt{p} = \mathtt{\&}(\mathtt{q} o \mathtt{i}) \; or \mathtt{p} = \mathtt{\&q}[\mathtt{i}]$
LOADSTMT	$\ell: p = *q$
STORESTMT	$\ell: *p = q$

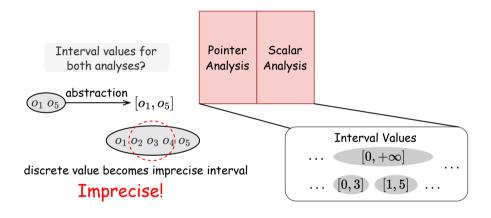
Combined Analysis



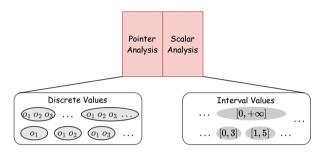
Combined Analysis Using Discrete Values



Combined Analysis Using Interval Values



Abstract Interpretation Over a Combined Domain

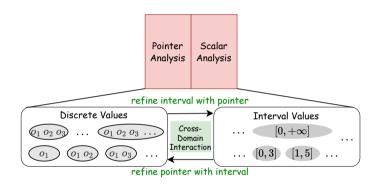


```
p = malloc(m*sizeof(int)); // p points to an array of size m
q = malloc(n*sizeof(int)); // q points to an array of size n
```

```
\mathbf{m} = \mathbf{r}[\mathbf{i}];
```

- The discrete values for points-to set of p, q depend on interval values of m and n.
- The interval value of m depends on the pointer aliasing between p, q and &r[i].
- Cyclic dependency between two domains requiring a bi-directional refinement. (variables highlighted in blue and red denote the discrete values and interval values dependent),

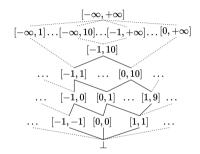
Abstract Interpretation Over a Combined Domain



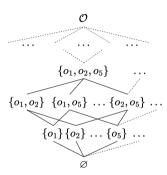
We require a combination of interval and memory address domains to precisely and efficiently perform abstract execution on SVFIR in the presence of pointers.

Abstract Interpretation over Interval and MemAddress Domains

A Combined Domain of Intervals and Discrete Memory Addresses



Interval domain for scalar variables



MemAddress domain for discrete memory address values

SVF Program Variables (SVFVar)

	Program Variables	Domain	Meanings
	SVFVar	$\mathbb{V} = \mathbb{P} \cup \mathbb{O}$	Program Variables
	ValVar	\mathbb{P}	Top-level variables (scalars and pointers)
FIObjVar $o \in (\mathbb{S} \cup \mathbb{G} \cup \mathbb{H})$ A single (base) memory objectGepObjVar $o_i \in (\mathbb{S} \cup \mathbb{G} \cup \mathbb{H}) \times \mathbb{P}$ i-th subfield/element of an (aggregate) objectConstantData \mathbb{C} Constant data (e.g., numbers and strings)	ObjVar	$\mathbb{O}=\mathbb{S}\cup\mathbb{G}\cup\mathbb{H}\cup\mathbb{C}$	Memory Objects (constant data, stack, heap, global)
Gep0bjVar $o_i \in (\mathbb{S} \cup \mathbb{G} \cup \mathbb{H}) \times \mathbb{P}$ <i>i</i> -th subfield/element of an (aggregate) object ConstantData \mathbb{C} Constant data (e.g., numbers and strings)			(function objects are considered as global objects)
Constant Data $\mathbb C$ Constant data (e.g., numbers and strings)	FIObjVar	$o \in (\mathbb{S} \cup \mathbb{G} \cup \mathbb{H})$	A single (base) memory object
	GepObjVar	$o_i \in (\mathbb{S} \cup \mathbb{G} \cup \mathbb{H}) imes \mathbb{P}$	i-th subfield/element of an (aggregate) object
Program Statement $\ell \in \mathbb{L}$ Statements labels	ConstantData	\mathbb{C}	Constant data (e.g., numbers and strings)
- region statement v c = statements table	Program Statement	$\ell \in \mathbb{L}$	Statements labels

Abstract Trace for The Combined Domain

- For top-level variables \mathbb{P} , we use $\sigma \in \mathbb{L} \times \mathbb{P} \to \mathit{Interval} \times \mathit{MemAddress}$ to track the memory addresses or interval values of these variables.
- For memory objects \mathbb{O} , we use $\delta \in \mathbb{L} \times \mathbb{O} \to \mathit{Interval} \times \mathit{MemAddress}$ to track their abstract values

	Notation	Domain	Data Structure Implementation
Abstract trace	σ	$\mathbb{L} imes \mathbb{P} o$ Interval $ imes$ MemAddress	preAbsTrace, postAbsTrace
	δ	$\parallel \mathbb{L} imes \mathbb{O} o \mathit{Interval} imes \mathit{MemAddress} \parallel$	
Abstract state	σ_{L}	$\mathbb{P} o \mathit{Interval} imes \mathit{MemAddress}$	AbstractState.varToAbsVal
δ_L		$\mathbb{O} o \mathit{Interval} imes \mathit{MemAddress}$	AbstractState.addrToAbsVal
Abstract value	$\delta_L(p)$ $\delta_L(o)$	Interval × MemAddress	AbstractValue

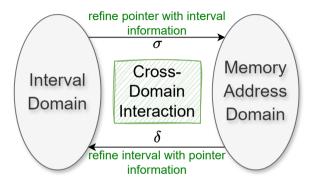
- *Interval* is used for tracking the interval value of **scalar variables** \mathbb{P} .
- MemAddress is used for tracking the memory addresses of memory address variables

 .

Implementation of Abstract Trace and State in Assignment-3

- For a program point *L*, the abstract state is an instance of class *AEState*, consisting of:
 - Top-level variable, *varToAbsVal* : $\sigma_L \in \mathbb{P} \to Interval \times MemAddress$
 - Memory object, addrToAbsVal : $\delta_I \in MemAddress \rightarrow Interval \times MemAddress$
- The abstract trace has two maps, preAbsTrace and postAbsTrace, which
 maintains abstract states before and after each ICFGNode respectively.
 - For an ICFGNode ℓ , $preAbsTrace(\ell)$ retrieves the abstract state $\langle \sigma_{\overline{\ell}}, \delta_{\overline{\ell}} \rangle$, and $postAbsTrace(\ell)$ represents $\langle \sigma_{\underline{\ell}}, \delta_{\underline{\ell}} \rangle$.
 - For each abstract state $\langle \sigma_{\overline{\ell}}, \delta_{\overline{\ell}} \rangle$ we use as [varId] to operate $\sigma_{\underline{\ell}}$ and use storeValue and loadValue to operate δ_{ℓ} .
 - Each variable's AbstractValue (e.g., as [VarId]) is initialized as \perp in an AbstractState before assigned a new value.
 - Each AbstractValue (e.g., as [VarId]) is a 2-element tuple consisting of an interval as [VarId] .getInterval() and an address set as [Varid] .getAddrs().
 - Print out SVFVars and their AbstractValues in an AbstractState by invoking as.printAbstractState()

Abstract Trace for The Combined Domain



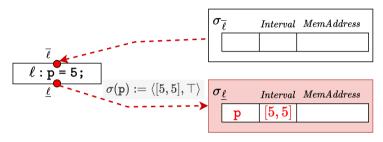
Abstract Execution Rules on SVFIR in the Presence of Pointers

Now let's use the *Interval* \times *MemAddress* abstract domain to update σ and δ based on the following rules for different SVFSTMT:

SVFSTMT	C-Like form	Abstract Execution Rule
CONSSTMT	$\ell: \mathtt{p} = \mathtt{c}$	$\mid \; \sigma_{\underline{\ell}}(\mathtt{p}) := \langle [\mathtt{c},\mathtt{c}], \perp angle$
COPYSTMT	$\ell: \mathtt{p} = \mathtt{q}$	$\mid \ \sigma_{\underline{\ell}}(\mathtt{p}) := \sigma_{\overline{\ell}}(\mathtt{q})$
BINARYSTMT	$\ell: \mathtt{r} = \mathtt{p} \otimes \mathtt{q}$	$\mid \ \sigma_{\underline{\ell}}(r) := \sigma_{\overline{\ell}}(p) \hat{\otimes} \sigma_{\overline{\ell}}(q)$
РніЅтмт	$\ell: \mathtt{r} = \mathtt{phi}(p_1, p_2, \ldots, p_n)$	$\mid \ \sigma_{\underline{\ell}}(r) := \bigsqcup_{i=1}^n \sigma_{\overline{\ell}}(p_i)$
BRANCHSTMT	$\ell_1:$ if($x < c$) then ℓ_2 else ℓ_3	$ \begin{vmatrix} \sigma_{\overline{\ell_2}}(x) := \sigma_{\underline{\ell_1}}(x) \sqcap [-\infty, c-1], & \text{if } \sigma_{\underline{\ell_1}}(x) \sqcap [-\infty, c-1] \neq \bot \\ \sigma_{\underline{\ell_3}}(x) := \sigma_{\underline{\ell_1}}(x) \sqcap [c, +\infty], & \text{if } \sigma_{\underline{\ell_1}}(x) \sqcap [c, +\infty] \neq \bot \end{vmatrix} $
SEQUENCE	$\ell_1;\ell_2$	$\left \begin{array}{c} \delta_{\overline{\ell_2}} \sqsupseteq \delta_{\underline{\ell_1}}, \sigma_{\overline{\ell_2}} \sqsupseteq \sigma_{\underline{\ell_1}} \end{array}\right $
ADDRSTMT	$\ell: p = \mathtt{alloc_{o_i}}$	$\mid \sigma_{\underline{\ell}}(\mathtt{p}) := \langle \top, \{o_i\} \rangle$
GEPSTMT	$\ell: \mathtt{p} = \&(\mathtt{q} \to \mathtt{i}) \ or \mathtt{p} = \&\mathtt{q}[\mathtt{i}]$	$\left \begin{array}{c} \sigma_{\underline{\ell}}(\mathbf{p}) := \bigsqcup_{\mathbf{o} \in \gamma(\sigma_{\overline{\ell}}(\mathbf{q}))} \bigsqcup_{j \in \gamma(\sigma_{\overline{\ell}}(\mathbf{i}))} \langle \top, \{\mathbf{o.fld}_j\} \rangle \end{array} \right.$
LOADSTMT	$\ell: p = *q$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \bigsqcup_{o \in \{o \mid o \in \sigma_{\overline{\ell}}(q)\}} \delta_{\overline{\ell}}(o)$
STORESTMT	$\ell:*p=q$	$\big \ \ \delta_{\underline{\ell}} := (\{ o \mapsto \sigma_{\overline{\ell}}(\mathtt{q}) o \in \gamma(\sigma_{\overline{\ell}}(\mathtt{p})) \} \sqcup \delta_{\underline{\ell}})$

Abstract Execution on CONSSTMT

SVFSTMT C-Like form		Abstract Execution Rule	
CONSSTMT	$\ell: \mathtt{p} = \mathtt{c}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \langle [\mathtt{c},\mathtt{c}], \perp \rangle$	



Algorithm 13: Abstract Execution Rule for CONSSTMT

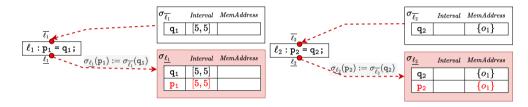
```
1 Function updateStateOnAddr(addr):
```

```
node = addr -> getICFGNode();
```

- as = getAbsStateFromTrace(node);
- 4 initObjVar(as, SVFUtil :: cast $\langle \text{ObjVar} \rangle (\text{addr} \rightarrow \text{getRHSVar}()));$
- $= as[addr \rightarrow getLHSVarID()] = as[addr \rightarrow getRHSVarID()];$

Abstract Execution on COPYSTMT

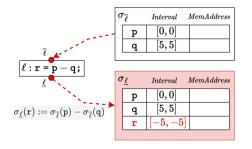
SVFSTMT	C-Like form	Abstract Execution Rule
СОРУЅТМТ	$\ell: \mathtt{p} = \mathtt{q}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \sigma_{\overline{\ell}}(\mathtt{q})$



Algorithm 14: Abstract Execution Rule for COPYSTMT

Abstract Execution on BINARYSTMT

SVFSTMT	C-Like form	Abstract Execution Rule
BINARYSTMT	$\ell: \mathtt{r} = \mathtt{p} \otimes \mathtt{q}$	$\sigma_{\underline{\ell}}(r) := \sigma_{\overline{\ell}}(p) \hat{\otimes} \sigma_{\overline{\ell}}(q)$



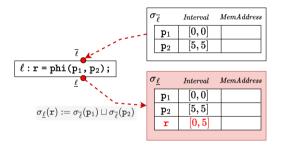
Algorithm 15: Abstract Execution Rule for BINARYSTMT

```
| Function updateStateOnBinary(binary):
| node = binary → getICFGNode();
| as = getAbsStateFromTrace(node);
| op0 = binary → getDpVarID(0);
| op1 = binary → getDpVarID(1);
| res = binary → getResID();
| as[res] = as[op0] sas[op1]
```

Operands op0 and op1 are assumed to be properly initialized (no uninitialized variables or randomization).

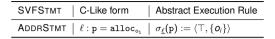
Abstract Execution on PHISTMT

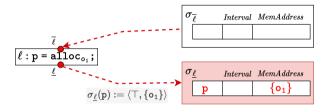
SVFSTMT	C-Like form	Abstract Execution Rule	;
РніЅтмт	$\ \ \ \ \ell : \texttt{r} = \texttt{phi}(\texttt{p}_1, \texttt{p}_2, \ldots, \texttt{p}_n)$	$\sigma_{\underline{\ell}}(r) := \bigsqcup_{i=1}^n \sigma_{\overline{\ell}}(p_i)$	



```
Algorithm 16: Abstract Execution Rule for PHISTMT
```

Abstract Execution on ADDRSTMT

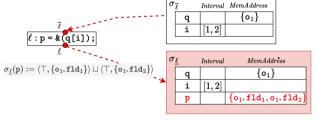




Algorithm 17: Abstract Execution Rule for ADDRSTMT

Abstract Execution on GEPSTMT

SVFSTMT C-Like form	Abstract Execution Rule
$\label{eq:general_general} GEPSTMT \ \ \big \ \ \ell : \mathtt{p} = \&(\mathtt{q} \to \mathtt{i}) \ \ or \ \mathtt{p} = \&\mathtt{q}[\mathtt{i}]$	$ \mid \ \sigma_{\underline{\ell}}(\mathtt{p}) := \bigsqcup_{\mathtt{o} \in \gamma(\sigma_{\overline{\ell}}(\mathtt{q}))} \bigsqcup_{j \in \gamma(\sigma_{\overline{\ell}}(\mathtt{i}))} \langle \top, \{\mathtt{o.fld}_j\} \rangle $

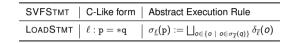


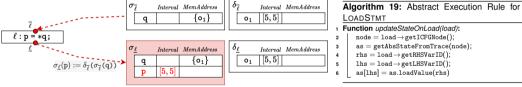
Algorithm 18: Abstract Execution Rule for GEPSTMT

function updateStateOnGep(gep):

- $\texttt{lhs} = \texttt{gep} \rightarrow \texttt{getLHSVarID()};$
- as[lhs] = as.getGepObjAddrs(rhs, as.getElementIndex(gep));;

Abstract Execution on LOADSTMT



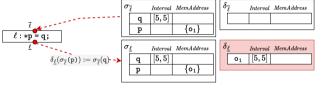


LOADSTMT Function updateStateOnLoad(load): $node = load \rightarrow getICFGNode();$ as = getAbsStateFromTrace(node);

 $rhs = load \rightarrow getRHSVarID();$ $lhs = load \rightarrow getLHSVarID();$ as[lhs] = as.loadValue(rhs)

Abstract Execution on STORESTMT

SVFSTMT	C-Like form	Abstract Execution Rule
STORESTMT	$\ell:*\mathtt{p}=\mathtt{q}$	$\mid \delta_{\underline{\ell}} := (\{o \mapsto \sigma_{\overline{\ell}}(\mathtt{q}) \mid o \in \gamma(\sigma_{\overline{\ell}}(\mathtt{p}))\} \sqcup \delta_{\underline{\ell}})$



Algorithm 20: Abstract Execution Rule for STORESTMT

Function updateStateOnStore(store):

- $rhs = store \rightarrow getRHSVarID();$ $lhs = store \rightarrow getLHSVarID();$
- as.storeValue(lhs, as[rhs])

Overall Algorithm of Abstract Interpretation in Assignment-3

```
Algorithm 3: Handle Singleton WTO
                                                                                 Function handleSingletonWTO(singletonWTO):
Algorithm 1: Analyse from main function
                                                                                     node := singletonWTO \rightarrow node():
Function analyse() // driver function to start the analysis:
                                                                                     feasible := mergeStatesFromPredecessors(node.preAbsTrace[node]);
   initWTO():
                                                                                     if feasible then
  handleGlobalNode():
                                                                                        postAbsTrace[node] := preAbsTrace[node];
  if getSVFFunction (main) then
                                                                                     else
      wto := funcToWTO[main]:
      handleWTOComponents(wto → getWTOComponents());
                                                                                        return:
                                                                                     foreach stmt ∈ node → getSVFStmts() do
                                                                                        updateAbsState(stmt);
Algorithm 2: Handle WTO components
                                                                                        bufOverflowDetection(stmt);
Function handleWTOComponents (wtoComps):
                                                                                     if callnode = SVFUtil :: dvn_cast(CallICFGNode)(node) then
   for wtoNode ∈ wtoComps do
                                                                                        \texttt{funName} := \texttt{callnode} \rightarrow \texttt{getCallSite()} \rightarrow \texttt{getCallee()} \rightarrow \texttt{getName()}
      if node = SVFUtil :: dvn_cast(ICFGSingletonWTO)(wtoNode) then
                                                                                          if funName == "OVERFLOW" && funName == "syf assert" then
         handleSingletonWTO(node)
                                                                                           // Handle svf_assert and OVERFLOW stub function for
      else if cycle = SVFUtil :: dyn_cast(ICFGCycleWTO)(wtoNode) then
                                                                                            correctness validation:
         handleCvcleWTO(cvcle)
                                                                                           handleStubFunctions(callnode):
                                                                               14
      else
                                                                                        else
         assert(false&&"unknownWTOtype!")
                                                                                            // Does not analyze recursive functions in this course:
                                                                                           handleCallSite(callnode):
```

Overall Algorithm of Abstract Interpretation in Assignment-3

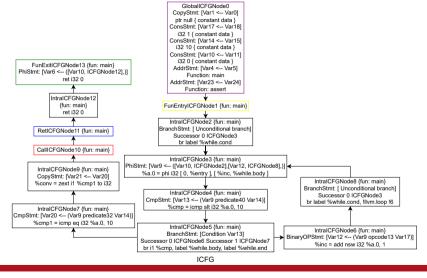
```
Algorithm 4: Handle Cycle WTO
1 Function handleCycleWTO (cycle):
     feasible := mergeStatesFromPredecessors(cycle_head.preAbsTrace[cycle_head]);
     increasing := true:
     if Ifeasible then
        return:
     else
        cur iter := 0'
        while true do
           if cur iter >= Options.WidenDelay() then
               prev_head_as := postAbsTrace[cycle_head];
              handleSingletonWTO(cycle.head());
11
               cur_head_as := postAbsTrace[cycle_head];
12
              if increasing then
13
                  postAbsTrace[cvcle_head] := prev_head_as.widening(cur_head_as);
                  if postAbsTrace[cycle_head] == prev_head_as then
                     increasing := false:
17
                     Continue:
18
               else
                  postAbsTrace[cycle_head] := prev_head_as.narrowing(cur_head_as);
                  if postAbsTrace[cycle_head] == prev_head_as then
20
                     Break:
21
22
              handleSingletonWTO(cvcle.head()):
23
24
            cur_iter + +:
```

```
extern void assert(int);
int main(){
    int a = 0;
    while(a < 10) {
        a++;
    }
    assert(a = 10);
    return 0;
}</pre>
```

```
Compile to LLVM IR
```

```
define dso local i32 @main() {
entry:
  br label %while.cond
while.cond:
  %a.0 = phi i32 [ 0, %entry ], [ %inc, %while.body ]
  %cmp = icmp slt i32 %a.0. 10
  br i1 %cmp. label %while.body. label %while.end
while.body:
  %inc = add nsw i32 %a.0. 1
  br label %while.cond.
while end:
  %cmp1 = icmp eq i32 %a.0. 10
  %conv = zext i1 %cmp1 to i32
  call void @assert(i32 noundef %conv)
  ret i32 0
```

LLVM IR



GloballCFGNode0
CopyStmt: [Var1 <-- Var0]
ptr null { constant data }
ConsStmt: [Var17 <-- Var18]
i32 1 { constant data }
ConsStmt: [Var14 <-- Var15]
i32 10 { constant data }
ConsStmt: [Var10 <-- Var11]
i32 0 { constant data }
AddrStmt: [Var4 <-- Var5]
Function: main
AddrStmt: [Var2 <-- Var24]
Function: assert

FunEntryICFGNode1 {fun: main}

IntralCFGNode2 (fun: main) BranchStmt: [Unconditional branch] Successor 0 ICFGNode3 br label %while.cond

ICFG

Algorithm 5: Abstract execution guided by WTO

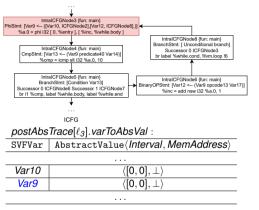
| Function handleStatement(\ell):
| tmpAS := preAbsTrace[\ell];
| if \ell is CONSSTMT or ADDRSTMT then
| updateStateOnAddr(\ell);
| else if \ell is COPYSTMT then
| updateStateOnCopy(\ell);
| ...;

$postAbsTrace[\ell_0].varToAbsVal:$

poor 100 11 de 101 100 1 de 1	
AbstractValue(Interval, MemAddress)	
$\langle \perp, \{0x7f00\} \rangle$	
$\langle \perp, \{0x7f00\} \rangle$	
$\langle [1,1], \perp \rangle$	
$\langle [1,1], \perp angle$	
$\langle [10,10], \perp angle$	
⟨[10, 10], ⊥⟩	
$\langle [0,0], \perp angle$	
$\langle [0,0], \perp angle$	

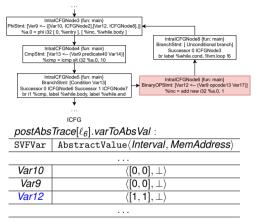
Print out the table via as.printAbstractState(). The AbstractValue can either be an interval or addresses. but not both!

Widen Delay Phase (cur_iter is 0)



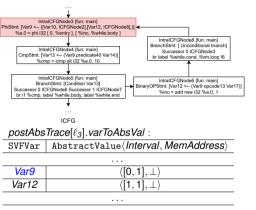
```
Algorithm 12: Handle Cycle WTO
  Function handleCycleWTO(cycle):
      cycle head := cycle -> head() -> node() :
      increasing := true :
      cur.iter := 0 :
      while true do
          // cur_iter ≡ 0. Options :: WidenDelay() ≡ 2
          if cur_iter > Options :: WidenDelay() then
              prev_head_state := postAbsTrace[cvcle_head];
              handleICFGNode(cvcle→head()):
              cur_head_state := postAbsTrace[cvcle_head]:
10
              if increasing then
11
                  postAbsTrace[cvcle_head] := prev_head_state.widen(cur_head_state) :
12
                  if postAbsTrace[cvcle_head] == prev_head_state then
13
                      increasing := false;
                      continue;
15
16
                  postAbsTrace[cvcle_head] := prev_head_state.narrow(cur_head_state) :
17
                  if postAbsTrace[cycle head] == prev_head_state then
18
                      break:
20
          else
21
              handleICFGNode(cvcle-head()):
          handleWTOComponents(cvcle->getWTOComponents()):
22
23
          cur_iter++:
```

Widen Delay Phase (cur_iter is 0)



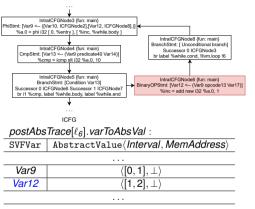
```
Algorithm 12: Handle Cycle WTO
1 Function handleCycleWTO(cycle):
      cycle_head := cycle \rightarrow head() \rightarrow node();
      increasing := true :
      cur_iter := 0 :
      while true do
          // cur iter = 0. Options ·· WidenDelay() = 2:
          if cur_iter > Options :: WidenDelay() then
               prev_head_state := postAbsTrace[cycle_head];
8
               handleTCFGNode(cvcle-head()):
              cur_head_state := postAbsTrace[cycle_head];
10
              if increasing then
11
                   postAbsTrace[cvcle_head] := prev_head_state.widen(cur_head_state) :
12
                   if postAbsTrace[cycle head] == prev_head_state then
13
                       increasing := false:
                       continue:
16
                   postAbsTrace[cvcle_head] := prev_head_state.narrow(cur_head_state);
17
                   if postAbsTrace[cycle_head] == prev_head_state then
18
19
                       break:
20
              handleICFGNode(cvcle→head());
21
22
          handleWTOComponents(cvcle-) aetWTOComponents());
23
          cur_iter++:
```

Widen Delay Phase (cur_iter is 1)



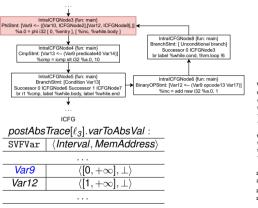
```
Algorithm 12: Handle Cycle WTO
  Function handleCycleWTO(cycle):
      cycle head := cycle -> head() -> node() :
      increasing := true :
      cur.iter := 0 :
      while true do
          // cur_iter ≡ 1. Options :: WidenDelay() ≡ 2:
          if cur_iter > Options :: WidenDelay() then
              prev_head_state := postAbsTrace[cvcle_head];
              handleICFGNode(cvcle→head()):
              cur_head_state := postAbsTrace[cvcle_head]:
10
              if increasing then
11
                  postAbsTrace[cvcle_head] := prev_head_state.widen(cur_head_state) :
12
                  if postAbsTrace[cvcle_head] == prev_head_state then
13
                      increasing := false;
                      continue;
15
16
                  postAbsTrace[cvcle_head] := prev_head_state.narrow(cur_head_state) :
17
                  if postAbsTrace[cycle head] == prev_head_state then
18
                      break:
20
          else
21
              handleICFGNode(cvcle-head()):
          handleWTOComponents(cvcle->getWTOComponents()):
22
23
          cur_iter++:
```

Widen Delay Phase (cur_iter is 1)



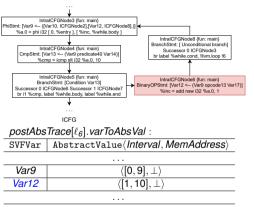
```
Algorithm 12: Handle Cycle WTO
  Function handleCycleWTO(cycle):
      cycle head := cycle -> head() -> node() :
      increasing := true :
      cur.iter := 0 :
      while true do
          // cur_iter ≡ 1. Options :: WidenDelay() ≡ 2:
          if cur_iter > Options :: WidenDelay() then
              prev_head_state := postAbsTrace[cvcle_head];
 8
              handleICFGNode(cvcle→head()):
              cur_head_state := postAbsTrace[cvcle_head]:
10
              if increasing then
11
                  postAbsTrace[cvcle_head] := prev_head_state.widen(cur_head_state) :
12
                  if postAbsTrace[cvcle_head] == prev_head_state then
13
                      increasing := false;
                      continue;
15
16
                  postAbsTrace[cvcle_head] := prev_head_state.narrow(cur_head_state) :
17
                  if postAbsTrace[cycle head] == prev_head_state then
18
                      break:
20
          else
              handleICFGNode(cvcle→head());
21
          handleWTOComponents(cvcle-)getWTOComponents()):
22
23
          cur_iter++:
```

Widen Phase (cur_iter is 2)



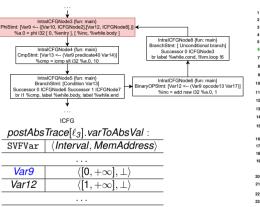
```
Algorithm 12: Handle Cycle WTO
  Function handleCycleWTO(cycle):
      cvcle_head := cvcle -> head() -> node() :
      increasing := true :
      cur iter := 0 :
      while true do
          // cur iter = 2. Options ·· WidenDelay() = 2
          if cur_iter > Options :: WidenDelay() then
7
              prev_head_state := postAbsTrace[cycle_head];
              handleICFGNode(cvcle→head()):
0
              cur_head_state := postAbsTrace[cycle_head];
10
              if increasing then
11
                  postAbsTrace[cvcle_head] := prev_head_state.widen(cur_head_state);
12
                  if postAbsTrace[cycle head] == prev_head_state then
13
                      increasing := false:
                      continue:
15
16
                  postAbsTrace[cvcle_head] := prev_head_state.narrow(cur_head_state);
17
                  if postAbsTrace[cycle_head] == prev_head_state then
18
                      break:
          موام
20
              handleICFGNode(cvcle→head());
          handleWTOComponents(cvcle-) getWTOComponents());
22
          cur_iter++:
23
```

Widen Phase (cur_iter is 2)



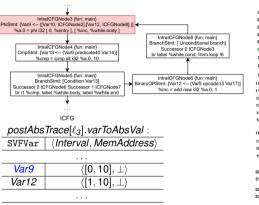
```
Algorithm 12: Handle Cycle WTO
  Function handleCycleWTO(cycle):
      cycle head := cycle -> head() -> node() :
      increasing := true :
      cur.iter := 0 :
      while true do
          // cur_iter ≡ 2. Options :: WidenDelay() ≡ 2
          if cur_iter > Options :: WidenDelay() then
              prev_head_state := postAbsTrace[cvcle_head];
 8
              handleICFGNode(cvcle→head()):
              cur_head_state := postAbsTrace[cvcle_head]:
10
              if increasing then
11
                  postAbsTrace[cvcle_head] := prev_head_state.widen(cur_head_state) :
12
                  if postAbsTrace[cvcle_head] == prev_head_state then
13
                      increasing := false;
                      continue;
15
16
                  postAbsTrace[cvcle_head] := prev_head_state.narrow(cur_head_state) :
17
                  if postAbsTrace[cycle head] == prev_head_state then
18
                      break:
20
          else
              handleICFGNode(cvcle→head());
21
          handleWT0Components(cvcle-aetWTOComponents())
22
23
          cur_iter++:
```

Widen Phase Fixed Point



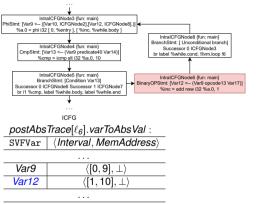
```
Algorithm 12: Handle Cycle WTO
  Function handleCycleWTD(cycle):
      cycle head := cycle -> head() -> node() :
      increasing := true ;
      cur iter := 0:
      while true do
          // cur_iter ≡ 3. Options :: WidenDelay() ≡ 2
          if cur_iter > Options :: WidenDelay() then
              prev_head_state := postAbsTrace[cvcle_head];
              handleICFGNode(cvcle-head()):
 9
              cur_head_state := postAbsTrace[cycle_head];
10
              if increasing then
11
                  postAbsTrace[cycle_head] := prev_head_state.widen(cur_head_state);
12
                  if postAbsTrace[cvcle_head] == prev_head_state then
13
                      increasing := false;
                      continue:
17
                  postAbsTrace[cvcle_head] := prev_head_state.narrow(cur_head_state);
                  if postAbsTrace[cvcle_head] == prev_head_state then
18
                      break:
          موام
20
              handleICFGNode(cvcle→head());
21
          handleWTOComponents(cvcle-) getWTOComponents());
22
          cur_iter++:
23
```

Narrow Phase



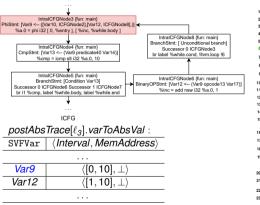
```
Algorithm 12: Handle Cycle WTO
1 Function handleCycleWTD(cycle):
      cycle head := cycle -> head() -> node() :
      increasing := true ;
      cur iter := 0 :
      while true do
          // cur_iter ≡ 3. Options :: WidenDelay() ≡ 2
          if cur iter > Options .. WidenDelay() then
7
              prev_head_state := postAbsTrace[cycle_head];
              handleICFGNode(cvcle \rightarrow head()) // increasing \equiv false;
9
              cur_head_state := postAbsTrace[cycle_head];
10
              if increasing then
11
                  postAbsTrace[cycle_head] := prev_head_state.widen(cur_head_state);
12
                  if postAbsTrace[cycle_head] == prev_head_state then
13
                      increasing := false:
                      continue:
16
                  postAbsTrace[cvcle_head] := prev_head_state.narrow(cur_head_state) :
17
18
                  if postAbsTrace[cycle_head] == prev_head_state then
                      break :
20
              handleICFGNode(cvcle-head());
          handleWTOComponents(cvcle->getWTOComponents()):
22
          cur_iter++:
23
```

Narrow Phase



```
Algorithm 12: Handle Cycle WTO
  Function handleCycleWTO(cycle):
      cycle head := cycle -> head() -> node() :
      increasing := true :
      cur.iter := 0 :
      while true do
          // cur_iter ≡ 3. Options :: WidenDelay() ≡ 2
          if cur_iter > Options :: WidenDelay() then
              prev_head_state := postAbsTrace[cvcle_head];
 8
              handleICFGNode(cvcle -> head()):
              cur_head_state := postAbsTrace[cvcle_head]:
10
              if increasing then
11
                  postAbsTrace[cvcle_head] := prev_head_state.widen(cur_head_state) :
12
                  if postAbsTrace[cvcle_head] == prev_head_state then
13
                      increasing := false;
                      continue;
15
16
                  postAbsTrace[cvcle_head] := prev_head_state.narrow(cur_head_state) :
                  if postAbsTrace[cycle head] == prev_head_state then
18
                      break:
          else
              handleICFGNode(cvcle→head());
21
          handleWT0Components(cvcle-) aetWTOComponents()):
22
23
          cur_iter++:
```

Narrow Phase Fixed Point



```
Algorithm 12: Handle Cycle WTO
  Function handleCycleWTO(cycle):
      cvcle_head := cvcle -> head() -> node() :
      increasing := true ;
      cur iter := 0 :
      while true do
          // cur_iter ≡ 4. Options :: WidenDelay() ≡ 2
          if cur_iter > Options :: WidenDelay() then
              prev_head_state := postAbsTrace[cycle_head];
9
              handleTCFGNode(cvcle > head()) // increasing = false:
              cur_head_state := postAbsTrace[cvcle_head]:
10
             if increasing then
11
                  postAbsTrace[cvcle_head] := prev_head_state.widen(cur_head_state);
12
                  if postAbsTrace[cycle_head] == prey_head_state then
13
                     increasing := false:
                      continue:
15
              else
16
                  postAbsTrace[cvcle_head] := prev_head_state.narrow(cur_head_state) :
17
                  if postAbsTrace[cvcle_head] == prev_head_state then
                     break:
20
             handleICFGNode(cvcle-head()):
21
          handleWTOComponents()):
22
23
          cur_iter++:
```

