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AP-266 Final Homework

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Contents

1 ASA Implementation and differentiating the code

The block diagram for the Aerostructural Analysis (ASA) module can be seen in Figure 1. The parameters that are not altered in optimization (assumed to be constant) are inserted as global values and called within the program (in the block, these are marked as blue with dotted connection lines).

The first test, to check if the ASA module is correctly implemented, is presented with the results of *test1.py* code. The results obtained corresponded to those expected in the provided documentation.

The results for the code differentiation is in the file *test2.py*. For the first test (Finite diff. test), we obtain, for a $h = 1E-7$, a result near what we expect (paying attention to the fact that the LLT residuals obtained in the previous test are of $1E8$ order, which explains the relatively high values for the reslltD). As for the second test (dot product), we also confirm the validity of the differential codes. The results for both tests are presented in the following text.

The results of the FD test (ubuntu-terminal-like):

FD TEST

```
FD test results (0.0 expected)
For reslltD we expect about 1.0

reslltD = [-0.05602474
            0.62212879
            0.74495198
            4.04575461]

resfemD = [ 1.55191738e-05
1.63494719e-05  1.01694945e-06
5.54263688e-05  -3.99580285e-13
-3.99580285e-13  -6.17267392e-06
-4.62449293e-06  -2.24523396e-05
2.15854980e-05]

liftExcessD = 1.5197342523309842e-08

marginsD = [-8.67884438e-10
-8.67884438e-10  -8.67884462e-10
-8.67884438e-10  -8.67884504e-10
-8.67884507e-10  8.01389971e-09
-8.67884452e-10]

KSmarginD = -8.678844656828666e-10

FBD = -7.610387982026623e-06

WeightD = -2.483773005224066e-05
```

DP TEST

```
Dot product results (0.0 expected)
dotp = -8.204551704693586e-09
```

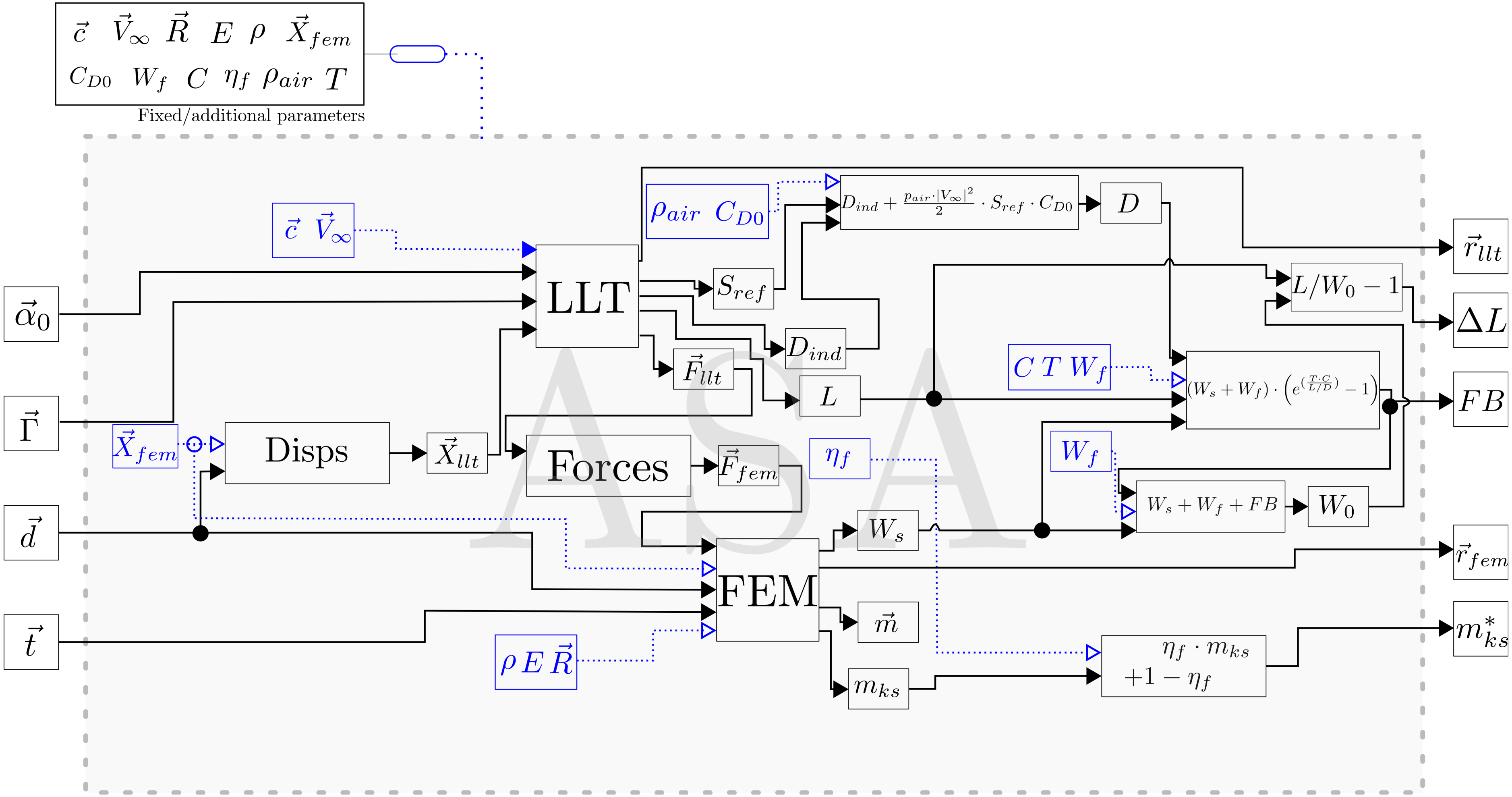

Figure 1: Block diagram for the ASA module.

2 Testing the ASA program (resfunc + adjfunc)

For this analysis, it is important to notice how we can obtain $\frac{dKS}{d\vec{\alpha_0}}$ and $\frac{dKS}{d\vec{t}}$. When we first run the reverse ASA, with KS seed equal 1.0 (all the others fixed output seeds are 0.0), while solving the desired gradients as 0.0 (when we use scipy solve function), we obtain the adjoint variables of our adjoin program. This is a direct application of the formula, as we found the values of input seeds that satisfies our $KS = 1.0$.

The results for this analysis are in the *test3.py* and are:

```
------------------------------ RES TEST ------------------------------
----------------------------------------------------------------------

Gama =  [12.99803608 14.28790619 14.28790619
 12.99803608]

d =  [ 2.69493469e-02 -8.96491634e-03
 9.57299674e-03 -7.85795127e-03
  -9.26179639e-34 -2.77656736e-60
  9.57299674e-03  7.85795127e-03
   2.69493469e-02  8.96491634e-03]
----------------------------------------------------------------------

------------------------------ ADJ TEST ------------------------------
----------------------------------------------------------------------

psi =  [-61.93477178 -10.79842249 -10.74184745
 -78.55769598]

lambda = [ 1.76989719e-04 -4.41232718e-05
 8.87470713e-05 -4.41174285e-05
  -1.53738245e+02 -1.30721517e-07
  8.87470714e-05  4.41174285e-05
   1.76989720e-04  4.41232719e-05]

dKSda0 = [-2.63697857 -1.16166158 -1.16166158 -2.63697857]

dKSdt =  [1.11410770e-03 6.61735632e+01
 6.61735633e+01 1.11410770e-03]
----------------------------------------------------------------------
```

3 Fuel burn minimization

The optimization is obtained in the archive *fb_min.py*. First, we have the analysis from the initial parameters and final values of the same, as follows (.4 precision):

3.1 Optimization results

```
------------------------------ OPTIMIZATION ------------------------------
------------------------------ START POINT ------------------------------
------------------------------ OBJ FUNC. ------------------------------
FB = 1449.0075
------------------------------ DES VARS. ------------------------------
a0 = [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0.]
t0 = [0.005 0.005 0.005 0.005 0.005 0.005 0.005
0.005 0.005 0.005 0.005 0.005
 0.005 0.005 0.005 0.005 0.005 0.005 0.005 0.005]
------------------------------ CON FUNC. ------------------------------
DL = 0.5942
KS = -1.1712
--------------------------------------------------------------------------
------------------------------ END POINT ------------------------------
------------------------------ OBJ FUNC. ------------------------------
FB = 1911.0287
------------------------------ DES VARS. ------------------------------
a0 = [-0.0568 -0.0475 -0.0408 -0.0356 -0.0314
-0.028  -0.0254 -0.0234 -0.0221
 -0.0215 -0.0215 -0.0221 -0.0234 -0.0254 -0.028
 -0.0314 -0.0356 -0.0408
 -0.0475 -0.0568]
t0 = [0.001  0.001  0.001  0.001  0.0014
0.0021 0.0029 0.004  0.0052 0.0066
 0.0066 0.0052 0.004  0.0029 0.0021 0.0014
 0.001  0.001  0.001  0.001 ]
------------------------------ CON FUNC. ------------------------------
DL = 0.0000
KS = -0.0000
--------------------------------------------------------------------------
```

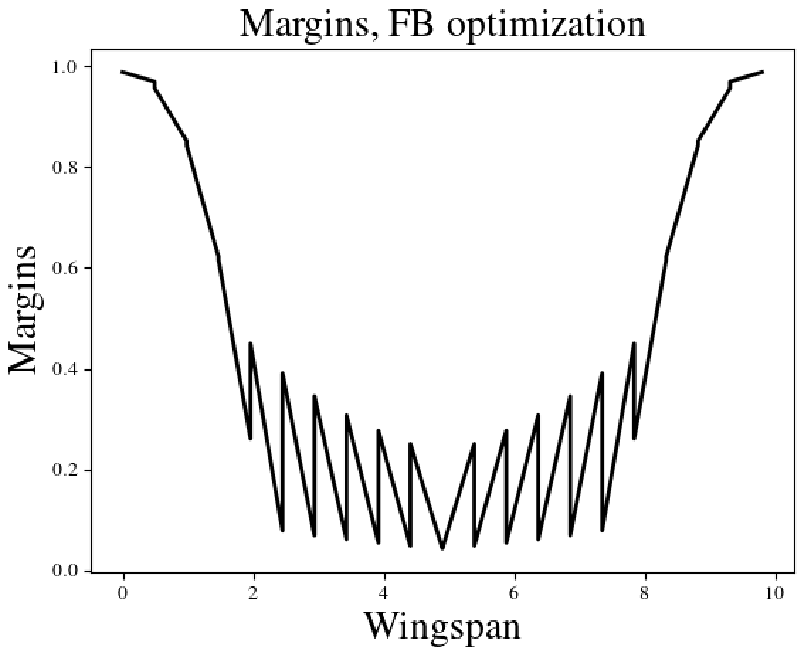

Figure 2: Margins from failure.

3.2 Margins, circulation and weight distribution

The margins distribution is plotted (with following nodes superposed) in Figure 2. We can see that around the middle of the Wingspan the nodes are nearer the failure, with a symmetric distribution along half the wing.

The circulation around the optimized wing is show in Figure 3 shows how close to the elliptical distribution the optimized result is. The FB value grows when the drag grows, which explain why this optimization gives this lift distribution. From previous analysis, we know that the elliptical value is the desired one for drag minimization, therefore this result is coherent with the aerodynamics analysis.

For the weight distribution analysis, let's display the ratios from the total weight within the structure. From the optimized result we have:

```
---------------------- WEIGHT RATIO ----------------------
----------------------------------------------------------
FB = 20.93 %

FM = 75.14 %

SM = 3.93 %
----------------------------------------------------------
```

where FB is the Fuel ration, FM is the fixed mass ratio and SM is the structure mass

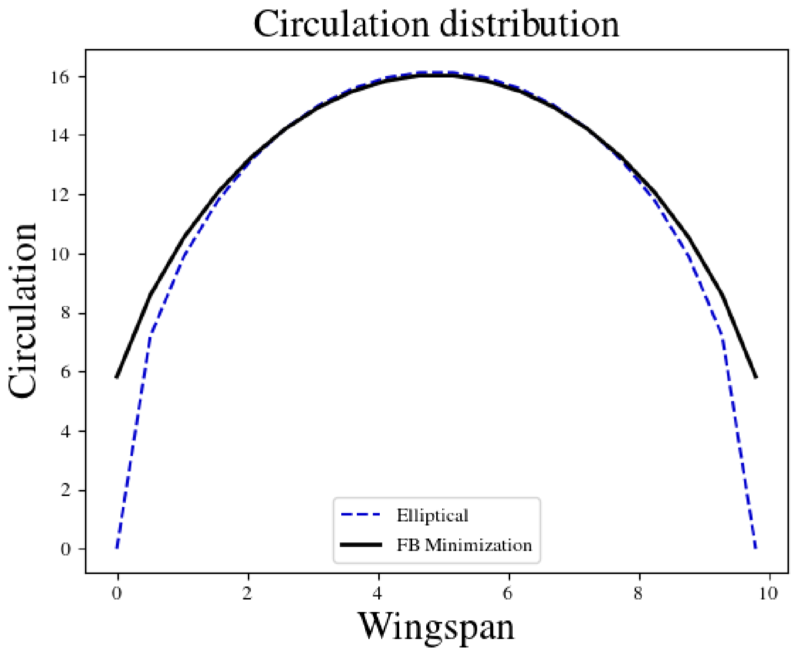

Figure 3: Circulation distribution around the wingspan.

ratio (all from the total mass value).

3.3 Optimization history

In Figure 4 we can see the evolution of this optimization, with some outliers from the optimization omitted in order to better observe the final distribution. The progress is shown from how "blue" and relatively big the markers are. Initial values are highlighted as their markers is a black diamond with red edge-color. Final values are highlighted with a red edge-color around the blue circle.

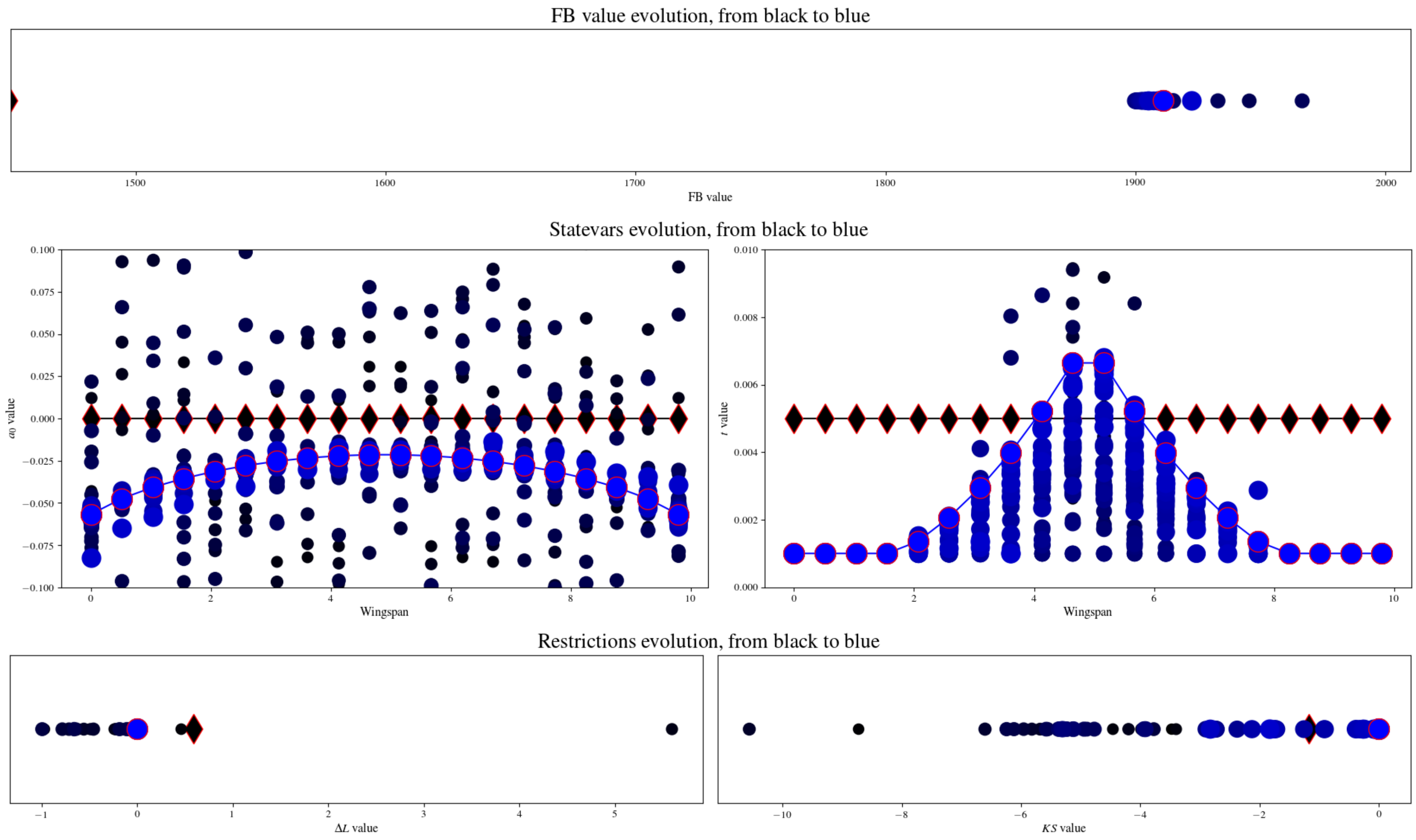

Figure 4: FB minimization history.

4 Weight minimization

The optimization is obtained in the archive *we_ min.py*. First, we have the analysis from the initial parameters and final values of the same, as follows (.4 precision):

4.1 Optimization results

```
------------------------- OPTIMIZATION -------------------------
------------------------- START POINT --------------------------
------------------------- OBJ FUNC. ----------------------------
W0 = 8993.7239
------------------------- DES VARS. ----------------------------
a0 = [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0.]
t0 = [0.005 0.005 0.005 0.005 0.005 0.005 0.005 0.005
0.005 0.005 0.005 0.005
 0.005 0.005 0.005 0.005 0.005 0.005 0.005 0.005]
------------------------- CON FUNC. ----------------------------
DL = 0.5942
KS = -1.1712
----------------------------------------------------------------
------------------------- END POINT ----------------------------
------------------------- OBJ FUNC. ----------------------------
W0 = 9099.3162
------------------------- DES VARS. ----------------------------
a0 = [-0.0824 -0.0707 -0.0592 -0.0478 -0.0367 -0.0261
-0.0168 -0.0094 -0.0043
 -0.0014 -0.0014 -0.0043 -0.0095 -0.0168
 -0.0261 -0.0367 -0.0478 -0.0592
 -0.0707 -0.0824]
t0 = [0.001  0.001  0.001  0.001  0.001  0.0015
0.0023 0.0032 0.0044 0.0058
 0.0058 0.0044 0.0032 0.0023 0.0015 0.001
 0.001  0.001  0.001  0.001 ]
------------------------- CON FUNC. ----------------------------
DL = -0.0000
KS = -0.0000
----------------------------------------------------------------
```

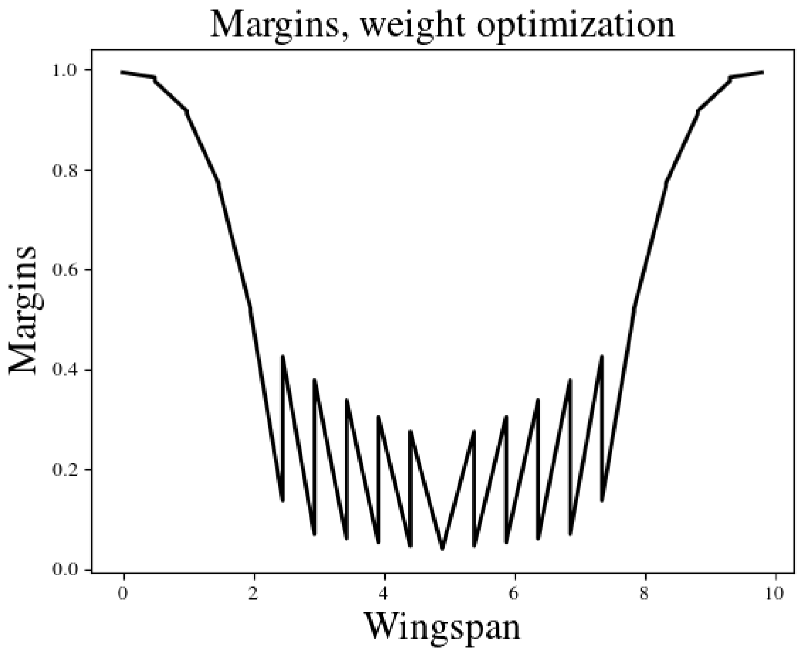

Figure 5: Margins from failure.

4.2 Margins, circulation and weight distribution

The margins distribution is plotted (with following nodes superposed) in Figure 5. We can see here a similar distribution to the previous section result, with symmetry along the middle of the wing and this region nearer the failure (0.0 value).

The circulation around the optimized wing is show in Figure 6 shows how a relatively new distribution for the circulation values. This is a worse result in aerodynamics terms, as the elliptical curve has the lesser drag. As the weight optimization values more about the total weight, it is according to expected not to obtain such a similarity as here the module also tries to minimize the structure mass.

For the weight distribution analysis, we confirm the previous discussion, as this time the structure mass has a lesser percentage in comparison with the previous section. The results are the following:

```
———————————— WEIGHT RATIO ————————————
——————————————————————————————————————
FB = 21.27 %

FM = 75.39 %

SM = 3.34 %
——————————————————————————————————————
```

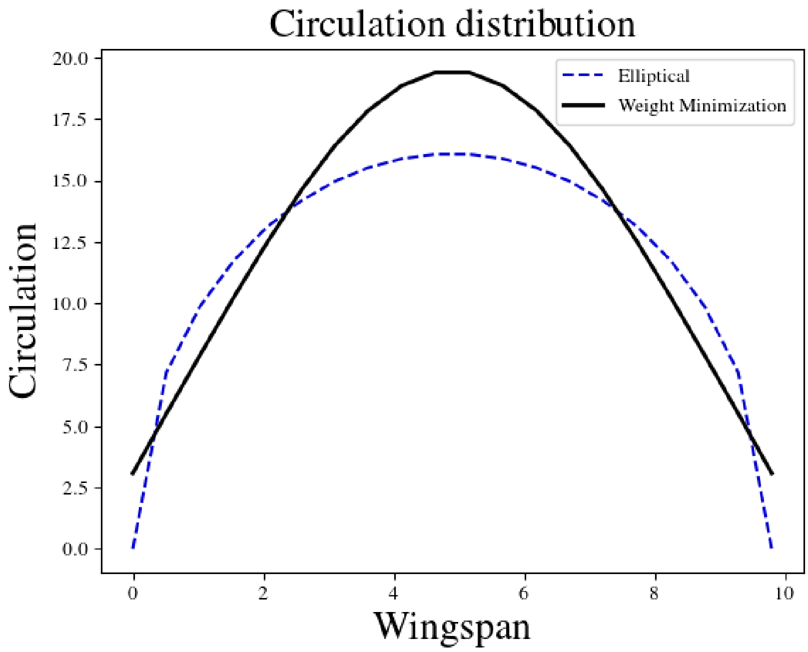

Figure 6: Circulation distribution around the wingspan.

where FB is the Fuel ration, FM is the fixed mass ratio and SM is the structure mass ratio (all from the total mass value).

4.3 Optimization history

In Figure 7 we can see the evolution of this optimization, with some outliers from the optimization omitted in order to better observe the final distribution. The progress is shown from how "blue" and relatively big the markers are. Initial values are highlighted as their markers is a black diamond with red edge-color. Final values are highlighted with a red edge-color around the blue circle.

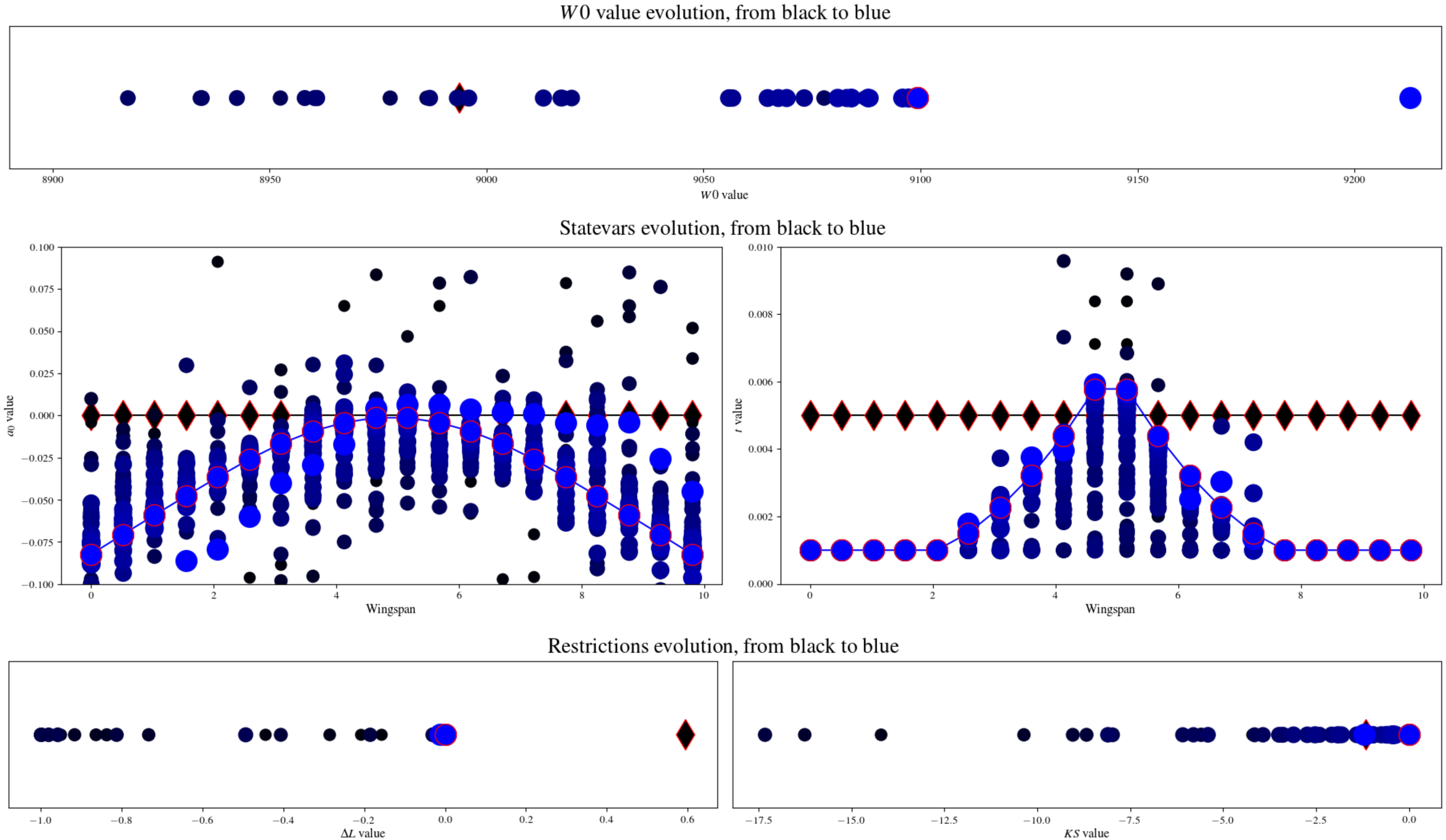

Figure 7: Weight minimization history.

5 Aspect ratio

The results of this section are on the *ar.py* file. We mainly focus on comparing the results for two different Aspect Ratios (AR).

5.1 Optimizations main results

We have four optimizations, each with the following results (identified as FB when referring to Fuel Burn minimization and W0 when referring to Weight minimization).

$AR = 6.0$, FB minimization:

```
------------------------------ OBJ FUNC. ------------------------------
FB = 1911.0287
------------------------------ DES VARS. ------------------------------
a0 = [-0.0568 -0.0475 -0.0408 -0.0356 -0.0314 -0.028
-0.0254 -0.0234 -0.0221 -0.0215 -0.0215 -0.0221 -0.0234
-0.0254 -0.028 -0.0314 -0.0356 -0.0408 -0.0475 -0.0568]
t0 = [0.001 0.001 0.001 0.001 0.0014 0.0021
0.0029 0.004 0.0052 0.0066 0.0066 0.0052 0.004
0.0029 0.0021 0.0014 0.001 0.001 0.001 0.001 ]
------------------------------ CON FUNC. ------------------------------
DL = 0.0000
KS = -0.0000
-----------------------------------------------------------------------
```

$AR = 6.0$, W0 minimization:

```
------------------------------ OBJ FUNC. ------------------------------
W0 = 9099.3162
------------------------------ DES VARS. ------------------------------
a0 = [-0.0824 -0.0707 -0.0592 -0.0478 -0.0367 -0.0261
-0.0168 -0.0094 -0.0043 -0.0014 -0.0014 -0.0043 -0.0095
-0.0168 -0.0261 -0.0367 -0.0478 -0.0592 -0.0707 -0.0824]
t0 = [0.001 0.001 0.001 0.001 0.001 0.0015
0.0023 0.0032 0.0044 0.0058 0.0058 0.0044 0.0032
0.0023 0.0015 0.001 0.001 0.001 0.001 0.001 ]
------------------------------ CON FUNC. ------------------------------
DL = -0.0000
KS = -0.0000
-----------------------------------------------------------------------
```

$AR = 10.0$, FB minimization:

```
------------------------------ OBJ FUNC. ------------------------------
FB = 1850.8001
------------------------------ DES VARS. ------------------------------
a0 = [-0.0637 -0.0539 -0.0464 -0.0402 -0.0352 -0.031
-0.0277 -0.0252 -0.0236 -0.0227 -0.0227 -0.0236 -0.0252
-0.0277 -0.031 -0.0352 -0.0402 -0.0464 -0.0539 -0.0637]
t0 = [0.001 0.001 0.001 0.0017 0.0029 0.0044
0.0063 0.0086 0.0113 0.0145 0.0145 0.0113 0.0086
0.0063 0.0044 0.0029 0.0017 0.001 0.001 0.001 ]
------------------------------ CON FUNC. ------------------------------
DL = 0.0000
KS = -0.0000
-----------------------------------------------------------------------
```

$AR = 10.0$, W0 minimization:

```
------------------------------ OBJ FUNC. ------------------------------
W0 = 9198.4944
------------------------------ DES VARS. ------------------------------
a0 = [-0.1199 -0.1081 -0.0912 -0.0716 -0.0508 -0.0296
-0.0089 0.0087 0.0214 0.0285 0.0285 0.0215 0.0089
-0.0085 -0.0291 -0.0501 -0.0709 -0.0904 -0.1073 -0.1192]
t0 = [0.001 0.001 0.001 0.001 0.001
0.001 0.0018 0.0034 0.0057 0.0086 0.0087 0.0058
0.0036 0.0019 0.001 0.001 0.001 0.001 0.001 0.001 ]
------------------------------ CON FUNC. ------------------------------
DL = -0.0000
KS = -0.0000
-----------------------------------------------------------------------
```

From a first analysis we are able to see that while a greater AR allowed for a lesser FB, the general weight of the structure increased. With a greater AR, we have a greater wingspan and, therefore, a greater wing structure, making the total weight gain to be expected. As for the FB difference, it opens some possibilities to conclusions, that might be clearer when we analyse the weight distribution of this total weight.

In the following text, it's presented the weight ratio for each optimization (OM reads as optimization mode). The first interesting and expected result is that when increasing the AR, while minimizing FB, the total weight increases considerably, while the FB portion (from the previous analysis) does not change in the same rate. This allows for a lesser percentage of FB in the total weight, as shown.

```
---------------------------- OM  AR ----------------------------
---------------------------- FB 6.0 ----------------------------
W0 =  9130.007564170764
FB = 20.93 %
FM = 75.14 %
SM = 3.93 %
----------------------------------------------------------------
---------------------------- OM  AR ----------------------------
---------------------------- WE 6.0 ----------------------------
W0 =  9099.316234931597
FB = 21.27 %
FM = 75.39 %
SM = 3.34 %
----------------------------------------------------------------
---------------------------- OM  AR ----------------------------
---------------------------- FB 10.0 ---------------------------
W0 =  9432.975782544781
FB = 19.62 %
FM = 72.72 %
SM = 7.66 %
----------------------------------------------------------------
---------------------------- OM  AR ----------------------------
---------------------------- WE 10.0 ---------------------------
W0 =  9198.494440761622
FB = 21.59 %
FM = 74.58 %
SM = 3.83 %
----------------------------------------------------------------
```

Other important result is that, when we try to minimize the total weight, the percentages of each participant does not have a great variation, relative to the FB analysis. This confirms the general idea from the previous section that the weight minimization does an intermediate analysis in minimizing the structure and fuel weights.

This results hints towards to the expectation that a FB analysis gives a relatively better performance in aerodynamic terms, as the change in FB minimization clearly focus on FB minimization (with a great increase in the structure mass, as it tries only to diminish the drag). Those expectations will be further studied in the following subsection.

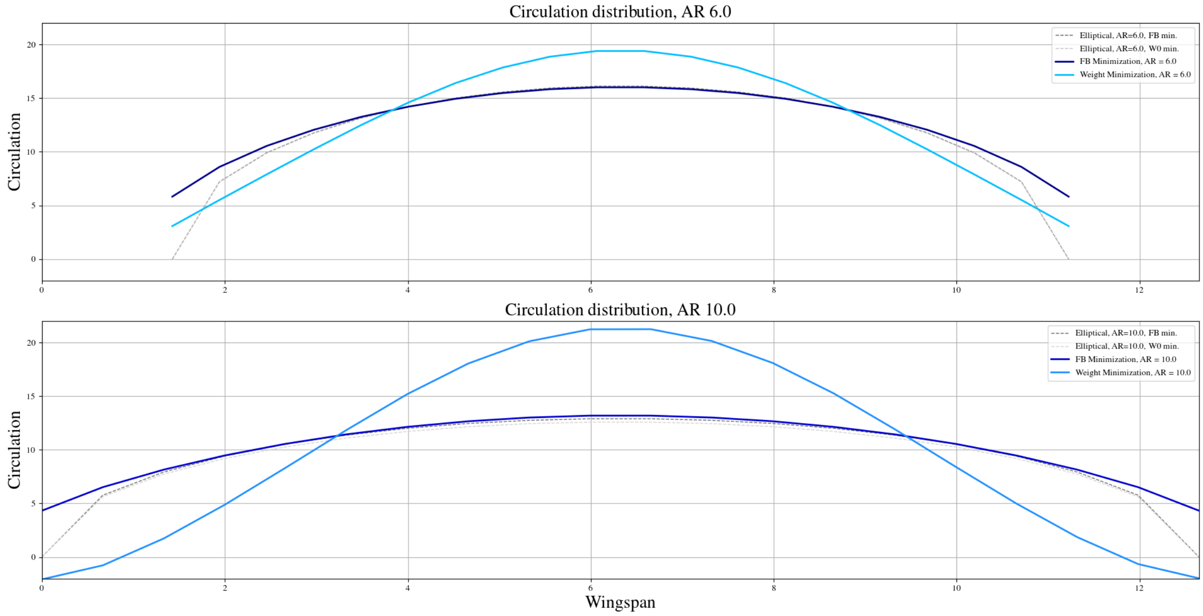

Figure 8: Lift distribution for two values of AR, two different minimization runs, centered around the same value.

5.2 Lift distributions

From Figure 8, we can see that the elliptical format (ideal form for lesser drag), that relies on C_L, changes relatively little when we compare FB or total weight minimization.

As for different AR values, the FB optimization, as expected from previous analysis, keeps following the ideal ellipse for a lesser drag, while the weight minimization program tends to a different direction, as it tries to diminish the weight structure. This seems to have a dramatically negative result, as we see from the Circular distribution with AR of 10.0.

This negative aerodynamic result may be exploited from the FB ration within those optimizations, which confirms that there is a greater FB and, therefore, a greater drag. A more reliable optimization to the problem should indicate an intermediate solution, the present analysis being useful for possible starting points in more complex projects.