Designing a Traffic Circle

February 10, 2009

Abstract

We model Bottleneck Section and Queuing Theory for the optimal choice among traffic-control designs of yield sign, stop sign and traffic lights at a roundabout.
The first model is based on maximizing the flow rate of the roundabout limited by the capacity of the Bottleneck Section-capacity of roundabout under a centain saturation, which is the lowest capacity of all Critical Sections. When a sign is in the incoming road, the flow rate of Critical Sections can be calculated by Wardrop Formula. While traffic lights are set, the capacity of the Critical Section can be given by effective green ratio and the maximum entry flow rate which is estimated via Kimber's equation. Combing those situations, whether sign or traffic lights the better alternative is given by a 0-1 integer programming model. If sign is better, the following step is to choose a yield sign or a stop sign. When traffic lights are recommended, the green time is discussed. Naturally, the whole model is divided into three sections, namely, using signs or traffic lights, using yield sign or stop sign and calculating the duration of the traffic lights.
The second model, based on queuing theory, compares four systems-No signal System, Yield Sign System, Stop Sign System and Traffic Lights System-with objectives which including the average sojourn time and the saturation of system. Assuming each incoming road is an information desk providing service for vehicles, and let the capacity of roundabout be the largest number of vehicles it could serve per hour, we then calculate the two objectives under different situations by varying the value of average rate of arrival vehicles as well as the number of incoming roads and circle lanes. Let 85 percent of the capacity of each situation be the upper limit of the traffic volume in reality, the conditions under which each type of traffic-control method should be used is available.

Contents

1 Introduction

A traffic circle is an intersection with a circular shape and, usually, a central island. In some traffic circles two-way traffic is allowed within the circle. It is much more common, however, that traffic is allowed to go in one direction only around a central island. Traditionally, traffic entering a circle has the right-of-way, although some circles give right-of-way to the primary roads. In roundabouts and often rotaries, as opposed to traffic circles, entering traffic must yield to traffic already in the circulatory roadway[1].

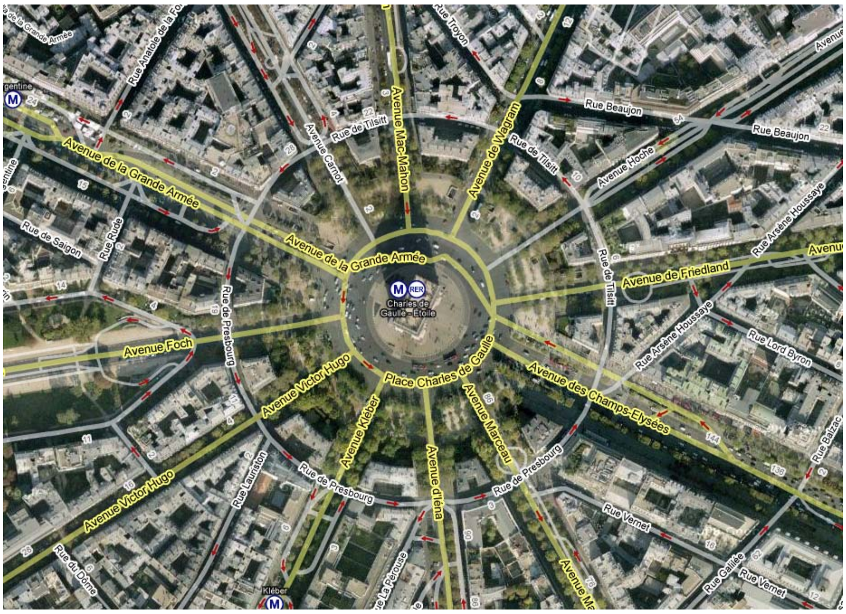

Fig 1: Example of Traffic Circle: Arc de Triomphe in Paris, Snapped from Google Maps.

A STOP SIGN is a traffic sign, usually erected at road junctions, that instructs drivers to stop and then to proceed only if the way ahead is clear. While a YIELD (Canada, Ireland, and the United States) or GIVE WAY (most current or former Commonwealth countries) SIGN indicates that a vehicle driver must slow down and prepare to stop if necessary ł usually while merging into traffic on another road ł but needn't stop if there is no reason to do so[2][3].
A driver who stops before a yield sign is said to have yielded the right of way to traffic on the main road. In contrast, a stop sign always requires a full stop.

1.1 Classification

Traffic circle can be divided into at least three groups according to *Roundabouts: An Information Guide*[4].

- **Rotaries**
  Rotaries are old-style circular intersections common to the United States prior to the 1960s. Rotaries are characterized by a large diameter, often in excess of 100 m (300 ft). This large diameter typically results in travel speeds within the circulatory roadway that exceed 50 km/h (30 mph). They typically provide little or no horizontal deflection of the paths of through traffic and may even operate according to the traditional yield-to-the-right rule, i.e., circulating traffic yields to entering traffic.

- **Neighborhood traffic circles**
  Neighborhood traffic circles are typically built at the intersections of local streets for reasons of traffic calming and/or aesthetics. The intersection approaches may be uncontrolled or stop-controlled. They do not typically include raised channelization to guide the approaching driver onto the circulatory roadway. At some traffic circles, left-turning movements are allowed to occur to the left of (clockwise around) the central island, potentially conflicting with other circulating traffic.

- **Roundabouts**
  Roundabouts are circular intersections with specific design and traffic control features. These features include yield control of all entering traffic, channelized approaches, and appropriate geometric curvature to ensure that travel speeds on the circulatory roadway are typically less than 50 km/h (30 mph). Thus, roundabouts are a subset of a wide range of circular intersection forms.

As Rotaries and Neighborhood traffic circles are becoming obsolete, we will focused on roundabout in this paper.

2 Terminology

Since the problem involves Highway Construction, we will apply some technical terminologies and definations, it will be helpfull to introduce natational conventions. We take four-leg roundabout for example in figure 2, which shows a number of key dimensions that are described in Table 1.

Capacity Denoted by C is the maximum flow rate that can be accommodated at a particular section of the road. It is typically expressed in passenger car equivalents per hour(pce/h), for a specified 15-minutes analysis period[4].

Entry Capacity Denoted by E is the capacity of the entry in a particular roundabout.

Flow Rate Denoted by Q means the number of vehicles per time unit given of one particular section of the road.

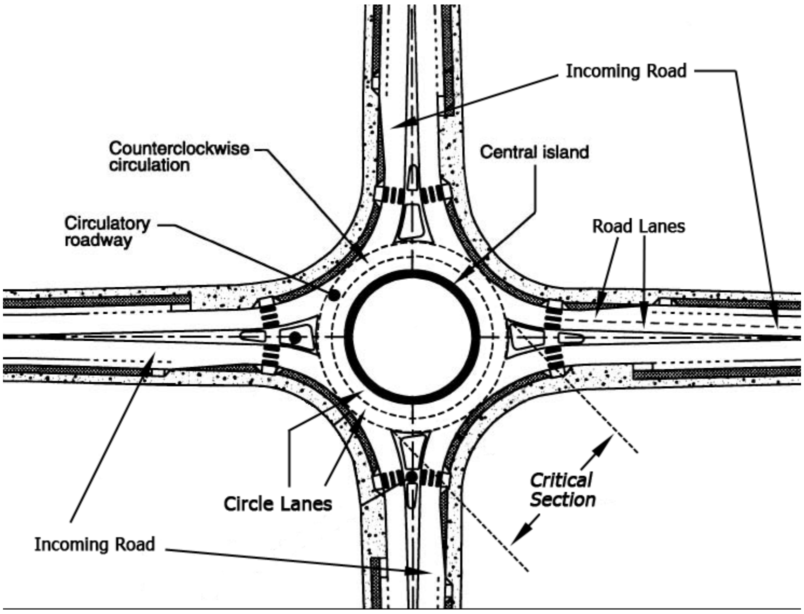

Fig 2: Drawing of Traffic Circle with Key Features

Tab 1: Description of key roundabout Features.

Features	**Description**
Central island	The central island is the raised area in the center of a roundabout around which traffic circulates.
Circulatory roadway	The circulatory roadway is the curved path used by vehicles to travel in a counterclockwise fashion around the central island.
Circle Lanes	A portion of circular road which is intended and marked for a single path of vehicles.
Incoming road	or entry lanes, at which traffic enters the roundabout
Road lanes	A portion of a roadway intended or marked to accomodate a single line of vehicles.
Critical section	always known as weaving section, a particular section of roundabout between neighboring incoming roads, in which weaving occurs, and is very essential, because the capacity of a roundabout is affected mainly by such section.

Saturation Denoted by S_a, which means the ratio of circulating flow rate to circulating capacity.

Delay Denoted by d is an essential parameter that was be widely used to measure the performance of a roundabout. The following is an excerpt from the *Roundabout: an information guide*[4]: Control delay is the time that a driver spends queuing and then waiting for an acceptance gap in the circulating flow while at the front of the queue.

Bottleneck Section A Critical Section which has the lowest capacity and its capacity equals to that of a roundabout at a certain saturation.

Right-turning/Outside lane one of the circle lanes in which cars turn right at the exit lane upstream next to the entry, valid when the number of circle lanes is greater than or equal to 2.

Inside lanes the lanes of circle except right-turn one, valid when the number of circle lanes is greater than or equal to 2.

Cycle/Period time Denoted useing T_p, the total time of the respective time of flashing once of green, yellow and red lights.

3 Analysis

The method to handle Vehicle Types is Converting All Types into Cars All kinds of vehicles could be converted into cars using the table in *Roundabout:an information guider*[4], then the vehicles in that essay is assumed to be cars.

Volumes are typically expressed in passenger car vehicles per hour (vph), for a specified 15-minute analysis period. To convert other vehicle types to passenger car equivalents (pce), use the conversion factors given in Table 2.

Tab 2: Conversion factors for passenger car equivalents (pce).

Vehicle Type	Passenger Car Vehicle Type Equivalent (pce)
Car	1.0
Single-unit truck or bus	1.5
Truck with trailer	2.0
Bicycle or motorcycle	0.5

Eliminating bicycles and pedestrians, our models will focus on typical traffic control methods-using yield sign, stop sign and traffic lights. Signs in the circle are eliminated since that method seldom appeared now. Then our task is to determine whether signs or traffic lights should be used in the incoming road, when a roundabout is given. Distinguished by its safety, the roundabout system is usually weak in transportation, that is why we choose capacity under different saturation as the main objectives. With varied traffic control methods, number of lanes, number of incoming roads, and saturation, programming method should be chosen to model capacity of roundabout under different conditions.

4 Assumptions

- *Each vehicle never passes its incoming entrance more than once.*
  Each car exits this system within a circle and it is quite reasonable in our daily life.
- *Ignore non-motor vehicles.*
  The bicycles, pedestrians are eliminated from our models.
- *Vehicles in the system work well and accidents never happen.*
- *Every Driver Obey the Traffic Rules in the System Considered*
  This assumption could simplify our models as well as help us to focus on the main impact of the introductory facilities.
- *No Influence Comes from the Environments*
  We do not take the weather into consideration, though it is recommended. In *Roundabout: an information guide*, two kinds of road–the urban road and the rural road are take into account separately, but in order to simplify the models we ingores the difference, though the urban road has a larger traffic volume per hour but a lower velocity.
- *No Signs be Settled in the Circle*
  As introduced before, signs set in the circle is the method of traditional roundabout; the modern roundabout usually set them in the incoming road, since it has many advantages.

5 Bottleneck Section Model

The capacity of a roundabout under a certain Saturation is determined by the minimun traffic flow of critical section i.e. the capacity of bottleneck section, on which the following model is based. Our model is divided into three sections: the first is to decide whether signs or traffic lights are needed on the incoming road; the second section explains which sign will be used on the condition that signs are need; while traffic lights are needed, the method for calculating duration of traffic lights is given in the third section.

5.1 FIRST SECTION: Using Signs or Traffic Lights

We assume that the maximum capacity of roundabout suggests best traffic control. Decided by the capacity of bottleneck section as the capacity of roundabout, It comes the objective function

$$\max Q = \min_{1 \leqslant i \leqslant n} Q_i, \tag{5.1}$$

where

Q_i = the largest traffic flow rate of the ith critical section (pce/h).

Attention has been taken to two situations: signalized and nonsignalized roundabout.

5.1.1 Nonsignalized Condition

Nonsignalized roundabout requires entering traffic to give way, or yield, to circulating traffic.This rule prevented circular intersections from locking up, by not allowing vehicles to enter the intersection until there were sufficient gaps in circulating traffic. Stop sign and Yield sign are important methods for traffic control. Wardrop formula offers a way to calculate traffic capacity of critical section

$$Q_{m,i} = \frac{354W_i(1+\frac{e}{W_i})(1-\frac{p_i}{3})}{1+\frac{W_i}{l_i}}, \tag{5.2}$$

where

$Q_{m,i}$ = the largest flow rate of the ith critical section (pce/h),
W_i = the width of the ith critical section (m),
l_i = the length of the ith critical section (m),
e = $\frac{e_1+e_2}{2}$, the average width of the circle intersect incoming road (m),
e_1 = the width of the incoming road (m),
e_2 = the width of the circle (m), and
p_i = the ratio of the number of vehicles in the ith critical section to that of the upstream critical section.

Please see Figure 3 for a detailed description of notations. Given a roundabout, W_i, l_i and e can be figured out. p_i is regarded as a variable related with the designing of traffic control. According to experiential tests, it's suggested that the actual value should be 80% of the result of Wardrop Formula, so rewriting (5.2) we obtain

$$f(p_i) = \frac{283W_i(1+\frac{e}{W_i})(1-\frac{p_i}{3})}{1+\frac{W_i}{l_i}}. \tag{5.3}$$

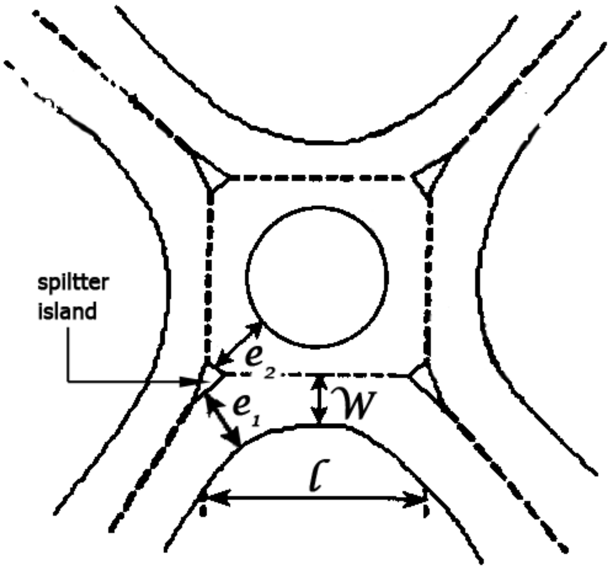

Fig 3: Drawing of detailed description of natations in Wardrop Formula

p_i is defined as

$$p_i = \frac{Q_{in,i}}{Q_{cir,i} + Q_{in,i}}, \tag{5.4}$$

where Q_{in} and Q_{cir} are shown in Figure 4, and their value can be estimated by

$$\begin{aligned} Q_{in,i} &= r_i \frac{3600}{d} T_a P_i, &(5.5)\\ Q_{cir,i} &= s' Q_s T, &(5.6) \end{aligned}$$

where r_i is the number of lanes of the ith incoming road, P_i is the ratio of the number of vehicles entering from the ith entry and running in the inside circle lanes to the number of vehicles entering from the ith entry, $Q_s = CS_a$, T_a is analysis time period, s' is the number of the Inside Circle Lanes, when there is only one lane in the circle, $s' = 1$, otherwise, $s' = s - 1$. That is

$$s' = \begin{cases} s & s = 1, \\ s - 1 & s \geqslant 2. \end{cases} \tag{5.7}$$

s is the number of lanes of circular road, usually, when there is more than one lane in the circle, the Outside Lane is preserved to vehicles that would turn right and get out of the roundabout in the next incoming road. And that is why addd holds.

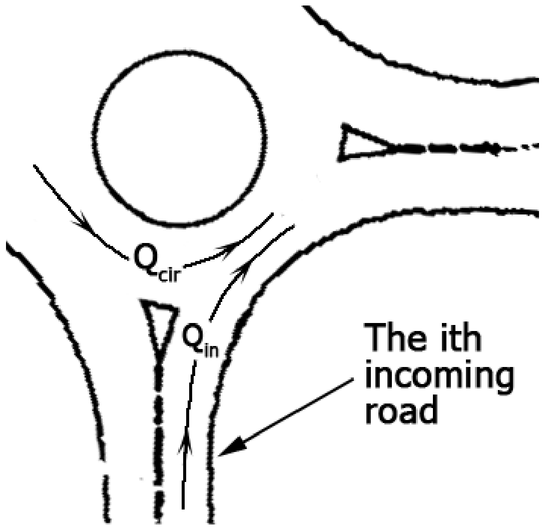

Fig 4: Drawing of Flow Rate, Q_{cir} and Q_{in} are the flow rate of the ith incoming road.

And delay time[4]

$$d = \frac{3600}{C} + 900T_a \left[\frac{Q_s}{C} - 1 + \sqrt{\left(\frac{Q_s}{C} - 1\right)^2 + \frac{\frac{3600}{C}\frac{Q_s}{C}}{450T}} \right] \tag{5.8}$$

where

d	=	average control delay (sec/veh),
Q_s	=	circulating flow rate for one lane(veh/h),
C	=	capacity of roundabout (veh/h),
T_a	=	analysis time period, measured by the hour, (=0.25 for a 15-minute period).

5.1.2 Signalized Condition

Compared to nonsignalized roundabout, signalizing the roundabout will provide a more accurate traffic control.

Let k_i be effective green ratio for the ith incoming road, the number of vehicles in the circle during the red time in one cycle is

$$Q'_{cir,i} = sQ_s(1 - k_i)T_a, \tag{5.9}$$

while during the green time in one cycle

$$Q'_{in,i} = sEk_iT_a, \tag{5.10}$$

where E is entry capacity, which can be derived from kimber model:

$$E = F - f_c Q'_{cir}\ ,\tag{5.11}$$

where

E	=	entry capacity (pce/h), which is the capacity of this entry,
Q'_{cir}	=	circulating flow rate (veh/h),
F, f_c		parameters defined by roundabout geometry.

Roundabout: An Information Guide[4] provides a way of calculating F, f_c, which is provided on Appendix A. E could be estimated by setting $Q'_{cir} = 0$ in (5.11). Combining (5.10) and (5.9) yields the flow rate of critical section, a function of k_i

$$g(k_i) = Q'_{in,i} + Q'_{cir,i},\tag{5.12}$$

and we simply assume that

$$k_i = \frac{Q'_{in,i}}{Q'_{cir,i} + Q'_{in,i}}.\tag{5.13}$$

Since we don't know which condition to choose, let $x_i be 0-1 variable so that Q_i$ can be expressed as

$$Q_i = (1 - x_i) f(p_i) + x_i g(k_i).\tag{5.14}$$

where

$$x_i = \begin{cases} 0, & \text{there is a sign on the ith incoming road,} \\ 1, & \text{there is a traffic light on the ith incoming road.} \end{cases}$$

This model can be used to choose a traffic-control method between signs and traffic light, and the green ratio is given by the model if using traffic light.

5.2 SECOND SECTION: Using Yield Sign or Stop Sign

Supposing there is a yield sign or stop sign,the following discussion is about judging whether a yield sign or a stop sign is needed.

When a vehicle in the incoming road approaches the roundabout, the driver should make the decision on how to get into the roundabout. Let v_m be the mean velocity of the vehicles in the incoming road. Let v_0 be the mean velocity of the vehicles in the roundabout. If the driver need not want to stop his car before he could find a gap to let his car get into in the flow of the vehicles in the roundabout, he should judge the situation in a particular time span. Judge Time denoted by DT is the time left for the driver to make a decision, and the following equation is used by us to calculate DT.

$$DT = \frac{v_m - v_0}{a},\tag{5.15}$$

Where a is the maximum deceleration of the vehicle, and it could be calculated by the equation: $a = \mu g$ or bound by traffic regulations. Indicating the average time a driver needs to find a gap and get his car in the roundabout, Delay time(d) could be used to measure the maximum time a driver have to make a decision. Because d indicates that a driver takes about a time span of d to get his vehicle in the roundabout. when his vehicle approaches the roundabout, he has a time span of DT to slow down his vehicle and decide whether he should stop or not. If $DT \leqslant d$, a yield sign is needed because the driver could chose a reasonable deceleration to get his vehicle in. If $DT > d$, a stop yield sign is needed because usually the driver have to stop his vehicle. Then we get

$$\begin{cases} DT \leqslant d, & \text{a yield sign is needed,} \\ DT > d, & \text{a stop sign is needed.} \end{cases} \tag{5.16}$$

When the traffic becomes heavier, a yield sign seems more likely to be recommended, and that fact conforms to natural thoughts: when the traffic is heavier, the driver should be more careful when passing through the intersection, then stopping his vehicle and looking around the roundabout before passing, which is better to himself as well as the others. The geometric conditions also have a significant effect on the choice. Because the maximum deceleration is limited by the slide friction factor, the time liven for a driver to make a decision is limited(which is called Judge Time by us). When slide friction factor increases, the Judge Time decreases, then a stop sign is more likely to be recommended because Judge Time may be greater than Delay Time resulting in more likely to queue in the incoming road.

5.3 THIRD SECTION: Calculating Duration of Traffic Lights

Although how to estimate the green time is not easy, the model about how to estimate the yellow time is provided by a few models, the following is a better one[5], which is useful in estimating the green time.

When a vehicle passes through the intersection, that paradox should be avoided-whether he wants to stop or not, his vehicle is in the intersection when the red light is on. The following works are based on that. Let v_0 be the maximum velocity the vehicle can travel by law in the road. Let e be the width of the intersection and L be the average length of the vehicle, which can be measured according to geometric design. Then the time a vehicle needed to pass the intersection is $\frac{e+L}{v_0}$. Then the minimum space a vehicle needed to stop denoted by S is given by

$$S = \frac{v_0^2}{2\mu g}, \tag{5.17}$$

Where μ is the slide friction factor, g is the acceleration due to gravity. Then yellow time denoted by t_y is

$$t_y = \frac{v_0}{2\mu g} + \frac{e+L}{v_0} + T_r, \tag{5.18}$$

Where T_r is the mean time a driver responds to a particular events.
Then, the ratio of the effective green time to the period denoted by k is subjected to the following equation

$$k = \frac{t_g + \frac{t_y}{2}}{T_p}, \tag{5.19}$$

Where t_y is divided into two pieces, one is added to the effective green time and another is added to the effective red time. P is the period time. During one period time, vehicles that can pass through the intersection is determined by the Circle Flow Rate and Incoming Flow Rate(We had discussed that in the first section of this model). Let n be the number of vehicles passing through the intersection during one period time, then we get

$$n = kQ_{in}T_p + (1-k)Q_{cir})T_p, \tag{5.20}$$

During one period time, vehicles permitted into the intersection are subject to the length of the critical section. Then the following inequality holds.

$$n \leq \frac{W}{L+L'}s' \tag{5.21}$$

Where L is the mean length of the vehicle; L' is the minimum length two vehicles should keep; W is the length of the critical path through which the vehicles getting into the roundabout are passing; s' is the number of the Inside Circle Lanes.

It is manifest that

$$T_p = t_g + t_r + t_y, \tag{5.22}$$

Combining (5.18)-(5.22), and maximum the green time, the following equations yield

$$\begin{aligned} t_y &= \frac{v_0}{2\mu g} + \frac{e+L}{v_0} + T_r, && (5.23) \\ t_g &= \frac{kWs'}{(kQ_{in} + (1-k)Q_{cir})(L+L')} - \frac{t_y}{2}, && (5.24) \\ T_p &= \frac{2t_g + t_y}{2k}, && (5.25) \\ t_r &= T_p - t_y - t_g. && (5.26) \\ & && (5.27) \end{aligned}$$

Then, we can use equations above to determine the duration of the traffic lights.

5.4 Example

In order to show how our model works, we give an example. The roundabout is symmetric and (3) shows some parameters of it(The geographic parameters

Tab 3: The Geographic Parameters

Parameters	Value	Description
l	34 m	the length of the critical section
W	4 m	the width of the critical section
e_1	4 m	entry width
e_2	6 m	approach half width
l'	40 m	effective flare length
D	40 m	inscribe circle diameter
ϕ	30 degrees	entry angle
r	40 m	entry radius
s	1	the number of the Circle Lanes
r	1	the number of the Incoming Lanes
μ	0.25	slide friction factor

are obtained from one design in the *Roundabout: an information guide, which is regarded as a typical Single-Lane roundabout*). Then the following equation holds[4]

$$Q_e = 1212 - 0.5447Q_c. \tag{5.28}$$

Let $Q_c = 0$, then obtain

$$E = 1212 \quad (\text{pce/h}), \tag{5.29}$$

(5.29) is manifest, because the maximum Entry Capacity denoted by E will be achieved when no vehicles coming from the circle against the vehicles coming from the incoming road.

Since the roundabout is symmetric, $C_i = C_j(i,j = 1,2,3,4)$ holds. We assume $C_i = 2500 \quad (pce/h)$. Let $Q_e = 0$, then

$$Q_c = 2251 \quad (\text{pce/h}), \tag{5.30}$$

The Circle Capacity is bigger than Q_c when $Q_e = 0$, then

$$C_i \geq 2251 \quad (\text{pce/h}), \tag{5.31}$$

That is why $C_i = 2500\,(\text{pce/h})$ was chosen.
The Circle Rate denoted by V_{cir} is regarded as a variable which varies from 0 to C_i, could be observed in real roundabout and will be used to judge whether signs or traffic lights are needed. For convenience, Saturation denoted by Sa is introduced as the following equation shows.

$$\text{Saturation} = \frac{\text{Cricle Rate}}{\text{Cricle Capacity}} \qquad or \qquad Sa = \frac{V_{cir}}{C_i} \tag{5.32}$$

Using the parameters upper, the Critical Section Model was solved in three steps.

STEP1: Determining Using Signs or Traffic Lights
The First section of Critical Section Model is to determine whether signs or traffic lights should be used in every incoming road which was solved by software LINGO. The following is the pith of the solution.
4 shows that signs are better alternative when $S_a \leq 0.86$, because at these cases, the inequality $V_1 \geq V_2$ holds, then the Capacity of the roundabout could reach its maximum when signs are used. Similarly, when $V_1 > V_2$, traffic lights are the better choice. Also, that facts are told by x: when $x = 0$, signs are needed; when $x = 1$, traffic lights are needed. When $S_a \to 1$, the traffic becomes heavier, and the traffic lights seems more likely to be used. When $S_a \geq 0.98$, the traffic is so heavy that the roundabout is unstable and crowed with vehicle1s, then further improvement is needed. If traffic lights are chosen, Effective Green Ratio denoted by k decreases along with the increment of S_a.
For convenience to analyze, some figures are showed.

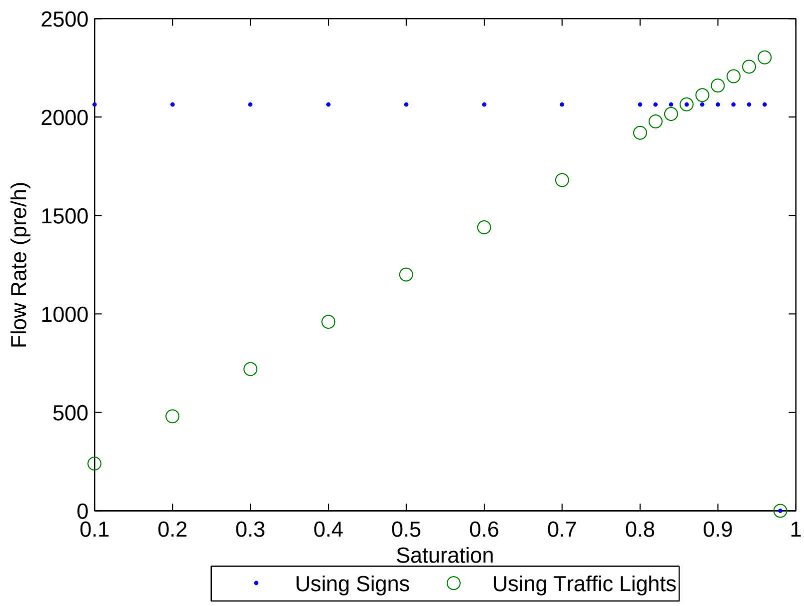

Fig 5: Comparison Between Using Signs and Using Traffic Lights (Measured by Capacity)

Figure 5 indicates that the saturation have no significant effects on the capacity of the roundabout, so we could not account for signs when the traffic of the roundabout is heavy; the capacity of the roundabout is linear with the saturation when traffic lights are used, but it is better to remove them when the traffic is not so heavy, because they have negative effects on the traffic. Following is a figure about delay time.
Figure 6 shows the relation between delay time and saturation. It indicates that

Tab 4: Solution For Variant S_a

S_a	$C\,(pce/h)$	$V_1\,(pce/h)$	$V_2\,(pce/h)$	$d\,(s)$	x	k
0.1	2064	2064	240	2.05	0	0.827
0.2	2064	2064	480	2.81	0	0.706
0.3	2064	2064	720	3.77	0	0.615
0.4	2064	2064	960	5.02	0	0.545
0.5	2064	2064	1200	6.72	0	0.489
0.6	2064	2064	1440	9.19	0	0.444
0.7	2064	2064	1680	13.01	0	0.406
0.8	2064	2064	1920	19.71	0	0.375
0.82	2064	2064	1978	21.68	0	0.369
0.84	2064	2064	2016	23.97	0	0.363
0.86	2064	2064	2064	26.68	0	0.358
0.88	2112	2064	2112	29.9	1	0.353
0.90	2160	2064	2160	33.76	1	0.347
0.92	2208	2064	2208	38.47	1	0.342
0.94	2256	2064	2256	44.26	1	0.338
0.96	2304	2064	2304	51.47	1	0.333
0.98	0	0	0	Inf	1	0.333

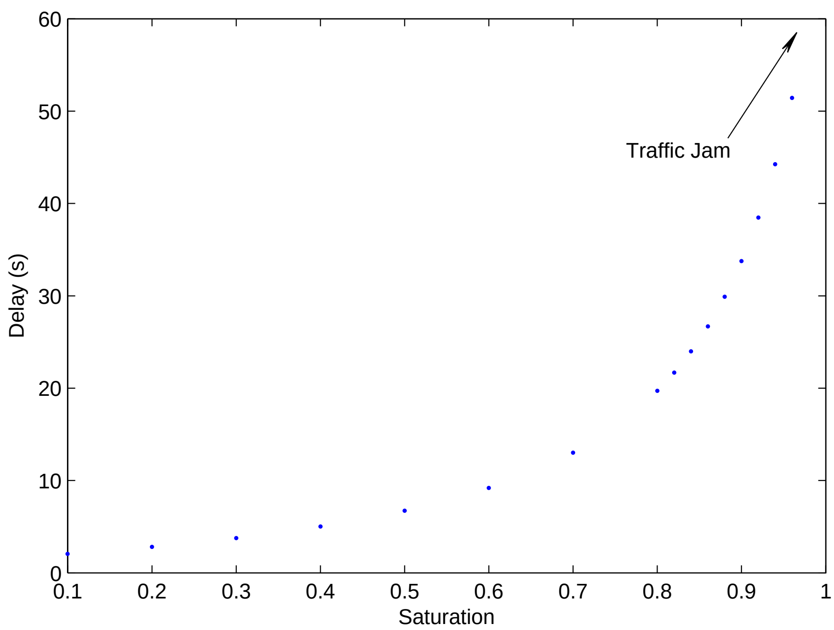

Fig 6: Delay Time

delay time increases exponentially with the increment of saturation.

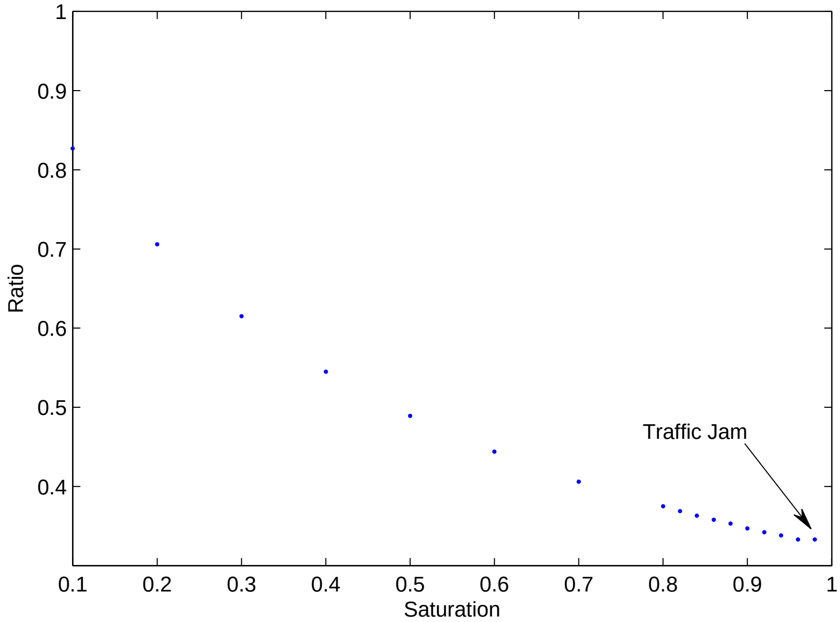

Fig 7: the Ratio of the Effective Green Time

Figure 7 shows that, green time should be cut when the traffic becomes heavier.

STEP2: Using Yield Sign or Stop Sign
When $S_a \leq 0.86$, signs are advised. Then using the Second Section of the Critical Section Model, how to choose the signs-a yield sign or stop sign-is settled.
The following equation could be used to calculated the design speed which we regarded as the mean speed, for a given path radius[4].

$$V_0 = \sqrt{127R(e+f)}, \tag{5.33}$$

where:

V_0	= Design speed, km/h	
R	= Radius, m	Then, V_0 could be calculated:
e	= Superelevation, m/m	
f	= Side friction factor	

$$V_0 = \sqrt{127 \times 40 \times (0.25+0.2)} \simeq 37.0\,(km/h) = 10.28\,(m/s). \tag{5.34}$$

Researches show how radius of inscribed circle affected design velocity: design velocity V increases with the increasing of radius R, but the rate of in-

crement decreases. When R is big enough, there should be no significant difference between driving in roundabout and straight highway. So we assume: $V_m = 80\,(km/h) = 22.22\,(m/s)$. (Exhibit 6-10 in *Roundabouts: An Information Guide*[4])
Then

$$\begin{aligned} DT &= \frac{V_m - V_0}{\mu g} \\ &= \frac{22.22 - 10.28}{0.25 \times 9.8}\,(s) \\ &= 4.87\,(s), \end{aligned}$$

when Delay denoted by d satisfies the inequality

$$d \leq 4.84, \tag{5.35}$$

A yield sign is needed, otherwise a stop sign is needed.

STEP3: Determining the Duration of Green Light
When traffic lights are needed, green time is determined by the third section of the Critical Section Model.
The average length of vehicles is assumed to be $3\,m$, that is $L = 3$. And the minimum length between two vehicles when passing through intersection is $1\,m$. Then, the equations (5.23)-(5.26) give the green time:

6 Queuing Theory Model

One intuition for modeling the traffic circle problem is to think of the arrival of vehicles as a stochastic process. A stochastic process is a collection of random variables that must take on a value at every state, where states are indexed by some parameter[7]. To analyze this stochastic process formulation, we use queuing theory. Queuing theory deals with analyzing the way that random variables in stochastic processes interact.

Tab 5: Traffic Light Time For Variant S_a

S_a	t_g	t_y	t_r
0.88	26.02	3.88	49.32
0.90	25.94	3.88	50.52
0.92	25.86	3.88	51.54
0.94	25.87	3.88	52.36
0.96	25.66	3.88	53.37

Assume the whole incoming vehicles as customers , and coming from different road randomly but well balanced . Suppose that incoming vehicles yield to random Poisson distribution . An incoming entrance is treated as an information desk, and the serving time is that a vehicle takes passing the intersection. While the waiting time represents the time that takes a vehicle at the incoming entrance for queuing.

6.1 Variables

i	system status, the number of cars in the queuing system,
λ	the average arrival rate of new vehicles of the system; is also the average number of vehicles that enter the whole traffic system, including the traffic circle as well as all the incoming road during the period of time T (pce/h),
μ	the average serving rate at one service desk,
μ'	the serving rate of all service desk,
$\frac{1}{\mu}$	the average serving time per vehicle; is also the time one vehicle spends entering the traffic circle,
n	the number of information desks, i.e. the number of incoming roads,
ρ	the saturation degree of the system, (only when , the system is stable, and naturally, the lower the better)
L_s	the average number of vehicles in the system,
L_q	the average length of the queue in the system,
W_s	the average time of sojourn (from entering the incoming road to entering the circle, s),
W_q	the average waiting time (from entering the incoming road to receive service, s).

6.2 Equations

In this problem, we choose to build an M/M/s queuing model[8].
The average service rate of the whole queuing system is

$$\mu' = \begin{cases} i\mu, & i < n, \\ n\mu, & i \geqslant n. \end{cases}$$

Let $\rho = \frac{\lambda}{n\mu}$, the probabilities of queuing system of diffferent status can be descripted by the following equations:

$$\begin{cases} P_0 = \left[\sum_{k=0}^{n-1} \frac{1}{k!} (\frac{\lambda}{\mu})^k + \frac{1}{n!} \frac{1}{1-\rho} (\frac{\lambda}{\mu})^n \right]^{-1}, \\ P_i = \begin{cases} \frac{1}{i!} (\frac{\lambda}{\mu})^i P_0, & (i \geqslant n) \\ \frac{1}{n! n^{i-n}} (\frac{\lambda}{\mu})^i P_0, & (i < n) \end{cases} \end{cases} \tag{6.1}$$

and the average queue length

$$\begin{cases} L_s = L_q + \frac{\lambda}{\mu}, \\ L_q = \sum_{i=n+1}^{\infty} (i-n) Pi = \frac{(n\rho)^n \rho}{n!(1-\rho)^2} P_0, \end{cases} \tag{6.2}$$

According to Little Theory, the average time

$$W_q = \frac{L_q}{\lambda}, \quad W_s = \frac{L_s}{\lambda}.$$

6.3 Objective(s) in the Model for Optimal Choice

To determine whether signs or traffic lights should be used, we have to compare some characteristics of each system. In this model, the average time of sojourn W_s and the saturation degree of the system ρ are the two factors that we use to evaluate each system.

W_s is how long each vehicle stays in the system on average, if W_s is long, it indicates that individual tends to spend more time in the system and this roundaout is less satisfactory. But if W_s is shorter, it takes each vehicle shorter time in the system on average. In other words, the shorter W_s is, the better the system is. ρ implies the number of vehicles a system could bear . First of all, assuming $\mu' = C$, so that the largest number of vehicles could be served per hour in the system is equal to the largest traffic capacity of the circle. And then, using $\rho = \frac{\lambda}{n\mu}$ to calculate ρ of each system with the same λ. In general, $\rho < 1$, and the lower ρ is, the better traffic capacity of the system is.

In order to compare these four system-No signal System, Yield Sign System, Stop Sign System and Traffic Lights System- in the queuing model, it is necessary to introduce a variable to identify them. The variable we choose is μ'. This queuing method is aimed at comparing those four systems, then what we need is the data in allowable range. We calculate μ' with formulas from *Capacity Analysis of Signalized Roundabout*[6].

6.4 Example

6.4.1 Methodology to Obtain μ'

- Type 1–No Signals
  Considering the vehicles in the circle lanes could either turn left or go straight. According to the interweave theory and the traffic flow of each entry $Q_{left} = Q_{right} = \frac{1}{2}Q_{ahead}$,we could cursorily evaluate the whole capacity of the traffic circle without signals to be

$$C^1 = \frac{a-1}{2}nN_{itw} = \frac{a-1}{2}n\frac{3600}{t_{itw}}, \quad \text{(pce/h)} \tag{6.3}$$

  where

  N_{itw} is the capacity of circle lanes of the traffic circle,
  t'_{itw} is the average headway between the two vehicles on the circle lanes (If car is used as the standard in calculation, let t'_{itw} to be 2.5s),
  n is the number of incoming road,
  a is the number of circle lanes.

- Type 2–Yield Sign
  Under the same road condition with Type 1, but yield signs are added into this system which often lead to lower velocity in the circle. Hence, the t'_{itw} in this system has to be somewhat lower than that in the No Signal System. Experientially, let $t'_{itw} = 2.44'$. Similarly, according to the interweave theory and the traffic flow of each entry which holds to $Q_{left} = Q_{right} = \frac{1}{2}Q_{ahead}$we could cursorily evaluate the whole capacity of the traffic circle with yield signs to be

$$C^2 = \frac{a-1}{2}nN'_{itw} = \frac{a-1}{2}n\frac{3600}{t'_{itw}}. \quad \text{(pce/h)} \tag{6.4}$$

- Type 3–Stop Sign
  Under the same road condition with Type 1, but stop signs are added in this system. Imitating the traffic light system, which will be discussed following, we could get $T' = t_{go} + t_{stop} = 2s + 3s = 5s$. Let $t_{start} = 2.3s$ and $t_{ahead} = 2.44s$, since $Q_{left} = Q_{right} = \frac{1}{2}Q_{ahead}$, it yields

$$\begin{aligned}
\beta_{left} &= \frac{Q_{left}}{Q_{left} + Q_{ahead}} = \frac{1}{3}, \\
N'_{ahead} &= \frac{3600}{T'}\left(\frac{t_{go} - t_{start}}{t_{ahead}} + 1\right), \\
N'_{ahead_left} &= N'_{ahead}(1 - \alpha\beta_{left}), \quad (\text{let } \alpha = 0.28)
\end{aligned}$$

  where

t_{start} is the average interval of a car from starting to totally passing the stop line,
t'_{ahead} is the average interval of a car from queuing to totaly passing the stop line,
β_{left} is the ratio of left-turning cars to the total number of vehicles.

The capacity of one entry

$$C'_{ave} = 2N'_{ahead_left} + N'_{right} = 2N'_{ahead_left} + C'_{ave}\beta_{right} = \frac{2N'_{ahead_left}}{1-\beta_{right}},$$

So that the capacity of the traffic circle with stop sign is

$$\begin{aligned} C^3 &= \frac{a-1}{2}nC'_{ave} = \frac{a-1}{2}n\frac{2N'_{ahead_left}}{1-\beta_{right}} \\ &= \frac{a-1}{2}n\frac{2\frac{3600}{T'}(\frac{t_{go}-t_{start}}{t_{ahead}}+1)(1-\alpha\beta_{left})}{1-\beta_{right}}. \quad \text{(pce/h)} \end{aligned} \tag{6.5}$$

- Type 4–Traffic Lights
  Under the same road condition with Type 1, but traffic lights are added to the system. Suppose the traffic lights is designed according to frequently-used patterns and the cycle of each traffic light is $T = t_{red} + t_{green} + t_{yellow} = 50s + 47s + 3s$. Here is similar to Type 3, except for T. Let $t_{start} = 2.3s$ and $t_{ahead} = 2.44s$, since $Q_{left} = Q_{right} = \frac{1}{2}Q_{ahead}$, we can calculate β_{left}, N_{ahead}, N_{ahead_left}, and then get a similar equation

$$\begin{aligned} C^4 &= \frac{a-1}{2}nC_{ave} = \frac{a-1}{2}n\frac{2N_{ahead_left}}{1-\beta_{right}} \\ &= \frac{a-1}{2}n\frac{2\frac{3600}{T}(\frac{t_{go}-t_{start}}{t_{ahead}}+1)(1-\alpha\beta_{left})}{1-\beta_{right}}. \quad \text{(pce/h)} \end{aligned} \tag{6.6}$$

6.4.2 Results

Capacity of every type could be obtained based on (6)(6.5)(6.5)(6.6).

Let $\mu'^1 = C^1$, $\mu'^2 = C^2$, $\mu'^3 = C^3$, $\mu'^4 = C^4$, according to queuing theory, the average sojourn time could be estimated.

For Table 7:

when $\lambda = 5849$, the ρ of each design is NULL, 0.8696,0.9912, 0.9577,obviously the best choice is to add traffic lights at each entry,

when $\lambda = 4522$, the ρ of each design is 0.7851, 0.6723, 0.7662, 0.7404, all these four plans are reasonable. Given W_s, we suggest that it is better to add a yield sign or a stop sign,

Tab 6: Capacity of 4 Tyeps of Design.

Number of Circle Lanes (a)	Nomber of incoming roads (n)	Capacity			
		Type 1	Type 2	Type 3	Type 4
3	4	5760	6726.3	5901.6	6107.3
3	5	7200	8408.1	7377	7634.1
2	4	2880	3363.2	2950.8	3053.7
2	5	3600	4204.1	3688.5	3817.1

Tab 7: Average sojourn time with 4 incoming roads and 3 circle lanes.

Type	λ(pce/h)	Capacity(pce/h)	$\rho(<1)$	W_s(s)
1	5849	5760	NULL	NULL
	4522	5760	0.7851	2.5509
	2875	5760	0.4991	2.5191
2	5849	5901.6	0.9912	2.5152
	4522	5901.6	0.7662	2.4874
	2875	5901.6	0.4872	2.4576
3	5849	6107.3	0.9577	2.4264
	4522	6107.3	0.7404	2.4007
	2875	6107.3	0.4707	2.3734
4	5849	6726.3	0.8696	2.1934
	4522	6726.3	0.6723	2.1728
	2875	6726.3	0.4274	2.1519

when $\lambda = 2875$, the ρ of each design is 0.4991, 0.4274, 0.4872, 0.4707, all these four plans are reasonable. Given W_s, the system performs quite well without any signal.

Tab 8: Average sojourn time with 5 incoming roads and 3 circle lanes.

Type	λ(pce/h)	Capacity(pce/h)	$\rho(<1)$	W_s(s)
1	5849	7200	0.8124	2.5281
	4522	7200	0.6281	2.5157
	2875	7200	0.3993	2.5043
2	5849	7377	0.7929	2.4661
	4522	7377	0.6130	2.4544
	2875	7377	0.3897	2.4439
3	5849	7634.1	0.7662	2.3813
	4522	7634.1	0.5923	2.3706
	2875	7634.1	0.3766	2.3612
4	5849	8408.1	0.6956	2.1580
	4522	8408.1	0.5374	2.1498
	2875	8408.1	0.3419	2.1430

For Table 8:

when $\lambda = 5849$, ρ of each design is 0.8124, 0.6956,0.7929, 0.7662, we conclude that it is better to add a yield sign or a stop sign,

when $\lambda = 4522$, ρ of each design is 0.6281, 0.5374, 0.6130, 0.5923, all these four plans are reasonable. Given W_s , the system performs quite well without any signals,

when $\lambda = 2875$, ρ of each design is 0.3993, 0.3419, 0.3879, 0.3766, all these four plans are reasonable. Given W_s , the system performs quite well without any signals.

For Table 9:

When $\lambda = 5849$ or $\lambda = 4522$, whichever method can not help this traffic circle survive from so many vehicles, thus, reconstruction is necessary, such as adding circle lanes, increasing the number of incoming roads with the combination of traffic lights.

When $\lambda = 2875$, the system survives, however, its service ability is limited, so we strongly recommend other ameliorations.

Tab 9: Average sojourn time with 4 incoming roads and 2 circle lanes.

Type	λ(pce/h)	Capacity(pce/h)	$\rho(<1)$	W_s(s)
1	5849	2880	NULL	NULL
	4522	2880	NULL	NULL
	2875	2880	0.9983	5.1559
2	5849	2950.8	NULL	NULL
	4522	2950.8	NULL	NULL
	2875	2950.8	0.9743	5.0262
3	5849	3053.7	NULL	NULL
	4522	3053.7	NULL	NULL
	2875	3053.7	0.9415	4.8488
4	5849	3363.2	NULL	NULL
	4522	3363.2	NULL	NULL
	2875	3363.2	0.8548	4.3836

6.5 Conclusions

Given the examples stated above, we could draw some conclusions listed as follows:

- The capacity of No Signal System is the lowest, Yield Sign System is the next, then is the Stop Sign System, while Traffic Lights System is the best. That is to say, traffic light system is more likely to survive from the pressure of large traffic volumn.

- Given a specific λ, more incoming roads or circle lanes save the time that the circle needs to serve each vehicle on average. Especially adding circle lanes could increase the ability significantly and is even more effective than using traffic lights.Besides, if the traffic circle contains four or more circle lanes, the traffic capacity is so high that no facilities are needed. When the traffic is not heavy, traffic lights neednt to be used, because the difference between those systems is not significant.That means in rural condition (which receive few vehicles per hour) or a circle with few or many circle lanes, traffic lights are rarely used. These suggestions accord with real situations perfectly.

- The results reveals that the most vital factor is the total number of arrival vehicles, the number of circle lanes comes second, followed by the number of incoming road , and facilities like signs or traffic lights is the last factor.

- In reality, the upper limit of traffic volume per hour is usually 85% of theoretical traffic capacity (μ) of the traffic circle[4]. Applying that, Table 10 shows the range of traffic volume per hour for common roubdabouts.

Tab 10: Range of traffic flow rate for different designs. (pce/h)

Geometric design / Type	4 entries, 2 circle lanes	4 entries, 3 circle lanes	5 entries, 2 circle lanes	5 entries, 3 circle lanes
1	0∼2448	0∼4896	0∼3060	0∼5016.36
2	2448∼2508.2	4896∼5016.4	3060∼3135.2	6120∼6270.5
3	2508.2∼2595.6	5016.4∼5191.2	3135.2∼3244.5	6270.5∼6489
4	2595.6∼2858.7	5191.2∼5717.4	3244.5∼3573.5	6489∼7146.9

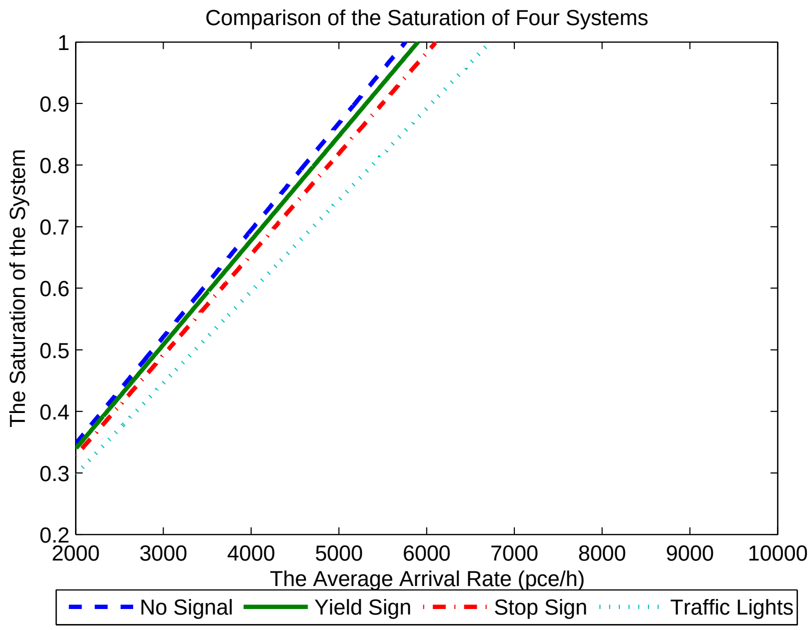

Fig 8: Saturations of four systems with four incoming roads.

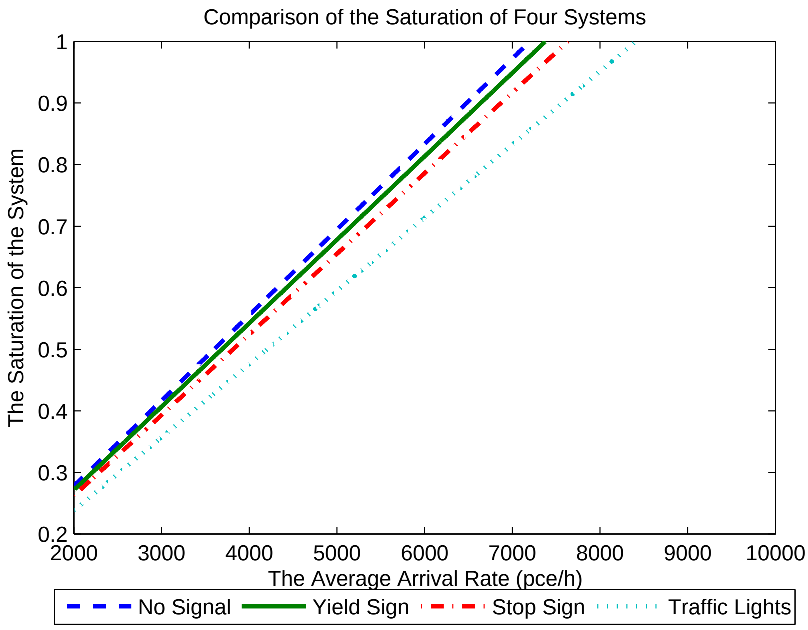

Fig 9: Saturations of four systems with five incoming roads.

- When the value of ρ of any system reaches 1, it means that this system is saturated so that even one more vehicle' entering the system would cause serious consequence, like traffic jam or accidents. Figures 8 and 9 show that No Signals System reaches saturation firstly, and the Traffic Lights System last. So that the Traffic Lights System is the most effective one with upper limit of λ being almost 8000(pce/h), while the others varies from 5500(pce/h) to 7000(pce/h). In traffic control designing, a Traffic Lights System would be more suitable for urban condition where the traffic flow rate is absolutely high.

7 Strengths and Weaknesses

7.1 Strengths of Critical Section Model

- The Critical Section Model can tell whether signs or traffic lights is needed in a given roundabout which is of single-lane or many lanes in circle. When signs are needed, the model could judge whether a yield sign or a stop sign is the better choice based on the geometric conditions and the saturation of the roundabout. While traffic lights are needed, the model could tell how to calculate the green time.

- Since this model is based on the capacity of the bottleneck section which can well indicate the capacity of roundabout, it is able to test whether the capacity of a roundabout suffices for given traffic conditions.

- Also, the model can be improved and applied to other situations based on the objectives focused, for example maximizing inflow rate or the outflow rate.

7.2 Strengths of Queuing Theory Model

- Queuing Theory Model results in clear consequence, which makes it stands out.
- Not only travel-control devices is taken into consideration, but grometric design of roads becomes an important factor in this model.

7.3 Weaknesses

- Since the Critical Section Model does not take the weather conditions into consideration, it may result in exaggerating the capacity of a given roundabout. The traffic of urban roundabouts and rural roundabouts have different characteristics, and it is better to consider how different types of roundabouts works.
- The Queuing Theory Model only take the capacity of traffic circle as the objective of judging traffic systems, so it is not comprehensive.

8 Conclusion

8.1 Conclusion of Bottleneck Section Model

In the first section of Bottleneck Section Model, we discussed about how to choose between signs and traffic lights in roundabouts. We concluded that when the traffic is not heavy, signs work better than traffic lights and it was strongly recommended by us. While the traffic is so heavy that traffic lights act better, the green time should be cut down along with the increment of the saturation of the roundabout. Though signs are the better choice in most cases (Our example shows only when the saturation is lager than 0.88, will the traffic lights are recommended), they have less significant effects on the capacity of the roundabout than traffic lights, since capacity correlates with the saturation linearly while traffic lights are used. The delay time, which is a standard for measuring the traffic system, increase exponentially along with the increment of the saturation in the roundabout.
In the second section, choosing a yield sign or a stop sign is discussed while signs work better than traffic lights. We conclude that: the choice is related to

the saturation as well as the geometric condition especially the slide friction factor.
Through the third section, we know that when the traffic becomes heavier the green time should be cut down.

8.2 Conclusion of Queuing Theory Model

The Queuing Theory Model helps us to quantify the differences between four roundabout systems. We concluded that:

- The capacity of No Signal System is the lowest, yield sign system is the next, and then comes the stop sign system, while the traffic light system is the highest. That is to say, traffic light system is more likely to survive from a larger traffic flow.
- Any facilities can not improve the capacity of a roundabout significantly who only have one or two lanes, then for design, a roundabout should have more lanes.
- The results reveals that the most vital factor is the total number of arrival vehicles, the number of circle lanes comes second, followed by the number of incoming road , and facilities like signs or traffic lights is the last factor.

9 Technical Summary

How to choose the type of flow-control method is presented in our papers for any specific circle and saturation of roundabout. Before using our models, detailed geographical parameters as well as saturation in different times of the specific roundabout should be prepared through measuring or statistic.
Those facts come with choosing flow-control methods in a specific roundabout with one circle lane and four incoming roads: signs are recommended in most cases especially when traffic is not very heavy (signs may be better choice when saturation reaches up to 0.88 [1]) though signs seem have no significant effect in improving Bottleneck Capacity of a roundabout under a particular saturation (The Bottleneck Capacity remains 2064(pce/h) though saturation changes). And that is why choosing signs could regardless of their effects on the capacity. The effective ratio is relatively small (less than 0.35), when the saturation becomes big enough that traffic lights are recommended. And the Bottleneck Capacity increases linearly with the increasing of the saturation, what may account for traffic lights are likely to be used in cities, because they usually have variable saturation and heavy traffic. With Delay Time approaching infinity and effective green time changing small when the saturation varies, traffic jams are likely to happen.
The decision on flow-control method using Bottleneck Section Model calls for three steps. Firstly, whether signs or traffic lights are better alternative is discussed. Capacity of the roundabout under a specific saturation is determined by Bottleneck Section which is one Critical Sections having the lowest capacity. Then optimal objective is to maximum capacity of the Bottleneck Section. When signs are used, Delay Time, which is a mean time span that calls for a driver to find a gap and make his car into the roundabout, and can be calculated by gap-acceptance theory, is used to measure the capacity of entry flow. Then capacity of Critical Section is given by Wardrops formula. While traffic lights are used, effective green ratio is equal to the ratio of Entry Flow Rate to Circle Flow Rate, so capacity of Critical Section in the downstream is determined, with maximum flow rate of the downstream of one entry be calculated by Kimmers equation by simply letting Circle Rate be 0, which shows the relation between Entry Flow Rate and Circle Flow Rate. To sum up, a 0-1 integer programming model is given and whether signs or traffic lights are needed in every entry is settled by solving the model. Secondly, using a yield sign or a stop sign when signs are recommended derived from Delay Time which is related to Circle Flow Rate, and Judge Time which is determined by deceleration of a vehicle. A yield sign is needed while Judge Time is less than Delay Time what indicates a driver has enough time looking around before entering the roundabout; otherwise a yellow sign is needed. Thirdly, when traffic lights are recommended, green time could be given through estimating yellow time and period time since effective

[1]This number comes from an example in secition 5.4

green time ratio had been given in the first step. Yellow time is given by avoiding the paradox-whether a driver chooses to stop or not, he must have been in the intersection, and period time is given by simply letting all average vehicles arrived during one period pass through the intersection and go to the Critical Section in the downstream. Period time is restrained since space of Critical Section is limited.

Based on Queuing Theory Model, two objectives-the average time of sojourn and the saturation of system-are used to compare the contribution of different traffic facilities to the traffic capacity of roundabout. After supposing each incoming road is an information desk providing service for vehicles, the traffic capacity of roundabout is regarded as the average serving rate which is often fixed for a specific roundabout. Calculating the traffic capacity by the formulas with independent variables of the number of entries and circle lanes for each type of traffic-control method, and then those two objectives will be finally gained according to a given average arrival rate of vehicles.

Through the results, you can choose the best design after comparing those two objectives in any specific traffic circle. What is more, let 85 percent of the capacity of each situation to be the upper limit of the traffic volume per hour in reality, the conditions, which include the range of real traffic volume per hour as well as the number of entries and circle lanes, could be easily differentiated by each type.

Followed are some data archived from Queuing Theory Model.

Range of traffic flow rate for different designs. (pce/h)

Geometric design / Type	4 entries, 2 circle lanes	4 entries, 3 circle lanes	5 entries, 2 circle lanes	5 entries, 3 circle lanes
1	0~2448	0~4896	0~3060	0~5016.36
2	2448~2508.2	4896~5016.4	3060~3135.2	6120~6270.5
3	2508.2~2595.6	5016.4~5191.2	3135.2~3244.5	6270.5~6489
4	2595.6~2858.7	5191.2~5717.4	3244.5~3573.5	6489~7146.9

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Appendix .A Capacity forcast model–Germen equations

$$Q_e = \begin{cases} k(F - f_c Q_c), & f_c Q_c \leqslant F \\ 0, & f_c Q - C > F \end{cases} \tag{.1}$$

where

$$\begin{aligned} Q_e &= \text{entry capacity, pce/h} \\ Q_c &= \text{circulating flow, pec/h} \end{aligned}$$

$$k = 1 - 0.00347(\phi - 30) - 0.978(\tfrac{1}{r} - 0.05)$$

$$F = 303x_2$$

$$f_c = 0.210t_D(1 + 0.2x_2)$$

$$t_D = 1 + \frac{0.5}{1+e^{\frac{D-60}{10}}}$$

where:

$$x_2 = v + \frac{e-v}{1+2S}$$

$$S = \frac{1.6(e-v)}{I'}$$

$$\begin{aligned} e &= \text{entry width, m} \\ v &= \text{approach half width, m} \\ I' &= \text{efective flare length, m} \\ S &= \text{sharpness of flare, m/m} \\ D &= \text{inscribed circle diameter, m} \\ \phi &= \text{entry angle, degrees} \\ r &= entryradius, m \end{aligned}$$