

Homework 1

Rey

CSCI 5636 - Numerical Solution of Partial Differential Equations

September 16, 2018

Example derivation for getting second order error for forward difference:

Forward difference using x_i and x_{i+1}

$$u(x_i + x_{i+1}) = u(x_i) + (x_{i+1} - x_i)u'(x_i) + \frac{(x_{i+1} - x_i)^2}{2!}u''(x_i) + \frac{(x_{i+1} - x_i)^3}{3!}u'''(x_i) \quad (1)$$

Forward difference using x_i and x_{i+2}

$$u(x_i + x_{i+2}) = u(x_i) + (x_{i+2} - x_i)u'(x_i) + \frac{(x_{i+2} - x_i)^2}{2!}u''(x_i) + \frac{(x_{i+2} - x_i)^3}{3!}u'''(x_i) \quad (2)$$

Let $x_{i+1} - x_i = \frac{1}{2}(x_{i+2} - x_i) = h$.

equations 1 and 2 become 3 and 4:

$$u(x_i + h) = u(x_i) + hu'(x_i) + \frac{h^2}{2!}u''(x_i) + \frac{h^3}{3!}u'''(x_i) \quad (3)$$

and

$$u(x_i + 2h) = u(x_i) + 2hu'(x_i) + \frac{(2h)^2}{2!}u''(x_i) + \frac{(2h)^3}{3!}u'''(x_i) \quad (4)$$

Multiply equation 3 through by 4 and isolate for the error

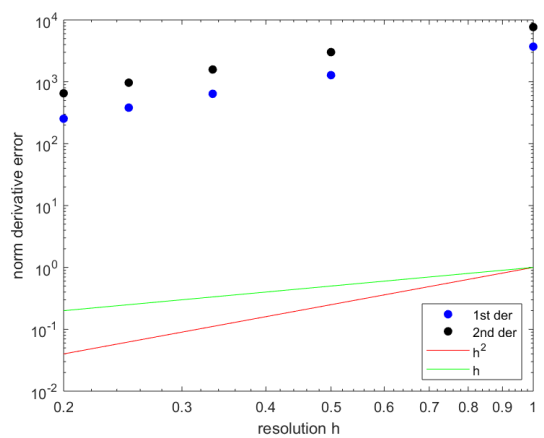
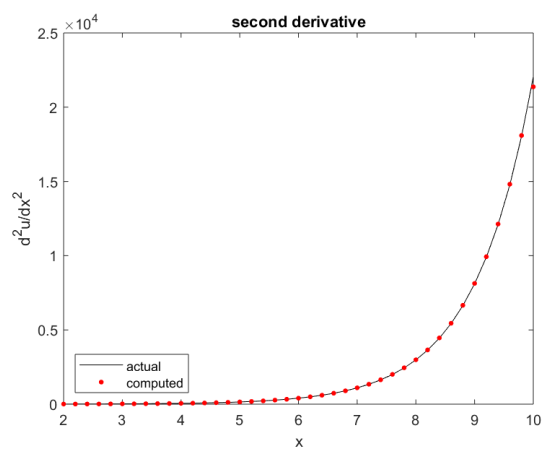
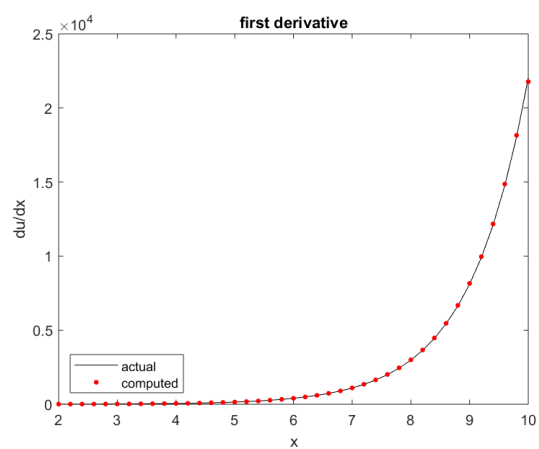
$$\frac{4u(x_i + h) - 4u(x_i)}{2h} = 2u'(x_i) + \frac{2h}{2!}u''(x_i) + \frac{2h^2}{3!}u'''(x_i) \quad (5)$$

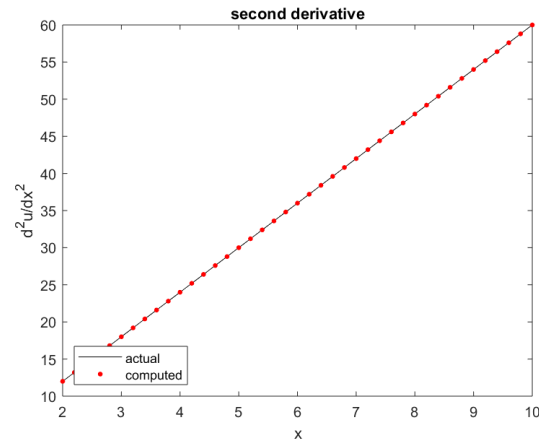
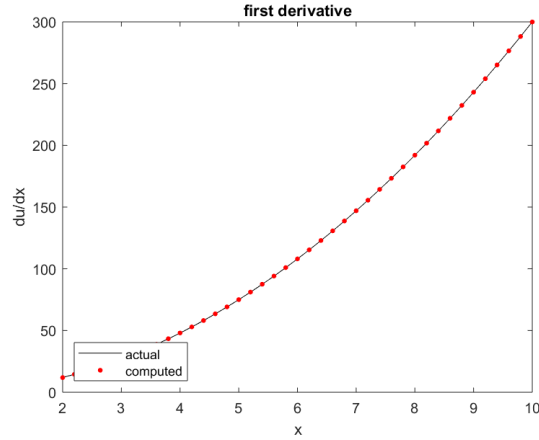
$$\frac{u(x_i + 2h) - u(x_i)}{2h} = u'(x_i) + \frac{2h}{2!}u''(x_i) + \frac{4h^2}{3!}u'''(x_i) \quad (6)$$

subtract equation 5 from 6

$$\frac{4u(x_i + h) - 3u(x_i) - u(x_i + 2h)}{2h} = u'(x_i) - \frac{2h^2}{3!}u'''(x_i) \quad (7)$$

I wanted to write out more derivations, but didn't have time.





I derived the first and second derivative formulae to give second order error. The results are not entirely what I'd expect. using the test of $u(x) = e^x$ I get the results from the first three images. The error seems to be to the second order.

Using the test equation $u(x) = x^3$ I get the results from the following three images. The second derivative error is not what I expected. I haven't figured out if it's something in my derivation of the equations used.

