Numerical Calculation Readme

First task:

Using a suitable computer program, determine the appropriate value of epsilon for variables with single (single) and double (double) precision, and compare it with the values calculated in class.

Second Task

The polynomial equation $f(x) = x^3 + 3x^2 - 2x - 4 = 0$ has a root close to $x = 10$. Use two relations given by $x = g(x)$ such that they converge to this root. Starting with an initial guess of $x_1 = 10$, find this root.

Write a MATLAB program to solve the above problem such that the root is found with six decimal place accuracy (i.e., $|x_{n+1} - x_n| < 10^{-6}$).

Third Task:

Write a program that solves the following system of equations with an accuracy of four decimal places. Use the Gauss-Seidel iterative method. Choose the initial guess arbitrarily.

$$
\begin{pmatrix}
4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
-1 & 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 \\
-1 & 0 & 0 & 4 & -1 & 0 & -1 & 0 & 0 \\
0 & -1 & 0 & -1 & 4 & -1 & 0 & -1 & 0 \\
0 & 0 & -1 & 0 & -1 & 4 & 0 & 0 & -1 \\
0 & 0 & 0 & -1 & 0 & 0 & 4 & -1 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & -1 & 4 & -1 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 4
\end{pmatrix}
\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{pmatrix}
=
\begin{pmatrix} 2 \\ 1 \\ 2 \\ 1 \\ 2 \\ 1 \\ 3 \\ 2 \\ 3 \end{pmatrix}
$$

Forth Task : Repeated as First Task

Fifth Task:

Using the error bounds of Simpson's rule, compute the integral below (using MATLAB) such that the error is less than 10^{-7}. Show the calculated error using the exact solution for the integral.

$$\int_0^1 e^x \, dx = e - 1$$

Sixth Task:

The equation governing the cooling of a body due to radiation is given as:

$$\frac{dT}{dt} = -\alpha(T^4 - T_a^4)$$

where T is the temperature of the body, T_a is the ambient temperature, and α is a constant that depends on the mass, surface area, heat capacity, and radiative properties of the body. Given the data for the problem as:

$$T(0) = 2500\,\mathrm{K}, \quad T_a = 250\,\mathrm{K}, \quad \alpha = 4 \times 10^{-12}\,\mathrm{K}^{-3}\mathrm{s}^{-1}$$

(A) Use the Euler method and second and fourth-order Runge-Kutta methods to solve the temperature changes up to $t = 10\,\mathrm{s}$ using MATLAB. Set the time step $\Delta t = 1/100$. Report the temperature values at $t = 5\,\mathrm{s}$ and $t = 10\,\mathrm{s}$ for each method.

(B) Plot the temperature changes over this period using the `` `plot` `` function and include your name and student number in the graph title. Print the charts and attach them to the exercise submission.