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T1 ________________	**4265**	F1 ________________
T2 ________________		F2 ________________
T3 ________________	Problem Chosen	F3 ________________
T4 ________________	**B**	F4 ________________

2009 Mathematical Contest in Modeling (MCM) Summary Sheet

Is it true, more cell phone ,less energy?

Summary

This article mainly discusses the energy needs for providing phone service and estimates the energy costs of this wasteful practice in the U.S. per day. Meanwhile, considering population and economic growth, we predict the American energy needs of phone service industry over the next 50 years.

Firstly, we establish a model to describe the consequences of this change for electricity utilization considering the cell phones may get lost or break. We finally find out that the energy needs mainly related to the number of a family members and the energy consumption ratio between landline and cell phone.

Secondly, through **AHP based Model**, we give a optimal way of providing phone service that 25% "pseudo American" population choose cell phone and the other choose the landline.

Thirdly, considering the cell phone's using time, charging time, energy consumption or other factors, we establish a model to estimate loss of energy in US per day, which equals to 1680~5040 barrels of oil. Additionally, we take into account other household appliances, which are commonly used in the current American families. Finally we find out that the U.S.A loses at least 1.06×10^5 barrels of oil per day.

Finally, we establish the **Logistics Model** to predict the population growth in the next 50 years, and then we establish the **Economic Growth Model** to predict the economic development. At last, we build the energy demand model to predict energy needs of the American phone service system in the next 50 years.

Contents

Is it true, more cell phone ,less energy?

1 Introduction

1.1 Background

In today's world, with the rapid development of science and technology, cell phones are widely used. Cell phone usage is mushrooming, and many people use cell phones and give up their fixed line. Every cell phone comes with a battery and a recharger. The increase of cell phone penetration will bring in enormous impact, especially on electricity use.

Nevertheless, cell phones need to be recharged periodically. However, many people keep their recharger plugged in whether they need to be recharge or not. And some are even charge their phones all night. It causes a huge energy waste. Furthermore, taking into account the energy wasted by other electrical equipments (TV, DVR, computer peripherals, etc.), it is enormous everyday.

Coincident with the development of future population and that of economics has been a growing concern on the tendency of change of the energy demanded by the telephone service, which needs our research and discussion.

1.2 Restatement of the Problem

- Consider that if all the telephones were replaced by cell phones. In other words, each family member has a cell phone. Model the change of electricity utilization in the current US when the cell phone gradually replace the landline and after that.

- From an energy perspective, what is the best way to provide phone services in this country? Meanwhile take into account the cell phone has a lot of social consequences and uses which landline doesn’t have.

- Estimates the amount of wasted energy of a Pseudo US due that the cell phones charge unreasonable.

- Estimates various recharger types (TV, DVR, computer peripheral equipment, etc.), the number of energy wasted by current United States every day.

- Consider that the growth of population and economic, predict the energy demand of phone services in the future.

2 Outline of Our Approach

- Firstly, according to the change in the number of cell phone holders in the United States these years, we analyze the characteristics of the numbers of cell phone holders’ change.

- Analyze the energy consumption between cell phones and landline telephones, thereby model the consequences of this change for electricity utilization in the current US, both during the transition and during the steady state.
- Consider energy and other community use of cell phones, we use the AHP method to discuss the best way of phone service

- According to Requirement 2, combine with relevant data, estimate the waste energy because of cell phone charging improperly.

- We use gray prediction model to predict the energy demand of phone service industry in the future based on the number of landlines and cell phones recent years.

- Taking into account that the economic development will exert a difference on the phone service industry. In order to improve the model, we make an analysis of the factors which affect the phone service industry, and forecast the future energy demand of the United States phone service industry.

The main structure of the paper can be summarized as following figure:

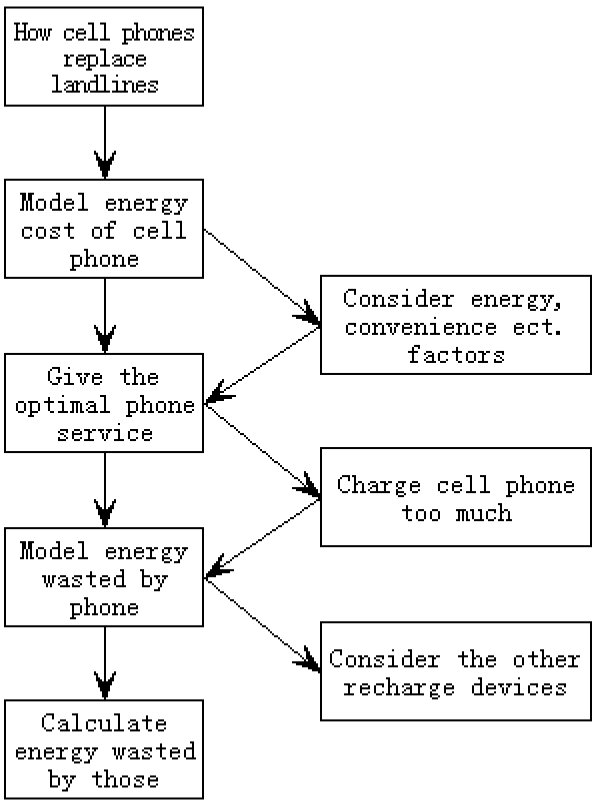

Figure1: The main structure of Requirement 1~4

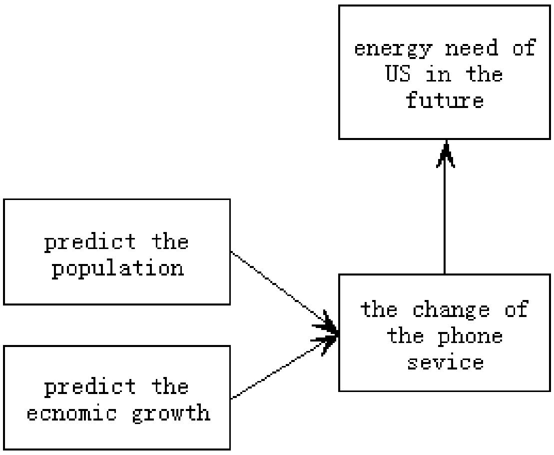

Figure 2: The main structure of Requirement 5

3 What brings us when the landlines are replaced by cell Phones

3.1 Analysis of the number of cell phone holders

In accordance with Requirement 1, we know that it is a time-related dynamic process that cell phone gradual replaces handset.

First of all, we want to study the law of how the number of cell phone holders changes. In order to get a reasonable law, through inspect the Statistical Abstract of the United States, and in accordance with the 1999 to 2004 six years of data, we use the SPSS16.0 software to map out the changing curves of cell phone using times which is followed by time, and then fit a linear equation.

The data of the Statistical Abstract of the US is in the following table:

Table 1: The number of American cell phone user from 1999 to 2004 [1]

Year	1999	2000	2001	2002	2003	2004
Mobile cellular subscribers (mil.)	490	740	955	1166	1414	1758

Drawing the number of American cell phone users change curve graph from1999 to 2004 as follows：

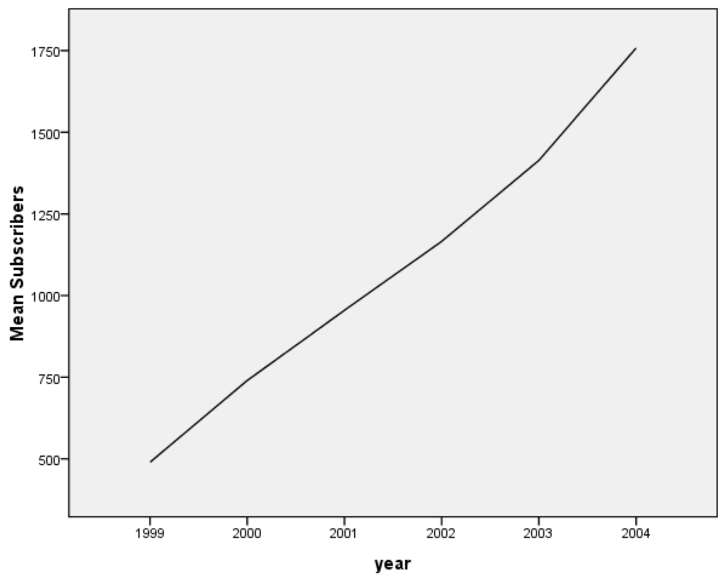

Figure 3: The number of American cell phone users changing trends

From the above figure, we can see the number of American cell phone users has showed a basic linear change, so we set up a linear regression model:

$$N_{cell\ phone} = aT_{year} + b \tag{1}$$

a ....variable factor
b ....constant term
T_{year} ....year
$N_{cell\ phone}$ ....the number of cell phone holder

We need to obtain the factor a and the constant term b, so we use ***spss16.0*** [2] to solve the model and we find out the answer as follow.

Table 2 :the result of solving the linear equations

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	-489165.962	20999.108		-23.295	.000	-547468.831	-430863.093
	year	244.943	10.492	.996	23.346	.000	215.813	274.072

a. Dependent Variable: Subscribers

From the table above, we can get

a = 244.943
b =- 489165.962

So (1) expression can be written by

$$N_{cell\ phone} = 244.943T_{year} + 489165.962 \tag{2}$$

Let $N_{cell\ phone}$ =0, we can find that T_{year} =1997,so in 1997,there is little American cell phone users, which can look to 0.

Next, we will test the model.

Table 3 : coefficient of correlation

Correlations

		Subscribers	year
Pearson Correlation	Subscribers	1.000	.996
	year	.996	1.000
Sig. (1-tailed)	Subscribers	.	.000
	year	.000	.
N	Subscribers	6	6
	year	6	6

From the above correlation of coefficient table can be seen that the correlation coefficient of cell phone users and the year is 0.996, which indicate there is a very close relationship between the two. The P (Sig.) value is less than 0.05, refuse the assumption that the two variables did not relevant.

Table 4: Model Summary

Model Summary[b]

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics					Durbin-Watson
					R Square Change	F Change	df1	df2	Sig. F Change	
1	.996[a]	.993	.991	43.890	.993	545.054	1	4	.000	1.544

a. Predictors: (Constant), year

b. Dependent Variable: Subscribers

In the Model Summary table, the square of correlation coefficient is 0.993, which indicate the year may explain the variable, which is the number of cell phone users has 99.3 percent variability. The value of Durbin-Watson existing detection is 1.554 <2,which can be known as between the adjacent residuals there exists positive correlation relationship .

Table 5: ANOVA

ANOVA[b]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	1049947.557	1	1049947.557	545.054	.000[a]
	Residual	7705.276	4	1926.319		
	Total	1057652.833	5			

a. Predictors: (Constant), year

b. Dependent Variable: Subscribers

We use the above Anova table to do the regression coefficient significance test. As the F value was significant probability is 0.000, less than 5%, so refuse the original assumption, consider that the regression coefficient is not zero, regression equation is meaningful.

3.2 Assumptions

According to 3.1, as the number of American cell phone holders change and consider requirement 1, we have made some assumptions:

- Suppose that all the landlines are replaced by cell phones.
- Each of the members of the household has a cell phone.
- The growth in the number of cell phone holder is linear, that is, it has the same annual growth rate.
- After each family member has a cell phone, they will no longer hold or use fixed line.
- Neglect the standby power consumption of fixed line.
- Members of a family share a landline phone.
- The number of cell phone holder is unchanged per year.

3.3 The electricity demand in the replacement process

Suppose that there are H family in the USA , each family has m members, each family uses fixed line an average time per year is $t_{landline}$ hours, each fixed line consumes $Q_{landline}$ joules per hour. And at the *i*th year，the number of cell phone users is N_i ,every cell phone users use cell phone an average time per year is $t_{cell\ phone}$,each cell phone consumes $Q_{cell\ phone}$ joules per hour.

So we can work out the energy demand of phone service per year.

$$Q_{total} = (H - {N_i}/{m})Q_{landline}t_{landline} + N_i Q_{cell\ phone}t_{cell\ phone} \tag{3}$$

$$(0 \le N_i \le Hm \text{ and } N_i \in Z)$$

Take account of the need for charging the batteries of the cell phones, as well as the fact that cell phones do not last as long as landline phones, for example, the cell phones get lost and break.

Assume the percent of cell phone subscribers whose cell phones get lost and break is α during a year .Then they buy a new cell phone after $c(c < 365)$ days. So there is αN_i cell phones cost no energy. We further improve the model mentioned above.

The energy needs for providing phone service based upon above analysis is given by

$$Q_{total} = (H - \frac{N_i}{m})Q_{landline}t_{landline} + (1-\alpha)N_i Q_{cell\ phone}t_{cell\ phone} + \alpha N_i Q_{cell\ phone}(1 - \frac{c}{365})t_{cell\ phone}$$

$$(0 \le N_i \le Hm \text{ and } N_i \in Z)$$

(4)

To simplify the formula ,we can get

$$Q_{total} = [(1-\frac{\alpha c}{365})Q_{cell\ phone}t_{cell\ phone} - \frac{Q_{landline}t_{landline}}{m}]N_i + HQ_{landline}t_{landline}$$

$$(0 \le N_i \le Hm \text{ and } N_i \in Z) \tag{5}$$

According to the analysis of **3.1**, we know that $N_i \infty T_{year}$. To find out the impact on electricity utilization while fixed line gradually replaced by cell phone. We need to analyze the sign of this formula $[(1-\frac{\alpha c}{365})Q_{cell\ phone}t_{cell\ phone} - \frac{Q_{landline}t_{landline}}{m}]$.

- Therefore, we make some assumptions according to practical experiences, assuming that the work hour of landline and cell phone is equal.

$$t_{cell\ phone} = t_{landline}$$

- Based on practical experiences, α and $\frac{c}{365}$ are very small, we will treat $\frac{\alpha c}{365}$ similar to 0 (Much closer to 0).
- At the same time we introduce a new parameter d, which stands for the energy consumption of fixed line per hour $Q_{landline}$ is d times the energy consumption of cell phone per hour $Q_{cell\ phone}$.

$$Q_{landline} = dQ_{cell\ phone} \tag{6}$$

Through simplify and assumptions above, we can get

$$Q_{total} = (1-\frac{d}{m})N_i Q_{cell\ phone}t_{cell\ phone} + Q_0 \tag{7}$$

where $Q_0 = HQ_{landline}t_{landline}$ as a constant, show how much energy is needed if there only exists fixed line.

Therefore, we should discuss the sign of the coefficient $(1-\frac{d}{m})$ as follows:

Ⅰ When $1-\frac{d}{m} > 0$,we can conclude phone service changes in energy demand as the figure below

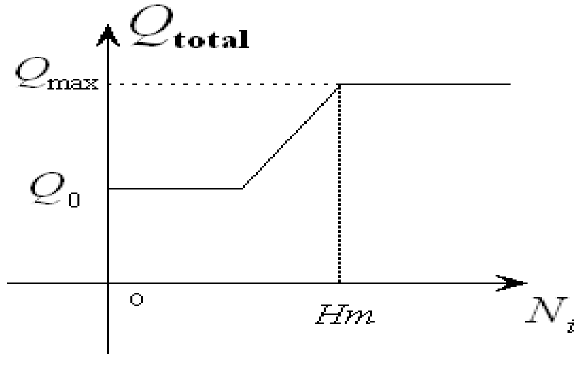

Figure 4 : the change for electricity utilization when $m > d$

When the number of average American family members is greater than the ratio of energy consumption between landline and cell phone.

$$m > d$$

The electricity demand on phone service industry is gradually rising. When the industry were serviced by landlines, energy consumption is Q_0.With the popularity of cell phones, there is a gradual increase in energy demand, and at last when cell phones replace fixed line the energy consumption will reach the maximum, that is

$$Q_{max} = (m-d)Q_{cell\ phone}t_{cell\ phone} + Q_0 \tag{8}$$

II When $1-\frac{d}{m}=0$,we can conclude phone service changes in energy demand as figure below

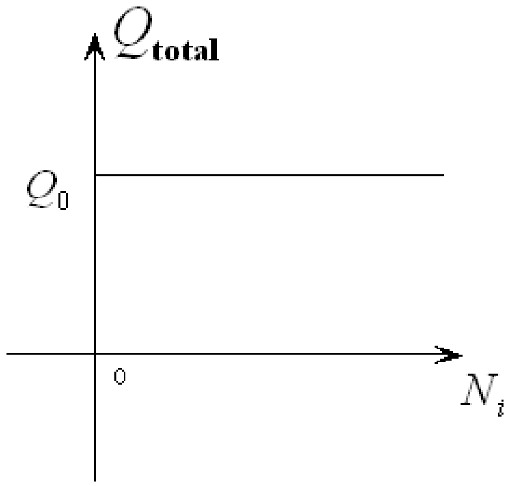

Figure 5: the change for electricity utilization when $m = d$

When the number of average American family members is equal to the ratio of energy consumption between landlines and cell phones, that is :

$$m = d$$

The electricity demand on phone service industry will not change. When the industry were serviced by landlines, energy consumption is Q_0.With the popularity of cell phones, the energy demand is unchanged, and at last when cell phones replace fixed line the energy consumption will be Q_0.

III When $1-\frac{d}{m}<0$,we can conclude phone service changes in energy demand as figure below

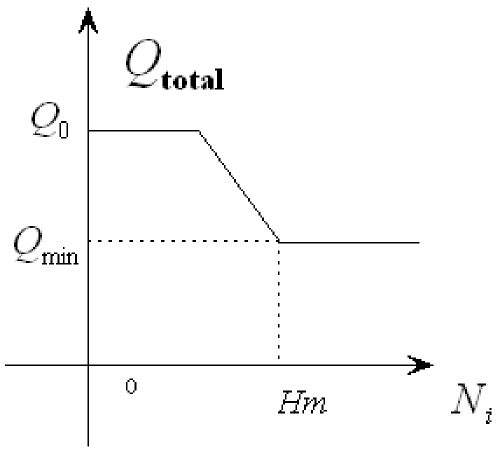

Figure 6 : the change for electricity utilization when $m < d$

When the number of average American family members is less than the ratio of energy consumption between landlines and cell phones ,that is:

$$d > m$$

The electricity demand on phone service industry is gradually reducing. When the industry were serviced by landlines, energy consumption is Q_0.With the popularity of cell phones, there is a gradual decrease in energy demand, and at last when cell phones replace fixed line the energy consumption will reach the minimum.
That is

$$Q_{min} = Q_0 - (d - m) Q_{cell\ phone} t_{cell\ phone} \tag{9}$$

4 the optimal way of providing phone service

4.1 which factor effect the choose of people ?

- **Energy**: One of the main problems the world is facing today is the energy crisis. With the development of industry and agriculture, the world has consumed a great amount of energy. But all this energy has gone forever. Therefore, energy supplies are becoming short. What is the optimal way of providing phone service to "Pseudo US" from an energy perspective. Energy needs can not be ignored.

- **Convenience** : Cell phone usage is with the goal of helping life to be more enjoyable and less stressful. Please, grab a cup of coffee, pull up a comfy chair, take out a cell phone and make a call to your friends or families. Therefore, we also take account of this factor . .

- **Safety** : If cell phones played a limited role in society there would be no debating their safety. Wireless communication is packaged into hip, little slide and flip phones that defy location. Talk when and where you want, take business on the road and enjoy a growing range of extra features that deliver music, video, messaging and Internet capabilities. There is no doubt that the factor is very important ,too.

- **Price** : Most people will take a account of the price when they choose the optimal way of phone service. So we also think about the factor .

4.2 Assumptions

Suppose that the best phone service is just impacted by several respects, such as the energy consumption, convenience, security level, price and talking performance.

4.3 Definitions and Symbols

A.............stands for the Pairwise Comparison Matrix of criteria layer acting on the target layer.
B.............stands for the maximum eigenvalue corresponding eigenvector matrix.

$b_i(i=1,2,3,4,5)$ ......stands for the various criteria Pairwise Comparison Matrix of program layer acting on criteria layer.
$P_j(j=1,2,3,4,5)$ .....stands for the five programs phone services to choose from

4.4 AHP[3] based model for decision making on phone service

(1) the issue of stratification

First of all, we divide this decision-making problem into three levels, the top is target layer, which choose the best way of phone service; the bottom is the program layer, there are five kinds of options, respectively, e.g 100% the use of cell phones, 25% of landline users, 50% of landline users, 75% of landline users and 100% of landline users, the rest of landline users are cell phone users; Middle layer is criterion layer, according to the model assumptions, there are the energy consumption, convenience, security level, price, and talking performance in the criteria layer . The phone service-level analysis figure is as follows:

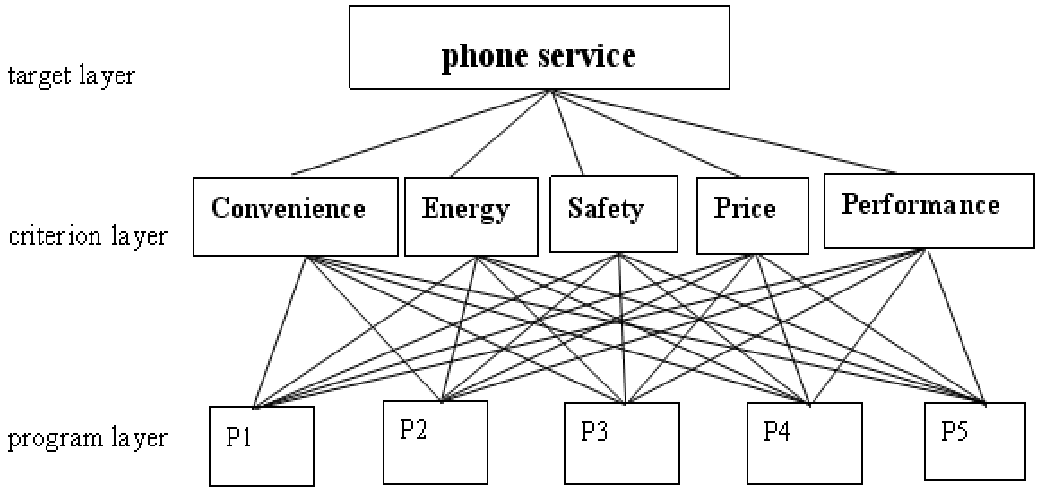

Figure 7 : The phone service-level analysis

(2) Solving methods of the Pairwise Comparison Matrix and weight vector

By comparing various criteria in order to determine the target weight, meanwhile the program guidelines for each criteria's weight, we will use the Pairwise Comparison Matrix and weight vector to analyze the quantitative problem.

Assume we need to compare one certain layer of n factors ($C_1, C_2, \cdots\cdots, C_n$) impacts on the factor O in the layer just above the certain layer. Taking two factors C_i and C_j at a time, a_{ij} is the impact of C_i and C_j to O ratio (based on comparative scale 1-9).All the comparison results can be expressed by the Pairwise Comparison Matrix.

$$A = (a_{ij})_{n\times n}, a_{ij} > 0, a_{ji} = \frac{1}{a_{ij}} \tag{10}$$

In general, if a Pairwise Comparison Matrix A satisfy

$$a_{ij}a_{jk} = a_{ik}, i, j, k = 1, 2, \cdots, \quad n \tag{11}$$

A is said to be a consistent matrix , referred to as line array.

If the **Pairwise Comparison Matrix** we get is a consistent matrix, naturally check corresponds to the eigenvalue n, normalized eigenvector (that is the sum of weight is one) that show the factors $C_1, \cdots, C_n$ act on the upper weight of O, the vector is called weight vector. Eigenvalue method can be used to find out the weight vector, if the A matrix has a largest eigenvalue λ just equal n, then A is a consistent matrix. If the largest eigenvalue λ of A matrix is not less than n, the larger λ ,the more inconsistency of A. So consistency test can be used to determine whether matrix A is a consistent matrix.

Table 6 : The specific meaning of 1-9 comparison scale

Aij scale Aij	meaning
1	the impact between C_i and C_j is the same
3	the impact of C_i is slightly than C_j
5	the impact of C_i is strong than C_j
7	the impact of C_i is significance strong than C_j
9	Cthe impact of C_i is absolutely strong than C_j
2,4,6,8	the impact between C_i and C_j is in the two adjacent grades
1,1/2,⋯,1/9	the impacts of C_i and C_j is multiplicative inverse

Consistency index (CI):

$$CI = \frac{\lambda - n}{n-1} \tag{12}$$

Random consistency index(RI):

Table 7: Random consistency index (RI)

n	1	2	3	4	5	6	7	8	9	10	11
RI	**0**	**0**	**0.58**	**0.9**	**1.12**	**1.24**	**1.32**	**1.41**	**1.45**	**1.49**	**1.51**

Consistency ratio:

$$CR = \frac{CI}{RI} < 0.1 \tag{13}$$

The inconsistencies in the degree of A are in the permission range this time, its eigenvectors can be used as a weight vector.

At this point we get weight vector of the second layer acting on the first layer, and then find out the weight vector of the third layer acting on the second layer 2, finally we get portfolio weight vector.

Record the weight vector 1,2 of the second and the third layer acting on the first and the second layer respectively

$$\begin{aligned} \omega^{(2)} &= (\omega_1^{(2)}, \cdots, \ \omega_n^{(2)})^T \\ \omega_k^{(3)} &= (\omega_{k1}^{(3)}, \cdots, \ \omega_{km}^{(3)})^T, k = 1, 2, \cdots, \ n \end{aligned} \tag{14}$$

We see $\omega_k^{(3)}$ as column vectors to constitute a matrix

$$W = [\omega_1^{(3)}, \cdots, \ \omega_n^{(3)}] \tag{15}$$

The portfolio weight vector of the third Layer acting on the first layer is

$$\omega^{(3)} = W^{(3)} \omega^{(2)} \tag{16}$$

Finally, according to portfolio weight vector to determine the feasibility of various options.

(3) Solution of the AHP based model

We can get the matrix A as followers:

$$A=\begin{bmatrix} 1 & 3 & 3 & 7 & 3 \\ \frac{1}{3} & 1 & 2 & 1 & 1 \\ \frac{1}{3} & \frac{1}{2} & 1 & 1 & \frac{1}{2} \\ \frac{1}{7} & 1 & 1 & 1 & 1 \\ \frac{1}{3} & 1 & 2 & 1 & 1 \end{bmatrix} \tag{17}$$

To solve this matrix eigenvalue λ =5.1661>4,now $CI=\dfrac{\lambda-5}{5-1}=0.0415$

By looking up table, we can get RI=1.12.By calculating we can find out CR=0.03705<0.1, meet the consistency test. And let its eigenvector be the weight vector of the program$(1.0000 \quad 0.3135 \quad 0.2118 \quad 0.2356 \quad 0.3135)^T$.

The following were the Pairwise Comparison Matrix of program layer acting on criteria layer:

$$b_1=\begin{pmatrix} 1 & 2 & 3 & 6 & 7 \\ \frac{1}{2} & 1 & \frac{2}{5} & \frac{9}{2} & \frac{13}{2} \\ \frac{1}{3} & \frac{2}{5} & 1 & \frac{2}{7} & 6 \\ \frac{1}{6} & \frac{2}{9} & \frac{2}{7} & 1 & \frac{11}{2} \\ \frac{1}{7} & \frac{2}{13} & \frac{1}{6} & \frac{2}{11} & 1 \end{pmatrix} \qquad b_2=\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{4} & \frac{1}{7} & \frac{1}{9} \\ 2 & 1 & \frac{1}{2} & \frac{1}{6} & \frac{1}{8} \\ 4 & 2 & 1 & \frac{1}{4} & \frac{1}{6} \\ 7 & 6 & 4 & 1 & \frac{1}{5} \\ 9 & 8 & 6 & 5 & 1 \end{pmatrix}$$

$$b_3=\begin{pmatrix} 1 & 2 & 4 & 6 & 8 \\ \frac{1}{2} & 1 & 3 & \frac{11}{2} & 7 \\ \frac{1}{4} & \frac{1}{3} & 1 & 4 & 3 \\ \frac{1}{6} & \frac{2}{11} & \frac{1}{4} & 1 & 3 \\ \frac{1}{8} & \frac{1}{7} & \frac{1}{5} & \frac{1}{3} & 1 \end{pmatrix} \qquad b_4=\begin{pmatrix} 1 & \frac{1}{3} & \frac{1}{4} & \frac{1}{8} & \frac{1}{9} \\ 3 & 1 & \frac{1}{2} & \frac{1}{6} & \frac{3}{8} \\ 4 & 2 & 1 & \frac{2}{5} & \frac{1}{2} \\ 8 & 6 & \frac{8}{3} & 1 & \frac{2}{3} \\ 9 & \frac{8}{3} & 2 & \frac{3}{2} & 1 \end{pmatrix}$$

$$b_5 = \begin{pmatrix} 1 & 3 & 5 & 8 & 9 \\ \frac{1}{3} & 1 & 4 & 6 & 7 \\ \frac{1}{5} & \frac{1}{4} & 1 & 5 & 6 \\ \frac{1}{8} & \frac{1}{6} & \frac{1}{5} & 1 & 3 \\ \frac{1}{9} & \frac{1}{7} & \frac{1}{6} & \frac{1}{3} & 1 \end{pmatrix} \tag{18}$$

According to b_i , we calculate maximum eigenvalue of the matrix and its corresponding eigenvector and consistency of indicators CI_k, the results shown in the table below

Table 8

k	1	2	3	4	5
	0.0758	0.0594	1.0000	0.1039	1.0000
	0.1056	0.0912	0.6804	0.2576	0.5600
	0.1759	0.1602	0.3433	0.4403	0.2757
$\omega_k^{(5)}$	0.3790	0.4180	0.1451	1.0000	0.1031
	1.0000	1.0000	0.0799	0.9705	0.0612
	5.3566	5.3157	5.2442	5.1249	5.4343
CI_k	0.0892	0.0789	0.0611	0.0312	0.1086

Through the random consistency index test of layer 3 of the calculation results of the optimal decision-making problems, the table can be accessed through all the consistency test.

Based on the weight vector in table above we can find the right combination of vector is

$$(0.6442 \quad 0.5146 \quad 0.4890 \quad 1.5783 \quad 0.8087)$$

So, P3 should be used, namely the use of 75% of the landline and 25% of cell phones, in the "pseudo-American" there are 225 million people use the fixed line, there are 75 million of people use cell phones.

(4) Analysis of model results

On the basis of the most consideration on the energy conservation, the model comes up with the criteria identified layer acting on the target layer is reciprocal matrix, so we get the smallest power consumption of phones use programs. In addition, we have quantized convenience, security level, price, and talking performance so as to get the best option.

5 Model the energy costs of the wasteful practice

5.1 Definitions and Symbols

- $T_{phone-total}$ ....the total time of a cell phone charged per day
- $T_{phone-waste}$ ....the waste time of a cell phone charged per day
- $T_{recharger}$ ....the time of recharger plugged in
- $Q_{recharger\&phone}$ ....the waste energy costs of a cell phone plugged in
- $Q_{recharger}$ ....the energy costs of a cell phone charged
- N_{oil} ....the energy wasted in terms of barrels of oil per day.
- η ....the efficiency of oil for electricity generation
- β ....the efficiency of power transmission
- Q_{waste}.......................the energy wasted per day
- $N_{population}$ ....the population of United States
- E_{oil} ....the energy of a barrel of oil contains

5.2 The Foundation of Model

On the basis of the most consideration on the energy conservation, the model comes up with the criteria identified layer acting on the target layer is reciprocal matrix, so we get the smallest power consumption of phones used programs. In addition, we have quantized convenience, security level, price, and talking performance so as to get the best option.

(1) the energy loss of this practice：

On the basis of the most consideration on the energy conservation, AHP based model comes up with the criteria identified layer acting on the target layer is reciprocal matrix, so we get a optimal program.，namely the use of 75% of the landlines and 25% of cell phones.

- Firstly ,we calculate the total energy costs of this wasteful practice that many people charge their phones every night without considering whether it needs to be recharged or not in the current US per day.

$$Q_{waste} = \frac{1}{4} N_{population} [T_{phone-waste} Q_{recharger\&phone} + (T_{recharger} - T_{phone-total}) Q_{recharger}] \tag{19}$$

- The amount of energy contained by N_{oil} barrel of oil is $N_{oil}E_{oil}$.

- To take account into the efficiency of oil for electricity generation and power transmission， we get the following formula:

$$Q_{waste} = \eta \beta N_{oil} E_{oil} \tag{20}$$

So that

$$N_{oil} = \frac{N_{population}[T_{phone-waste} Q_{recharger\&phone} + (T_{recharger} - T_{phone-total}) Q_{recharger}]}{4\eta\beta E_{oil}} \tag{21}$$

(2) Solution and Result

After checking out the relevant information， we get the following useful data.

$N_{population}$ =303,824,640 [4]

$Q_{recharger\&phone} = 3.6\times10^3 \sim 1.08\times10^4$ J [5]

$Q_{recharger} \approx 0$

$T_{recharger}$ =24 小时

According to American magazine PewResearchCenter statistics, a barrel of oil can power about: 587.78KWh. [3]

$$\eta E_{oil} = 2.17\times10^9 J$$

The efficiency of power transmission is about 60%~70%. [5]

β =0.6~0.7 [6]

Now we can calculate the result of this task.

The energy costs of this wasteful practice for the Pseudo US is equal to the amount of energy contained by about ***1680~5040*** barrels of oil based upon the data and model in Requirement 2.

6 How much energy do we waste per day?

6.1 Background

- **What is the standby power consumption?**

"Nowadays,more and more products have the standby function,such as the Telecontrol switch,the Wake-on LAN,the Timed switch.When we use these features,a certain electrical power will be consumed,that is what we called the Standby Power Consumption(SPC). Standby function greatly facilitate our lives,but it wastes a great deal of energy. Studies of some related institutions abroad show that the SPC is 3% to 13% of the National Organization for Economic Co-operation (OECD) civil electronical consumption.Especially in recent years, with the rapid development of network and the increase of electrical equipment , this ratio will continue to rise if we do not control the SPC. Therefore, appealling to paying attention to the SPC, generalising and applying skills of the SPC and products are the problems remained to discuss for every nation in this world."

- **The impact of the standby power consumption**

According to a household survey of the Oxford University about the SPC, in kinds of products, the SPC of the audio-visual products is the 68.6% of the total consumption , the cookers is 13.2% , the telephones is 7.8%, the refrigerant equipment is 7.7% , others are 2.7%. Average household SPC is the 8% of the total household electrical consumption in UK..

Recent studies show that the SPC is 3% to 13% of the National Organization for Economic Co-operation (OECD) civil electrical consumption. In 2001, a survey about the SPC of the urban families and the potential of the energy-saving is conducted by the Certification Center (the former China Energy Conservation Product Certification Center) shows that , SPC of urban families is about 10% of the average electrical consumption of the urban families in China, and the SPC of kinds of household electrical products ranges from 15W to 30W . [3]

6.2 The model of the SPC

6.2.1 Model assumptions

- Suppose that each family has only one TV, one air-condition, one washing machine, one printer and one other household appliances.
- Suppose that each mobile phone is equipped with only one charger.

6.2.2 Definitions and Symbols

- K（K=1，2，3，4，5，6）represents TV, DVR, printers, air conditions, washing machines and mobile phone chargers
- W_k（W/h）（k=1，2，3，4，5，6） indicates the unit standby energy consumption of the household appliance k ;
- T_k（h）（k=1，2，3，4，5，6） indicates the standby time-consuming per day of the household appliance k
- E_k（J）（k=1，2，3，4，5，6） indicates the standby power consumption(SPC) per day of the household appliance k;
- M_k（k=1，2，3，4，5，6）indicates the total number of the household appliances k in America;
- E（J）indicates the sum of standby power consumption(SPC) per day of the six kinds of household appliances in the United States;
- μ indicates the generated output of the oil;
- n indicates the total number of barrels of oil waste caused by the sum of the electrical loss of the SPC per day of the six kinds of household appliances

6.2.3 The Foundation of Model

$$E_k = W_k T_k$$
$$E = \sum_{k=1}^{6} M_k E_k \qquad (22)$$
$$n = E / \mu$$

By looking up some materials, the average SPC of some common household appliances and the total standby time of the household appliances per day in USA is the just as the following table.

[1]

Table 9 : Data of the recharge device

Household appliance	SPC per unit（W）	standby time per day（h）	Electrical consumption per uint（J）	The sum of users in US（million）
TV	0.07	18.5	4662	287
DVR	43.46	22	3534696	49
Printer	5.40	21.6	419904	54
Air-condition	0.9	18	58320	72.629
Washing machine	2.46	21	185976	89.287
Mobile phone charger	1.34	20	96480	75

According to the above table,we can draw a Column to show the different waste of electrical energy of the various kinds of household appliance in the standby mode per day.

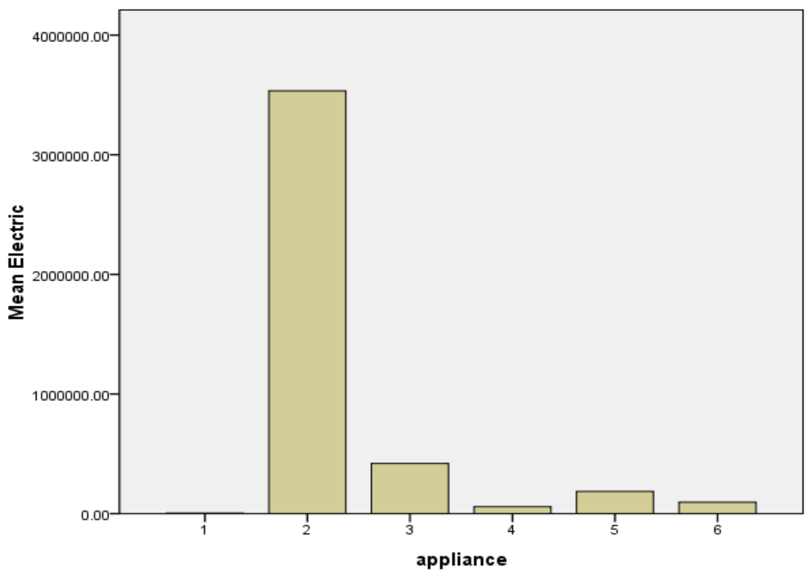

Figure 8: Waste of electrical energy of household appliance Column

Through the column of waste of electrical energy of household appliance,we can draw a conclution that, DVR's standby power consumption is the largest, about 3,534,696 joules, a waste of about 0.02 barrels of oil; followed by the Printer and Washing machine, about a day wasted 400,000 electric and 200,000 joules joules. Therefore, in their daily lives, we must pay special attention to these appliances, especially the DVR, Printer and Washing machine, in order to reduce standby energy waste.

According to the data mentioned above, we can get the function E=2.25*10^14J According to statistics converyed by the American magazine PewResearchCenter, one barrel of oil can generate about: 587.78KWH, namely 2.1160e +009 J.

Therefore, the total number of oil barrels used for the SPC of these six household appliances is

$$n = 1.06 \times 10^5 \quad \text{barrels.}$$

7 How much energy do we need in the future ?

7.1 Analysis of the problem

- In the next 50 years, "pseudo-American" population and the economy will change constantly. In the relatively abundant conditions of resources and environmental, the rate of population growth gets larger, population growth becomes faster, but when the population reach a certain number, the resources will be correspondingly reduced, the population that environment can hold will be gradually reduced, so the rate of population growth will be correspondingly reduced.

- American economic growth is mainly embodied in the growth of GDP. In the past few decades, how does the United States GDP grow and what will be the principle of GDP growth in the next 50 years? We have to do statistical analysis with our previous data, so that we can get the law of economic growth and predict the economic growth in the next 50 years.

- Demographic change will lead to the total energy demand change in the United States, meanwhile, economic growth may cause the change of phone service system's electricity demand. This article establish two basic models to illustrate the principle of "pseudo-American" demographic change and economic growth at first, and then based on the assumption of requirement 1 ,requirement 2 and requirement 3, predict the energy demand every 10 year in the next 50 years. Finally predict the oil demand through the power generation efficiency of oil .

7.2 Population growth model (Logistic Model) [3]

7.2.1 Definitions and Symbols

x(t)........stands for the population at t moment.
x_0.........stands for the population at the initial moment

r ....stands for the population growth rate

x_m ....stands for the largest population that natural resources and environmental conditions can bear, which is population capacity.

7.2.2 The Foundation of Model

Consider the increse of the population from t to t+Δt ,there exists

$$x(t+\Delta t)-x(t)=rx(t)\Delta t \tag{23}$$

Let Δt→0， we can get x (t) satisfied the differential equation

$$\frac{dx}{dt}=rx, x(0)=x_0 \tag{24}$$

By solving the model mentioned above, we can find

$$x(t)=x_0e^{rt} \tag{25}$$

7.2.3 Improve Model

The model above can be obtained based on the assumption of the population growth rate is unchanged, but, when the population growth to a certain extent, because natural resources, environmental conditions and other factors, population growth rate will be reduced, hence we improve the exponential growth model above. Use r (x) to replace r, then we get

$$\frac{dx}{dt}=r(x)x, x(0)=x_0 \tag{26}$$

Suppose that r(x) is a linear function of x, that is

$$r(x)=r-sx \ (r>0,s>0) \tag{27}$$

Here r is the stable growth rate which shows that the growth rate of a small population (in theory, x = 0) .

When x = x_m ,the population no longer increase, that is, the growth rate r (x_m) = 0, substituting into (5), so we can find (5) is

$$r(x)=r(1-\frac{x}{x_m}) \tag{28}$$

Let (6)-type substitution into (4) type is

$$\frac{dx}{dt}=rx(1-\frac{x}{x_m}), x(0)=x_0 \tag{29}$$

The factor rx in the equation (7)'s right side embodies the population its own growth trends, factor $(1-{x}/{x_m})$ embodies the role of resources and environment block population growth, obviously, the greater x is , the greater the former factor is, the smaller the latter factor is, the population growth is the results of the two factors .

Solving the (7) by separating variables, we can get that,

$$x(t)=\frac{x_m}{1+(\frac{x_m}{x_0}-1)e^{-rt}} \tag{30}$$

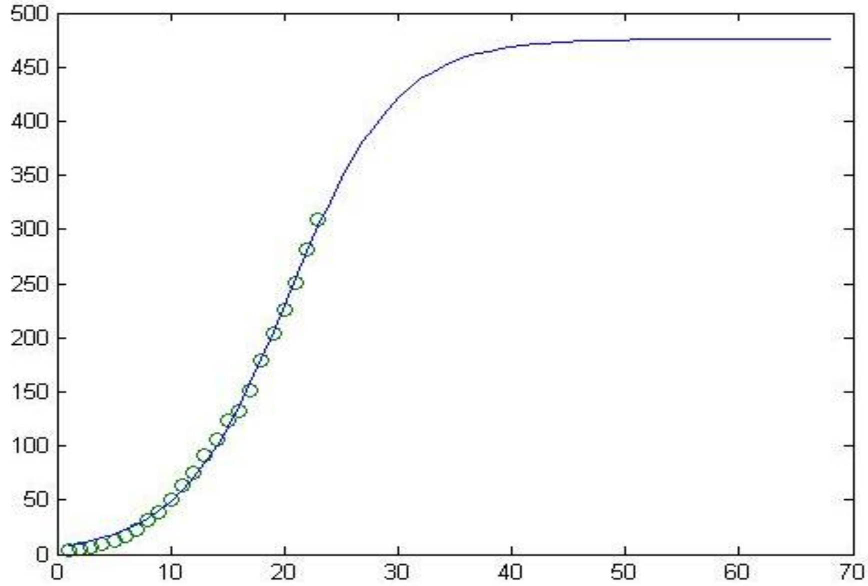

Figure 9 : Population growth trends

7.2.4 Solution of Model

Based on the American population data from 1790~ 2010,as follows :

Table 10: the United States demographic data

Year	1790	1800	1810	1820	1830	1840	1850	1860
population	3.9	5.3	7.2	9.6	12.9	17.1	23.2	31.4
Year	1870	1880	1890	1900	1910	1920	1930	1940
population	38.6	50.2	62.9	76	92	106.5	123.2	131.7
Year	1950	1960	1970	1980	1990	2000		
population	150.7	179.3	204	226.5	251.4	281.4		

We use least squares method to estimate the model parameters r, x_m and x_0 ,and find out

x_m =475.9762 (million)

x_0 =6.4915 (million)

r = 0.2107 (million/decade)

7.2.5 Test the Model

We use this model to estimate the population of the United States in 2010,let $t = 2010$ substitute into (8) and find out

$$X(2010)=309.2 \text{ (million)}$$

The current United States, a country of about 300 million people, it is manifest that the model result is very close to real data, which can be considered that the model confirms to the reality.

According to this model the population of the US in the next fifty years is

Table 11 : the population of the US in the next fifty years

year	2020	2030	2040	2050	2060
Population(mil.)	326019	346779	365637	382485	397314

7.3 Economic Growth Model (Linear regression model)[2]

7.3.1 Definitions and Symbols

t .... stands for time, namely the year

GDP(t) ....stands for the total GDP of the US at the t moment, which is caculated by billion

7.3.2 Statistical data analysis

By searching the *Statistical Abstract of the United States*, we find out that the GDP data from 1994-2006 is as follows

Table 12 : the Statistical Data of GDP in the U.S. [1]

year	1994	1995	1996	1997	1998	1999	2000
GDP（billion）	7072	7398	7817	8304	8747	9268	9817
year	2001	2002	2003	2004	2005	2006	
GDP（billion）	10128	10470	10961	11713	12456	13247	

We use *SPSS16.0* to draw the following linear graph with the data in the table.

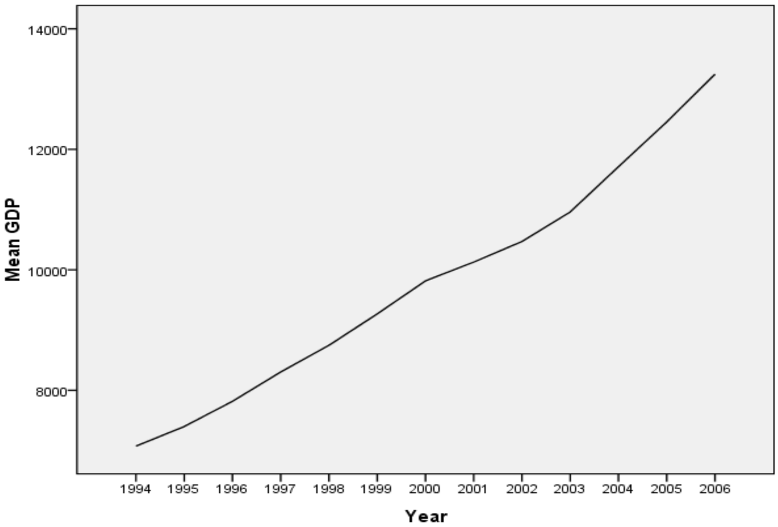

Figure 10: GDP trend

Through analyzing the statistics, we can see that the trend of United States' GDP from 1994 to 2006 is in linear shape, so we establish a linear regression model firsrly.

7.3.3 The Foundation of Model

Suppose

$$GDP(t) = at + b \tag{31}$$

We use SPSS 16.0 to solve the model, available that,

a = 495.610
B = -981419.934

Therefore, we let

$$GDP(t) = 495.610t - 981419.934 \tag{32}$$

By using the SPSS statistical software, we can get the following Model Summary

Table 13 : Model Summary

Model Summary[b]

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics					Durbin-Watson
					R Square Change	F Change	df1	df2	Sig. F Change	
1	.994[a]	.987	.986	227.561	.987	863.285	1	11	.000	.549

a. Predictors: (Constant), Year

b. Dependent Variable: GDP

The table lists the model's related coefficient (R)、 the square of related coefficient（R Square）、Adjusted R Square、Std.Error of the Estimate , etc.According to the data from the table, =0.994 stands for that variable t can be interpreted the variability of the variable GDP99.4%. Durbin-Watson value of the existing detection is 0.594<2, therefore,we can treat that the adjacent residuals exits positive correlation.

Table 14 : Model Summary

ANOVA[b]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	4.470E7	1	4.470E7	863.285	.000[a]
	Residual	569625.995	11	51784.181		
	Total	4.527E7	12			

a. Predictors: (Constant), Year

b. Dependent Variable: GDP

At the same time, we can convey the significant testing to regression coefficient by using the deviation analysis table.Table lists some statistics such as regression and residual etc.For the fact that the value of significant probability (F) is 0.000, which is less than 5%, therefore, we can consider that the regression coefficient is non-zero,and the regression equation is meaningful.

the results of economic growth of the United States in the next 50 years .Through the above-mentioned model, the GDP changes of the United States in the next 50 years are shown in the following table

Table 15 : GDP of the US in the next 50 years

Year	GDP（billion）
2020	19712.266
2030	24668.366
2040	29624.466
2050	34580.566
2060	39536.666

7.4 Energy demand prediction model

7.4.1 Model assumptions

- We suppose that the "pseudo-American"demographic change will not affect the proportion of the mobile phone users in the country's use;
- We suppose that "pseudo-American" demographic change will only affect the total energy demand of the telephone service;
- We suppose that the extent of economic growth and the progress of scientific and technological have the relationship of linear in "pseudo-American";
- We suppose that "pseudo-American" economic growth will make cell phones and landlines more energy-efficient in the using state and the standby mode.

7.4.2 Definitions and Symbols

（1） w_{11}（J/h）indicates that the power consumption when the mobile phone is used;

（2） w_{12} （J/h) indicates the power consumption when the phone is in the standby mode;
（3） w_{21}（J/h) indicates the power consumption of the landline telephone when it is used;
（4） w_{22}（J/h) indicates the power consumption of the landline when it is in standby mode;
（5） t_{11} （h） indicates the annual average amount of time used by a cell phone;
（6） t_{12}（h）indicates the average time of a cell phone when it is in standby state;
（7） t_{21} （h） indicates the average time used by a landline in per year;
（8） t_{22} （h） indicates the average standby time of a landline per year;
（9） n_1 indicates the average usage of Mobile phone per year;
（10） n_2 indicates the average usage of landline per year;
（11） W_1 indicates all the energy needs of mobile phones in a year in America;
（12） W_2 indicates all the energy needs of landline phones in a year in America;
（13） W indicates the electrical power demand of telephone service in a year in America.

7.4.3 The Foundation of Model

Without taking the impact of energy demand on which economic growth and demographic change make into account,we build the following model of energy demand.
All the energy needs of mobile phones in a year in America is

$$W_1 = n_1 \sum_{i=1}^{2} w_{1i} t_{1i} \tag{33}$$

All the energy needs of landline phones in a year in America is

$$W_2 = n_2 \sum_{i=1}^{2} w_{2i} t_{2i} \tag{34}$$

The electrical power demand of telephone service system in a year in America is

$$W = \sum_{i=1}^{2} W_i \tag{35}$$

7.4.4 Improve Model

Now considering the confluence to the electrical power demand of the telephone service system in a year in America caused by the economic growth..

U.S. economic growth will lead to development of science and technology.According to the model assumptions,development of science and technology will reduce the energy consumption of mobile phones and landline.At the same time, the economy grows , the standard of the people's living improves,and the use of mobile phone and landline will increase. Taking the fact that GDP is the main indicator to measure the economic growth into account , We build the mathematical model to determine the energy consumption of the following changes with the GDP.

$$w_{ij} = w_{ij}(GDP), (i = 1,2, j = 1,2) \tag{36}$$

The number of mobile phone users and landline users will become larger and larger because of the economic growth,located

$$n_k = n_k(GDP),(k=1,2) \tag{37}$$

Take the (14) and the (15) into (11) and (12) ,we will get

$$W_1(GDP) = n_1(GDP)\sum_{i=1}^{2} w_{1i}(GDP)t_{1i} \tag{38}$$

$$W_2(GDP) = n_2(GDP)\sum_{i=1}^{2} w_{2i}(GDP)t_{2i} \tag{39}$$

$$W(GDP) = \sum_{i=1}^{2} W_i(GDP) \tag{40}$$

7.4.5 Solution of Model

By looking up to some materials, the power consumption of the current American cell phone and landline in used and standby mode and the use of time, the use of statistical data such as shown in the table below

According to some information,we have got the time of

Table 16 : Data of cell phone and landline

Kind	State	Power（w）	Time of using （h）	User（mil）
Cell phone	Work	3.53	2.3	2679.6
	Standby	2.92	21.7	2679.6
Landline	Work	2.25	1.5	172
	Standby	2.01	22.5	172

Supposed that with the development of the economy, lower power consumption and the increase of the number of users are in linear change,that is

$$w_{ij}(GDP) = w_{ij}^{(0)} - \alpha GDP(t) \tag{41}$$

Where $w_{ij}^{(0)}$ indicates the current power cost

$$n_k(GDP) = n_k^{(0)} + \beta GDP(t) \tag{42}$$

Where $n_k^{(0)}$ indicates the current number of users

Obtained by least square method, α =0.025, β =0.12,so in the next 50 years,every 10 years, electricity demand shown in the table below

Table 17 : the Energy cost in the future

Year	2020	2030	2040	2050	2060
Energy(10^{18} J)	2.63	2.51	2.50	2.37	2.28

According to American magazine PewResearchCenter statistics, a barrel of oil can power about: 587.78KWH namely 2.116×10^9 J.Therefore, in the next 50 years, Every 10 years in the telephone service system, the number of barrels of oil needed for the

Table 18: the Energy cost in terms of barrels of oil

Year	2020	2030	2040	2050	2060
Oil (10^9 barrels)	1.2414	1.1884	1.1863	1.1201	1.0767

From the above calculations we can see that, with growth of the "pseudo-American" population and development of economy, the number of barrels of oil consumption reduced gradually in every 10 years. The fact shows that when the population has grown to a certain extent, population will show a stable trend, and population growth rate will gradually tends to 0.However, economic development led to the development of science and technology, making electronic products, such as mobile phones and fixed phones reduce energy consumption gradually, and thus lots of oil resource will be saved.

8 strengths and weaknesses

Strengths

- On the basis of statistical analysis of the actual data ,this model quantify the impact of economic development and population growth of the telephone service system on the energy consumption.
- In order to predict the energy needs of the "pseudo-American" in the next 50 years every 10 years, and to derive specific requirements for the number of barrels of oil.

Weaknesses

- Model is much straightforward, but in considering the impact of economic growth, only suppose that the energy consumption and the number of users change is a linear relationship.
- We neglect the restricting factors of the development of science and technology, so this model should be further improved.

9 Appendix

9.1 The number of cell phone users and landline users from the Statistical Yearbook of the United States

Table 1353. **Key Global Telecom Indicators for the World Telecommunication Service Sector: 1995 to 2004**

[**In billions U.S. dollars (779 represents $779,000,000,000), except as indicated.** All data were converted by annual average exchange rates. Country fiscal year data was aggregated to obtain calendar year estimates]

Indicators	1995	1999	2000	2001	2002	2003	2004
Telecom market total revenue (bil. dol.)	779	1,123	1,210	1,232	1,314	1,426	(NA)
Telecom telephone services revenue [1] (bil. dol.)	428	476	477	479	478	475	552
Other statistics:							
Main telephone lines (mil.) [2]	689	905	983	1,053	1,086	1,140	1,207
Mobile cellular subscribers (mil.)	91	490	740	955	1,166	1,414	1,758
International telephone traffic minutes [3] (bil.)	63	100	118	127	131	142	145
Personal computers (mil.)	235	435	500	555	615	650	775
Internet users (mil.)	40	277	399	502	619	724	863

NA Not available. [1] Revenue from installation, subscription and local, trunk and international call charges for fixed telephone service. [2] See footnote 1, Table 1354. [3] Including traffic between countries of former Soviet Union.

Source: International Telecommunication Union, Geneva Switzerland, 2005; <http://www.itu.int/ITU-D/ict/statistics/at_glance/KeyTelecom99.html>. Reproduced with the kind permission of ITU.

9.2 Part of the procedure source code

Idempotent method in order to solve the problem of the matrix's largest eigenvalue and its corresponding eigenvector

```
function [z,m]=power_m(A,max_it,tol)
[n,nn]=size(A);z=ones(n,1);
it=0;error=100;
disp('it m z(1) z(2) z(3) z(4) Z(5)')
while it<max_it & error>tol
    w=A*z;ww=abs(w);%w=vk,ww=|w|
    [k,kk]=max(ww);%k=max(vk)
    m=w(kk);%k=w(kk),m=k
    z=w/w(kk);%z=uk
    out=[it+1 m z'];
    disp(out)
    error=norm(A*z-m*z);
    it=it+1;
end
Error
```

References

[1]U.S. Census Bureau. *Statistical Abstract of the United States*.2008

[2] Wang Jie. *Experimental design and SPSS application*. Chemical Industry Press.2007.1

[3]Jiang Qiyuan, Xie Jinxing, Ye Jun. *Mathematical model*. Higher Education Press. 2005.12, pp.9-15&224-244.

[4] CIA The World Factbook United States.*The World Factbook*. https://www.cia.gov/library/publications/the-world-factbook/print/us.html. 2009.2.6

[5]ID1261302866 .*How much energy does a cell phone charger use?* http://wiki.answers.com/Q/How_much_energy_does_a_cell_phone_charger_use.2009

[6] Standby Power. *Standby Power Summary Table*. http://standby.lbl.gov/summary-table.html. 2009.2.6

[7] China Bureau of Quality Supervision.*Standby power consumption - an energy blood sucker*. 2002.10

[8] CuiHua HanWei. *Energy point of view from the home audition Product watch standby power.* 2007

[9] Zhang Zongming. *Study on Electric-Magnetic Structure of the Contactless Power Transfer System*. University of Chongqing .2007.