MESUGA, REYMOND R.

BS PHYSICS 3-1

PROBSET 1

PROBLEM 1

1. Given the Inverse laplace Transformation

$$\mathcal{L}^{-1}\left[\frac{e^{-\sqrt{a p}}}{\sqrt{p}}\right] = \frac{1}{\sqrt{\pi x}} e^{-\frac{a}{4x}}$$

show that

$$\mathcal{L}^{-1}\left[\frac{e^{-\sqrt{\omega}\frac{(z-L)}{\sqrt{D}}}}{\sqrt{\omega}}\right] = \frac{1}{\sqrt{\pi t}} e^{-(z-L)^2/4Dt}$$

Using the given laplace transform, let

$$\frac{e^{-\sqrt{p}}}{\sqrt{p}} = \frac{e^{-\sqrt{\omega}}}{\sqrt{\omega}} \quad \text{and} \quad \sqrt{a} = \frac{(z-L)}{\sqrt{D}} = \frac{(z-L)^2}{D} \quad \text{and} \quad x=t$$

So we can get

$$\mathcal{L}^{-1}\left[\frac{e^{-\sqrt{\omega}(z-L)/\sqrt{D}}}{\sqrt{\omega}}\right] = \frac{1}{\sqrt{\pi t}} e^{-\frac{(z-L)^2/D}{4t}}$$

$$= \frac{1}{\sqrt{\pi t}} e^{-(z-L)^2/4Dt}$$

where $\mathcal{L}^{-1}[F(\omega)] = f(t)$.

PROBLEM 2 A

(2) a. $y'' + y' = e^t$ where $y(0) = 1$ \& $y'(0) = 0$ b.

$\mathcal{L}\{y\} = Y(s)$

$\mathcal{L}\{y'\} = sY(s) - y(0)$

$\mathcal{L}\{y''\} = s^2Y(s) - sy(0) - y'(0)$

$\mathcal{L}\{e^t\} = \frac{1}{s-1}$

$[s^2Y(s) - sy(0) - \cancel{y'(0)}] + sY(s) - y(0) = \frac{1}{s-1}$

$Y(s)[s^2 - s] + sY(s) - 1 = \frac{1}{s-1}$

$Y(s)[s^2 - s] + sY(s) = 1 + \frac{1}{s-1} = \frac{s-1+1}{s-1} = \frac{s}{s-1}$

$Y(s)[s^2 - s + s] = \frac{s}{s-1}$

$Y(s)[s^2] = \frac{s}{s-1} \Rightarrow Y(s) = \frac{1}{(s-1)s}$

$\frac{1}{(s-1)s} = \frac{A}{s-1} + \frac{B}{s}$

$1 = A(s) + B(s-1)$

if $s = 1$ $A = 1$

if $s = 0$ $B = -1$

$Y(s) = \frac{1}{s-1} - \frac{1}{s}$

$y(t) = \mathcal{L}^{-1}\{Y(s)\}$

$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\{\frac{1}{s-1} - \frac{1}{s}\}$

$\boxed{y(t) = e^t - 1}$ //

PROBLEM 2 B.

b. $y' + y = \sin 3t$ where $y(0) = 0.$

$\mathcal{L}\{y\} = Y(s)$

$\mathcal{L}\{y'\} = sY(s) - y(0)$

$\mathcal{L}\{\sin 3t\} = 3/s^2 + 9$

$$sY(s) - \cancel{y(0)}^{\to 0} + Y(s) = \frac{3}{s^2+9}$$

$$Y(s)\left[\cancel{s-1}\right] = \frac{3}{(s-1)(s^2+9)}$$

$$\frac{3}{(s-1)(s^2+9)} = \frac{A}{(s-1)} + \frac{Bs + C}{s^2+9}$$

$$3 = A(s^2+9) + (Bs+C)(s-1)$$

$$3 = A(s^2+9) + (Bs+C)(s-1)$$

if $s = 1$ the $\boxed{A = \frac{3}{10}}$

$$3 = As^2 + 9A + Bs^2 - Bs + Cs - C$$

$$3 = (A+B)s^2 + (C-B)s + 9A - C$$

$0 = A + B \Rightarrow \boxed{A = \frac{3}{10}}, \boxed{B = -\frac{3}{10}}$

$0 = C - B \Rightarrow \boxed{C = \frac{3}{10}}$

$3 = 9A - C \Rightarrow$

$A = \frac{3+C}{9}, \quad C = 9A - 3$

$$Y(s) = \frac{(3/10)}{s-1} + \frac{(-3/10)s + 3/10}{s^2+9}$$

$$Y(s) = \frac{3}{10}\left(\frac{1}{s-1}\right) + \left(\frac{-3}{10}\right)\left(\frac{s}{s^2+9}\right) + \frac{3}{10}\left(\frac{1}{s^2+9}\right)$$

$$\boxed{y(t) = \frac{3}{10}\left(e^{t}\right) \cancel{+} - \frac{3}{10}\left(\cos 3t\right) + \frac{1}{10}\left(\sin 3t\right)}$$