# Designing a Traffic Circle

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## Summary

In our paper we construct an optimation model to determine how best to control traffic flow in, around, and out of a traffic circle. We suppose that the stop sign or the yield sign has the same effect as a red light. So here we only research one method of traffic control: the traffic light. The overall scheme is to find the cycle of traffic light and green ratio.

Considering the total delay in the system of traffic circle, we construct the first model. By learning from models[3] which are used to compute delay at signalized intersections and modifying them, we obtain a new model which is applicable for traffic circle. The total delay at signalized intersections is mainly composed of delay d1 resulted from wait before passing the traffic light. While the total delay for traffic circle is the summation of d1 and delay in passing the circle d2. According to the formula in computing traffic capacity on the incoming road in the Gap-Acceptance theory model[6] and the equation of traffic volume and vehicles' velocity V in the Greenshield model[3] [7], we deduce the formula of V and x. Based on this formula, we construct a optimization model whose object function is to make the total delay reach its minimum. Decision variables of this model are c and k. It's limited by constraint condition that c belongs to a certain interval and has an relational expression with k. When trying to apply this model to practical traffic circle, we classify various traffic circles according to its size and traffic volume of the incoming road. Then by taking the parameters of each case into the model and making a program to solve it, we get the optimal value of c and k.

Above, we use the traversal method to find the best solution, which will consume a lot of time. In order to save time, we consider building another assistant model to find a good initial solution which will make the search faster. After some analysis, we build another optimation model with two objects without constraint condition.

At last, we work out a Technical Summary based on the above models and solutions.

# Introduction

Traffic circle first appeared in the early 20th century in England、France and so on[1]. It is an important form of traffic which can be considered when several lanes cross at one point, especially for more than 4 lanes. A traffic circle is formed as following: A center island is set up in the central of the intersection; a circular road, which is the so-called circle, is then built around the island as shown in Fig. 1. Only one-way travel is allowed in the circle. Vehicles in the circle travel counterclockwise in most countries of the world while clockwise in others. Under normal circumstances, various vehicles could pass through the traffic circle continually without periodic congestions and this is one of the strengths of the traffic circle. In addition, since the likely conflict would be avoided, this form of traffic is preferable to intersections that cross directly for its better safety. Furthermore, we can plant trees or do some landscaping on the center island, which is helpful for protecting the environment.

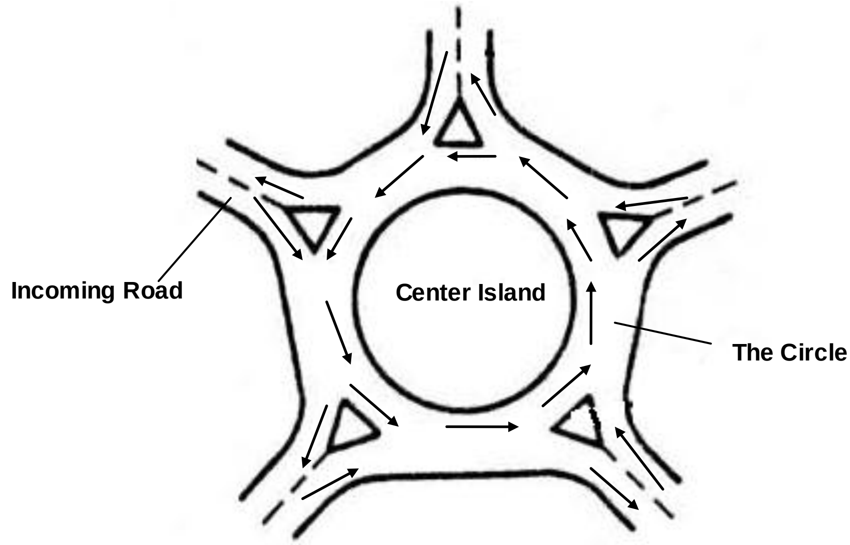


**Figure 1: A Sample of Traffic Circle with 5 Incoming Roads**

Since the traffic circle appeared, people had been researching how to estimate the traffic flow of the circle and methods of controlling it. When vehicles on the incoming road try to enter the circle, whether vehicles on the coming road or vehicles already in the circle get the priority to continue travelling without stopping or yielding should be determined and showed explicitly to the driver. There are several ways to show signs representing the priority. For example, if it is proper to give priority to vehicles on the coming road, we can place a yield sign (generally a stop sign is not allowed in the circle) in the direction of the coming vehicles already in the circle. Thus drivers already in the circle will pay attention when they pass the entry to observe whether there are vehicles in the coming road entering the circle and prepare to yield to them if there are. That is, vehicles trying to enter the circle get the priority. On the contrary, we can place a stop sign or a yield sign on the incoming lane to give priority to vehicles already in the circle.

However, with the increase of traffic quantity, collisions are likely to occur if no measures are taken to control traffic flow. So it's necessary to formulate a scheme to control traffic flow, making sure that the traffic of the traffic circle runs smoothly. In our paper we construct two models in two different aspect to determine how best to control traffic flow in, around, and out of a traffic circle.

## Assumptions

1. Along the traffic circle there are N incoming roads, and the distribution of these roads is symmetry.
2. The vehicles traveling in the traffic circle are identical.
3. The in-flow and out-flow traffic volume on each incoming road are equal.
4. The probability for a vehicle to get out from each of the N incoming roads is equal and equals to 1/N.
5. As long as the traffic volume doesn't reach the value of the traffic capacity of the circle lane, the incoming vehicles can enter the circle directly without slowing down or stop.
6. The system of traffic light is similar to the system of random service, so we assume random variable T obeys the law of negative exponential distribution.
7. All the drivers obey traffic rule.

## Notations and Definitions

- Green ratio: It refers to the ratio of the green light time in a cycle of the traffic light to the time of the total cycle.

- Traffic volume[2]: It refers to the number of vehicles pass a certain lane or a certain section of a lane in unit time.

- Traffic capacity[2]: It refers to the maximum traffic volume(using an hour as the unit time) during a certain period of time under normal condition of road、traffic and rules. Its definition is under the assumption that the weather is fair and the road surface is fine.

## A Model in terms of delay

## 1. Notations

| | |
|---|---|
| $d_1$ | delay on the incoming road before entering the circle(s) |
| $d_2$ | delay in the circle from enter to leave(s) |
| $s$ | the ratio of passing vehicles to total vehicles during the time of green light |
| $g$ | effective green interval duration (s) |
| $c$ | the time of traffic light cycle(s) |
| $C$ | the traffic capacity of the incoming road (veh/s) |
| $x$ | the ratio of in-flow traffic volume to in-flow traffic capacity |

| | |
|---|---|
| $T$ | the observing time(s) |
| $q_a$ | the practical in-flow traffic volume before passing the traffic light(veh/s) |
| $L$ | a random variable representing the travelling distance of vehicles in the circle(m) |
| $V$ | velocity（m/s） |
| $V_f$ | the maximum of the vehicles' velocity when the traffic is smooth（m/s） |
| $Q_c$ | the traffic volume in the circle(veh/s) |
| $l$ | minimum safe distance between two adjacent parking vehicles（m） |
| $\alpha$ | the ratio of number of free vehicles to number of total vehicles |
| $T_0$ | time that spent by the following vehicle to enter the circle when the former has already made a judgment（s） |
| $T_l$ | the critical value of headway in terms of time when entering the circle（s） |
| R | total losing time of traffic light signal (s) |
| $\Delta$ | maximum headway in terms of time of vehicles travelling in queue(s) |

# 2.Basic theory obtained by predecessor

## 2.1 Analysis

There is another factor which is frequently-used in traffic control problem. Delay is an important parameter that is used in the optimization of traffic signal timings and the estimation of the level of service at signalized intersection approaches. However, delay is also a parameter that is difficult to estimate. While many methods are currently available to estimate the delays incurred at intersection approaches. Among them, we select a classical one--- "Delay at signalized intersections" to illustrate as below:

Delay at signalized intersections is computed as the difference between the travel time that is actually experienced by a vehicle while going across the intersection and the travel time this vehicle would have experienced in the absence of traffic signal control. The diagram of Fig. 1 further indicates that the total delay experienced by a vehicle can be categorized into deceleration delay, stopped delay and acceleration delay. Typically, transportation professionals define stopped delay as the delay incurred when a vehicle is fully immobilized, while the delay incurred by a decelerating or accelerating vehicle is categorized as deceleration and acceleration delay, respectively. In some cases, stopped delay may also include the delay incurred while moving at an extremely low speed. For example, the 1995 Canadian Capacity Guide for Signalized Intersections (ITE, 1995) defines stopped delay as any delay incurred while moving at a speed that is less than the average speed of a pedestrian (1.2 m/s).

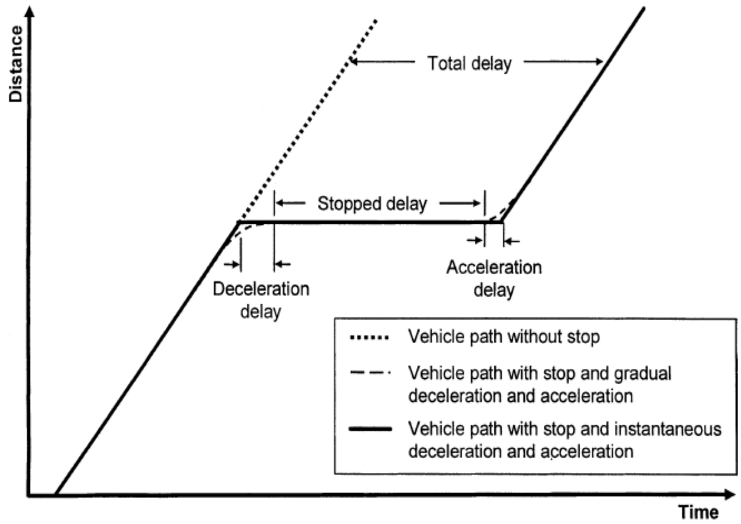


Figure2. Definition of total, stopped, deceleration and acceleration delays

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## 2.2 A famous model to compute delay---Akeclik-model

The Akeclik-model is a Time-Dependent Stochastic Delay Models for under-saturated and over-saturated Conditions.

A main consequence of the stochastic delay modeling described in the previous section is that the estimated delays tend to infinity as traffic demand approaches saturation (v/c ratio of 1.0). This was considered as a weakness by many researchers (Akcelik, 1988; McShane and Roess, 1990; Fambro and Rouphail, 1997). For low v/c ratios, the models should produce delay estimates that are similar to those produced by deterministic queuing delay models assuming constant uniform arrivals. As the load increases, a larger proportion of the delay is caused by the randomness of vehicle arrivals and attributed to the inability to clear all queued vehicles in some cycles. As the v/c ratio approaches 1.0, the models should finally not tend to infinity, but should instead produce estimates that become tangent to the deterministic oversaturation model.

According to this model, we can get expressions as below:

$$d_1 = \begin{Bmatrix} \frac{c(1-g/c)^2}{2[1-(g/c)x]} & x < 1 \\ (c-g)/2 & x \geqslant 1 \end{Bmatrix} + \frac{Q_0}{c}$$

$$Q_0 = \begin{cases} \frac{CT}{4}\left[(x-1)+\sqrt{(x-1)^2+\frac{12(x-x_0)}{CT}}\right] & x > x_0 \\ 0 & x \geqslant x_0 \end{cases}$$

$$x_0 = 0.67 + \frac{sg}{600}$$

# 3. The modified model

## 3.1 Analysis

The models in the part of basic theory can be used in intersections but not the traffic circle. So we make some modify on them to get the model which is applicable for the traffic circle. In face, a intersection is a special traffic circle when the radius of the circle is zero and left-turn is forbidden. So the delay resulted by the traffic light is the same for intersection and traffic circle. In the system of intersection, generally there is no delay in passing the intersection after waiting in the front of traffic light because vehicles travel a very short linear distance(width of the road). While in traffic circle, vehicles travel around the circle. Thus the length of travelling is longer and the velocity is lower. So the delay when passing the circle can not be neglected. Based on the above analysis, the total delay for traffic circle is the summation of delay that resulted from waiting the traffic light and delay in passing the circle and the critical in the latter. So the key is to find delay in passing the circle.

## 3.2 Deduction

As we assumed that the probability for a car to get out from each of the N incoming roads is equal and equals to 1/N, we can compute the expectation of the $L$ as follow:

$$E(L) = \sum_{i=1}^{n} \frac{1}{n}\frac{i}{n}L = \frac{n+1}{2n}L \qquad (10)$$

The minimum time of a vehicle traveling across the traffic circle equals the ratio of the length it has traveled and the full-traveling velocity. And the real time of the vehicle traveling across the traffic circle is the ratio of the length it has traveled and the full-traveling velocity. So the delay of a vehicle across the traffic circle can be expressed as below:

$$d_2 = \frac{E(L)}{V} - \frac{E(L)}{V_f} \qquad (11)$$

In the Greenshield model, we can get the expression which indicated the relationship between the velocity of the vehicle and the traffic flow on the road[9]. The equation is shown below:

$$Q_c = \frac{V}{l}(1 - \frac{V}{V_f}) \qquad (12)$$

Using the gap-acceptance theory we can get the relationship between the traffic capacity $C$ and the traffic flow inside the traffic circle[3]:

$$C = \frac{\alpha Q_c}{1 - e^{\lambda} T_0} e^{-\lambda(\Delta - T_l)} \qquad (13)$$

$$\lambda = \frac{Q_c \alpha}{1 - Q_c \Delta} \tag{14}$$

As the definition of $x$ we can get the equation:

$$x = \frac{q_a}{C} \tag{15}$$

Here we can see that if the $x$ and the $q_a$ is given, we can get the $C$, and evaluate the $C$ in the equation (4) and solve it we can get the value of $Q_c$,
Then solve the equation (12) we get the value of $V$. At last take the value of $V$ into the equation (11) we can get the delay time.
We can see the changing of the argument with the same variable $V$as below

$$V = V(Q_c) = V(C) = V(x)$$

The delay we have analyzed above is the value of one vehicle, so in order to get the total delay we need to multiple our result above with the traffic flow which is concerned. In conclusion the model can be described as below:

$$\min z = q_a d_1 + Q_c d_2$$

$$s.t. \begin{cases} g + R = c \\ c_a \leqslant c \leqslant c_b \end{cases}$$

## A model to find the initial value

## 1 Notations

| | |
|---|---|
| $g$ | time of green light in a cycle of traffic light |
| $N$ | the number of incoming road as shown in Fig. 2 |
| $T$ | a random variable that represents the interval of arriving in the place of the traffic light (Fig.2) between two adjacent coming vehicles in the incoming road; |
| $M$ | the maximum of $Q_a$ |
| $K$ | the green ratio |
| $Q_0$ | the in-flow traffic volume on each incoming road before passing the traffic light as shown in Fig. 2 |
| $Q_{in}$ | the in-flow traffic volume on each incoming road after passing the traffic light as shown in Fig. 2; |
| $Q_i$ | the around-flow traffic volume on the circle lane between the $i$th incoming road and the $(i+1)$th incoming road for $i$=1,2,…N |
| $Q_{out}$ | the out-flow traffic volume on each incoming road |
| $Q_a$ | the around-flow traffic volume on the circle lane |

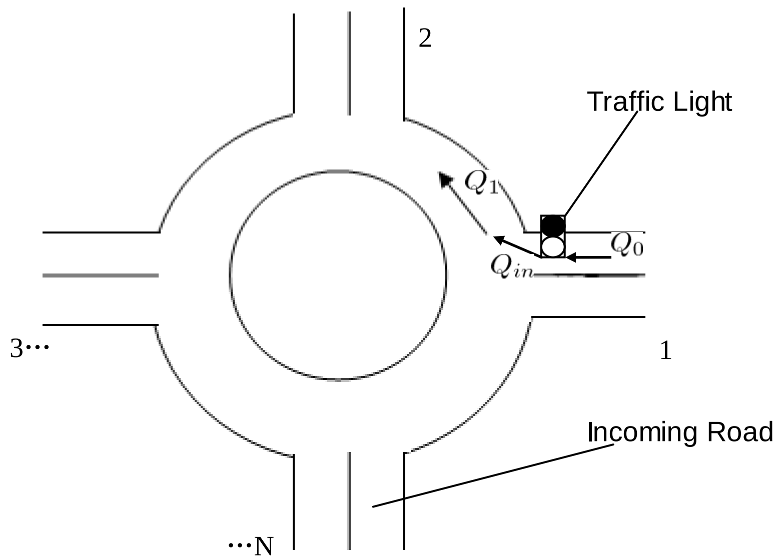


Figure3: A ideal symmetry traffic circle with N incoming

# 2 Illustration of The Model

## 2.1 Analysis

In total, there are three kinds of traffic flow that need to be controlled in the problem, which we call the in-flow、the around-flow and the out-flow respectively here for short. It's obvious that the critical one is the around-flow. When the around-flow is fluent, then the in-flow and out-flow will be fluent absolutely. Under normal circumstances, vehicles in the circle could get out the circle smoothly. So there is no need to control the out-flow. Based on this fact, we assume that the out- flow always keeps fluent.

## 2.2 Deductions

Now let's see the in-flow. Before vehicles pass the traffic light, the traffic volume of the incoming road is $Q_0$. By introducing the random variable T, we can formulate the expression of $Q_0$ as below:

Since the sign T represents the interval of arriving in the place of the traffic light between two adjacent coming vehicles in the incoming road, then we can deduce that 1/T means the number of vehicle arriving in the traffic light in a unit time. According to the definition of traffic volume, it’s easy to get that the traffic volume arriving in the traffic light $Q_0$ is just equal to the value of 1/T, that is:

$$Q_0 = \frac{1}{T}$$

Next let’s consider the traffic volume after passing the traffic light $Q_{in}$. It’s obvious that there is some relationship between $Q_0$ and $Q_{in}$. When there are only a few vehicles in the circle (we mean it is far from the maximum capacity of the circle M), the traffic light can keep green and directly $Q_{in}$ will be equal to $Q_0$ as a result of assumption 5. Otherwise, when the traffic volume increases, we need to use the traffic light to control the in-flow. Under this condition, $Q_{in}$ will certainly be changed and not equals to $Q_0$. Specifically, $Q_{in}$ will decrease. Here we simply assume the relationship between the in-flow and the arrival-flow is linear and the expression of $Q_{in}$ can be derived as following:

In a cycle of the traffic light :

$$\frac{Q_{in}}{t_r + t_g} = Q_0 \cdot t_g$$

$$Q_{in} = Q_0 \cdot \frac{t_g}{t_r + t_g}$$

Here $t_g$ represents the time of green light in a cycle and $t_r$ represents the time of red light.

According to the definition of green ratio, we know:

$$k = \frac{t_g}{t_r + t_g}$$

$$Q_{in} = k\ \frac{1}{T}$$

Now we deduce the expression of the around-flow traffic volume on the circle lane between the $i$th incoming road and the $(i+1)$th incoming road $Q_i$.

$$\begin{aligned} Q_i &= 1 \times Q_{in} + \frac{n-1}{n} \times Q_{in} + \frac{n-2}{n} \times Q_{in} + \dots + \frac{1}{n} \times Q_{in} \\ &= (1 + \frac{n-1}{n} + \frac{n-2}{n} + \dots + \frac{1}{n}) \times\ Q_{in} \\ &= \frac{n+1}{2} \times\ Q_{in} \\ &= \frac{n+1}{2} k \frac{1}{T} \end{aligned}$$

Then we turn to consider how to best control the traffic in-flow by regulating the green ratio $k$ and what's the objective of the control. Firstly it's easy to find that we hope the expectation of the traffic volume of the circle lane to be as high as possible in order to enhance the traffic capacity, that is:

$$max\{E(Q_i)\} \tag{1}$$

Another factor that needs to be taken into account is the total time of being full loaded in the circle. When the circle is full loaded, the around-flow gets its maximum value and any new incoming flow will lead to a fall of the around-flow. So the shorter the time during which the circle is full loaded is, the better the traffic flow is. We use the concept of probability to express this objective:

$$max\{P(Q_i) < M\}. \tag{2}$$

In order to make the dimensions of these two items same, we refine item(1) in this way:

$$max\{\tfrac{E(Q_i)}{M}\} \tag{3}$$

Since the operations of item(2) and item(3) are both $max$,so we can get the normalized addition of this two items :

$$max\{\omega\frac{E(Q_i)}{M} + (1-\omega)P(Q_i < M)\} \tag{4}$$

Then we try to simplify the item(4). According to assumption6, we know:

$$F(T) = \begin{cases} 1 - e^{-\lambda T} & \text{for } T \geqslant 0 \\ 0 & \text{for } T < 0 \end{cases} \quad \text{and} \quad \lambda = E(Q_0) = E(\tfrac{1}{T}) \tag{5}$$

The first item in the brackets can be deduced as below:

$$\begin{aligned} E(Q_i) &= E(\frac{n+1}{2}k\frac{1}{T}) \\ &= \frac{n+1}{2}kE(\frac{1}{T}) \\ &= \frac{n+1}{2}k\lambda \\ \frac{E(Q_i)}{M} &= \frac{n+1}{2}\frac{k}{M}\lambda \\ &= kc \quad \text{(here we notate } x = \frac{n+1}{2}\frac{\lambda}{M}\text{).} \end{aligned} \tag{6}$$

The second item in the brackets can be deduced as below:

$$\begin{aligned} &P(Q_i < M) \\ &= P(\frac{n+1}{2}k\frac{1}{T} < M) \\ &= P(T > \frac{n+1}{2}\frac{k}{M}) \\ &= 1 - F(\frac{n+1}{2}\frac{k}{M}) \\ &= e^{-\frac{n+1}{2}\frac{\lambda}{M}k} \\ &= e^{-kx} \end{aligned} \tag{7}$$

Take formula(6) and formula(7) into item(4), we obtain a model which can be used to find out the value of $k$ for a certain traffic circle:

$$max\{\omega kx + (1-\omega)e^{-kx}\} \tag{8}$$

We can obtain the value of $k$ when the item in the brackets reaches its maximum. Then we can arrange the condition of the traffic light based on $k$.

## 2.3 Solutions and discussions

Here we notate the item in the brackets of formula(8) $Y$:

$$Y = \omega kx + (1-\omega)e^{-kx}$$

By $\frac{dY}{dk} = 0$,we get: $$k = \frac{ln(\frac{1-\omega}{\omega})}{x} \quad (9)$$

（The related program of MATLAB7.0 is given by Appendix Ⅰ)

To make this solution more simple and intuitive, we try various expressions of the solution to check the reasonableness of this model and formulate some conclusions as below.

**The first expression**

According to the meaning of the variables $k, x$, and $\omega$,they are all between 0 and 1. For all the cases in this interval of (0,1), the relationship of this three variables can be expressed by the following three-dimensional space graph:

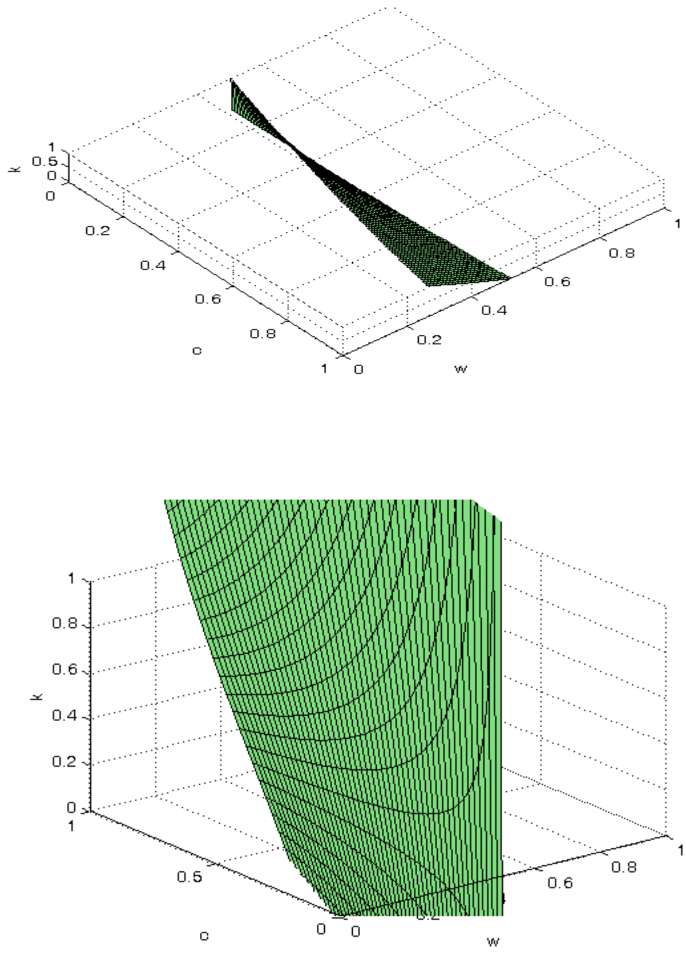

Figure 4 : the three-dimensional space graph of $k, x$, and $\omega$

**The second expression**

Although the three-dimensional space graph contains all of the cases, it's not intuitive, simple and useful enough. For this, we try to give the two-dimensional curve when $\omega$ is given certain value. Several graphs of different value for $\omega$ are given as below:

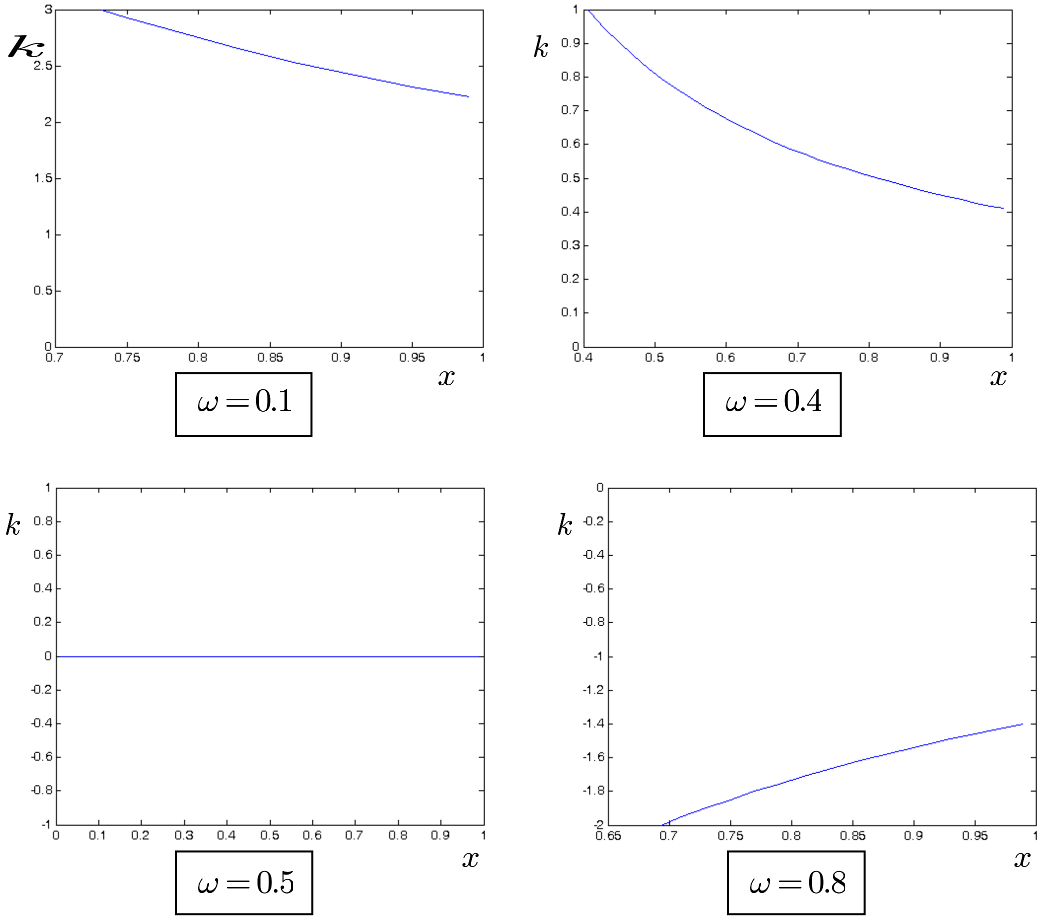


Figure 5: $k-x$ graphs of different value for $\omega$

In the above graphs, when $\omega = 0.1$, there is no content when $k$ is in the interval of (0,1); when $\omega = 0.5$, the value of $k$ keeps 0.5; when $\omega = 0.8$, the value of $k$ is negative. By observing the above graphs, we can get that the value of $\omega$ should be limited in a certain interval. Then we try to find this interval as below:

Let $k = 1, x = 1$, from the equation(9), we get:

$$\omega = \frac{1}{1+e} \approx 0.2689$$

So the interval of $\omega$ is $(\frac{1}{1+e}, 0.5)$. When we use this model to control the traffic flow of a practical traffic circle, we should see to it that the value of $\omega$ is proper.

When the value of $\omega$ is in the required interval, for example, $\omega = 0.4$ as shown in the above figure, the value of $k$ decreases when $x$ is increasing. That is to say, if the in-flow traffic volume is high, the traffic condition in the circle may be heavy, then we should let less vehicles enter the circle, so the time of green light should be shorter. As the time of green light is shorter, then the green ratio will decrease. Thus the shape of the graph (when the value of $\omega$ is proper) accords with the practical trend. So to some extent, this model is reasonable.

**The third expression**

Let $\omega = 0.45$, for a series of value for $x$, we can get the corresponding value of $k$ as below:

Table 1 ($\omega$=0.45,the cycle of traffic light is 60s)

| $x$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| $k$ | 2.0 | 1.0 | 0.67 | 0.50 | 0.40 | 0.33 | 0.29 | 0.25 | 0.22 | 0.20 |
| $g$ | 120 | 60 | 40 | 30 | 24 | 20 | 17 | 15 | 13 | 12 |

From the above table, if $x$ is too small ($x$=0.1 and $x$=0.2), the value of $k$ will be more than 1,which is obviously unreasonable. However, when $x$ is too small, all vehicles in the incoming road can enter the circle smoothly and under this circumstances traffic light is no longer needed.

## Technical summary

In our study, we recommend using the traffic light first. We classify the various traffic circles according to its size, traffic volume of the incoming road, and the volume-to-capacity ratio. To use our method to choose the appropriate flow-control method, you need to follow the steps below:

Step1. You need to prepare the statistic on the three terms talked above. Make sure that all the variables are in SI.

Step2. You need to run our first program (Appendix Ⅱ.g ), just input the traffic volume of the incoming road and you will get an answer of the green ratio.

Step3. Run our second program (Appendix Ⅱ), and input the three parameters which are prepared above and the green ratio got from step2.

After that you will get an answer which include the cycle of traffic light and the effective green light time. The cycle of the traffic light means the time during which the traffic light change from red to green and change back. The effective green time is not the real green time, you need to multiply the effective green time with a const to get the real green time . Note that when the value of effective green time is zero, we suggest you to put a stop or yield sign on incoming road, and when it almost equals one, we suggest you to put a yield sign in the circle.

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**Appendix**

I

a.
```
>> syms w c k
>> f = w*c*k + (1-w)*(1/exp(c*k))
f =
w*c*k+(1-w)/exp(c*k)
>> dfdk = diff(f, k)
dfdk =
w*c-3/2*(1-w)/exp(3/2*c*k)*c
>> g = solve(dfdk, k)
g =
2/3*log(-3/2*(-1+w)/w)/c
```

**b.**
```
function fig1
syms w c k
f = w*c*k + (1-w)*(1/exp(c*k));
dfdk = diff(f, k);
g = solve(dfdk, k);
w = 0.01:0.02:0.99;
c = 0.01:0.02:0.99;
[w, c] = meshgrid(w, c);
G = subs(g);
figure;
xlabel('w');
ylabel('c');
zlabel('k');
```

**c.**
```
function fig2
w = 0.45;
c = 0.1:0.1:1;
k = 0;
a = 1;
for c = 0.1:0.1:1
    k(a) = (log((1-w)/w)/c)*60
    a = a + 1;
end
figure;
plot(c, G);
xlabel('c');
ylabel('k');
```

II

a.
```
function out = fun_C()
alpha = 0.9;
To = 2;
delte = 1.5;
Tl = 4;
x = 0.2;
C ＝ 0.78；
syms Qc;
f = solve('((alpha*Qc) / (1 -(exp(-(Qc*alpha / (1 - Qc*delte))*To)))) *
    exp(-(Qc*alpha / (1 - Qc*delte))*(delte-Tl)) - C');
out = f;
```

**b.**
```
function out = sol_Qc()
alpha = 0.9;
To = 2;
delte = 1.5;
Tl = 4;
C = 10;
syms Qc
f = @fun_C;
% solving manually
out = solve(f);
```

**c.**
```
function out = sol_V(Qc)
Vf = 60/3600;
l = 6.5;
n = 4;
syms V
E = (n+1)*l/(2*n);
%E/V - E/Vf = Qc;
V = E/(Qc + E/Vf);
out = V;
```

**d.**
```
function d2
E = (n+1)*l/(2*n);
V = sol_V;
d2 = E/V + E/Vf;
function out = d2()
```

**e.**
```
function d = Akcelik(g, c, x, s, T, CAP)
if x<1
    delay1 =   (c*(1 - g/c)^2 / (2*(1 - (g/c)*x)));
else
    delay1 =   ((c-g)/2);
end
x0 = 0.67 + s*g/600;
if x>x0
    delay2 =   ((CAP*T/4) * ((x-1) + sqrt((x-1)^2 + 12*(x-x0)/(CAP*T))));
else
    delay2 =   0;
end
d = delay1 + delay2/c;
```

**f.**
```
function d = delay(g, c, x, s, q, T, CAP)
d = Akcelik(g, c, x, s, T, CAP)* q + d2;
```

**g.**
```
function [gm, cm, dm] = opm(a, stp1, b, stp2, x, s, q, T, CAP)
c = a;
u = 0.2;
g = c - r;
d_tmp = delay(g, c, x, s, q, T, CAP);
cm = c;
gm = g;
dm = d_tmp;
for c = (a+stp):stp1:b
    for g = 0:stp2:c
        d = delay(g, c, x, s, q, T, CAP);
        if d < d_tmp
            cm = c;
            gm = g;
            dm = d;
            d_tmp = d;
        else
            continue;
        end
    end
ends
```