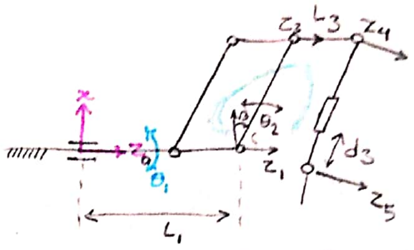
**Project :**

1. DH Parameters : first way, Hard way!

| Link | $a_i$ | $\alpha_i$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | 0 | 0 | $L_1$ | $\theta_1$ |
| 2 | 0 | $-\pi/2$ | 0 | $\theta_2+\beta$ |
| 3 | $L_2$ | 0 | 0 | $\pi/2-(\theta_2+\beta)$ |
| 4 | $L_3$ | 0 | 0 | $-\beta$ |
| 5 | $L_2$ | 0 | 0 | 0 |

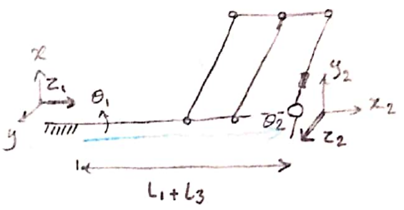
$$H_n^{n-1} = \begin{bmatrix} C_\theta & -S_\theta C_\alpha & S_\theta S_\alpha & a C_\theta \\ S_\theta & C_\theta C_\alpha & -C_\theta S_\alpha & a S_\theta \\ 0 & S_\alpha & C_\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$




$$H = \begin{bmatrix} C_{(\beta+\theta_2)} C_1 C(3+4) & C(\beta+\theta_2) S_1 C(3+4) & -S_1 & -S(\beta+\theta_2) C(3+4) \\ -C(\beta+\theta_2) C_1 S(3-4) & -C(\beta+\theta_2) S_1 C(3+4) & 0 & S(\beta+\theta_2) S(3+4) \\ \begin{matrix}[C_1 C(\beta+\theta_2) C(3+4)](L_2-d) \\ C_{(\beta+\theta_2)} C_1 L_2 + C_{(\beta+\theta_2)} C_1 C_3 L_3\end{matrix} & \begin{matrix}(C_{(\beta+\theta_2)} C_3 C_4 - S(\beta+\theta_2) S_3 S_4](L_2-d) \\ S(\beta+\theta_2) C_3 L_3 - S(\beta+\theta_2) L_2\end{matrix} & 0 & \begin{matrix} L_1 - [S_{(\beta+\theta_2)} C(3+4)](L_2-d) \\ -S(\beta+\theta_2)[C_3 L_3 - L_2]\end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2nd way:

| Link | $\alpha_i$ | $a_i$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | $-\pi/2$ | 0 | $L_1+L_3$ | $\theta_1$ |
| 2 | $-\pi/2$ | 0 | 0 | $\theta_2$ |
| 3 | 0 | 0 | $d$ | 0 |




$$H_n^{n-1} = \begin{bmatrix} C_\theta & -S_\theta C_\alpha & S_\theta S_\alpha & a C_\theta \\ S_\theta & C_\theta C_\alpha & -C_\theta S_\alpha & a S_\theta \\ 0 & S_\alpha & C_\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} C_1 C_2 & S_1 & -C_1 S_2 & 0 \\ S_1 C_2 & -C_1 & -S_1 S_2 & 0 \\ -S_2 & 0 & -C_2 & L_1+L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Successive Screw: $\dot{x}_E = \begin{bmatrix} 0 \\ 0 \\ L_1+L_3-d \end{bmatrix}$

| joint | $S_i$ | $S_{0i}$ |
|---|---|---|
| 1 | $(0,0,1)$ | $(0,0,0)$ |
| 2 | $(0,1,0)$ | $(0,0,L_1+L_3)$ |
| 3 | $(0,1,0)$ | $(0,0,L_1+L_3)$ |

$$A_3^0 = A_1^0 A_2^1 A_3^2 \; P_0 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & 0 & s_2 & -s_2(L_1+L_3) \\ 0 & 1 & 0 & 0 \\ -s_2 & 0 & c_2 & -(c_2-1)(L_1+L_3) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & (L_1+L_3-d) \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} c_1c_2 & s_1 & -c_1s_2 & 0 \\ s_1c_2 & -c_1 & -s_1s_2 & 0 \\ -s_2 & 0 & -c_2 & L_1+L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \text{Same as part 1. } ✓$$

3. workspace: matlab code attached.

$-\pi/4 \leq \theta_1 \leq \pi/4$ , $-\pi/4 \leq \theta_2 \leq \pi/4$ $\quad 0 \leq d \leq 3$ cm

4. Inverse kinematics: $T_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & L_1+L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\quad T_2 = \begin{bmatrix} c_2 & 0 & -s_2 & 0 \\ s_2 & 0 & c_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\quad T_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$T_2^{1^{-1}} \; T_1^{0^{-1}} \; T_0^{3} = \underset{I}{T_3^{2}} \rightarrow \begin{bmatrix} c_1c_2 & c_2s_1 & -s_2 & s_2(L_1+L_3) \\ s_1 & -c_1 & 0 & 0 \\ -c_1s_2 & -s_1s_2 & -c_2 & c_2(L_1+L_3) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$s_1P_x - c_1P_y = 0 \longrightarrow q_1 = \arctan\left(P_y/P_x\right)$$

$$-c_1s_2\,P_x - s_1s_2\,P_y - c_2\,P_z + c_2(L_1+L_3) = 0$$

$$c_1c_2\,P_x + c_2s_1\,P_y - s_2\,P_z + s_2(L_1+L_3) = 0$$

$$P_y = \frac{s_1}{c_1}\,P_x$$

$$-\frac{s_2}{c_2}\,P_x/c_1 = \frac{\cancel{c_2}\left((L_1+L_3)-P_z\right)}{\cancel{c_2}} \longrightarrow$$

$$\tan q_2 = (L_1+L_3-P_z) \times \frac{c_1}{P_x} \longrightarrow$$

$$q_2 = \arctan\left(\frac{(L_1+L_3-P_z)\,c_1}{P_x}\right)$$